IEOR 174 Project Final Report

Strategic Portfolio Optimization: A Monte Carlo Approach to Asset Allocation

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Presentation Link:

https://drive.google.com/drive/u/1/folders/1-ZOt3r1P3amzoWnYT44IGx5J8mjY6-mK

Detailed description of the problem and motivation

Our project's primary aim is to meticulously craft an optimal free cash portfolio strategy over a year, leveraging the analytical power of Monte Carlo simulations. The venture is grounded in historical data analysis for bonds and bank deposits, coupled with the simulation of stock prices, to paint a realistic picture of potential future market scenarios. This initiative is motivated by the challenge of managing investment risks and rewards within the confines of a predetermined budget, with a focus on maximizing returns and minimizing losses.

We are considering the following diverse asset classes for our portfolio:

- Bonds: Specifically, 1-year treasury bonds, where we intend to analyze their historical yield behavior to anticipate future trends and determine the best holding periods.
- Stocks: Our analysis includes both ETFs and individual stocks. For stocks, we employ a stochastic approach, simulating price movements using geometric Brownian motion to reflect the unpredictable nature of the stock market.
- Bank Deposits: Including various certificates of deposit (CDs), we plan to analyze and compare their historical interest rates to estimate future returns.

The motivation behind this project is the inherent complexity and unpredictability of investment decisions, especially in fluctuating market conditions. We aim to equip investors with a strategic tool that aids in navigating these complexities, offering a model that not only predicts potential returns but also advises on risk mitigation strategies. The Monte Carlo simulations will enable us to explore a vast array of possible market outcomes, thereby providing a comprehensive view of potential investment scenarios.

Our project's primary goal is to discern the optimal allocation and holding duration for various assets, aligning with different investment strategies ranging from conservative to aggressive. By meticulously analyzing historical data and simulating future market conditions, we aim to develop a nuanced understanding of how different asset allocations can impact overall portfolio performance. This detailed approach is designed to aid investors in making informed decisions that align with their financial goals and risk tolerance, all within a specified initial budget framework.

Analysis on the problem and analytic methods used

Given the data from Monte Carlo simulations, there are some metrics to compute and compare among portfolios with different weights of allocation.

- **Sharpe ratio**: A risk adjusted relative return, which can be used to compare portfolio performance. It calculates the change of return due to one unit change of risk.
    - **Formula:** (Rp - Rf) / σ;
    - where Rp is the return of the portfolio,
    - Rf is the risk free rate,
    - and σ is the standard deviation of the portfolio's excess return.
    - In general, a Sharpe ratio above 1 is considered as a good performance.
- **Expected Portfolio returns**: The expected amount of return on a portfolio by adding up weight-adjusted returns.
    - **Formula**: E(Rp) = w1*E(R1) + w2*E(R2) + … + wi*E(Ri),
    - i is the number of investment options included in the portfolio,
    - w is the weight of the particular investment option,
    - and R is each expected return.
- **Portfolio variance and volatility**: The measurement of dispersion of returns or a measure of risk, calculating the deviation of returns from the portfolio mean return. It determines whether the portfolio is at the appropriate level of risk.
    - **Formula**: for 2 options: w1^2*σ1^2 + w2^2*σ2^2 + 2*w1*w2*Cov.
    - The standard deviation is the volatility.
- **Trade-offs based on risk-aversion:** Analyze how the expected return would change based on the level of risk aversion. More risk-averse investors would prioritize lower portfolio volatility and more risk-like investors would maximize the portfolio return regardless of risk level.
- **Geometric Brownian Motions**: A stochastic process of asset price given by Brownian Motions.
    - **Formula**: dS(t) = u*S(t)dt + σS(t)dB(t)
        - where S(t) is the asset price
        - u is the drift
        - σ is the volatility
        - B(t) is the Brownian motion
    - Formula of S(t) by Ito's lemma: S(t) = S(0)*exp((u-1/2σ)^2*t)+σB(t))

A series of reasonable simulation algorithms or structures

To construct our investor's portfolio, we will assume that the individual has $100,000 they are willing to invest. His options are to invest in 1-year bonds, Certificate of Deposits (CDs), 4 Exchange-Traded Funds (ETFs), and 11 Fortune 500 individual stocks. The one year bond equates to the risk-free interest rate, and CDs are included so that the investor can consider quarterly or bi-quarterly interest rates.

ETFs are a type of investment fund that's traded on stock exchanges, similar to individual stocks. It pools together the money of multiple investors to buy a diversified portfolio of assets like stocks, bonds, commodities, or a combination. ETFs are designed to track the performance

of a specific index, sector, commodity, or asset class. We included these in the portfolio to spread risk across different securities within a single investment.
For our individual stocks, we hand selected 11 stocks from the Fortune 500 (top 500 publicly traded companies in the United States by total revenue each year), each from a separate market sector.

Here are the different rates and stock values we have compiled below.
Note: All numbers were collected from October 23rd, 2023

Since each bank has set up their CDs differently, we have decided to treat them as different investment options instead of calculating a mean interest rate for a specific length. Since our goal is to maximize portfolio returns, depending on the specified duration for the investor, we will use the maximum rate available. For example for one year we will use the one year bond rate and for half a year we would use .0488.

Names	Interest Rates
BOA(1-2 month)	0.03%
Wells Fargo(3 month)	4.4%
BOA(4-5 month)	3.93%
Chase(6, 8 month $1000 to $99999)	3.92%
Chase(6, 8 month above $100K)	4.88%
BOA(7 month below 100K)	4.89%
BOA(7 month above 100K)	5.27%
BOA(10 month)	0.05%
Citi Bank(11 month)	4.93%
BOA(12 month)	3.45%
One year bond	5.442%

Then, we found the initial stock prices of all the ETFS and individual stocks we are taking into consideration for our simulation. These values were collected from ***Yahoo Finance*** due to its accuracy, extensive coverage, user-friendly interface, and longstanding credibility in the finance industry.

ETF/Individual	Name	Initial Stock Price (USD), Price Since Prior Year
ETF	QQQ	355.67, increase

ETF	NULG	59.19, increase
ETF	IMCG	54.82, increase
ETF	VIOG	89.69, decrease
Individual	XOM	109.45, decrease
Individual	LIN	365.43, increase
Individual	UPS	148.16, decrease
Individual	AMZN	126.56, increase
Individual	WMT	161.01, increase
Individual	JNJ	151.39, decrease
Individual	V	231.53, increase
Individual	AAPL	173.00, increase
Individual	META	314.01, increase
Individual	NEE	51.52, decrease
Individual	PLD	100.81, decrease

Major Design Decisions

These are the ETFs to be considered: General (QQQ), Large Cap (NULG), Mid Cap (IMCG), Small Cap (VIOG)

In our simulation, we've intentionally incorporated various types of Exchange-Traded Funds (ETFs) to capture diverse market segments:

Nasdaq ETF: QQQ stands out by providing exposure to a diverse group of cutting-edge Nasdaq-100 companies (100 of the largest non-financial companies listed on the Nasdaq stock exchange) renowned for their innovation and market leadership. It offers a concentrated view of high-growth potential companies within sectors such as technology, biotechnology, and consumer discretionary industries.

Large-Cap ETF: Comprising stocks from established, large-scale companies, these ETFs typically offer stability and lower volatility, providing a solid foundation within a portfolio.

Mid-Cap ETF: Representing mid-sized companies, these ETFs strike a balance between stability and growth potential, suitable for portfolios with moderate risk tolerance.

Small-Cap ETF: Comprising stocks of smaller companies, these ETFs offer higher growth potential but carry higher risk due to the volatility associated with smaller firms. They are often considered for portfolios seeking aggressive growth opportunities.

These are the individual stocks to be considered: XOM Exxon Mobil Corp (energy), LIN Linde PLC (materials), UPS (industrials), AMZN Amazon (consumer discretionary), WMT Walmart (consumer staples), JNJ Johnson and Joshnson (healthcare), V Visa (financials), AAPL Apple (information technology), META Meta (communication services), NEE NextEra Energy (utilities), PLD prologis (real estate)

The stock market encompasses 11 major industries, each with its distinct characteristics, which is why we've chosen individual stocks from these sectors. Incorporating these diverse stocks allows us to capture a broad spectrum of market segments and analyze the unique dynamics influencing each industry within our simulations.

For each copy of 1-year simulation, we will simulate the prices for ETFs and individual stocks by assuming it follows Geometric Brownian Motion (GBM). The mean of GBM determines our mean drift of our stock. To assign the proper values to this mean, we looked at the stock value on Yahoo Finance from the year prior to see if it increased or decreased (as noted in the initial stock price column in the chart above). If it increased, we assigned a value of .001 and -.001 for the opposite. For variance, generally, ETFs tend to have lower volatility compared to individual stocks. This is primarily because ETFs are diversified investments that hold a basket of assets, such as stocks, bonds, or commodities, across various sectors or industries. By holding a mix of assets, ETFs spread risk, and any volatility associated with individual stocks within the fund may be offset by the performance of other assets in the ETF. Thus, we assigned the value of .01 to ETF's mean variance and .001 to individual stock's mean variance.

For our portfolio allocation, we generated an array of random weights for a specified number of elements, ensuring that the sum of these weights equals 1 by normalizing them. Given initial stock prices, mean drift, mean variance, and interest rates from historical financial data, we computed returns on this specific portfolio weight.

This computation will depend on our investment strategy of which we have three:

1. Invest to return maximum return (risk-seeking investor)
2. Invest to return maximum Sharpe ratio (risk neutral investor, wants to balance risk and return)
3. Halfway into the year, we take a look at how our investments are doing, if the return is below a certain value, we remove our investments from the stocks into the deposits, otherwise we keep them the same (risk-averse investor)

The reason why we included the last strategy is to see whether or not we would advise an investor to hold their assets for a year or move them around. This strategy is when we implement CD interest rates instead of the risk free bond rate. We decided to take out the very risk-seeking strategy because of time constraints and feasibility.

We considered 10,000 combinations of random weights of the portfolio, and for each random weight, we will conduct 10k Monte Carlo copies of 1-year simulation to compute the mean return and portfolio volatility for each combination and for each type of investor strategy. The optimal portfolio allocation in terms of maximum return and the sharpe ratio will be determined from simulation results. For the third method, we simply return the weighted average after implementing that strategy.

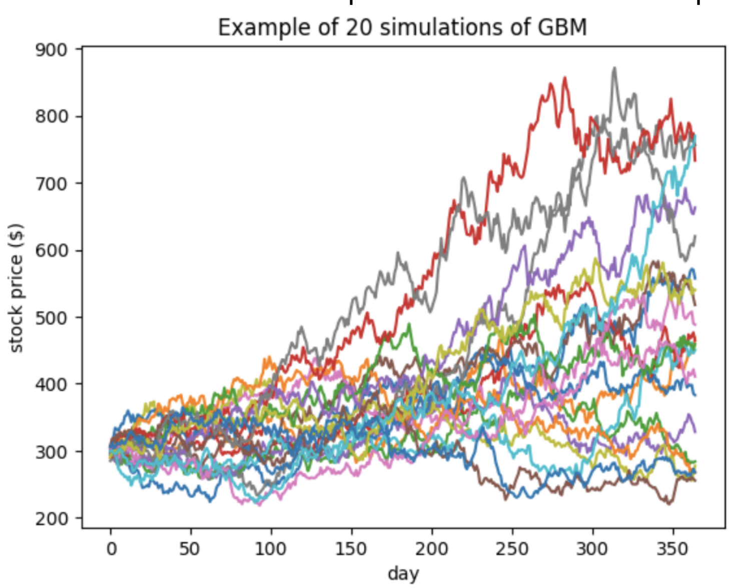
Softwares Used and Alternative Approaches

The main software packages that we used were the random module from Numpy. We sampled From the normal distribution to construct the geometric brownian motion simulations. We then used The random module again to draw from a uniform distribution between 0 to 1 to generate the random Portfolio weights. The package implements pseudo random generators for drawing random distributions.

Alternative approaches may include using the various density based methods covered in class like the inversion method based on the pdf of the normal distribution. The trade off is that since Numpy already has an interface built in that achieves the same purpose, it would be more efficient to directly call the Numpy package instead of enabling the simulation of the distribution ourselves.

Primary results

In the simulations, we use geometric brownian motion to simulate the stock price time series throughout the year. Below illustrates an example of 20 simulations of a specific stock.



For the **static allocation strategy** (where portfolio weights remain the same throughout the year) , the optimal weight with regards to returns, sharpe ratio, and volatility in the experiment for different random weights is as follows:

Max Return	Minimum Volatility	Max Sharpe Ratio
851.38%	30.62	0.9278
Weight no. 8388	Weight no. 1281	Weight no. 2074

We can see that the three metrics of evaluating the “optimal” portfolio weights all yield a **different** set of weights.

Conservative strategy: For the dynamic strategy (where stocks are sold in the middle of the year if return on stocks is below a certain threshold x), we tested out a variety of thresholds x to evaluate the optimal Threshold under this dynamic allocation policy in the middle of a year.

Threshold x	Max Return	Minimum Volatility	Max Sharpe Ratio
-10%	1.407 (no. 483)	0.2769 (no. 316)	0.725 (no. 514)
-50%	1.262 (no.53)	0.262 (no. 356)	0.889 (no. 356)
-100%	1.705 (no. 646)	0.295 (no. 558)	0.840 (no. 558)

We can see that consistently having a conservative strategy in the middle of the year significantly decreases the maximum return that is achieved. However, **volatility is greatly decreased** and the optimal sharpe ratio achieved is not much different to a static strategy. Another observation is that in table 2, the **minimum volatility portfolio weight has been the same set of weights** as the ones that attained the maximum sharpe ratio. This indicates that in a conservative strategy context, **lower volatility drives down the sharpe ratio greatly**.

Reasonability of The Results and What We Learned From the Experiments

The results do seem reasonable in many aspects. Firstly, the set of weights that attain the max return and the min volatility has always been different. This illustrates the **trade-off between return and volatility**, which we aim to balance by finding the max sharpe ratio weight.

Moreover, in the conservative strategy context, **max return becomes lower** than static strategy, which makes sense because stocks have larger risk which also makes possible much higher returns at a small chance. In the conservative context, the investor inhibits the possibility of attaining the extreme of the stocks’ highest returns because they sell them whenever losing a certain amount and put them all to low return deposits. The fact that **minimum volatility has decreased** significantly under the conservative context is also reasonable as the investors prioritize low risk deposits over losses in stocks.

Challenges Along the Way and for the Future

Our project met a few challenges that have required careful attention and reworking from our original strategy. First, we transitioned from mutual funds to Exchange Traded Funds (ETFs) since these funds can be bought and sold throughout the trading day, which is more liquid than the mutual funds, which we previously intended to use. It was also difficult to determine the risk-free interest rates we initially wanted to use because they were all so different, which led us to just pick either investing in bonds or CDs since they both have steady risk-free interest rates. In addition, determining precise risk-free interest rates and establishing accurate values for implementing Geometric Brownian Motion (GBM) was a bit difficult, requiring some research and critical thinking to ensure the fidelity of our simulations.

Furthermore, managing elements beyond our direct control has presented challenges, and ultimately we were unable to come up with solutions to take the following difficulties into account. Balancing the fidelity of our simulations with user-friendly accessibility was an ongoing objective, meaning we needed a blend of sophisticated modeling and intuitive interface design. Moreover, the incorporation of unforeseen real-world events, such as natural disasters, seemed out of scope from what we learned in the class. In navigating these challenges, our project underwent critical adaptations and refinements. While some hurdles remained unresolved, these experiences underscored the complexity of financial simulations and the importance of adaptability in addressing multifaceted investment scenarios.

Next Steps

If we were to continue advancing this project, our primary focus would revolve around a deeper exploration of diverse initial portfolio configurations and their influence on overall performance. Our intention would be to conduct a comprehensive analysis, delving into the effects of these configurations on both returns and associated risks. Additionally, our next phase would involve an in-depth examination of the higher-risk strategy outlined in Strategy 3. Expanding upon this strategy to incorporate quarterly adjustments is one example of doing so.

We would likely continue with refinement of strategies, informed by both theoretical frameworks and real-world insights and challenges that we mentioned in some of the sections above. Additionally, we would extend our analysis to encompass a broader range of variables and market conditions would enhance the depth and reliability of our simulations. Overall, the goal would be to create a more robust and adaptable model for investment decision-making.

Larger Context of the class and IEOR Field

Our IEOR 174 project, **aligned with core IEOR methodologies**, applies advanced data analysis, optimization techniques, risk management, and Monte Carlo simulations to develop an optimal portfolio strategy, **exemplifying the course's focus on "Simulation for Enterprise-Scale Systems."** By analyzing historical data and simulating future market scenarios, the project optimizes asset allocation and risk-return balance, adhering to IEOR's principles of systematic decision-making under uncertainty. The extensive use of simulations, particularly Monte Carlo methods, directly connects to the course's emphasis on understanding and optimizing enterprise-scale systems, showcasing how IEOR tools and concepts can be practically applied to complex financial market scenarios.

Conclusion and Takeaways

Through the implementation of the project, we learned to apply the Monte Carlo simulation technique to address a tangible issue in the financial industry that simulated the impact of portfolio weights and the level of risk aversion on investment decisions. In the process, we recognized the significance of pre-simulation groundwork, involving the collection and cleaning of historical data for stocks, CDs, and ETFs to identify constraints and distributions.
Although we encountered challenges during implementation, including issues related to runtime length and data selection, we successfully solved the challenges by reducing the model's complexity. This adjustment is particularly crucial in real-world scenarios where it is impractical to incorporate every parameter or unknown factor into decision-making processes. Overall, this project has prepared us with the approach of practical knowledge and problem-solving skills for solving complex financial decision-making problems.