TPK4450: Semester work 2020

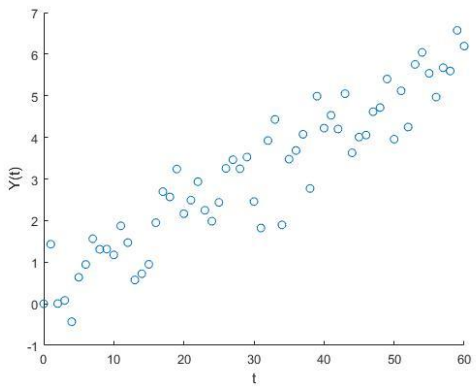
Information:

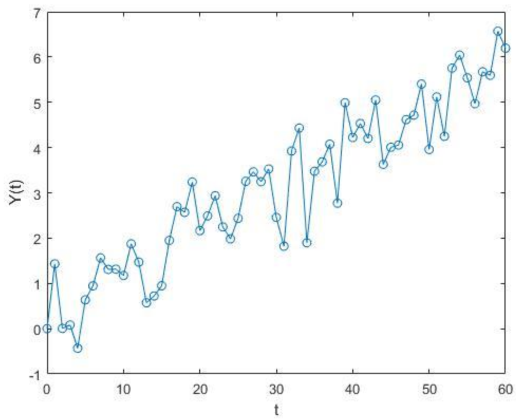
- The semester work is mandatory. It counts 30% to the final grade in the course.
- The semester work is individual. You must submit report individually.
- It consists in 3 parts: Fault diagnosis, failure prognosis, maintenance modelling
- You must submit a report for each part. You should include figures that the exercise asks for, with your comments, explanations and/or analysis.

Failure prognosis

An industrial system is monitored continuously. A data set shows that there is a deviation of its optimal performance as time progresses.

The data set is given in the file 'monitoring.txt'. The first column shows the time of the measurements and the second column is the difference to an optimal performance point, which can be taken as a degradation indicator. The data set is shown in the figures below.





We want a model describing such a degradation process that we can use for prognosis.

1. From the candidates: Linear trend, Gamma process and Wiener process (with drift), which one would you discard based on the observations and why?

Import the dataset to Matlab or Python

Part A

First try with a **linear trend model**:

2. What method can you use to estimate the parameter of a linear function $X(t)$ that best explains the observations $Y(t)$? Estimate the parameter a , for the function: $X(t) = a \cdot t$.
   Plot $X(t)$ along side the given observations $Y(t)$.
3. Considering that the error $\varepsilon(t) = Y(t) - X(t)$ is normally distributed, what method can you use to estimate the parameters of the distribution of the error $\varepsilon(t)$? Estimate the parameters (μ_L and σ_L)
   Does this look familiar? Have a look at problem c from the part I of the semester work… i.e. the mean evolves linearly with slope approximately 0.1 and the deviation of the sample mean is approximately. $\left(\sigma = {}^{2}/_{\sqrt{10}}\right)$
   You can then consider we are monitoring the process described in part I, but ***keep working with the estimated parameters****, consider you don't know the underlying true parameters as you did in part I, and* ***all you have to work with is the provided data set****.*

In part I we were trying to detect an early fault (with a mean of 2 °C) to raise an alarm. Consider now that despite faults, the process is required to continue in operation, as long as a strict threshold imposed at level $L = 10$ is not reached.

We now want to do prognosis and to estimate the remaining useful life of this system, in order to make all the preparations necessary for maintenance and production.

4. From the last observation $\left(t_j, y\left(t_j\right)\right) \approx (60, 6.19)$, simulate trajectories (1×10^4) of $Y(t)$ with the estimated parameters $(\hat{a}, \widehat{\mu_L}, \widehat{\sigma_L})$. You can choose the time horizon for the trajectories.
5. Estimate the time at which each of the trajectories passes the threshold $L = 10$. Then estimate the $RUL\left(t_j\right)$ for each trajectory and make a histogram.

Part B

Try now a **Wiener process with linear drift**.

For a stochastic process, we look at a distribution of the increments in the process $\mathrm{I} = \mathrm{Y}(\mathrm{t} + \Delta\mathrm{t}) - \mathrm{Y}(\mathrm{t})$

6. What is the distribution of the **increments** of a Wiener process with drift ?. Estimate the parameters from the given data set. $(\widehat{\mu_W}, \widehat{\sigma_W})$

Proceed now to simulate trajectories of the stochastic process.

7. From the last observation $\left(t_j, y(t_j)\right) \approx (60, 6.19)$, simulate trajectories (1×10^4) of $Y(t)$ with the estimated parameters of the increments. You can choose the time horizon for the trajectories.

   Hint: you may generate random increments by keeping $\Delta t = 1$, and use

   $Y(t) = y(t_j) + \sum_{k=1}^{(t-t_j)/\Delta t} I_k$ for $t > t_j$; with $I_k = Y(t_j + k\Delta t) - Y(t_j + (k-1)\Delta t)$

8. Estimate the time at which each of the trajectories passes the threshold $L = 10$. Then estimate the $RUL(t_j)$ for each trajectory and make a histogram.

9. Compute the theoretical distribution of the RUL with the estimated parameters and plot it on top of the histogram.

   Hint:

   It is known that the first passage time of a Wiener process with linear drift is Inverse Gaussian distributed, with probability density function:

   $$f_T(t; \mu, \lambda) = \sqrt{\frac{\lambda}{2\pi t^3}}\, e^{-\left(\frac{\lambda}{2\mu^2}\right)\left[\frac{(t-\mu)^2}{t}\right]} \quad \text{for } t > 0, \mu > 0, \lambda > 0$$

   Then, the pdf of $RUL(t_j)$ can be written as:

   $$f_{RUL}(t_j) = f(h; \mu_I, \lambda_I)$$

   The cumulative distribution of the $RUL(t_j)$, i.e. $F_{RUL}(t_j) = \Pr\left(RUL(t_j) \leq h\right)$ is given by:

   $$F_{RUL}(t_j) = \int_0^h f(u; \mu_I, \lambda_I)\, du$$

   with $\widehat{\mu_I} = \frac{L - y(t_j)}{\widehat{\mu_W}}$ and $\widehat{\lambda_I} = \left(\frac{L - y(t_j)}{\widehat{\sigma_W}}\right)^2$ for $\Delta t = 1$

Now consider that the system was monitored until its actual failure time (first passage time), which was recorded approximately at $t = 94$, (i.e. 34 shifts after the time at which you made the prognosis $t_j = 60$)

10. What is the $\Pr\left(RUL(t_j) \leq 34\right)$ that you obtained from:
    a. The linear trend model.
    b. The simulations of the stochastic process.
    c. The Inverse Gaussian distribution.

If the unit was monitored until $t = 80$ (i.e. you get the path of observations until t=80), and you made prognosis from this time point.

11. What would you expect about the histogram / distribution of the RUL(80) compared to the ones you obtained at t=60, RUL(60) ? (No calculations needed)