9.4

8

$x_1 = y$

$x_2 = y' = x_1'$

$x_3 = y'' = x_2'$

$x_4 = y''' = x_3'$

$y''' = y' - y + \cos t$

$x_1' = x_2$

$x_2' = x_3$

$x_3' = x_2 - x_1 + \cos t$

$$\vec{x}' = A\vec{x} + f$$

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \cos t \end{bmatrix}$$

24

$$\det \begin{vmatrix} e^t & \sin t & -\cos t \\ e^t & \cos t & \sin t \\ e^t & -\sin t & \cos t \end{vmatrix} = e^t(\cos^2 t + \sin^2 t) - e^t(\sin t\cos t - \sin t\cos t) + e^t(\sin^2 t + \cos^2 t) = 2e^t$$

$2e^t \neq 0$, $\therefore$ linear independent

fundamental matrix:
$$\begin{pmatrix} e^t & \sin t & -\cos t \\ e^t & \cos t & \sin t \\ e^t & -\sin t & \cos t \end{pmatrix}$$

Gen sol:
$$c_1\begin{pmatrix} e^t \\ e^t \\ e^t \end{pmatrix} + c_2\begin{pmatrix} \sin t \\ \cos t \\ -\sin t \end{pmatrix} + c_3\begin{pmatrix} -\cos t \\ \sin t \\ \cos t \end{pmatrix}$$

9.3 20D HW7

(12):

$$\begin{array}{c} ① \\ ② \\ ③ \end{array}\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[③-②]{②-① \text{ then}} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & -1 & 1 \end{array}\right] \xrightarrow[②-(-2\times③)]{①-②} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 2 & -1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right]$$

& ③ × -1

① + ③

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right] \qquad A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 2 \\ -1 & 1 & -1 \end{bmatrix}$$

(18):

$$X^{-1}(t) = \frac{1}{\det(X(t))}\begin{bmatrix} -2\sin 2t & -\cos 2t \\ -2\cos 2t & \sin 2t \end{bmatrix} = \frac{1}{-2\sin^2(2t) - 2\cos^2(2t)}\begin{bmatrix} -2\sin 2t & -\cos 2t \\ -2\cos 2t & \sin 2t \end{bmatrix}$$

$$= \frac{1}{-2}\left[\ \vdots\ \right] = \begin{bmatrix} \sin 2t & \frac{\cos 2t}{2} \\ \cos 2t & \frac{-\sin 2t}{2} \end{bmatrix}$$

(24)

$$\det = 1\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} - 0\begin{bmatrix} 0 & -1 \\ -1 & 1 \end{bmatrix} + 2\begin{bmatrix} 0 & 3 \\ -1 & 2 \end{bmatrix} = 1(3+2) + 2(0+3) = 5+6 = \boxed{11}$$

(28)

$$A - rI = \begin{bmatrix} 3-r & 3 \\ 2 & 4-r \end{bmatrix}, \quad \det(A - rI) = 0 \quad \text{so} \quad (3-r)(4-r) - 6 = 0$$

$$(3-r)(4-r) = 6 \Rightarrow 12 - 7r + r^2 - 6 = 0 \Rightarrow r^2 - 7r + 6 = 0$$

$$(r-1)(r-6) = 0 \qquad \boxed{\begin{matrix} r_1 = 1 \\ r_2 = 6 \end{matrix}}$$

(40)

a) $$\int A(t) = \begin{bmatrix} t & -\frac{e^{-2t}}{2} \\ 3t & -\frac{e^{-2t}}{2} \end{bmatrix}$$

b) $$\int_0^1 B(t)\,dt = \begin{bmatrix} -e^{-t}\big|_0^1 & -e^{-t}\big|_0^1 \\ e^{-t}\big|_0^1 & -3e^{-t}\big|_0^1 \end{bmatrix} = \begin{bmatrix} -e^{-1}+1 & -e^{-1}+1 \\ e^{-1}-1 & -3e^{-1}+3 \end{bmatrix} = \begin{bmatrix} \frac{e-1}{e} & \frac{e-1}{e} \\ \frac{1-e}{e} & \frac{3(e-1)}{e} \end{bmatrix}$$

c) $$\frac{d}{dt}(A(t)\cdot B(t)) = \frac{d}{dt}\begin{bmatrix} 1\cdot e^{-t} + e^{-2t}\cdot -e^{-t} & 1\cdot e^{-t} + e^{-2t}\cdot 3e^{-t} \\ 3e^{-t} + e^{-2t}\cdot -e^{-t} & 3e^{-t} + e^{-2t}\cdot 3e^{-t} \end{bmatrix}$$

$$= \frac{d}{dt}\begin{bmatrix} e^{-t} - e^{-3t} & e^{-t} + 3e^{-3t} \\ 3e^{-t} - e^{-3t} & 3e^{-t} + 3e^{-3t} \end{bmatrix} = \begin{bmatrix} -e^{-t} + 3e^{-3t} & e^{-t} - 9e^{-3t} \\ -3e^{-t} + 3e^{-3t} & -3e^{-t} - 9e^{-3t} \end{bmatrix}$$