Лабораторная работа №12

Решение краевых задач методом разностных аппроксимаций

Вариант 17

Задание 1. Составить разностную схему и получить численное решение краевой задачи с точностью 10^{-3} .

```
• y'' + (1+x^2)y = -1, -1 \le x \le -1, y(-1) = y(1) = 0
• ay'' + (1+bx^2)y = -1, -1 \le x \le -1, y(-1) = y(1) = 0, a = sin(17), b = cos(17)
```

```
import numpy as np
import math
import scipy as sp
from scipy import sparse
import matplotlib.pyplot as plt
```

```
In [4]: # variant number
K = 17
```

Напишем функцию, решающую линейную краевую задачу вида y''' = y''(y) y = f(y) y = 0 , y'' = y''(y) y = 0 , y'' = y''(y) y = 0

```
y'' - p(x)y = f(x), p(x) > 0, a \le x \le b, y(a) = A, y(b) = B.
```

Раздедим отрезок [a,b] на $n=rac{b-a}{h}$ частей. Заменим $y''(x_k)=rac{y_{k+1}-2y_k+y_{k-1}}{h^2}, k=1,n-1.$ Получаем $y_{k+1}-(2+h^2p(x_k))y_k+y_{k-1}=h^2f(x_k), k=1,n-1.$

Получили трехдиагональную систему из n-1 уравнений с n-1 неизвестными. Решая систему методом прогонки, найдем приближенное решение краевой задачи.

```
def solve_de(px, fx, a, b, n=2000):
In [5]:
            h = (b - a) / n
            X = np.linspace(a, b, n + 1) # строим сетку иксов
            P = px(X)
            # data содержит три вектора:
            # первый - значения диагонали ниже главной;
            # второй - значения главной диагонали;
            # третий - значения диагонали выше главной.
            data = np.array([[1] * (n-1), [-(h * h * P[i] + 2) for i in range(1, n)], [1] * (n - 1)
            offsets = [-1, 0, 1]
                                                # отступы относительно главной диагонали
            # строим трехдиагональную матрицу
            M = sparse.dia_matrix((data, offsets), shape=(n - 1, n - 1)).toarray()
            M = np.insert(M, 0, np.array([0] * (n - 1)), 0)
            M = np.insert(M, n, np.array([0] * (n - 1)), 0)
            M = np.insert(M, 0, np.array([1, 1] + [0] * (n - 1)), 1)
            M = np.insert(M, n, np.array([0] * (n - 1) + [1, 1]), 1)
            print(M)
```

```
F = fx(X) * h * h
F[0] = 0
F[-1] = 0

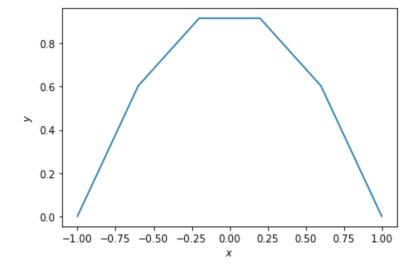
print(F)
solution = np.linalg.solve(M, F)
return X, solution
```

Найдем p(x) и f(x) для первого случая.

Очевидно, что $p(x) = -(1+x^2)$, f(x) = -1.

```
def px1(x):
 In [7]:
              return - 1 - x * x
          def fx1(x):
 In [8]:
              return -1 + (x - x);
 In [9]: X, sol = solve_de(px1, fx1, -1, 1, n=5)
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In [10]: plt.plot(X, sol)
          plt.xlabel('$x$')
          plt.ylabel('$y$')
```

Out[10]: Text(0, 0.5, '\$y\$')



Найдем p(x) и f(x) для второго случая.

Разделим все уравнение на a и преобразуем. Получим $y''+rac{1+bx^2}{a}y=-rac{1}{a}$. Тогда $p(x)=-rac{1+bx^2}{a}$, $f(x)=-rac{1}{a}$, или $p(x)=-rac{1+cos(k)x^2}{sin(k)}$, $f(x)=-rac{1}{sin(k)}$

```
In [11]: def px2(x):
    return - (1 + np.cos(K) * x * x) / np.sin(K)
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In [12]: def fx2(x):

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In [13]: X, sol2 = solve_de(px2, fx2, -1, 1)
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          plt.plot(X, sol2)
In [14]:
           plt.xlabel('$x$')
           plt.ylabel('$y$')
          Text(0, 0.5, '$y$')
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          np.max(sol)
In [15]:
          0.9163772797453755
Out[15]:
           np.min(sol2)
In [16]:
           -0.36532656231899147
Out[16]:
          nset = [10, 20, 100, 200, 500]
In [17]:
           for i in nset:
               X, solution = solve_de(px1, fx1, -1, 1, i)
               plt.plot(X, solution)
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return (x - x) - 1 / np.sin(K)

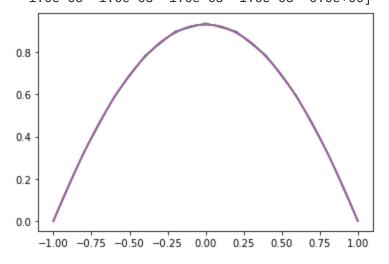
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In [18]:

for i in nset:

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     plt.plot(X, solution)
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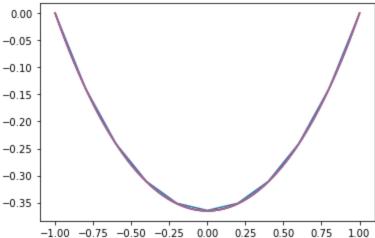
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• y'' + p(x)y' + q(x)y = f(x)

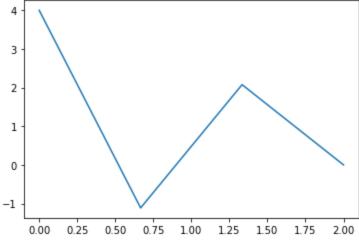
Задание 2. Найти приближенное решение краевой задачи методом конечных разностей:

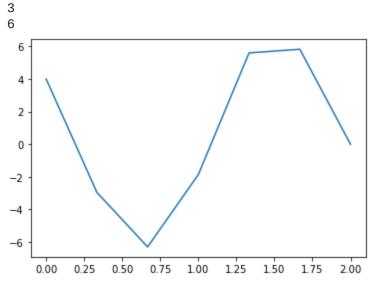
```
• a \le x \le b, y(a) = YAy(b) = YB
• p(x) = 0.5sin(x), q(x) = 7(1+sin^2(x)), f(x) = 6(1+x^2)
• a = 0, b = 2, YA = 4, YB = 0 с заданной точностью \varepsilon=0.05 и построить его график.
```

```
In [21]: def q(x) \rightarrow float:
             return 7* (1 + math.sin(x)**2)
         def f(x) -> float:
In [22]:
             return 6 * (1+x*x)
In [24]: def get_norm(values: list, step: float) -> float:
             # summ: float = 0.0
             # for value in values:
                   summ += value * value * step
             # return math.sqrt(summ)
             return max(*values[0:-1])
In [34]: def get_2_task_solution_with_number_of_subranges(n: int, Ua: float, Ub: float, interval:
             step: float = (interval[1] - interval[0]) / n
             y_{kprev} = lambda x: 2 - p(x) * step
             y_k = lambda x: q(x) * 2 * step * step - 4
             y_{knext} = lambda x: 2 + p(x) * step
             b_k = lambda x: 2 * step * step * f(x) # for free coefficients vector
             free_coefficients: list[float] = [Ua]
             main\_coefficients: list[list[float]] = [[0] * (n + 1)]
             main\_coefficients[0][0] = 1
             x_array: list[float] = [interval[0]]
             for k in range(1, n):
                 new_row: list[float] = [0] * (n + 1)
                 new_row[k - 1] = y_kprev(x_array[-1])
                 x_array.append(x_array[-1] + step)
                 new_row[k] = y_k(x_array[-1])
                 new_row[k + 1] = y_knext(x_array[-1] + step)
                 main_coefficients.append(new_row)
                 free_coefficients.append(b_k(x_array[-1]))
             x_array.append(interval[1])
             main\_coefficients.append([0] * (n + 1))
             main\_coefficients[-1][-1] = 1
             free_coefficients.append(Ub)
             y_array: list[float] = list(np.linalg.solve(main_coefficients, free_coefficients))
             plt.plot(x_array, y_array, mew=2, ms=10)
             plt.show()
              return x_array, y_array, get_norm(y_array, step)
         def get_2_task_solution(epsilon: float, Ua: float, Ub: float, interval: tuple) -> None:
In [351: |
             number_of_ranges: int = 3
             first_solution = get_2_task_solution_with_number_of_subranges(number_of_ranges, Ua,
             print(number_of_ranges)
             number_of_ranges *= 2
             print(number_of_ranges)
             second_solution = get_2_task_solution_with_number_of_subranges(number_of_ranges, Ua,
```

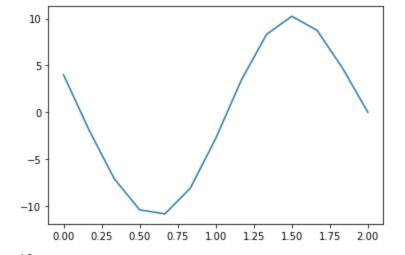
```
print(abs(first_solution[2] - second_solution[2]))
             while abs(first_solution[2] - second_solution[2]) > epsilon:
                 number_of_ranges *= 2
                 first_solution = second_solution
                 second_solution = get_2_task_solution_with_number_of_subranges(number_of_ranges,
                 print(number_of_ranges)
                 print(abs(first_solution[2] - second_solution[2]))
In [36]: def solve_task2() -> None:
             Ua, Ub = 4, 0
             interval: tuple[float, float] = (0, 2)
             epsilon: float = 0.05
             get_2_task_solution(epsilon, Ua, Ub, interval)
          3
          2
```

In [37]: solve_task2()

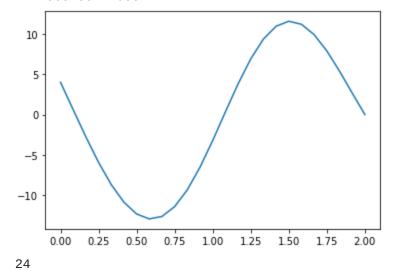




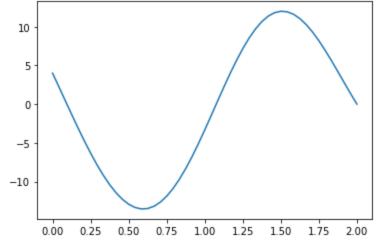
1.8289123023355218



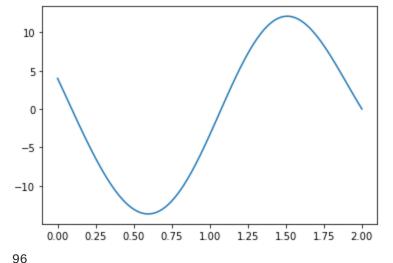
12 4.418032897770051



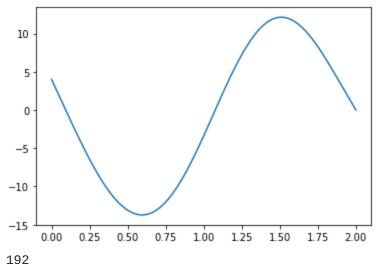
1.35133909060505



48 0.3935086792307896



0.10246147093541502



0.03292537030991305

Задание 3. Методом конечных разностей найти приближенное решение указанной в индивидуальном варианте краевой задачи с точностью є и построить его график. Решение системы разностных уравнений найти, используя метод прогонки.

```
• y'' - xy' + 0.2y = x + 1
```

•
$$y(0.9) - 0.5y'(0.9) = 2$$

•
$$y(2.9) = 1$$

• $\varepsilon = 0.04$

```
In [5]: INTERVAL: tuple = (0.9, 2.9)

def get_norm(values: list, step: float) -> float:
    summ: float = 0.0

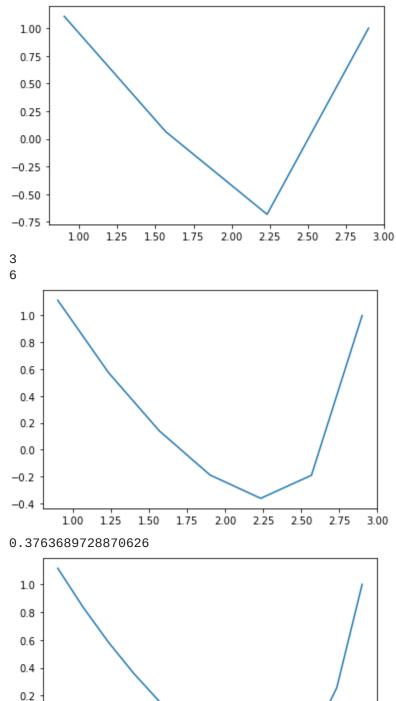
for value in values:
    summ += value * value * step

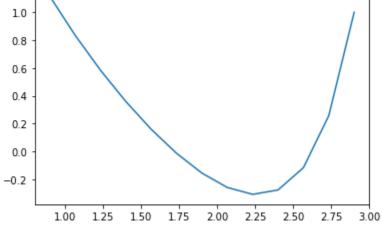
return math.sqrt(summ)
    return max(*values[0:-1])
```

```
In [6]: # initial conditions are "hardcoded" in this solution below:
    def get_3_task_solution_with_number_of_subranges(n: int) -> tuple:
        step: float = (INTERVAL[1] - INTERVAL[0]) / n

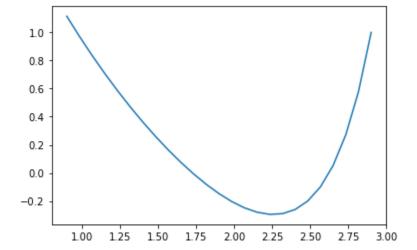
    y_kprev = lambda x: 2 + x * step
    y_k = lambda x: 0.4 * step * step - 4
```

```
y_{knext} = lambda x: 2 - x * step
             b_k = lambda x: (x + 1) * 2 * step * step # for free coefficients vector
             # coefficients matrices with first initial condition already
             free_coefficients: list[float] = [4 * step]
             main_coefficients: list[list[float]] = [[0] * (n + 1)]
             main\_coefficients[0][0] = 2 * step + 1.5
             main\_coefficients[0][1] = -2
             main\_coefficients[0][2] = 0.5
             x_array: list[float] = [INTERVAL[0]]
             for k in range(1, n):
                 new\_row: list[float] = [0] * (n + 1)
                 new_row[k - 1] = y_kprev(x_array[-1])
                 x_array.append(x_array[-1] + step)
                 new_row[k] = y_k(x_array[-1])
                 new_row[k + 1] = y_knext(x_array[-1] + step)
                 main_coefficients.append(new_row)
                 free_coefficients.append(b_k(x_array[-1]))
             x_array.append(INTERVAL[1])
             main_coefficients.append([0] * (n + 1))
             main\_coefficients[-1][-1] = 1
             free_coefficients.append(1)
             y_array: list[float] = list(np.linalq.solve(main_coefficients, free_coefficients))
             plt.plot(x_array, y_array, mew=2, ms=10)
             plt.show()
             return x_array, y_array, get_norm(y_array, step)
 In [7]: def get_3_task_solution(epsilon: float) -> None:
             number_of_ranges: int = 3
             first_solution = get_3_task_solution_with_number_of_subranges(number_of_ranges)
             print(number_of_ranges)
             number_of_ranges *= 2
             print(number_of_ranges)
             second_solution = get_3_task_solution_with_number_of_subranges(number_of_ranges)
             print(abs(first_solution[2] - second_solution[2]))
             while abs(first_solution[2] - second_solution[2]) > epsilon:
                 number_of_ranges *= 2
                 first_solution = second_solution
                 second_solution = get_3_task_solution_with_number_of_subranges(number_of_ranges)
                 print(number_of_ranges)
                 print(abs(first_solution[2] - second_solution[2]))
         def solve_task3() -> None:
In [8]:
             epsilon: float = 0.04
             get_3_task_solution(epsilon)
In [11]: | solve_task3()
```

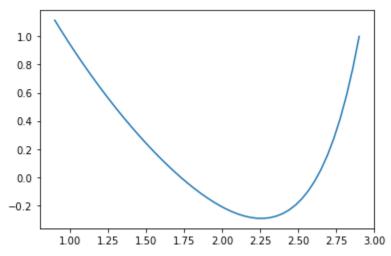




12 0.1711234168705309



24 0.08103112951943459



48 0.03889307894889582

Задание 4. Методом конечных разностей найти приближенное решение краевой задачи с тремя верными значащими цифрами. Решение системы разностных уравнений найти, используя метод прогонки.

```
• (-k(x)y')' + q(x)y = f(x), x = (a, b)
```

- -k(a)y'(a) + 0.5y(a) = 0
- k(b)y'(b) + 0.5y(b) = 0
- a=0,b=1.5,c=0.875,k(x)=0.5 при $\mathbf{x}=(a,c),k(x)=1.8$ при $\mathbf{x}=(c,b),q(x)=5.6$ при $\mathbf{x}=(a,c),k(x)=8.5$ при $\mathbf{x}=(c,b),f(x)=9x(3.5-x)$

```
In [12]: a: float = 0
b: float = 1.5
c: float = 0.875
```

```
In [13]: def K(x: float) -> float:
    if a <= x <= c:
        return 0.5
    else:
        return 1.8</pre>
```

```
In [14]: def Q(x: float) -> float:
    if a <= x <= c:
        return 5.6
    else:
        return 8.5</pre>
```

```
In [15]: def F(x: float) \rightarrow float:
             return 9 * x * (3.5-x)
In [17]: def get_norm(values: list, step: float) -> float:
             summ: float = 0.0
             for value in values:
                  summ += value * value * step
              return math.sqrt(summ)
             return max(*values[0:-1])
         # initial conditions are "hardcoded" in this solution below:
In [20]:
         def get_4_task_solution_with_number_of_subranges(n: int) -> tuple:
             step: float = (b - a) / n
             y_{kprev} = lambda x: K(x)
             y_k = lambda x: 2 * step * Q(x)
             y_{knext} = lambda x: -1 * K(x)
             b_k = lambda x: 2 * step * F(x) # for free coefficients vector
             # coefficients matrices with first initial condition already
             free_coefficients: list[float] = [0]
             main_coefficients: list[list[float]] = [[0] * (n + 1)]
             main\_coefficients[0][0] = 3 * K(a) + step
             main\_coefficients[0][1] = -4 * K(a)
             main\_coefficients[0][2] = K(a)
             x_array: list[float] = [a]
             for k in range(1, n):
                  new\_row: list[float] = [0] * (n + 1)
                 new_row[k - 1] = y_kprev(x_array[-1])
                 x_array.append(x_array[-1] + step)
                 new_row[k] = y_k(x_array[-1])
                 new_row[k + 1] = y_knext(x_array[-1] + step)
                 main_coefficients.append(new_row)
                 free_coefficients.append(b_k(x_array[-1]))
             x_array.append(b)
             main\_coefficients.append([0] * (n + 1))
             main\_coefficients[-1][-3] = K(b)
             main\_coefficients[-1][-2] = -4 * K(b)
             main\_coefficients[-1][-1] = 3 * K(b) + step
             free_coefficients.append(0)
             y_array: list[float] = list(np.linalg.solve(main_coefficients, free_coefficients))
             plt.plot(x_array, y_array, mew=2, ms=10)
             plt.show()
             return x_array, y_array, get_norm(y_array, step)
In [21]: def get_4_task_solution(epsilon: float) -> None:
             number_of_ranges: int = 3
             first_solution = get_4_task_solution_with_number_of_subranges(number_of_ranges)
             print(number_of_ranges)
             number_of_ranges *= 2
```

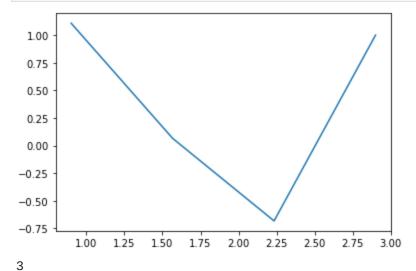
```
print(number_of_ranges)

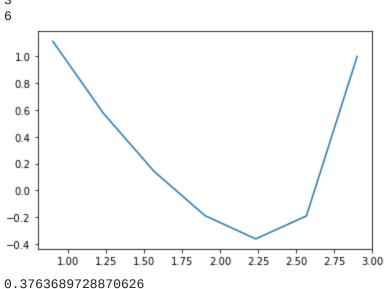
second_solution = get_4_task_solution_with_number_of_subranges(number_of_ranges)
print(abs(first_solution[2] - second_solution[2]))

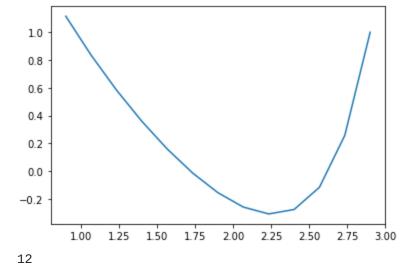
while abs(first_solution[2] - second_solution[2]) > epsilon:
    number_of_ranges *= 2
    first_solution = second_solution
    second_solution = get_4_task_solution_with_number_of_subranges(number_of_ranges)
    print(number_of_ranges)
    print(abs(first_solution[2] - second_solution[2]))
```

```
In [26]: def solve_task4() -> None:
    epsilon: float = 0.001
    get_3_task_solution(epsilon)
```

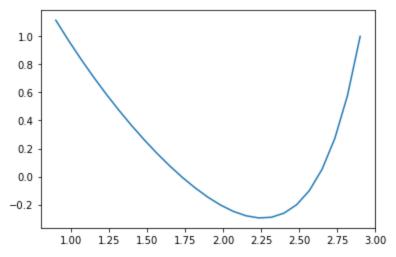
In [27]: solve_task4()



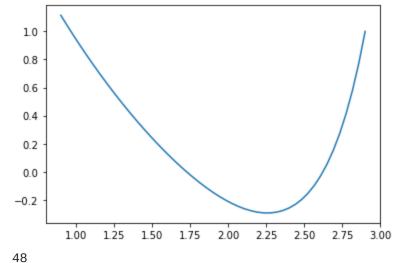




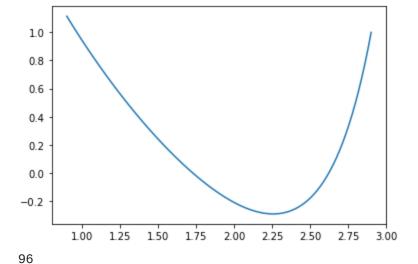
0.1711234168705309



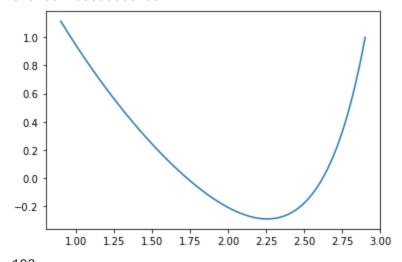
24 0.08103112951943459



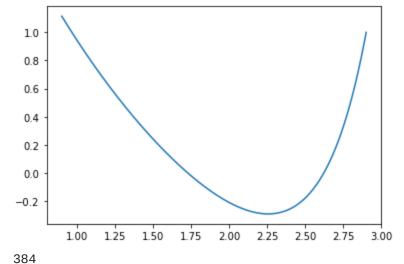
0.03889307894889582



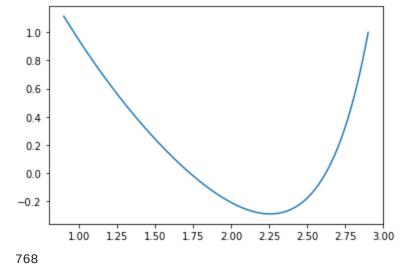
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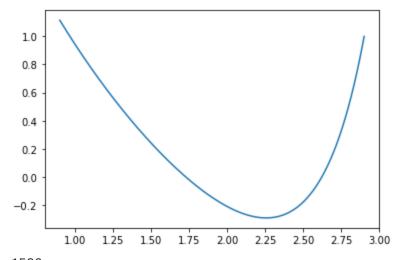
192 0.00933434142658951



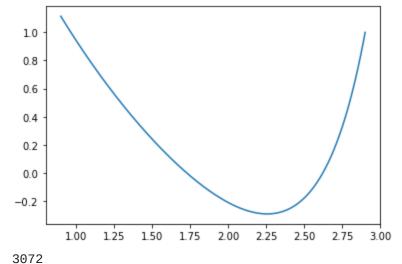
0.004630403877576628



0.0023057519770108836



1536 0.001150480450597624



0.0005746371510289006