SP 500 Recent Statistical Performance Analysis

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WorldQuant University MScFE Capstone Project

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Abstract

We have analyzed statistically the weekly adjusted close prices and returns of four ETFs (U.S. SP500, VNQ, XLE, and XLF) for the last ten years. The objective of this assignment is to revisit the performances of all the ETFs considering the SP500 as the benchmark. We would like to analyze the historical data in the perspective of market dynamics and COVID-19 pandemic impact. As part of analysis, the methods like data series stationarity, distributions, associations, serial autocorrelation behavior, mean-reversion relationship, dependencies, etc. have been applied on four selected ETFs. Furthermore, we plan to compare the performances and insights of these ETFs in detail so that we can emphasize on the S&P 500. This project is designed to consider the benchmark role of the S&P 500 and attempts to establish that the S&P 500 is a dependent ETF.

Keywords: *QQ-plot, ADF, CADF, PCA, k-Means, EDA, Multicollinearity, VIF, Stationarity, Serial Autocorrelation, Correlation, R-Squared Value, p-value*

1. Project Objective

Our main objective for this project is to conduct an extensive study on the SP500 ETF performances for a decade starting from June 2012 to June 2022. It's expected to find notable events in the 10 years duration like COVID-19 pandemic effects. Further work includes revealing the performance of other three ETFs from different industries. The plan is to consider the S&P 500 as a benchmark ETF and compare other three ETFs' significant movements in terms of distribution, joint distributions, correlation status, clustering, and stationarity. It is understood that the S&P 500 ETF as a benchmark is more appealing to the financial market, as the index of this ETF covers all the market ETFs and relatable parameters.

2. *Literature Review*

Meanwhile, a great deal of research has been done with ETFs under many different scopes and so far, the S&P 500 is one of the most attempted ETFs amongst all. The scopes of the works are diversified. Firstly, modeling data for predictions of ETFs performances under external conditions (macroeconomic indicators) and constructing economic indicators under certain circumstances is one kind of work. Kajal and Moore (1991) worked on this type of scope of economic indicators [1]. They constructed leading economic indicators for the purpose of analyzing recessions. Besides traditional analytical approaches a new and improved set of tools and approaches are being started to apply to model the economic indicator driven ETF index. In the recent past, the underlying tests and visualizations are being carried out by Machine Learning approaches to track appropriate indicators [9].

Another pattern of work are performance metrics of stocks in the case of abrupt phase shifting. This needs to be brought under study with regime shift identification for traders. Doing so, they can fix their trading strategy by having clear ideas on various asset classes and market sections in the identified regimes. In researching the instrument of the S&P 500 index from 2000-2017, is the Markov Autoregressive model in the paper of Sonam Srivastava, Mentor – Ritabrata Bhattacharyya [4].

Furthermore, Mark Babayev, Folakemi Lotun, Googwill Tatenda Mumvenge, Ritabrata Bhattacharyya [5] did distinct work emphasizing the black swan events. These identifying black swans might be important for a broad scale, especially for traders because a short-term Mean-Reversion system is helpful to get a risk-return profile picture for them. Based on this, risk mitigation techniques can be applied if the users have a clear understanding of trading performances. Hence, there is a possibility to construct an outperforming Mean-Reversion strategy.

As long as the accuracy of the work increased to a remarkable level and therefore the prediction of the performances of ETFs by Ligita Gaspareniene, Rita Remeikiene, Aleksejus Sodidko, and Vigita Vebraite [2]. In addition, there are some works by Sonam Srivastava, Mentor – Ritabrata Bhattacharyya [4] in relation to the identification of a regime shift. As such, the appropriate time selection for the investment of funds from a class of assets with the suitable timing to buy and sell of investments. Therefore, some works are relating to appropriateness, best fit, and accuracy of the model for a specific set of ETFs and timing.

There are some works for historical performances of ETFs done by many researchers. Some portfolio managers invested in the same S&P 500 in different markets in the different countries over the word. Gabjin Oh and Seunghwan Kim, Cheol-Jun Um (2006) work for this type of cross-border investors [6]. They addressed the statistical properties of daily index performance data (historical) of the S&P 500 with another ETF (KOSDAQ) of seven different countries. During the investigation, using the Detrended Fluctuation and Surrogate tests, found the returns of international stock market indices of those countries follow universal power laws. They observed that there was an exponent of approximately three while for the Korean market it is 0.3. In addition, the non-linearity of returns was traced from the magnitude time series that reveals emerging market ETFs (KOSDAQ) incurred a higher volatility than that of mature markets (S&P 500).

In this project, we were a bit different with ETFs' historical analysis and we paid our attention to the indigenous ETFs of the U.S. market. We picked one ETF from three different sectors including real estate, energy, and finance along with the S&P 500 for our work.

3. Background

Statistically, we are revisiting the S&P 500 performance of recent years. Our study on the SP500 performance shows the adjusted closing price for the past decade starting in July 2012. We aim to capture some rare events and their impact over the S&P 500. There are two points worth noting: First, is the symmetry, spreading, distributions, and inherited features. Second, the effect due to rare events. For instance, COVID-19 and the Russian attack on Ukraine. From a regime switching perspective, COVID-19 has the most significant impact for any market.

We chose to use an adjusted closing price. The adjusted closing price took stock splitting, offering, and dividends declared into the consideration. Therefore, it shows market sentiment of the trading day/week. An ETF is distinct from the S&P 500 index in many ways. Firstly, it weighs 80% of the stock market capital in the U.S. with the size of around 33.8 trillion USD in December 2022. Therefore, it reflects the U.S. economy to a reasonable extent. Our research focuses on the S&P 500 data from July 2012 to June 2022. We also considered three other ETFs weekly returns in our research: Vanguard's real estate index fund ETF (VNQ), an energy sector SPDR ETF (XLE), and a finance sector SPDR ETF (XLF).

4. Methodology

In this work, we applied some basic statistical measures, Exploratory Data Analysis (EDA) and Unsupervised Machine Learning (UML), tools to historical data. Both basic statistical measures and EDA are simple and well recognized by researchers' groups. Since we would like to avoid all complex methods, like the Monte Carlo Simulation, we applied basic statistics, EDA and some UML algorithms (PCA, k-Means Clusters). Since our intension was to investigate the insights of data patterns, we use both PCA and K-means. To be noted that the PCA and k-Means are appealing and acceptable to researchers. Historical data with common data fields is available and easy to compute. Therefore, we considered historical data.

4.1. Data Collection

We considered four ETFs in our work including Vanguard's real estate index fund ETF (VNQ), an energy sector SPDR ETF (XLE), and a finance sector SPDR ETF (XLF), and the S&P 500 (SPY). We picked the S&P 500 as our benchmark instrument since it is a stock market index tracker of the 500 largest companies listed in the U.S. These blue-chip companies represent 80% of the U.S. stock market capital which embodies the U.S. economy to a significant degree. This looks to be the most influential market reflecting indicators based on the most of the academicians' and researchers' interests. Our concentration was to evaluate the relationships from July 2012 to June 2022. We were interested in the weekly adjusted closing prices and returns, extracted from price. We used python programming language through Jupyter notebook in this work. We used yfinance to extract relevant data from Yahoo Finance with the appropriate API Key. As Datatype is pandas DataFrame we used pct_change to calculate weekly returns.

4.2. Statistical Tests

We did some basic statistical tests including mean, median, standard deviation, skewness, and kurtosis. The results of these tests helped us to figure out the symmetric features and tailing pattern of the distributions of the datasets. We visualized the data through Exploratory Data Analysis (EDA), such as Box-plot (outlier detection), Scatter-plots and QQ-plot (normal distribution tracing). We, further, conducted a Jarque-Bera test to check sample data Skewness and Kurtosis in terms of normal distribution [3]. We attempted the Augmented Dickey-Fuller (ADF) test to analyze the stationarity of the series (first order serial autocorrelation). We, therefore, calculate correlation coefficients to find out the degree and direction of association among each pair of the ETFs prices. Additionally, we did a joint distribution of prices of each pair of ETFs considering the S&P 500 as a benchmark.

4.3 Hyper-technical Algorithm

4.3.1 PCA

Principal Component Analysis (PCA) is categorized as unsupervised learning, which is a well-established field of statistical approaches. This method has been used for quite some time and serves a variety of beneficial functions. The core concept is to reduce high-dimensional data to a much smaller set of dimensions so that the variables are independent. The methods we demonstrated in our project include comparing the covariance matrices of SPY, VNQ, XLE, and XLF. The PCA method takes high dimensional data and purifies it to its most crucial components. The Eigen-Decomposition of the matrix, which is the process of using historical data, returns, correlations, and covariances breaking down a matrix of covariances into pieces. The Sparse Principal Component Analysis (Sparse PCA) limits the number of variables by projecting into fewer dimensions. It uses only part of the stock rather than taking the conventional matrix and mapping it into a reduced space where entire variables are used.

In summary, the PCA is a form of unsupervised learning without labels. It is extremely useful in helping us understand data through Eigen-decomposition of the matrix. We are then able to reduce the data dimensions from a high-dimensional space to a lower-dimensional space. If we have a collection of independent variables, we can use eigenvalues to gauge the relative importance of each. Furthermore, it provides us a way to determine the relative importance of different dimensions in order to conduct further time series data analyses.

4.3.2 K-Means

Data is divided into clusters based on degree of similarity in the unsupervised machine learning technique known as data clustering. The most widely used method is K-Means clustering.

There are five basic fundamental steps of K-means, which can be listed as follows: In the first step, we decide how many clusters to search (In K-means, this is the k). The algorithm then chooses k points at random from our data. These locations are also known as centroids. In the second step, each data point's distance from to centroids is determined. With the next step, each data point will be assigned to the closest centroid in order to create clusters. In the fourth step, a new centroid is determined by averaging the values of all its cluster members and discarding prior centroid values. During the final step, centroids are then calculated, again, by repeating steps two through four. The method is considered to converge when the centroids become stable.

Given enough data to train the algorithm, K-means may be used on any number of dimensions. However, in our project, we limit our use case with two dimensions in order to facilitate easy to read visualizations. The initial centroids chosen in step one are very critical in our project. For instance, even though it appears that the algorithm has discovered the optimal groupings, the fundamental steps were to be reinitiated with various initial centroids placements. In which case, a superior outcome might yet be discovered.

4.3.3 Markov Switching Model

The Markov Switching Autoregressive Model applies state-dependent parameters to dynamic regression models with varying characteristics throughout undetected states to account for structural breaks or multi-state occurrences. These models are referred to as Markov-switching models since a Markov chain is used to transition between the unobserved states. There are two types of models: Markov-switching autoregressive (MSAR) models ensuring regular adjustments, while regression (MSDR) models facilitate fast adjustments.

5. Evaluation

We applied some traditional statistical measures and Exploratory Data Analysis (EDA) techniques. Since basic statistical measures allow us to determine different order moments (mean, SD, Skewness, Kurtosis) of data series, we applied that. Also, under EDA we applied normality tests, Box-plots, QQ-plots, joint-distribution plots for pair data series and Scatter Plots. Since all of these assist us to have clear ideas about normality of the distribution and outliers' information. We, further, applied Augmented Dicky-Fuller test (ADF) t-statistic to get a picture about stationarity (presence of unit root). We, alongside, did first order serial autocorrelation. Additionally, we applied correlation heat map and Variance Inflation Factor (VIF) to determine the multicollinearity. We used the Correlation matrix to calculate the degree and direction of association between ETFs prices. We also calculate Cointegrated ADF (CADF) to study the Mean-reversion property before and after the COVID-19 pandemic. If the ML approach is of concern, we applied PCA and K-means so that we can figure out the number of components that capture the variance in the data set as well as the data of the similar patterns that are addressed effectively. We especially paid attention to the COVID-19 impact over our study. In that context we did mean-reversion property related work. We also identified the regime shift in the tenure of the study. We, therefore, applied the Markov Autoregressive Switching Model (MASM).

To justify the results generated from the model applications we relied upon R-Squared value for all regression methods. Usually, the higher R-Squared value indicates the more reliability of the model. But, in some event higher R-Squared value may misguide (suggests unreal events). So, while using R-Squared value as a mark of reliance, we need to understand the insights of the event clearly. We further use p-value to justify some other model generated results. In this assignment, we used p-value for the judgment of t-test statistics to detect the unit root of time series datasets.

The evaluation methods we consider in this project are R-Squared, Mean Squared Error (MSE), and p-value. R-Squared is a statistical fit metric that quantifies the proportion of a dependent variable's fluctuation that can be accounted for by the independent variable. A higher R-squared value indicates the more reliable the method is. The R-squared value listed in the figure below further confirms our model. The degree of inaccuracy in statistical models is gauged by the mean squared error (MSE). The p-value is the determining factor for consideration of unit root existence in our time series data. We applied an ADF test to check stationarity. For autoregressive models, we specifically took into account the probability ratio test for regime switching periods during pre and post COVID-19 periods. The two figures below illustrate the comparison of statistical tests of these periods. The test results can be found in our repo link provided in this paper.

OLS Regression Results

Dep. Variable:	SPY	R-squared:	1.000
Model:	OLS	Adj. R-squared:	1.000
Method:	Least Squares	F-statistic:	5.871e+29
Date:	Sat, 11 Mar 2023	Prob (F-statistic):	3.61e-45
Time:	13:43:48	Log-Likelihood:	296.46
No. Observations:	8	AIC:	-582.9
Df Residuals:	3	BIC:	-582.5
Df Model:	4		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-9.975e-18	1.45e-17	-0.687	0.541	-5.62e-17	3.62e-17
SPY	1.0000	2.95e-15	3.39e+14	0.000	1.000	1.000
VNQ	5.69e-16	9.89e-16	0.575	0.605	-2.58e-15	3.72e-15
XLE	4.718e-16	9.24e-16	0.510	0.645	-2.47e-15	3.41e-15
XLF	0	2.16e-15	0	1.000	-6.86e-15	6.86e-15

Omnibus:	0.401	Durbin-Watson:	1.202
Prob(Omnibus):	0.818	Jarque-Bera (JB):	0.321
Skew:	-0.361	Prob(JB):	0.852
Kurtosis:	2.336	Cond. No.	319.

OLS Regression Results

Dep. Variable:	SPY	R-squared:	1.000
Model:	OLS	Adj. R-squared:	1.000
Method:	Least Squares	F-statistic:	3.657e+30
Date:	Sat, 11 Mar 2023	Prob (F-statistic):	0.00
Time:	13:42:11	Log-Likelihood:	1862.2
No. Observations:	51	AIC:	-3714.
Df Residuals:	46	BIC:	-3705.
Df Model:	4		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	3.036e-17	5.26e-18	5.774	0.000	1.98e-17	4.09e-17
SPY	1.0000	4.64e-16	2.16e+15	0.000	1.000	1.000
VNQ	-2.637e-16	3.66e-16	-0.721	0.475	-1e-15	4.73e-16
XLE	1.943e-16	1.4e-16	1.387	0.172	-8.77e-17	4.76e-16
XLF	-2.567e-16	2.98e-16	-0.862	0.393	-8.57e-16	3.43e-16

Omnibus:	8.906	Durbin-Watson:	0.395
Prob(Omnibus):	0.012	Jarque-Bera (JB):	13.945
Skew:	0.438	Prob(JB):	0.000937
Kurtosis:	5.407	Cond. No.	115.

6. Results

6.1 Basic Statistical Methods:

XLF looks paid highest, 0.27%, amongst all while XLE paid lowest, 0.18%, on average. In all ETFs median values are higher than that of respective mean values indicating non-normal distribution and left skewed (asymmetric distribution).

VNQ and XLF look less skewed compared to other ETFs, indicating a nearly symmetric distribution. The greatest spreading is for XLE and the least for SPY around their respective mean. VNQ had a distinctively heavier tail amongst all return datasets. No kurtosis value was between -2 to +2, and hence all heavy tailed. As long as risky ETFs are concerned, XLE was riskier than all other ETFs since its SD value around mean was the highest.

	Mean	Median	SD	Skewness	Kurtosis
SPY	0.002595	0.003970	0.022172	-0.693357	8.879396
VNQ	0.001795	0.003787	0.029099	-0.275380	18.496385
XLE	0.001624	0.003133	0.038104	-0.702861	6.991102
XLF	0.002698	0.004165	0.029190	-0.110342	7.769409

Table 1: Basic statistical parameters of the weekly returns

6.2 Correlation

Correlation calculated pairwise linear relationship between ETFs. Both VNQ and XLF have significant correlation to SP which means strong associations of VNQ to SP and XLF to SP separately while XLE looks negatively associated to SP with a lower degree. Hence, a combination of XLF with SP creates a diversified portfolio (in case of 2-ETF portfolio) and less risk tolerance. We calculated correlations in several ways (Spearman, Pearson and Kendall).

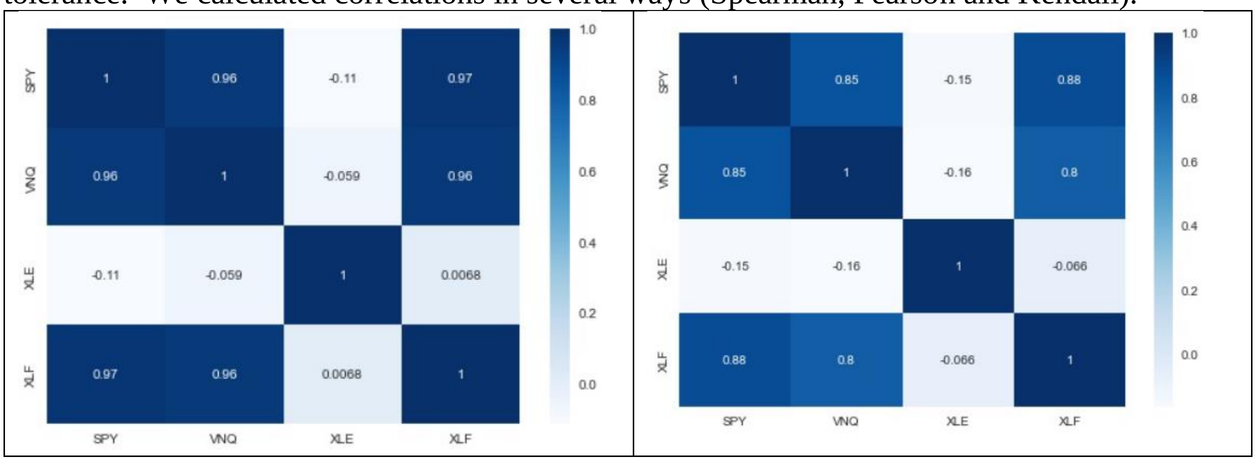


Table 2: Correlation matrix (Pearson) Left table, and Correlation matrix (Spearman)

6.3 Stationarity for time series

Stationarity process is a distinct process where unconditional probability distribution remains constant over time. Both ADF t-statistic and KPSS tests are used to check stationarity of a time series. We applied ADF in our case. The assessment process to check stationarity is to check p-values at 95% Confidence Interval. The null hypothesis states that there is a unit root in the time series. We could successfully be able to reject the null hypothesis since there is no p-value below 0.05 in our calculations. The lowest p-value is 0.29 for XLE. Therefore, our all-time series against all respective ETFs are with absence of unit root (not stationary).

```
spy adf: -0.6360366323614757 adf_pvalue: 0.8626295763634646
vnq adf: -1.2245898576702972 adf_pvalue: 0.6629293503461295
xle adf: -1.987259482871717 adf_pvalue: 0.292187159963141 8
xlf adf: -1.2924955283662656 adf_pvalue: 0.6325985867198329
```

Table 3: p-values for prices of all 4 ETFs in ADF test

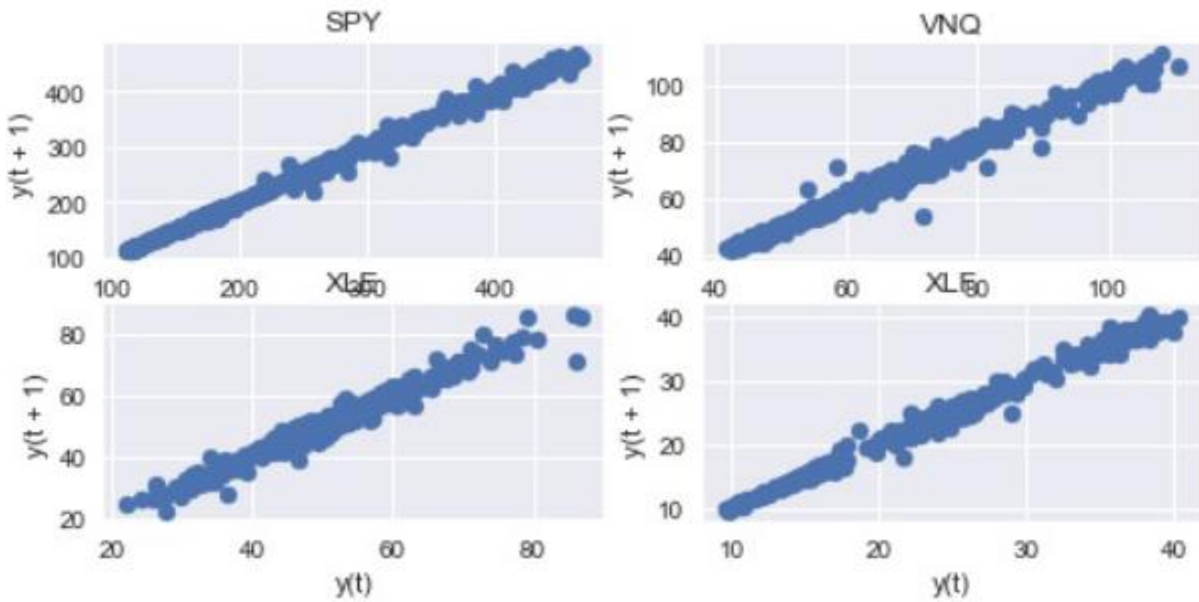


Table 4: First Order Serial Autocorrelation for all ETFs

If we consider the results from the above graphs with the outputs from QQ-plot, we will find that the returns don't follow the normal distributions. Any analysis assuming other than normal data distribution will be applicable to the return of instruments.

6.4 PCA

We had fit PCA methods (a linear combination of independent variables that is used for variable selection) to return datasets to reduce interrelated data to reduced dimensions through transformation of original variables to a new set of variables that are not correlated retaining most possible variations. We applied four PCA components to study the method. Our findings are PCA-1 explained 73% of the variance in the data which is the most and PCA-2 explained 25%. Therefore, two PCA components are needed to express 98.33%. PCA works on decomposition of covariance matrix.

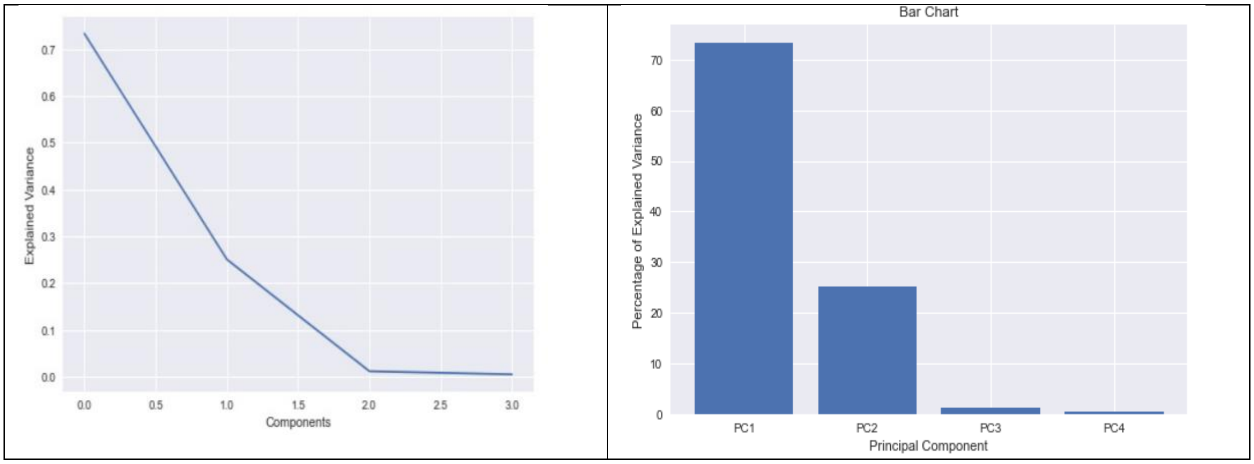


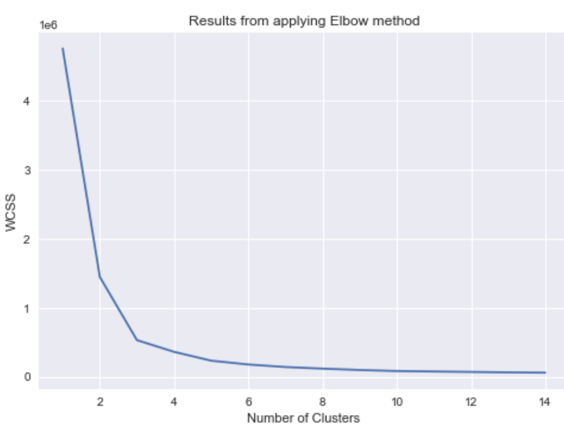
Table 5: PCA analysis table

6.5 K-means

We studied price data with another unsupervised ML into 3 different clusters. We found cluster 1 includes VNQ and XLE, whereas cluster 2 includes only SPY and cluster 3 includes XLF. Therefore, only VNQ and XLE are from the same group while SP and XLF are in the different and distinct groups.

	labels	companies
1	1	VNQ
2	1	XLE
0	2	SPY
3	3	XLF

Table 6: k-Means Clustering



Graph 1: K-means Clustering (Elbow Method)

6.6 Markov Autoregression

Markov Switching Model Results

Dep. Variable:	SPY	**No. Observations:**	521
Model:	MarkovRegression	**Log Likelihood**	1346.217
Date:	Sun, 12 Mar 2023	**AIC**	-2674.433
Time:	18:39:08	**BIC**	-2636.132
Sample:	07-09-2012	**HQIC**	-2659.430
	- 06-27-2022		
Covariance Type:	approx		

Regime 0 parameters

	coef	std err	z	P>|z|	[0.025	0.975]
sigma2	0.0002	1.89e-05	8.792	0.000	0.000	0.000

Regime 1 parameters

	coef	std err	z	P>|z|	[0.025	0.975]
sigma2	0.0007	0.000	5.062	0.000	0.000	0.001

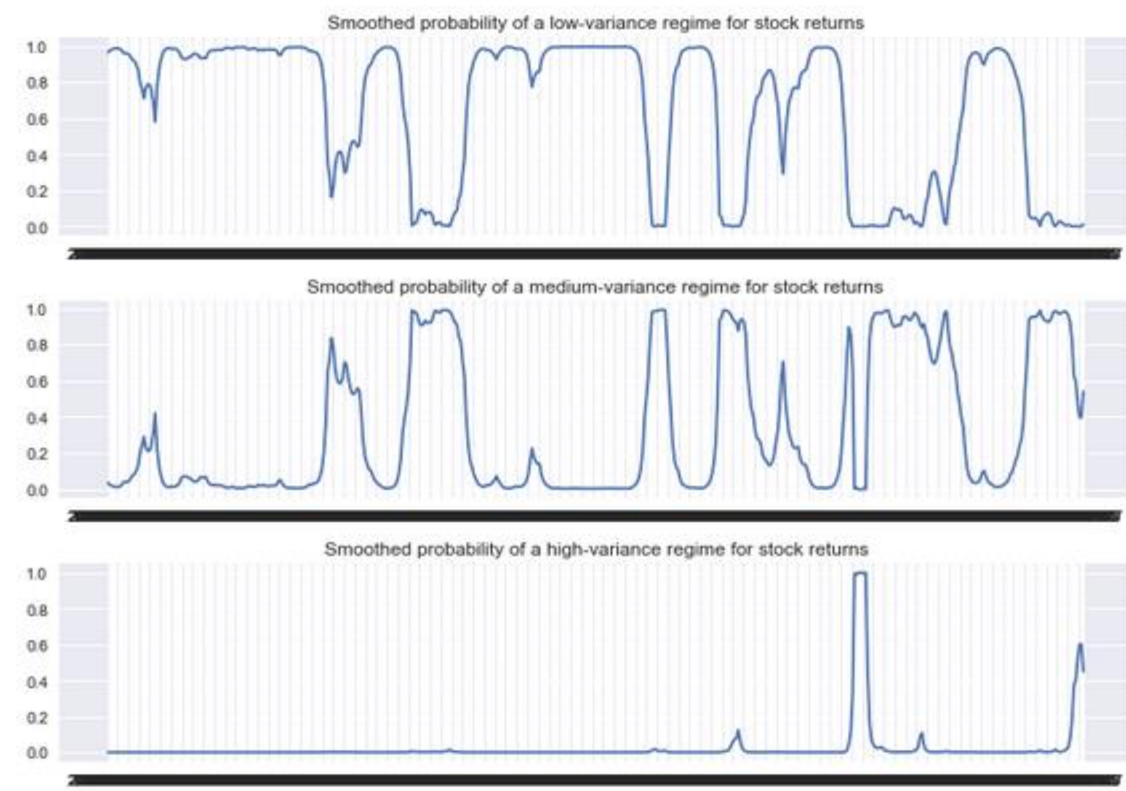
Regime 2 parameters

	coef	std err	z	P>|z|	[0.025	0.975]
sigma2	0.0065	0.004	1.753	0.080	-0.001	0.014

Regime transition parameters

	coef	std err	z	P>|z|	[0.025	0.975]
p[0->0]	0.9716	7.47e-05	1.3e+04	0.000	0.971	0.972
p[1->0]	0.0551	0.023	2.426	0.015	0.011	0.100
p[2->0]	1.209e-106	nan	nan	nan	nan	nan
p[0->1]	0.0284	2.4e-05	1183.150	0.000	0.028	0.028
p[1->1]	0.9319	0.027	35.003	0.000	0.880	0.984
p[2->1]	0.1449	0.128	1.129	0.259	-0.107	0.396

Table: For the S&P 500, the Markov Switching model results



Graph: Low, Medium, and High variance regimes

6.7 COVID-19

```
CADF(spy_before_covid19,vnq_before_covid19)
```

```
(-20.787078837303785, 0.0, 0, 364, {'1%': -3.4484434475193777, '5%': -2.869513170510808, '10%': -2.571017574266393}, -1943.415551191053)
```

```
# Modeling after a specified range of time
#Get data before that specific date
spy_after_covid19=spy_return["2022-03-31":]
vnq_after_covid19=vnq_return["2022-03-31":]
```

```
CADF(spy_after_covid19,vnq_after_covid19)
```

```
(-6.60004117182907, 6.7718128607800494e-09, 4, 8, {'1%': -4.6651863281249994, '5%': -3.3671868750000002, '10%': -2.802960625}, -91.59017688936535)
```

CADF is applied to identify the mean-reversion relation for all four ETFs. The weekly returns of VNQ, XLE, and XLF are X variables while the S&P 500 is our proxy (benchmark that generates market movement). Our aim is to predict any relationship individually pairwise. Therefore, we applied multiple regression models. The entire dataset was split into two categories (pre-COVID-19 and post-COVID-19). Surprisingly, both the regression model results with maximum accuracy (R-Squared value of 1 for both pre and post COVID-19 which looks a bit abnormally high). These abnormalities suggest us to attempt the Correlation matrix and VIF for covid-19 to seek the multicollinearity.

From the values of the above picture the test statistics are -20.787 (for before COVID-19) and -6.600 (for after COVID-19). Both values are smaller than standard Critical values of 1%, 5%

and 10%. This influences us to reasonably reject the null hypothesis. To be noted that CADF is used for identification of mean-reversion relation between datasets (here, we picked S&P 500 and VNQ). We found that there was a cointegrating relationship to a degree between these two ETFs. The other meaning of this indicates there was a mean reverting relationship between the S&P 500 and VNQ before and after the COVID-19 pandemic.

OLS Regression Results

Dep. Variable:	SPY	**R-squared:**	1.000
Model:	OLS	**Adj. R-squared:**	1.000
Method:	Least Squares	**F-statistic:**	3.625e+29
Date:	Sun, 12 Mar 2023	**Prob (F-statistic):**	7.44e-45
Time:	18:44:17	**Log-Likelihood:**	294.53
No. Observations:	8	**AIC:**	-579.1
Df Residuals:	3	**BIC:**	-578.7
Df Model:	4		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	-1.301e-17	1.85e-17	-0.704	0.532	-7.18e-17	4.58e-17
SPY	1.0000	3.75e-15	2.67e+14	0.000	1.000	1.000
VNQ	1.388e-17	1.26e-15	0.011	0.992	-3.99e-15	4.02e-15
XLE	-5.551e-17	1.18e-15	-0.047	0.965	-3.8e-15	3.69e-15
XLF	5.551e-16	2.74e-15	0.202	0.853	-8.17e-15	9.28e-15

Omnibus:	0.365	**Durbin-Watson:**	1.283
Prob(Omnibus):	0.833	**Jarque-Bera (JB):**	0.441
Skew:	-0.291	**Prob(JB):**	0.802
Kurtosis:	2.008	**Cond. No.**	320.

OLS Regression Results

Dep. Variable:	SPY	**R-squared:**	1.000
Model:	OLS	**Adj. R-squared:**	1.000
Method:	Least Squares	**F-statistic:**	2.733e+30
Date:	Sun, 12 Mar 2023	**Prob (F-statistic):**	0.00
Time:	18:44:17	**Log-Likelihood:**	1854.8
No. Observations:	51	**AIC:**	-3700.
Df Residuals:	46	**BIC:**	-3690.
Df Model:	4		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	3.534e-17	6.08e-18	5.812	0.000	2.31e-17	4.76e-17
SPY	1.0000	5.37e-16	1.86e+15	0.000	1.000	1.000
VNQ	-6.939e-17	4.23e-16	-0.164	0.870	-9.21e-16	7.82e-16
XLE	5.551e-17	1.62e-16	0.343	0.733	-2.71e-16	3.82e-16
XLF	1.735e-16	3.45e-16	0.503	0.617	-5.2e-16	8.67e-16

Omnibus:	6.572	**Durbin-Watson:**	0.255
Prob(Omnibus):	0.037	**Jarque-Bera (JB):**	9.449
Skew:	0.243	**Prob(JB):**	0.00887
Kurtosis:	5.052	**Cond. No.**	115.

Table: Pre and Post Covid 19 (3 months before and after the Covid 19)

Markov Switching Model Results

Dep. Variable:	SPY	**No. Observations:**	83
Model:	MarkovRegression	**Log Likelihood**	187.342
Date:	Sun, 12 Mar 2023	**AIC**	-356.685
Time:	18:43:36	**BIC**	-334.915
Sample:	06-03-2019	**HQIC**	-347.939
	- 12-28-2020		
Covariance Type:	approx		

Regime 0 parameters

	coef	std err	z	P>|z|	[0.025	0.975]
sigma2	0.0002	5.88e-05	3.512	0.000	9.13e-05	0.000

Regime 1 parameters

	coef	std err	z	P>|z|	[0.025	0.975]
sigma2	0.0007	0.000	3.798	0.000	0.000	0.001

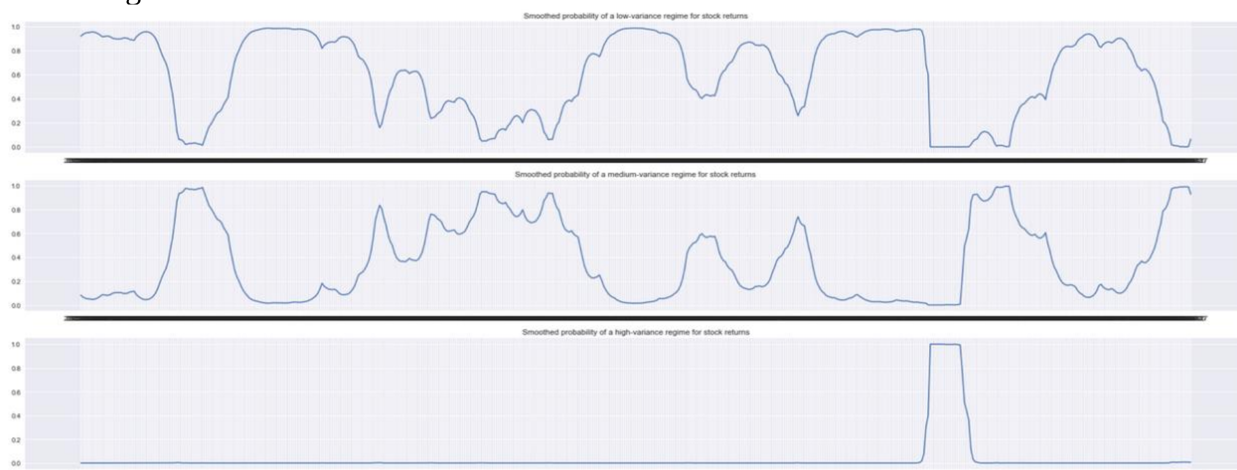
Regime 2 parameters

	coef	std err	z	P>|z|	[0.025	0.975]
sigma2	0.0093	0.005	1.900	0.057	-0.000	0.019

Regime transition parameters

	coef	std err	z	P>|z|	[0.025	0.975]
p[0->0]	0.9683	0.032	30.363	0.000	0.906	1.031
p[1->0]	0.0322	nan	nan	nan	nan	nan
p[2->0]	3.962e-19	nan	nan	nan	nan	nan
p[0->1]	3.262e-19	nan	nan	nan	nan	nan
p[1->1]	0.9678	nan	nan	nan	nan	nan
p[2->1]	0.1424	0.122	1.163	0.245	-0.098	0.383

Table: Regime Shift Identification in COVID-19



We applied regression analysis for both before and after COVID-19 and calculated the R-Squared value for both pre-COVID-19 and post-COVID-19. The values are incredibly 1 in both cases. This might be an amateur approach and there might be a misspecification in the model. Multicollinearity could be the main problem. We then applied traditional multicollinearity checks (Correlation coefficient calculation and VIF value calculation). Since we have very high VIF values for all four ETFs (except XLE) and the big correlation coefficients under our features. So, a simple question is here and that is the utility of the regression model. Therefore, further research is needed to handle this issue.

```
SPY:
 (-24.963080509697384, 0.0, 0, 520, {'1%': -3.4429882202506255, '5%': -2.8671142122781066, '10%': -2.569738849852071}, -2385.4559413478323)
VNQ:
 (-8.815778251180248, 1.936169671147854e-14, 7, 513, {'1%': -3.443161545965353, '5%': -2.8671904981615706, '10%': -2.5697795041589244}, -2130.760097799231)
XLE:
 (-5.558944087196891, 1.5576566380391623e-06, 17, 503, {'1%': -3.4434175660489905, '5%': -2.8673031724657454, '10%': -2.5698395516760275}, -1858.340334708856)
XLF:
 (-14.322828891243127, 1.1336375229344225e-26, 2, 518, {'1%': -3.443037261465839, '5%': -2.8671357972350493, '10%': -2.569750352856994}, -2112.244597435606)
```

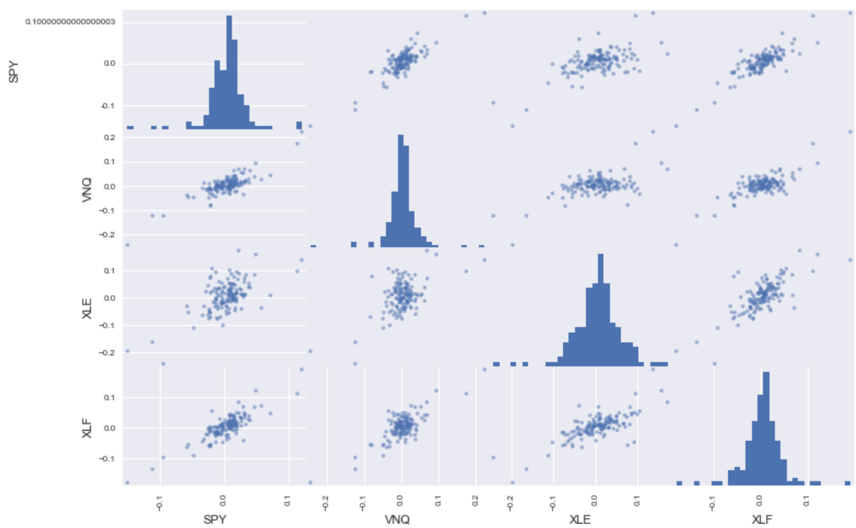
Table: Regime Shift test results (ADF value)

The T-statistic values for all four ETFs were calculated. From the values of the above picture the test statistics are -24.963 (S&P 500), -8.816 (VNQ), -5.559 (XLE), and -14.323 (XLF). All these values are smaller than standard critical values of 1%, 5% and 10%. This influences us to reasonably reject the null-hypothesis. This implies that there is less possibility of changes in course of time for every level for 1%, 5% and 10%.

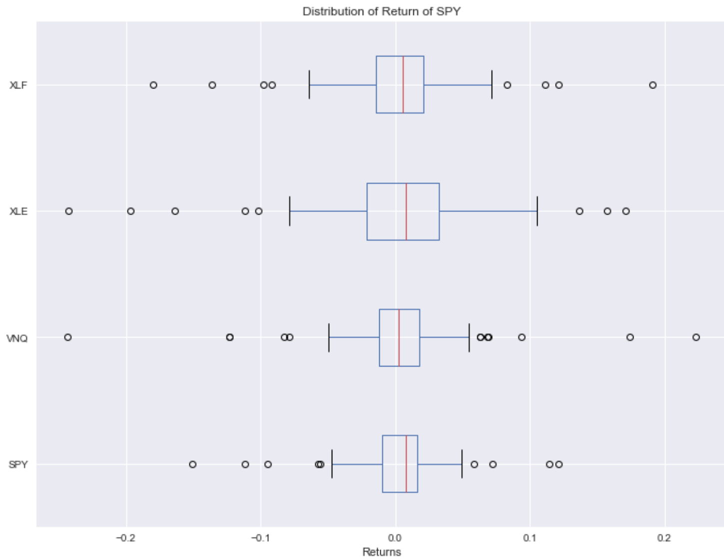
6.8 EDA:

Box plots are used for EDA. It is an efficient way to identify and compare extreme values of any dataset. Our box plot is assessing extreme values of returns. Boxplot displays distribution of data including IQR, Q1, median, Q3 and outliers. XLE indicates more volatile ETF during COVID-19 since it comprised of a widest box whereas SP had small range boxplot which indicates the less risky feature of the ETF. It is maybe due to SP tracks many companies of diversified sectors and hence the ETF had diversified properties.

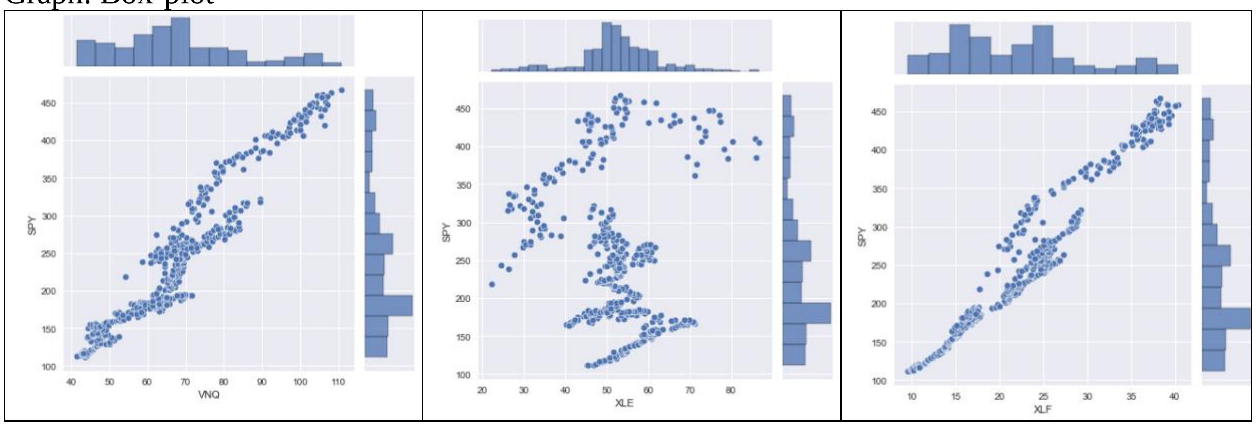
To be noted that price data series grouping doesn't change as a result of application of normalization. Idiosyncratically, the same application doesn't hold the return data series grouping.



Graph: Scatter-plot

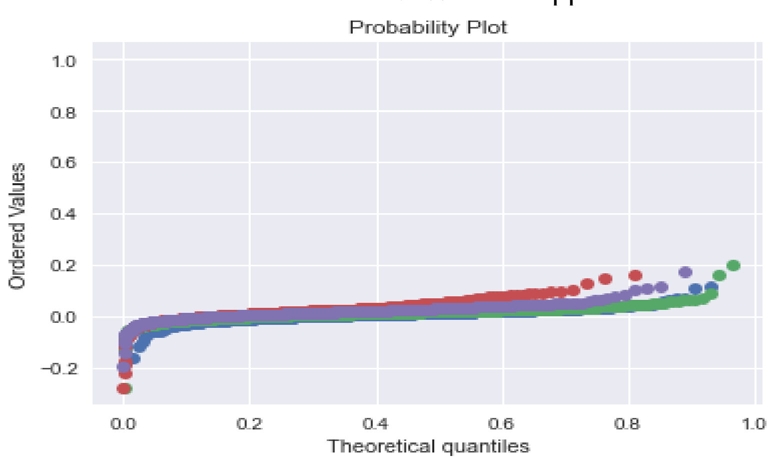


Graph: Box-plot

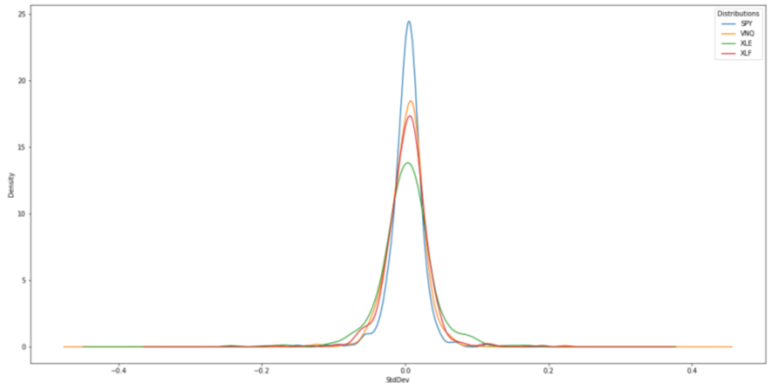


Graph: Joint distribution of prices of all ETFs against SP 500

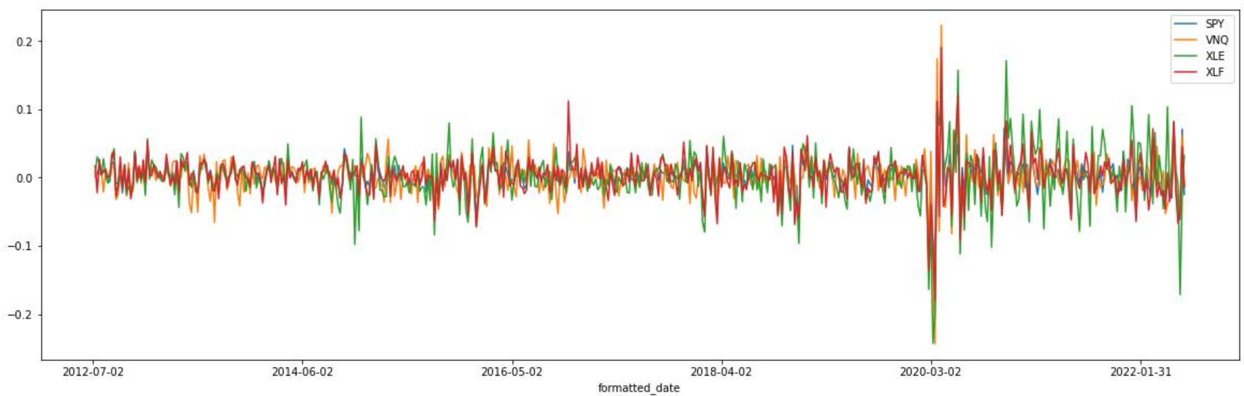
We drew a QQ-plot graph and calculated the p-value for the Jarque-Bera test and found that the p-value is 0 which shows that we reject null-hypothesis. This implies no data series distribution follows the normal distribution. We then applied the Density function as below.



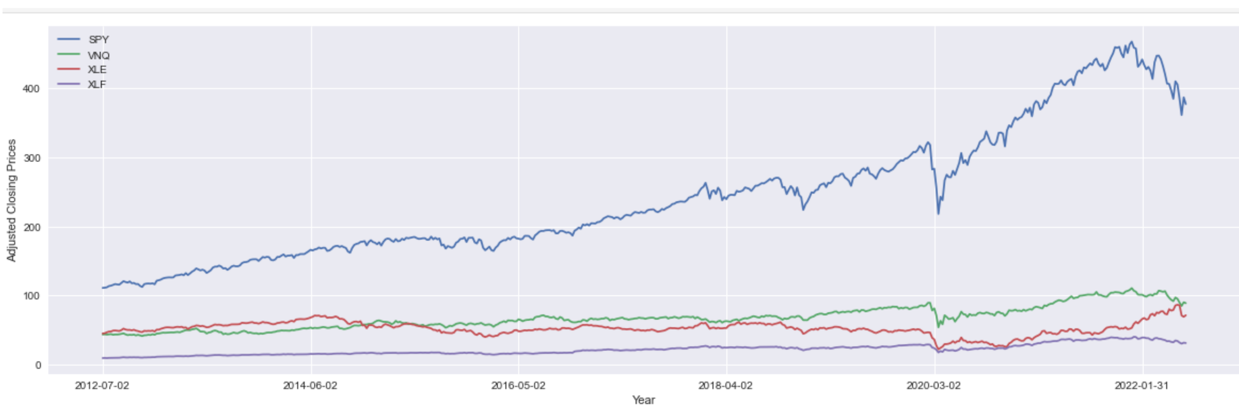
Graph: QQ-plot



Graph: Test for normality (Density function)



Graph: Return distributions



Graph: Adjusted Close Prices

6.9 Mean Reversion

We applied ADF t-test for all the ETFs and found all negative integer values on the return time series which indicates that all the returns have mean-reverting properties. Additionally, we further worked on prices of the ETFs alternatively to determine if there were any time series that exhibit trends. We found there were no trends that can be addressed as potential mean reverting properties. We did this with Cointegrated ADF. However, all the ETFs (individually) don't have any mean-reversion property because of statistically insignificant t-statistic value of the ETF's price.

	ETFs	ADF T-tests
0	SPY	-12.986614
1	VNQ	-15.589372
2	XLE	-10.008056
3	XLF	-12.868518

Table: ADF t-statistic values

	features	VIF
0	SPY	406.495133
1	VNQ	266.318000
2	XLE	84.407593
3	XLF	480.293167

Table: VIF figures

```
[['SPY', -12.986613791626338],
 ['VNQ', -15.589371608217544],
 ['XLE', -10.008056363548784],
 ['XLF', -12.86851784441295]]
```

Table: Returns' figures

6.10 Multicollinearity

We found from the cluster correlation heat matrix that correlation coefficients look high and hence there is a possibility of existence of multicollinearity. Multicollinearity is one of the main problems of linear regression. Multicollinearity indicates the dependency among several independent variables (inter correlations between independent variables). We further calculated VIF to get confirmation about this multicollinearity and found all the ETFs have this feature. Both SPY and XLF had severe issues while XLE had the nominal.

7. Conclusion

After through experimental analysis, it is observed that four ETFs weekly returns are left skewed distributions with significant data residing at all the tail area. This indicates the S&P 500 is dependent on the other three ETFs. Their serial autocorrelation looks nearly the same by eyeballing which shows evidence that the S&P 500 is dependable on the other three ETFs and statistical properties remain unchanged for the other three ETFs. It also highlights the synchronization of the S&P 500 to other ETFs. Correlations are strong and similar; Variance Inflation Factor (VIF) for all the four is high. The joint analysis of correlation and VIF shows that one variable is dependent on all other three ETFs. All this experimental evidence strongly proves that the S&P 500 is a dependable benchmark. It also says, for any other ETF performance, the S&P 500 ETF is a reliable benchmark indicator.

Our code repo can be viewed through the following link:
https://github.com/krishxx/wqumscfin/blob/main/src/MScFE_Capstone.ipynb

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