To what extent did Gödel's incompleteness theorem lead to the shift from pure formalistic mathematics to applied mathematics in 1930-1935?

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Question: To what extent did Gödel's incompleteness theorem lead to the shift from pure formalistic mathematics to applied mathematics in 1930-1935?

A: Identification and Evaluation of Sources

This investigation will explore the question: To what extent did Gödel's incompleteness theorem lead to the shift from pure formalistic mathematics to applied mathematics in 1930-1935? The half-decade in focus reflects the 1931 release of Austrian logician Kurt Gödel's incompleteness theorems.

The first source that will be evaluated in depth is Dr. Panu Raatikainen's journal article "On the Philosophical Relevance of Gödel's Incompleteness Theorems", written and published in 2005 in the International Journal of Philosophy. The author, Panu Raatikainen, is a Finnish scholar of philosophy. He published a book on Incompleteness in 1998 and 2016 respectively. Raatikainen was an adjunct professor at the University of Helsinki at the time. This article comes several decades after the publication of Godel's incompleteness theorems, painting a holistic picture of its influences decades after its release. There was no relevant historical context of Finland in the early 2000s which affected this article.

The purpose of this article is to inform and discuss the impacts of Gödel's incompleteness theorem, being directed at researchers and historians with the document's publication in a reputable journal. The audience is those who would like to better understand the philosophical implications of the theorems. His thesis is that the incompleteness theorem sent rippling shocks around the mathematical community. The paper's value is in elucidating the modern consensus surrounding the theorems. Raatikainen examines the case of Hilbert's program, which the author argues is stifled by Gödel's theorems, aiding the focus question. However, this paper does not entirely address the entirety of the question of research for this Historical Investigation as it limits its view to looking solely at the theorem's impacts, disconnected from the events taking place in the outside world. The relevance of this source to this investigation is for understanding the impacts of Godel’s theorems' shock value in the 21st century, contextualizing its long-term significance which may have played a role in the shifts in types of mathematics.

The second source that will be evaluated is Dr. Alonozo Church’s journal article “The Richard Paradox” published and written in 1934 in The American Mathematical Monthly. Church was an American scholar of mathematics, publishing several papers on the topic during his nearly forty-year professorship at Princeton University from 1929 to 1967. This article comes shortly after the publication of Godel’s incompleteness theorems and has the relevant context of the Great Depression. Mathematicians including Church may have been generally more discouraged due to the economic downturn.

The purpose of this article is to discuss ‘The Richard paradox’, describing functions in an axiomatic system. The article reflects on Godel’s incompleteness theorems and is useful to other scholars of mathematics who work in logic. Church argues these paradoxes are exacerbated by Godel’s theorems. The value is that this is a primary source, recounting a mathematician’s reaction to how the theorems reflect on his work. Church refers to the startling and “unpleasant” implications of the theorem, aiding the focus question. However, the author may be taking a disproportionately negative view of the theorem due to the Great Depression and Church does not specialize in the same areas as Godel. The relevance of this source to this investigation is for understanding the impacts of Godel’s theorems' shock in the short term, contextualizing 1930s mathematicians’ reactions who may have advocated for the shifts in types of mathematics.

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B: Investigation

The impacts of Gödel's incompleteness theorems on the shift from pure formalistic mathematics to applied mathematics in the period of 1930-1935 is a topic of debate among scholars. We first must describe the shift from pure formalistic mathematics to applied mathematics as well as the implications of Godel’s incompleteness theorems. There is evidence that there was a shift from pure formalistic mathematics to applied mathematics in 1930-1935. David E. Rowe and John McCleary conducted a study analyzing a sample of leading mathematical journals during the period, which found a significant decrease in the number of papers on foundational topics and a corresponding increase in papers on applied topics during this period, suggesting that there was indeed a notable shift during the 1930s. (Rowe and McCleary, 2000).

There were two prominent schools of thought in mathematics at the time: Intuitionism, which believed that every mathematical proof needs to be constructive and have concrete examples and that mathematics contains self-evident truths, and Formalism, which believed that all mathematics could be reduced to axioms and placed a large emphasis on using the basic building blocks of mathematics. In 1921, esteemed German mathematician David Hilbert (1862–1943) created a challenge for all mathematicians known as Hilbert’s Program. A subsector of this program called “for a formalization of all of mathematics in axiomatic form.” The Hilbert program was the centerpiece of formalistic mathematical thought in the early 20th century, while intuitionists did not take much part in the program (Zach, 2019). Godel’s released his two incompleteness theorems in 1931. His first theorem stated that a consistent set of axioms for elementary arithmetic always yields a true but unprovable statement. His second theorem stated that “no consistent formal system can prove its own consistency” (Raatikainen)

Intuitionists gained an upper hand against the school of formalism, as the incompleteness theorems “seemed to confirm the intuitionists’ misgivings about formalism.” (Raatikainen) Many mathematicians were stopped in their tracks and delayed their research as their goal of formalizing mathematics could never be achieved (Church).

Some argue that the incompleteness theorems played a major role in this shift. On the one hand, Gödel's incompleteness theorems dealt a severe blow to the formalistic program of Hilbert and his followers, who had sought to establish a complete and consistent foundation for all of mathematics. This result challenged the core assumption of formalism that mathematics could be reduced to a set of rules and symbols, and it led many mathematicians to question the value of pursuing pure formalistic research. Mathematicians stated that “Hilbert’s Consistency Program could not be carried out … [as Godel] had proven two theorems which were then considered moderately devastating and which still include nightmares among the informed.” (Smorynski) According to 1930’s mathematician Alonzo Church, the “unpleasant theorem of Kurt Godel to the effect that, in the case of any system of symbolic logic which has a claim to adequacy, it is impossible to prove it freedom from contradiction in the way projected in the Hilbert program.”, essentially stopping the program.

However, some historians argue that the impacts of Godel’s incompleteness theorems are overstated. For example, Zach argues that “although the impact of Godel’s incompleteness theorems for Hilbert’s program was recognized soon after their publication, Hilbert’s program was by no means abandoned,” citing the two mathematicians Bernays and Gentzen who continued and published work contributing to the Hilbert program (Zach, 2007). This may be influenced by the communication technologies available during the period. Not every mathematician had the resources to form a detailed opinion about the theorem, meaning that those who did may have publicly exclaimed the importance of results while others would be unaffected. Those who could study the theorems may have overstated the impacts of the result which they understood to be very important.

On the other hand, the shift towards applied mathematics and other alternative approaches was not solely the result of Gödel's incompleteness theorems. Other factors, such as the growing recognition of the importance of interdisciplinary research and the changing political, social, and economic climates of the time, also played a role in shaping the direction of mathematical research. This emphasis on practical applications led to the growth of applied mathematics. The changing political and social climate of the time also influenced mathematical research. During the 1930s, many large mathematical organizations had to be put on hold due to the political conditions in Europe. Schappacher argues that the International Mathematical Union was essentially absent and that mathematicians were scared for the safety of themselves and their colleagues. (Schappacher) The Great Depression and political upheavals caused mathematicians to focus on developing practical applications of mathematics that could help address social and economic problems. Servos argues that the reason for this shift was not due solely to needing for employment but reflected instead the completion of a period of mathematical development: “American mathematicians had earned the respect of academic colleagues …. Under the new circumstances, a desire to find a larger audience slowly supplanted the discipline's former passion for purity." (Servos)

However, Sawyer argues that the political, economic, and social conditions played a large role in the lull in mathematics in the 1930s. More specifically, he believed that unemployment was “the most dangerous of these [factors]”, arguing that it is “impossible for any subject to flourish triumphantly in an age of total decadence” (Sawyer). The precarious economic and political conditions led to a decrease in mathematics work in general during the time. However, the mathematicians who were able to continue in their profession had skills in applied mathematics which allowed them to reap the practical benefits of their knowledge. Furthermore, these conditions could also have decreased the ability to access materials to learn about Godel's incompleteness theorem, further driving the point that other factors contributed to the shift.

There are many examples of authors contradicting each other in this debate. The most prominent example would be the reactions of mathematicians during the time. The use of primary sources proves to be difficult for this investigation as during the period (1930-1935) immense panic was caused by the incompleteness of theorems. Raatikainen's paper provides the view of many of the mathematicians of the time, arguing for impacts in research areas in proportion to the immense panic analyzed. However, this panic seems to be overstated as other historians lean towards the political, social, and economic conditions of the time. Schappacher chooses to emphasize the external conditions, such as the lack of meetings for the International Mathematical Union, as the cause for this shift. By providing little focus on Godel's incompleteness theorem the source provides a unique perspective that is not influenced by the panic of mathematicians of the time but rather by the trends which followed in the surrounding period.

Taken together, these sources suggest that while Gödel's incompleteness theorem may have played a role in the shift towards applied mathematics, other factors such as economic and political considerations with the growing recognition of the practical applications of mathematics likely also played a significant role. These findings highlight the complexity of the historical context in which this shift occurred and the various factors that influenced it. Based on the research presented, the other factors played a more significant role in the shift to applied math away from formalistic math when compared to the impacts of the theorems. The economic and political climates alongside the increasing importance of technology and computer science contributed to the shift toward applied mathematics. Thus, Gödel’s incompleteness theorems only led to the shift from pure formalistic mathematics to applied mathematics in 1930-1935 to a minor extent.

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C: Reflection

My greatest challenge in this research was finding connections between the historical context of the 1930s and Godel's incompleteness theorems. Many times, sources would be either too general or too technical to be used in the investigation. This challenge was addressed by modifying the question to identify a shift to applied mathematics, which was explored by more sources when compared to intuitionist mathematics. The research question needed to go through multiple iterations as it was originally too specific to find evidence for. Another challenge was the technicality and specificity of mathematics literature. To find mathematician's opinions, I had to dig through detailed proofs or commentary on other theorems.

I learned the challenges faced in conducting research in the history of mathematical results. I had to piece together the data and the trends of mathematical history and learn to use the reactions of mathematicians buried under layers of technical writing. In addition to gaining experience with using advanced sources such as mathematical journals for the first time, I learned how to better parse primary sources for reactions and insights into their reaction to research. An insight that led me to my final thesis was the use of mathematics in science and how scientific and practical thinking and mathematics became more employable and valued in the years following the Great Depression. It gave me a direction to look in and allowed me to find more sources that supported the same idea that employability played a key role in the discouragement of pure mathematicians.

The consistent reaction among the primary sources caused me to reframe my research. The nature of mathematics research to be immensely focused caused the sources I had to yield an exaggerated view of the theorems. Those who had spent their lives working on pure formalistic mathematics in logic would be more deeply impacted by the theorems than the general mathematician. The diverging historians' opinions contradicting the consistent opinions of primary sources led me to take a more data-driven approach, as using qualitative accounts of the theorem was not as reliable. I found that mathematicians may be primarily concerned with the communication of only their mathematical results as opposed to the implications of their reactions (especially outside of their specialty). Researchers in mathematics and history will face the issue of finding sources that are both sufficiently technical while placing a result into a historical context.

(Word count: 395)

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