Quicksort

Prepared By:
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Sorting algorithms

- Insertion, selection and bubble sort have quadratic worst-case performance
- The faster comparison based algorithm ?

  O(nlogn)

- Quicksort

Quicksort Algorithm

Given an array of *n* elements (e.g., integers):

- If array only contains one element, return
- Else
  - pick one element to use as *pivot.*
  - Partition elements into two sub-arrays:
    - Elements less than or equal to pivot
    - Elements greater than pivot
  - Quicksort two sub-arrays
  - Return results

Quicksort Algorithm

- Quick Sort method is an efficient sorting method for larger List.
- It works fine for the list having large number of elements.
- It uses Divide and conquers strategy in which a list is divided into two smaller lists.
- First initialize LOW with index of the first element and HIGH with index of last element.

Quicksort Algorithm

- Now we scan elements from left to right and compare each element with PIVOT element. If the scanned element is less then the PIVOT element we scan next element and increment the value of LOW. Repeat same procedure until we found the element which is greater then the PIVOT element.
- Now we scan elements from right to left and compare each element with PIVOT element. If the scanned element is greater then the PIVOT element we scan next element and decrement the value of HIGH. Repeat same procedure until we found the element which is less then the PIVOT element.

Quicksort Algorithm

- Now we compare the value of LOW and HIGH. If LOW is less then HIGH then we interchange the element which are at the index LOW and HIGH.
- Increment the value of LOW and decrement the value of HIGH. Repeat above procedure while value of LOW less then or equal to value of HIGH.
- After the completion of first PASS the entire list of elements is divided in to two lists. First list contains elements which are less then the PIVOT element and second list contains elements which are greater then the PIVOT element.
- The above procedure is recursively repeated for the sub lists until all the elements in the lists are sorted.
- **The order of comparison for this method is o (nlogn).**

Example

We are given array of n integers to sort:

40	20	10	80	60	50	7	30	100

Pick Pivot Element

There are a number of ways to pick the pivot element.

In this example, we will use the first element in the array:

40	20	10	80	60	50	7	30	100

Partitioning Array

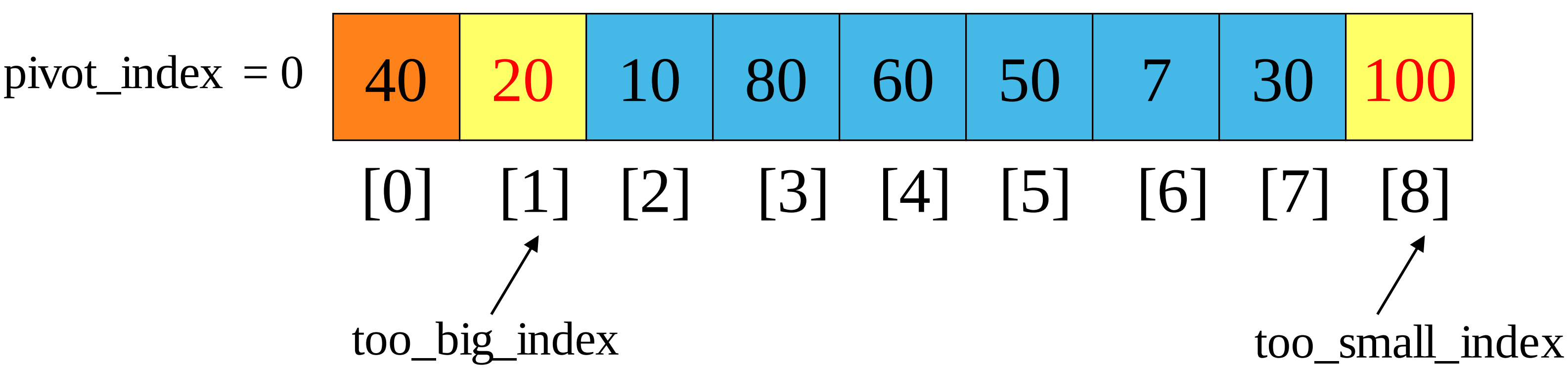
Given a pivot, partition the elements of the array such that the resulting array consists of:

1. One sub-array that contains elements >= pivot
2. Another sub-array that contains elements < pivot

The sub-arrays are stored in the original data array.

Partitioning loops through, swapping elements below/above pivot.

1. While data[too_big_index] <= data[pivot]
   ++too_big_index
2. While data[too_small_index] > data[pivot]
   --too_small_index
3. If too_big_index < too_small_index
   swap data[too_big_index] and
   data[too_small_index]
4. While too_small_index > too_big_index, go to 1.



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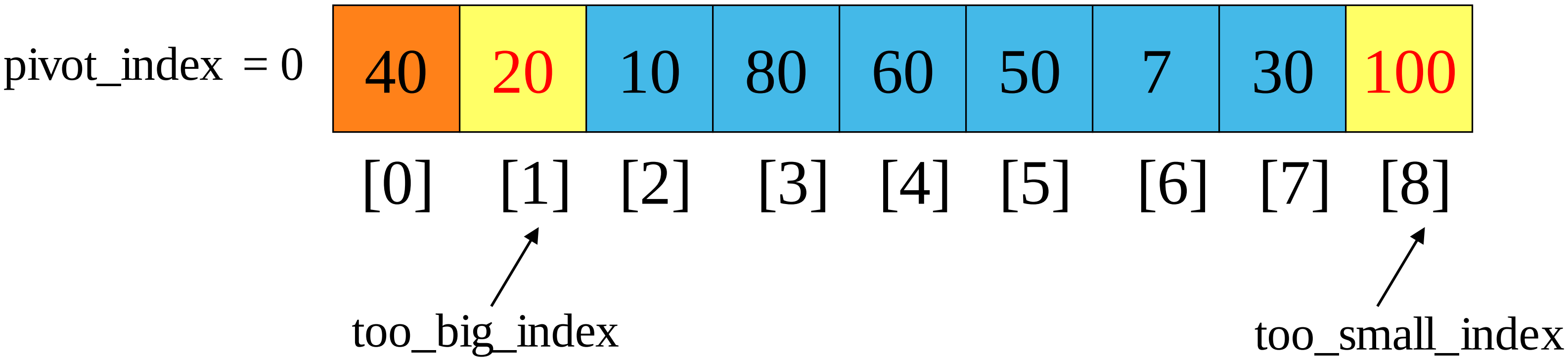

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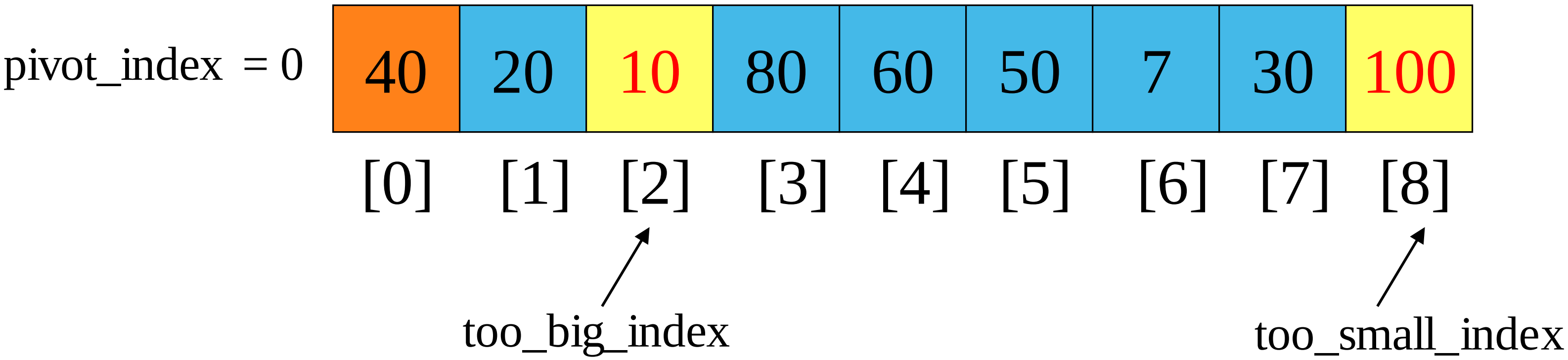

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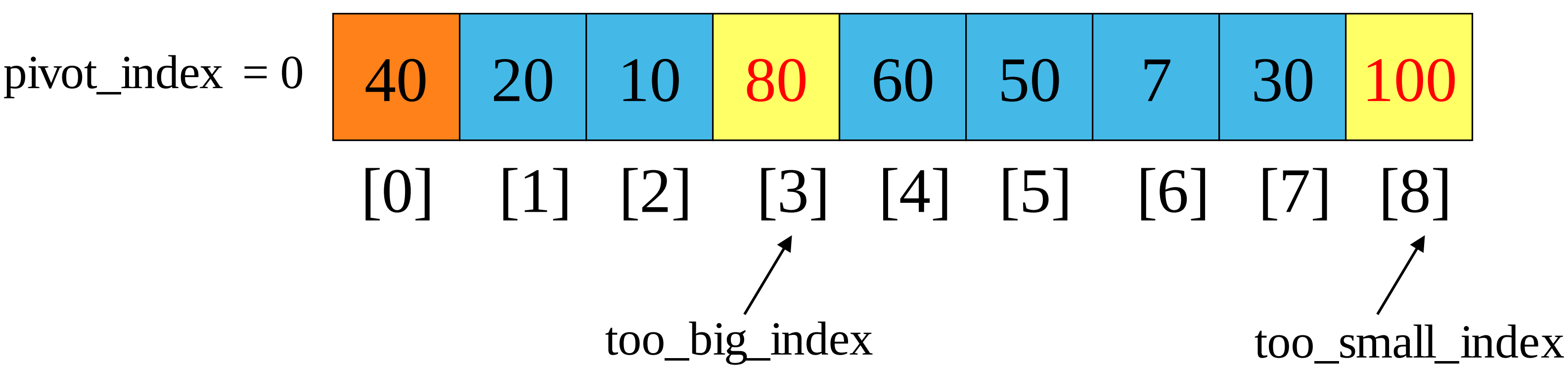

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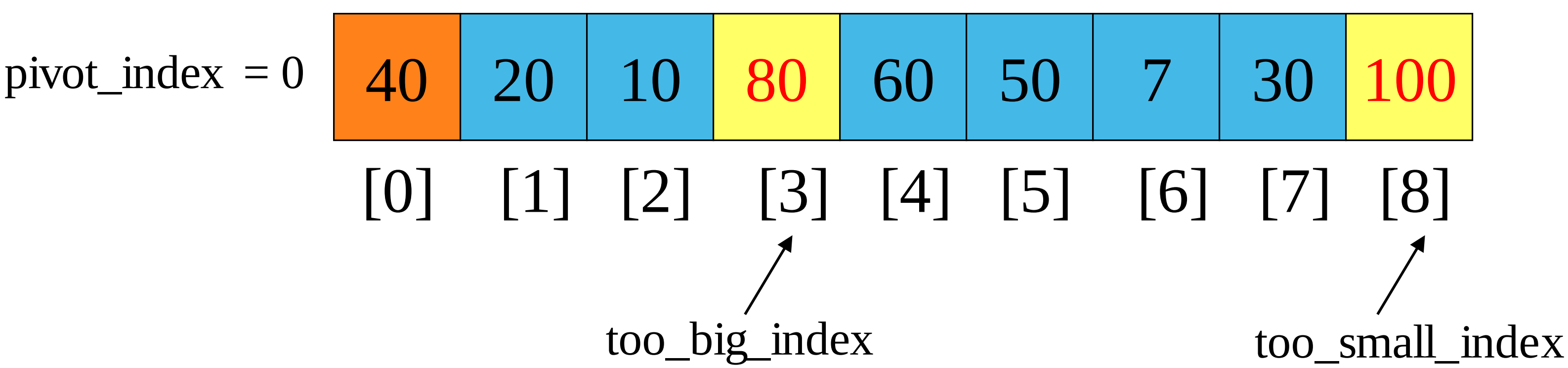

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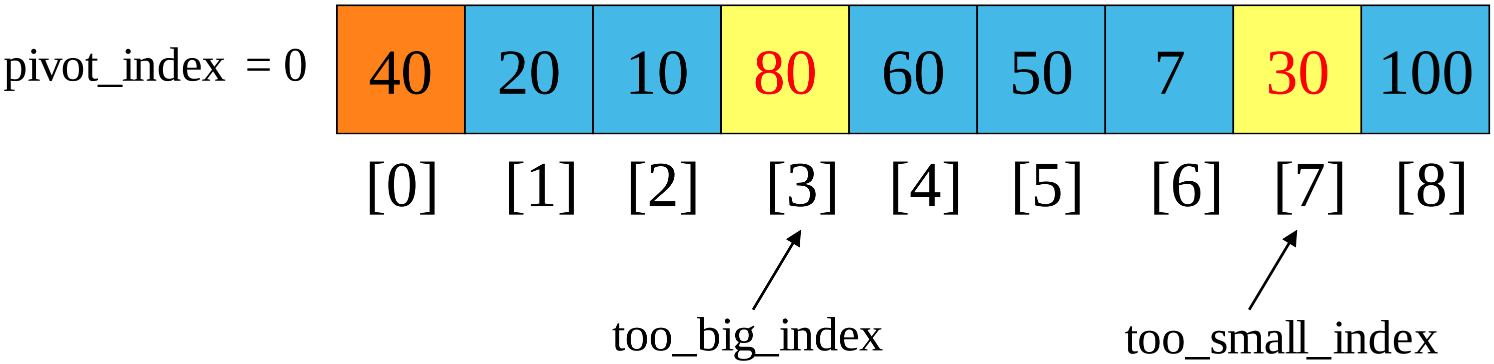

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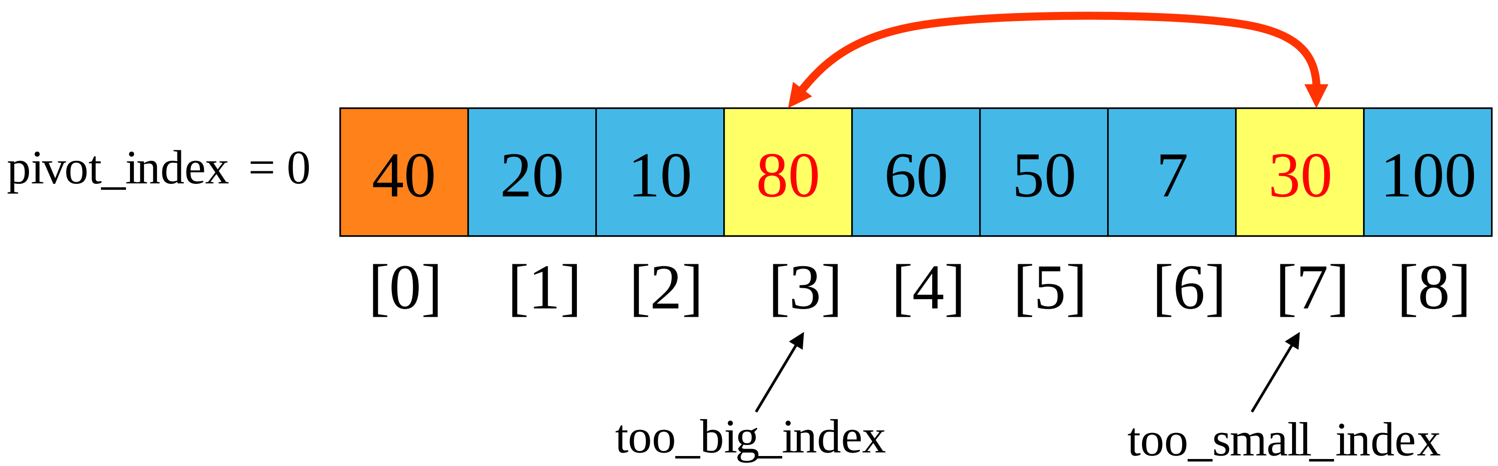

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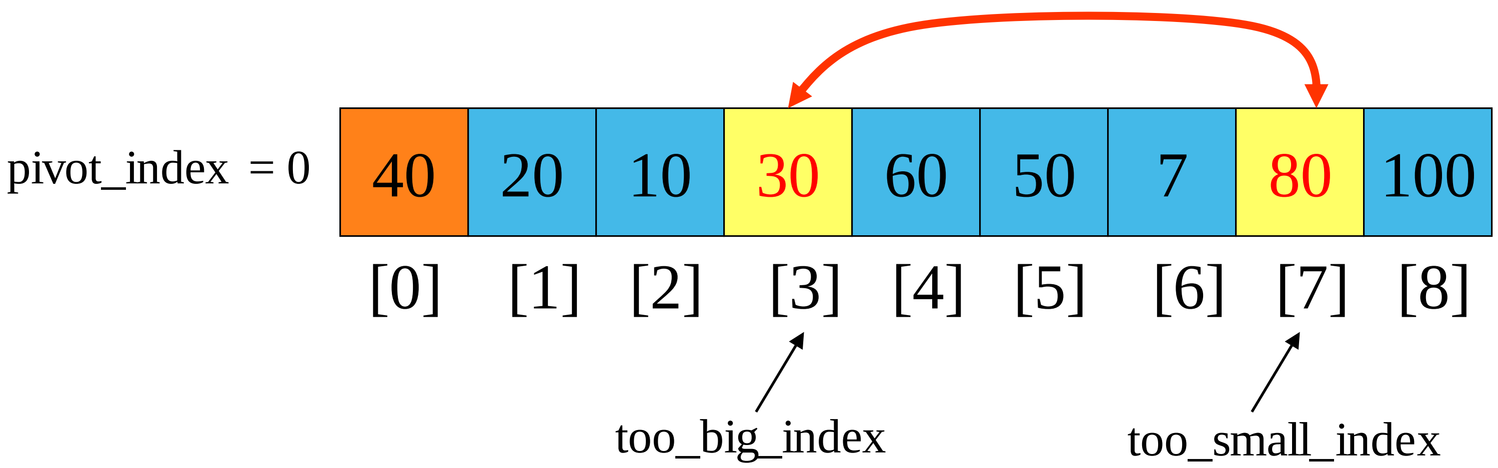

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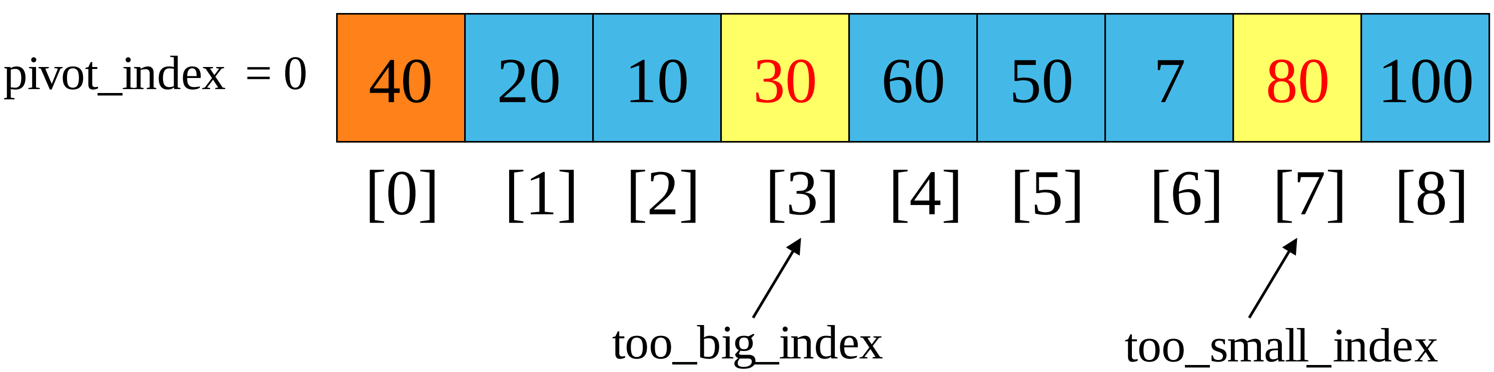

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too_big_index → [3]

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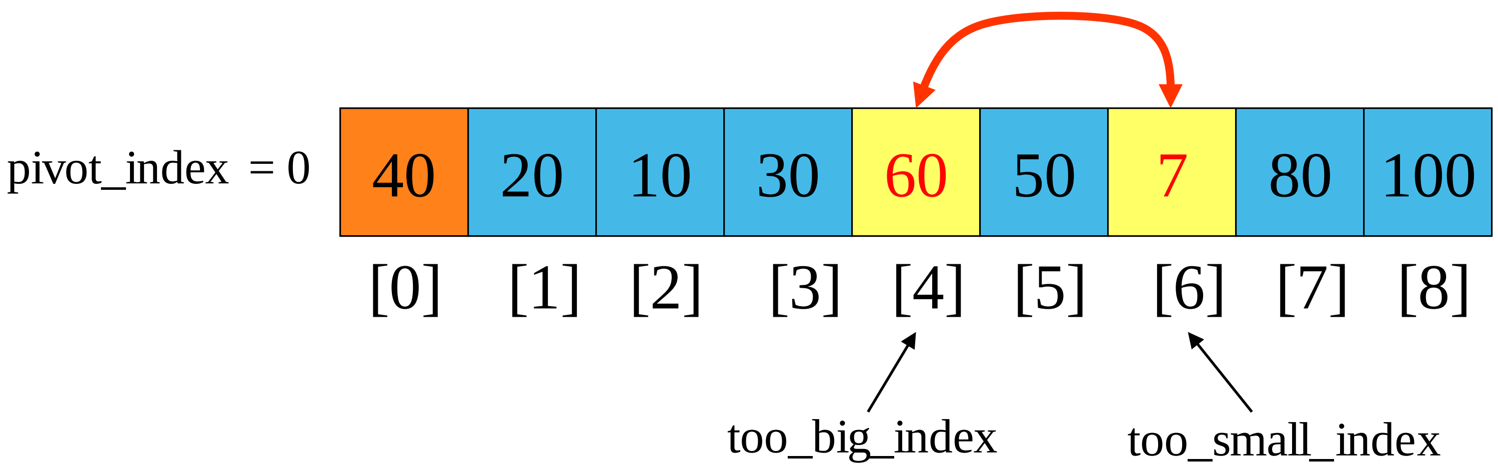

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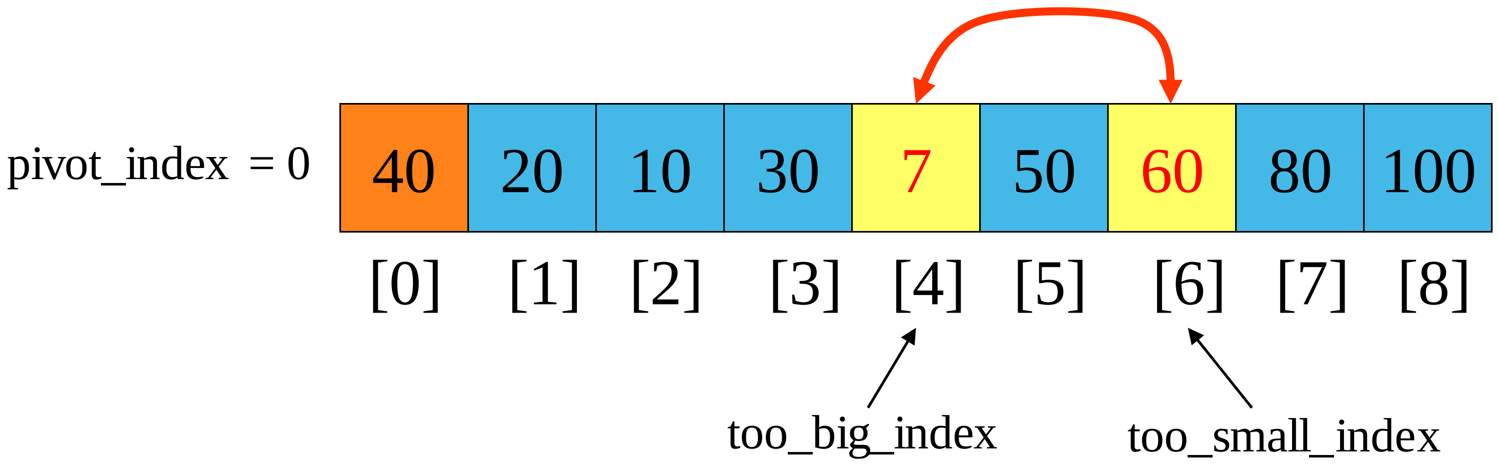

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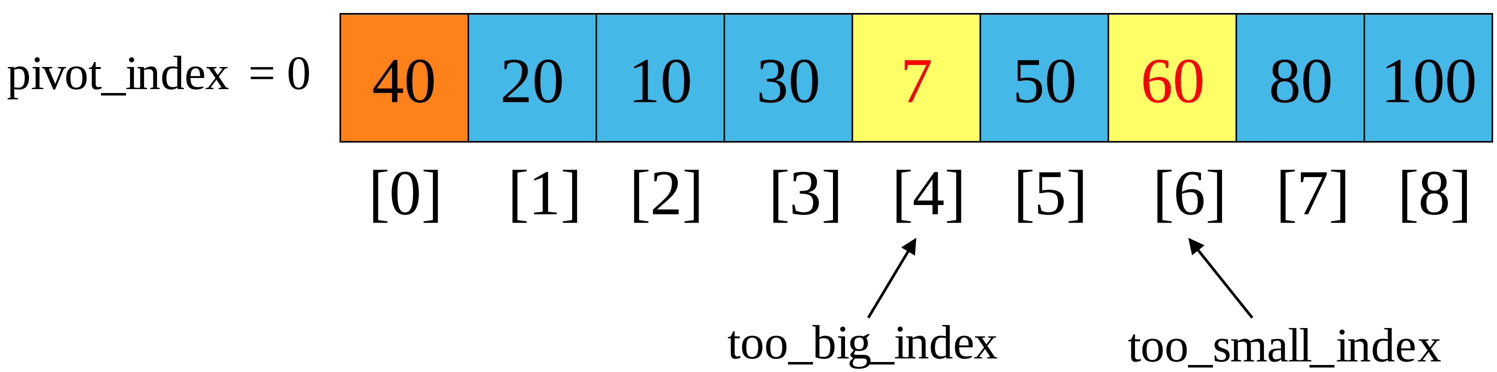

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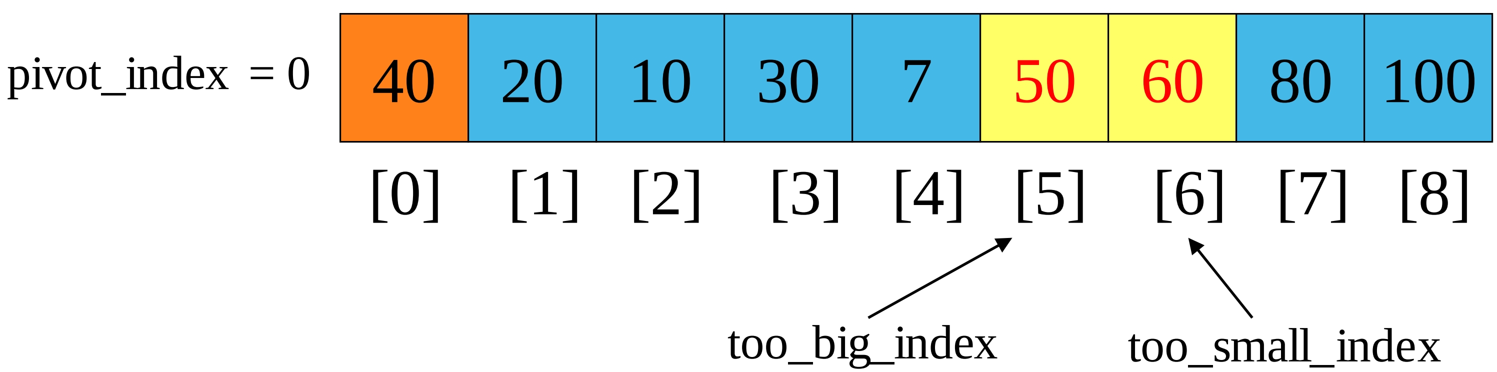

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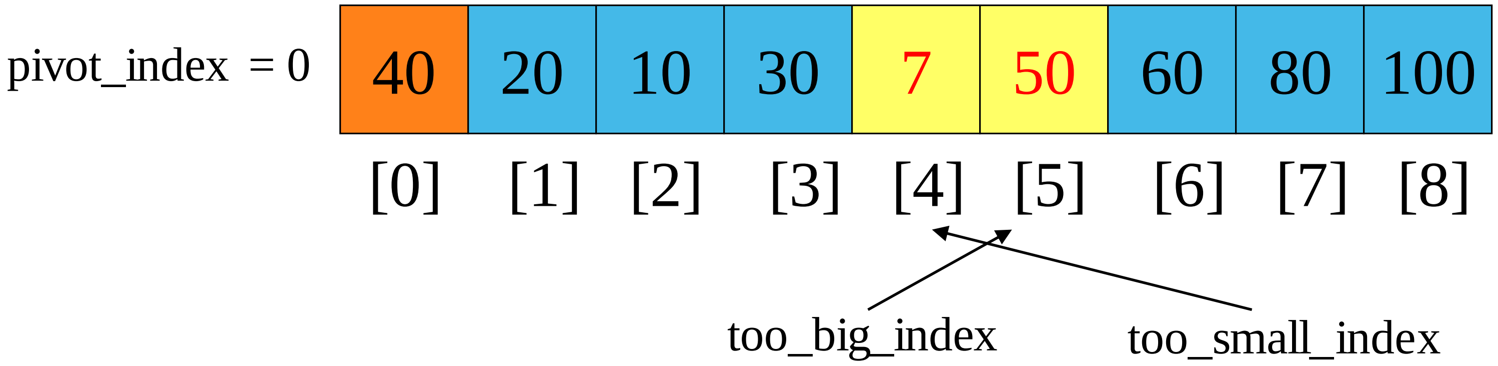

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pivot_index = 4

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too_big_index too_small_index

Partition Result

7	20	10	30	40	50	60	80	100
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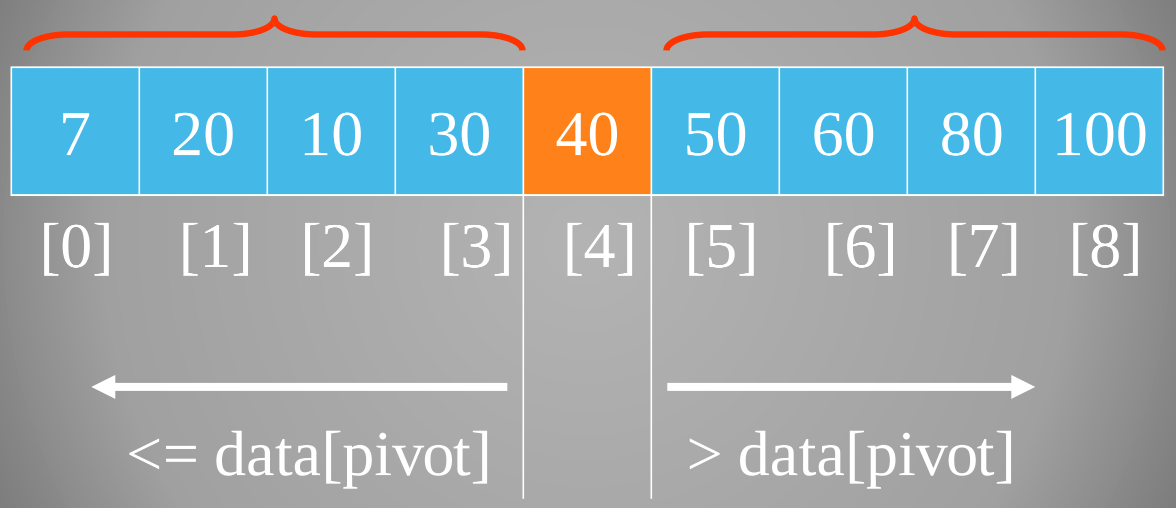
<= data[pivot]

> data[pivot]

Recursion: Quicksort Sub-arrays



Quicksort Algorithm

```
Step-1 : low<- first ,high <- Last
Step-2 : pivot <- (low +high)//2
Step-3 : Repeat through step-6 while low<high
Step-4 : Repeat While list1[low]<list1[pivot]
            low<-low+1
Step-5 : Repeat While list1[high]>list1[pivot]
            high<-high-1
Step-6 : if(low<=high) then
          temp=list1[low]
          list1[low]=list1[high]
          list1[high]=temp
          low<-low+1
          high<-high-1
Step-7 : if (first<high)
                 call quick_sort(list1,first,high)
          if(low<last)
                 call quick_sort(list1,low,last)
Step-8 : Exit
```

```python
def quick_sort(list1,first,last):
    low=first
    high=last
    pivot =(low+high)//2
    while (low<high):
        while(list1[low]<list1[pivot]):
            low=low+1
        while(list1[high]>list1[pivot]):
            high=high-1
        if(low<=high):
            temp=list1[low]
            list1[low]=list1[high]
            list1[high]=temp
            low=low+1
            high=high-1
    print(list1)
    if(first<high):
        quick_sort(list1,first,high)
    if(low<last):
        quick_sort(list1,low,last)
list1=[40,20,10,80,60,50,7,30,100]
print("list before sorting",list1)
quick_sort(list1,0,len(list1)-1)
print("list after sorting", list1)
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- What is best case running time?

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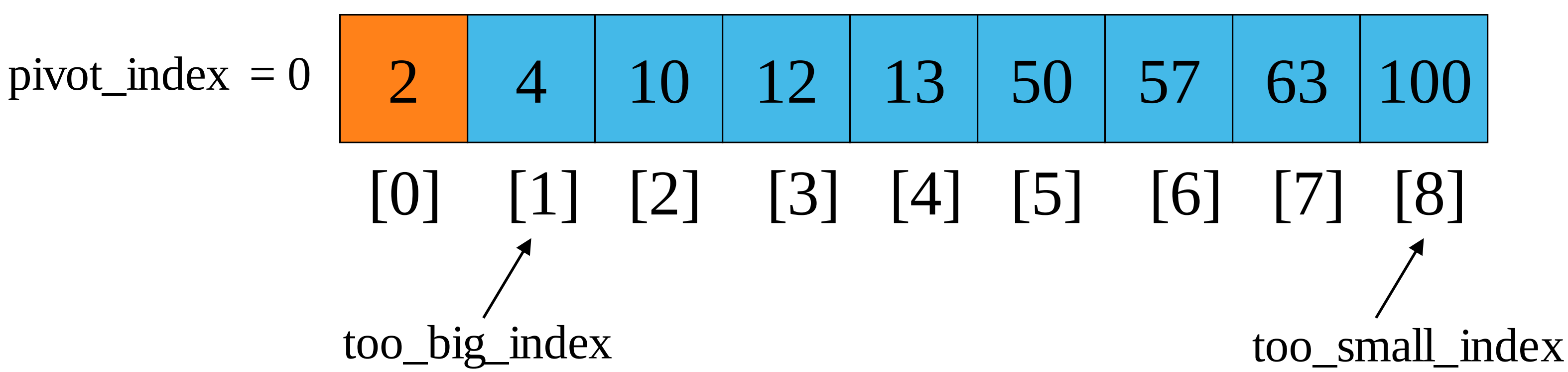
Quicksort Analysis

- Assume that keys are random, uniformly distributed.
- Best case running time: $O(n \log_2 n)$
- Worst case running time?

Quicksort: Worst Case

- Assume first element is chosen as pivot.
- Assume we get array that is already in order:



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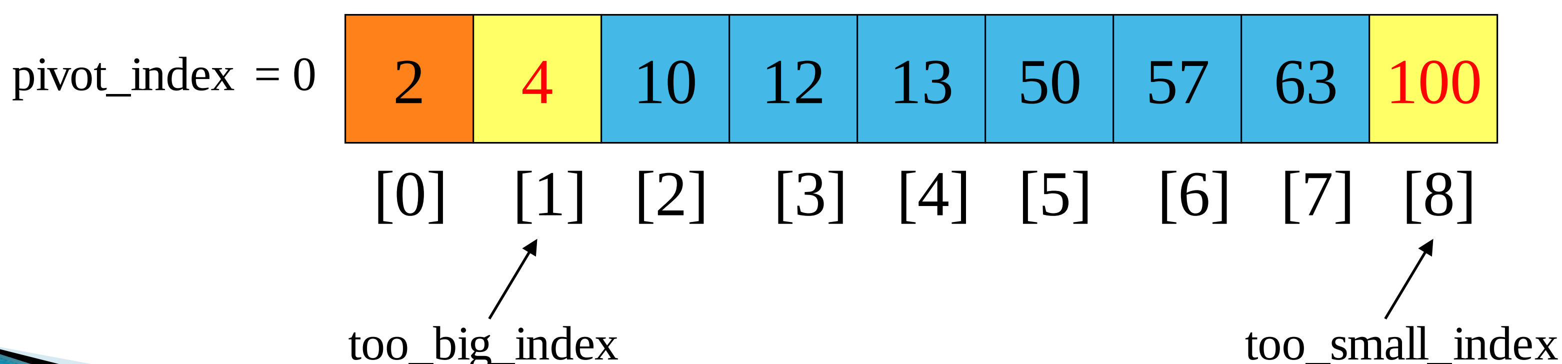

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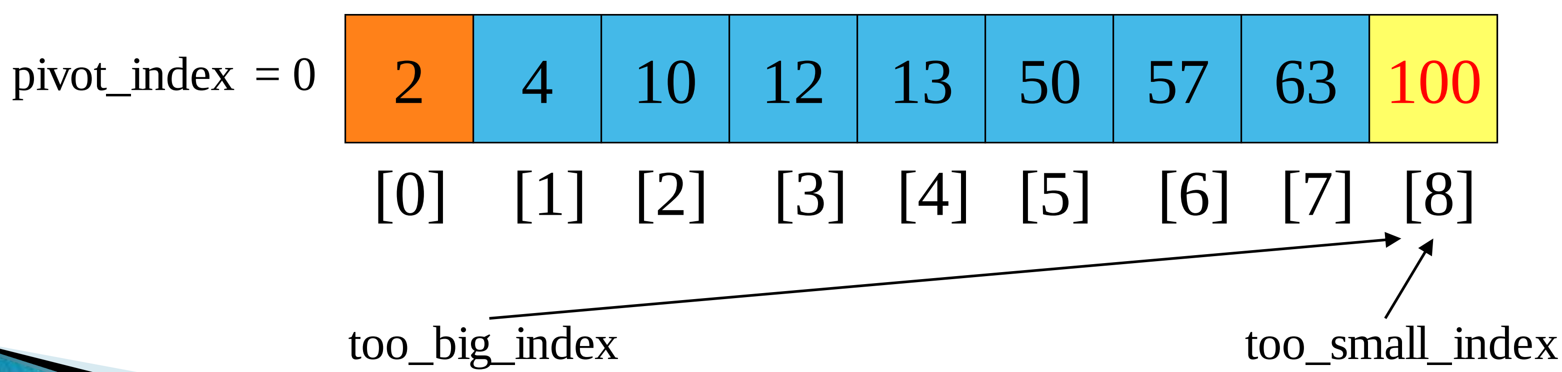

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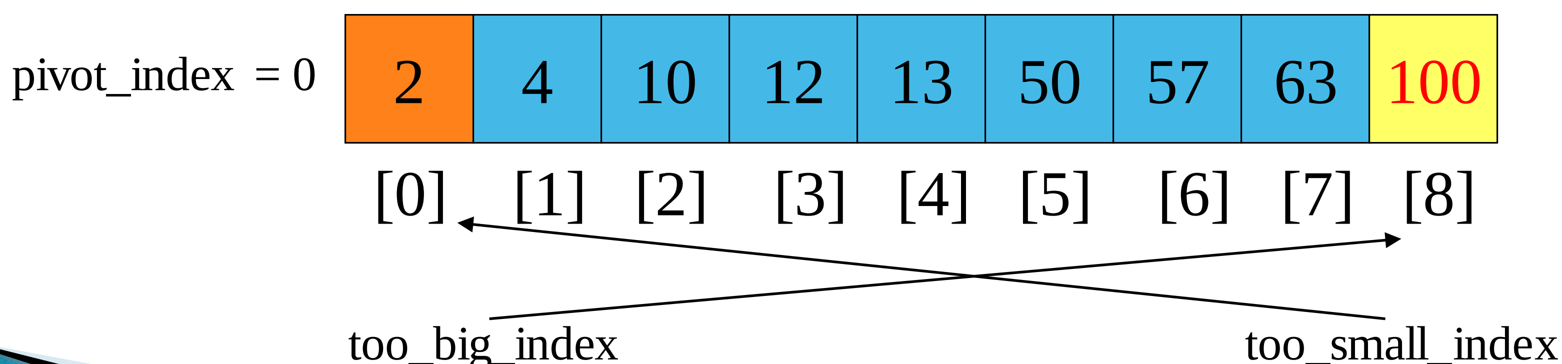

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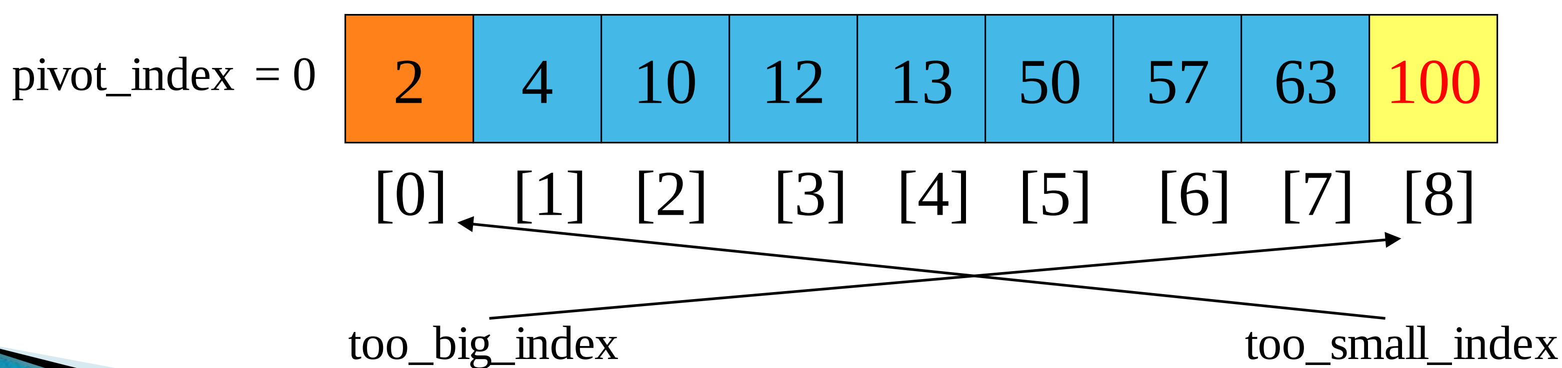

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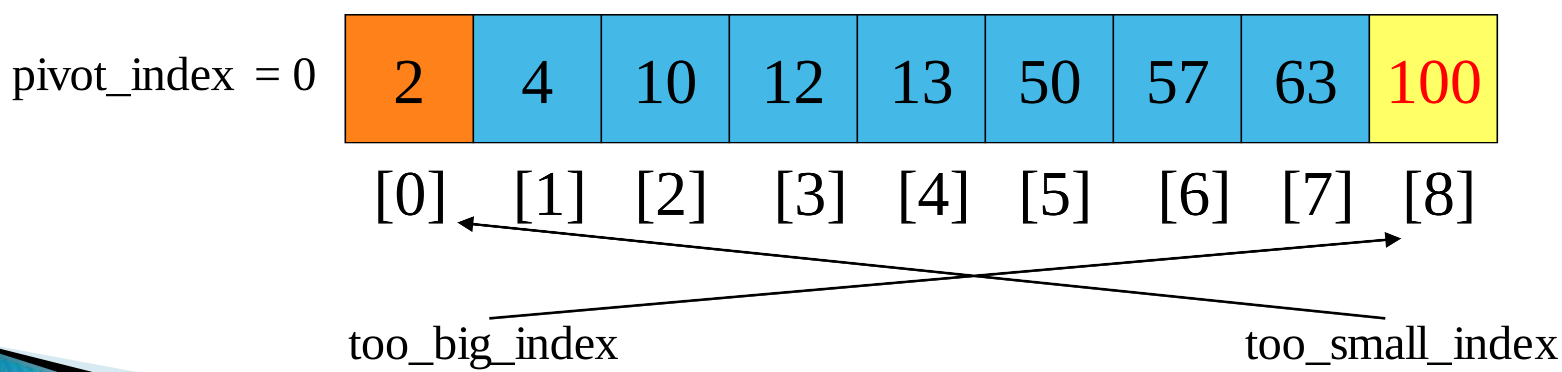

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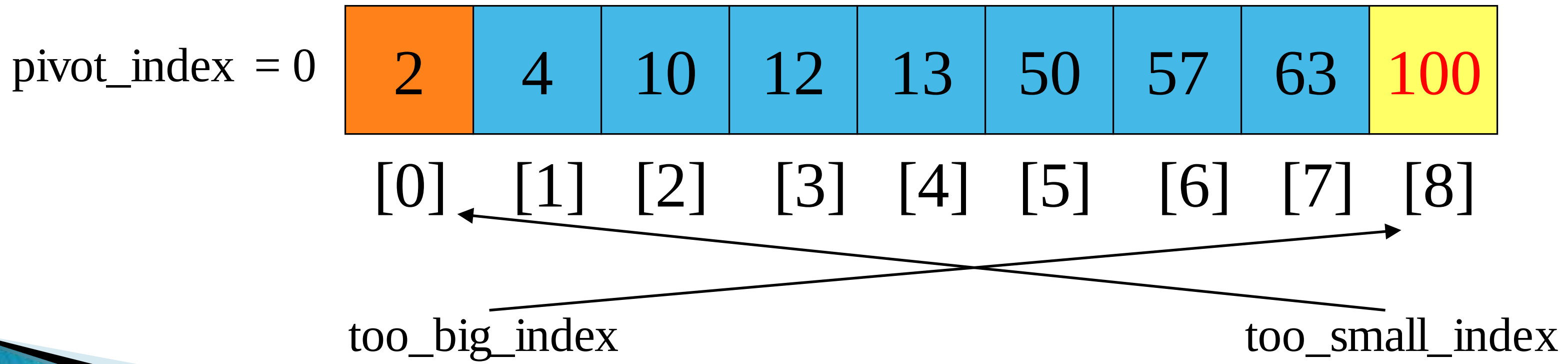

1. While data[too_big_index] <= data[pivot]
   ++too_big_index
2. While data[too_small_index] > data[pivot]
   --too_small_index
3. If too_big_index < too_small_index
   swap data[too_big_index] and data[too_small_index]
4. While too_small_index > too_big_index, go to 1.
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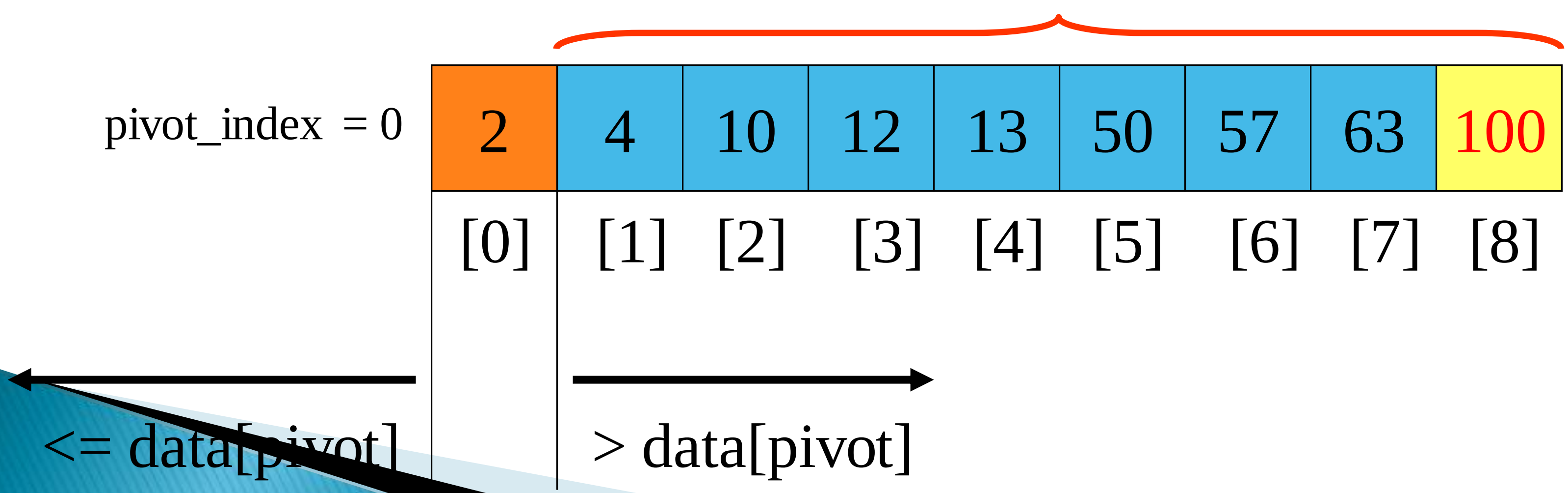

Quicksort Analysis

- Assume that keys are random, uniformly distributed.
- Best case running time: $O(n \log_2 n)$
- Worst case running time?
  - Recursion:
    1. Partition splits array in two sub-arrays:
       - one sub-array of size 0
       - the other sub-array of size n-1
    2. Quicksort each sub-array
  - Depth of recursion tree?

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- What can we do to avoid worst case?

Improved Pivot Selection

Pick median value of three elements from data array:

data[0], data[n/2], and data[n-1].

Use this median value as pivot.