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Lab 3
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Communication Systems

Sampling and Aliasing

1. Sampling a Cosine Signal

The Analog to Digital Conversion (ADC) sampling process is easy to visualize in the time domain. Evaluating a continuous-time signal *x(t)* at a sequence of uniformly spaced time intervals transforms *x(t)* into the discrete-time signal $x(kT_S)$.

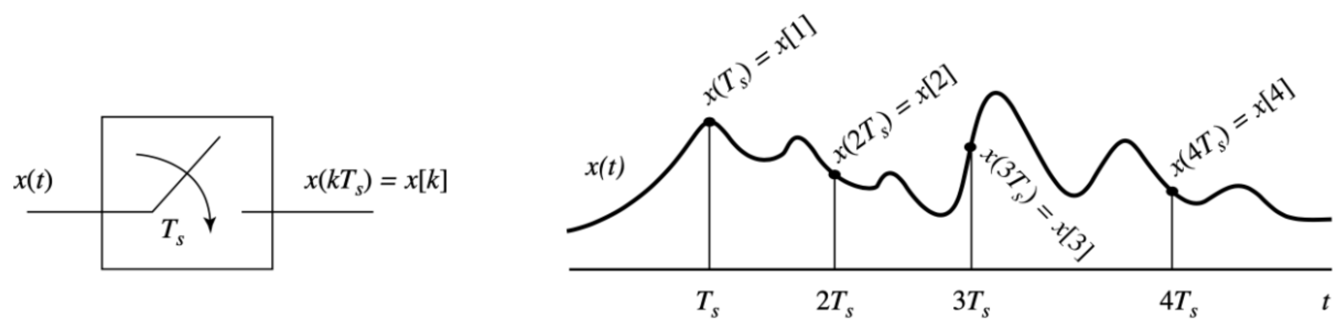


Figure 1
The sampling process shown as an evaluation of *x(t)* at the times T = T_S, $2T_S$, $3T_S$, $4T_S$, ...

Prompt

Generate a cosine signal *cos(2πft)* and plot its spectrum (Hint: refer tospeccos.m) with sampling interval T_S = 1/100.

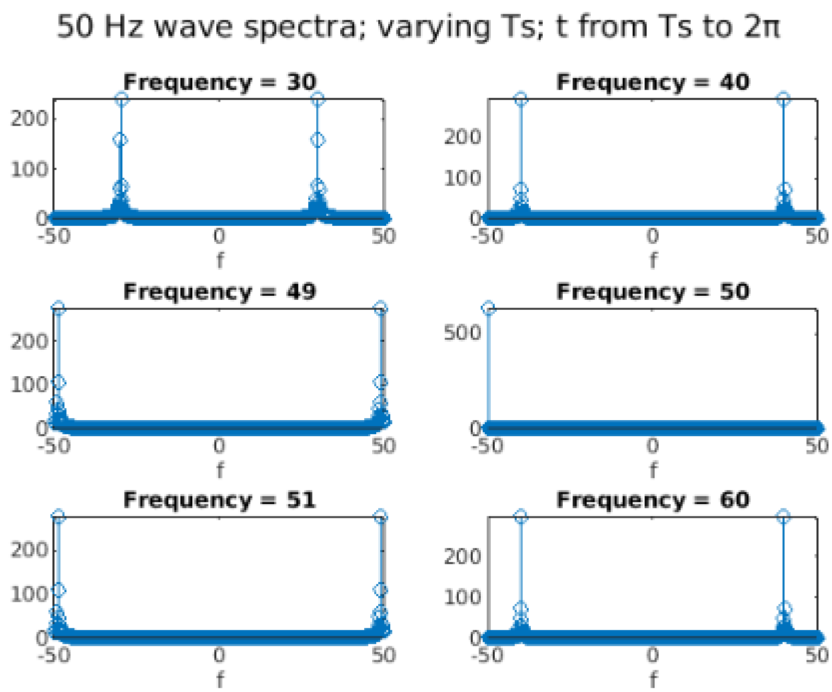
Find the spectrum of the cosine signal when f= 30, 40, 49, 50, 51, 60 Hz. Which of these show aliasing?

Expectation

I expect, given these frequencies, that only f=50 Hz will escape DFT leakage and aliasing therefore. Additionally, f = 51 and 60 Hertz will not find a T_S=1/100 satisfying their Nyquist rate.

Results

Exactly as expected.



The lower-frequency waves had bin leakage, the higher-frequency signals were aliased into other, lower bins. Only the 50 Hz signal was accurately captured, with a single spectra at f=-50.

2. Different Sampling Intervals

Prompt

Generate a cosine signal with frequency 50 Hz. Plot the spectrum when this signal is sampled at T_S = 1/50, 1/90, 1/100, 1/110, and 1/200. Which of these show aliasing?

Expectation

Given that 1/50 and 1/90 are below the Nyquist frequency of this signal, they are both guaranted to have aliasing. For T_S = 1/50, the signal will appear as a DC magnitude. For T_S = 1/90, the signal will appear time-varying but have DFT leakage. I expect T_S = 1/110 to experience DFT leakage as well, due to the frequency-bin equation

$$Bins = \frac{kF_s}{N} = \frac{110k}{N}$$

not yielding integer results when N = floor(2π*110) = 691 and k=(-N/2:1:N/2-1) = (-345:1:344). The closest bins to 50 will be k=±314, 315; yielding bins F=±49.99, ±50.14 Hz. Selecting an exact time interval of the measurement, to refine *N*, could potentially eliminate this leakage that will occur by taking *N* over exactly one period.

Sampling intervals of 1/100 and 1/200 are expected to perfectly represent the frequency spectrum of the 50 Hz cosine wave.

T_s	**Aliasing**
50	Yes (AC information lost)
90	Yes
100	No
110	Yes
200	No

Results

The results were exactly as expected. One noteworthy, but not surprising, result is that T_s = 100 Hz only had a spectra at the negative frequency i.e. N = -50. The positive spectra at N = 50 was not recorded.

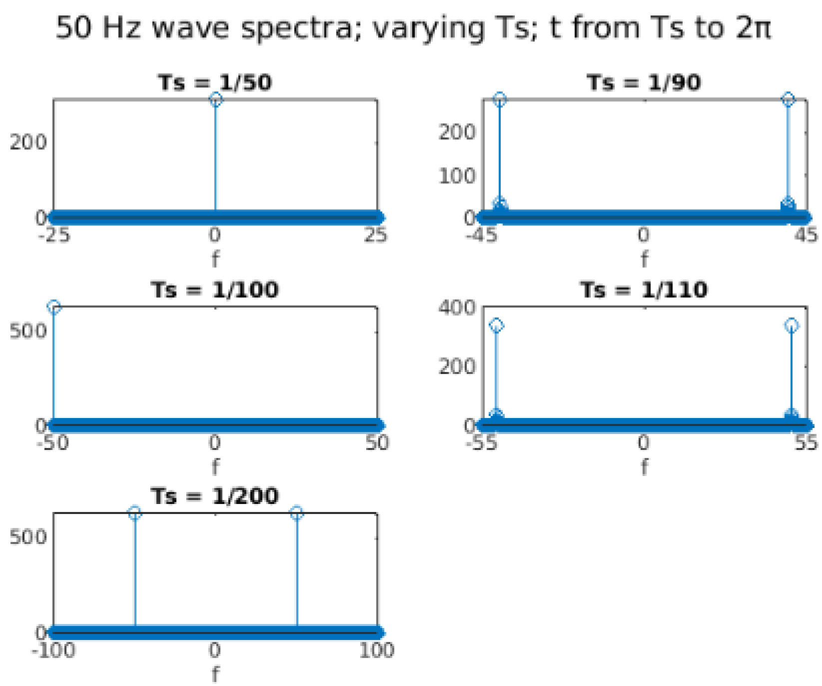


Fig. 3 - Results of experiment with t from Ts to 2pi.

Changing the Time Interval

Slightly changing the duration of the recording enormously affected the spectrograph.

For instance, going from 0 to (2pi)+T_s concentrated the leaky, false-positive bins of the f_s *high enough* to register an AC current but *not* high enough to satisfy the Nyquist principle

- 90 Hz - all collected into *one* bin around f = +/- 40 Hz. That is very surprising to me, and could strongly mislead someone analyzing this signal at such a sampling frequency who was not aware *a priori* of the fundamental frequency.

The loss of positive spectra for F_S = 100 Hz was unchanged, i.e. only the negative -50 Hz spectra was in the domain of the output, not the positive.

The DFT leakage at F_S = 110 Hz was actually *eliminated* by increasing t.

And finally, the changed timeline *introduced leakage* to F_S = 200 Hz.

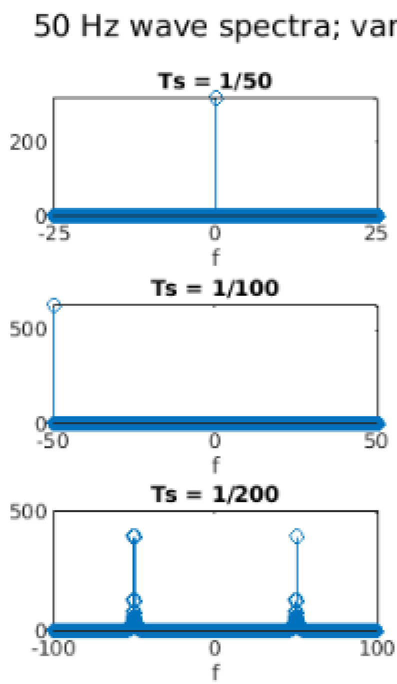


Fig. 3-2 - Results of experiment with t from 0 to 2pi + Ts.

3. Different Fundamental Frequencies

Prompt

Generate a cosine signal similar to Exercise #1, sampling it with interval T_S = 1/100 to find the spectrum of a square signal with fundamental f=10, 20, 30, 33, 43. Hz. Can you predict where the spikes will occur in each case? Which of the square signals show aliasing?

Expectation

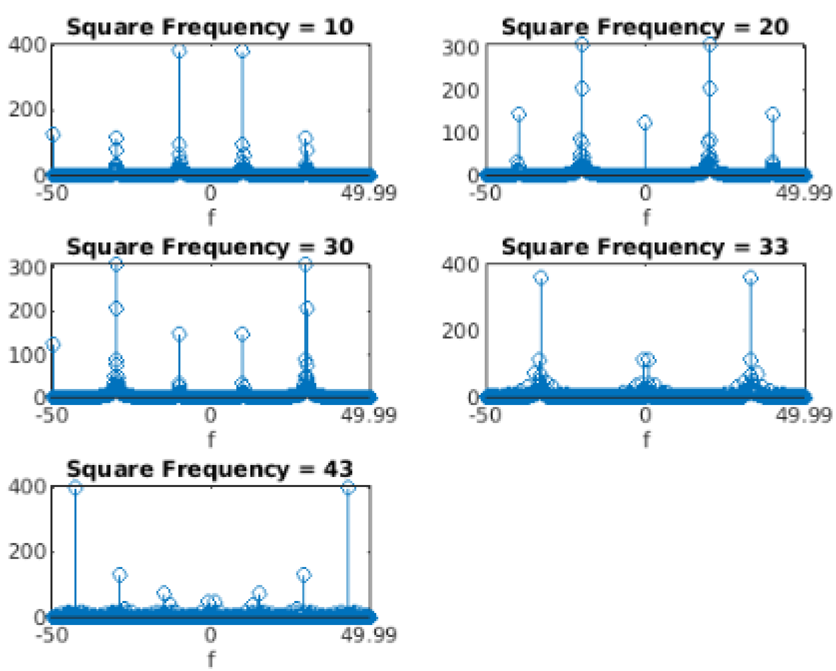
I took the simplest solution with this question and used `sign()` to generate a square wave, rather than adding harmonics as we have done in the past.

Knowing that f_s = 100, I expect the fundamental frequency f_0= 30, 33, and 43 will alias. Only f_0 = 10, 20 Hz should be contained within one +/- frequency bin. Additionally, the harmonics of the fundamental frequency will alias.

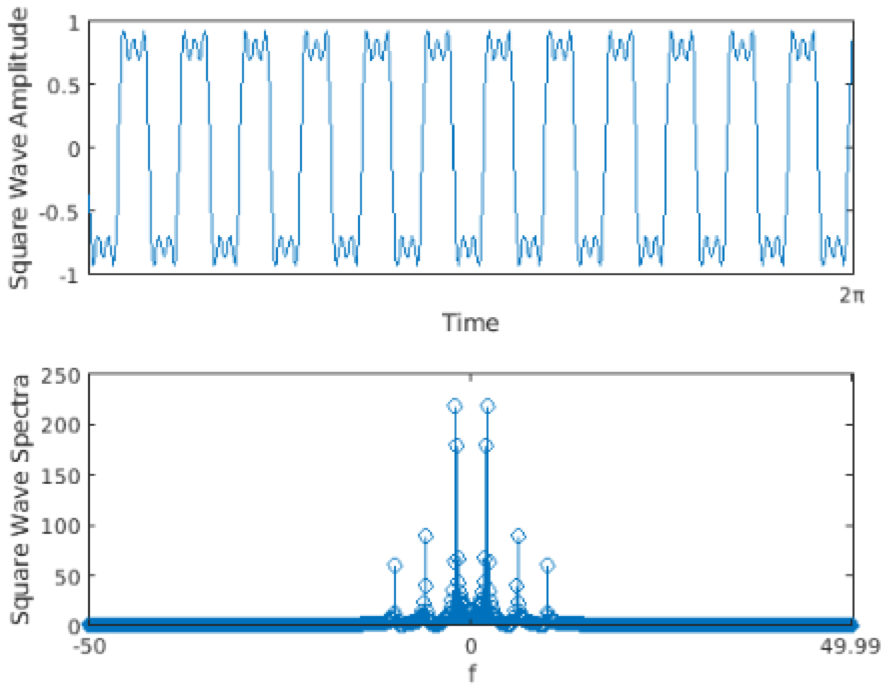
f_0	Aliasing
10	No
20	No
30	Yes
33	Yes
43	Yes

Results

Surprising to me, all fundamental frequencies and their harmonics showed aliasing. Additionally, there was a DC spectra for f=20, 33.



It's not surprising that the DFT showed leakage in all cases. The sign() function will create a perfectly sharp square wave, with only one transition point at t=0. A semi-square wave created from a limited number of harmonics, all of a frequency below the sampling rate, could be accurately captured by the DFT. For instance, below is shown a 3-harmonic square wave of f = 2, 6, 10 Hz is shown in the time-domain and frequency domain, retaining T_s = 1/100 s.



Surprisingly to me, even this carefully-constructed square wave resulted in a small amount of DFT leakage.

Discussion

These results have been thoroughly discussed in the Results section of each exercise. While there were a couple curveballs, overall the results are not surprising. The Discrete Fourier Transform is a powerful tool for dissecting the frequency content of time-varying signals.

One important factor in being able to fully rely on this analysis are some prior knowledge of the signal's likely content. For instance, in Exercise 2, the 50 Hz signal sampled at 90 Hz distilled all frequency content into a single sub-50 Hz bin. Without knowing the signal's origin, without seeing any leakage, I wouldn't suspect any leakage or aliasing.

Additionally, naming the exact frequency bin calculated in MatLab can be quite difficult. Generally in this lab I have refrained from labeling the exact frequency bins of peak spectra. This would require manually inspecting the data for the maximum values and their indices. That is fairly simple for a dual-peak spectrograph, but would get quite difficult for spectrographs of signals including multiple frequencies.