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CMSC 150 -B3L

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Exercise 09

Part 2: Word Problem

After creating your function, test the accuracy by performing the word problems below. Use macheps = $1*10^{-9}$, the maximum number of max iterations = 1000 and set the format of digits to 4 significant figures by this R command: options(digits=4).

To describe your answers:

1. include the last iteration's values of x0, x1, x2, x3 and their respective values over the function f, as well as the values of A,B,C and approximate error for each iteration;
2. include the values returned by your function;
3. attach a graph of the functions.

Problem: Robot-Positioning System

In control systems analysis, transfer functions are developed that mathematically relate the dynamics of a system's input to its output. A transfer function for a robotic-positioning system is given by

$$G(s) = \frac{C(s)}{N(s)} = \frac{s^3 + 12.5s^2 + 50.5s + 66}{s^4 + 19s^3 + 122s^2 + 296s + 192}$$

where $G(s)$ = system gain, $C(s)$ = system output, $N(s)$ = system input, and s = Laplace transform complex frequency.

Employ initial root estimates as **–5**, **–4.7**, and **–4.4**. Find a root for **[1]** $C(s)$ and **[2]** $N(s)$.

Prerequisites:
macheps = 1*(10^(-9))
max iterations = 1000
format of digits = 4 significant figures [R command: options(digits=4)]

```r
# prerequisites
macheps = 1 * (10 ^ (-9))
max = 1000
options(digits = 4)
```

Given mathematical functions:
C(s) = s^3 + 12.5*s^2 + 50.5*s + 66
N(s) = s^4 + 19*s^3 + 122*s^2 + 296*s + 192

```r
# given functions
csn = function(s) s^3 + 12.5*s^2 + 50.5*s + 66
nsn = function(s) s^4 + 19*s^3 + 122*s^2 + 296*s + 192
```

Given initial root estimates:
x0 = -5 ; x1 = -4.7 ; x2 = -4.4

```r
# given initial root estimates
x0 = -5
x1 = -4.7
x2 = -4.4
```

[1] Finding the root for $C(s)$

C(s) = s^3 + 12.5*s^2 + 50.5*s + 66

Function call:

```
> csn_root = MullerMethod(csn, x0, x1, x2, macheps, max, TRUE)
      x0     x1     x2     f(x0)     f(x1)     f(x2)        A      B         C     x3     f(x3)  Error(%)
1 -5.000 -4.700 -4.400 1.0000000 9.520e-01 6.160e-01 -1.60000 -1.600 6.160e-01 -4.103 1.589e-01 7.235e+00
2 -4.700 -4.400 -4.103 0.9520000 6.160e-01 1.589e-01 -0.70313 -1.748 1.589e-01 -4.015 2.312e-02 2.186e+00
3 -4.400 -4.103 -4.015 0.6160000 1.589e-01 2.312e-02 -0.01847 -1.548 2.312e-02 -4.000 6.128e-04 3.732e-01
4 -4.103 -4.015 -4.000 0.1589178 2.312e-02 6.128e-04  0.38112 -1.502 6.128e-04 -4.000 6.455e-07 1.020e-02
5 -4.015 -4.000 -4.000 0.0231214 6.128e-04 6.455e-07  0.48425 -1.500 6.455e-07 -4.000 2.672e-12 1.076e-05
6 -4.000 -4.000 -4.000 0.0006128 6.455e-07 2.672e-12  0.49945 -1.500 2.672e-12 -4.000 2.842e-14 4.452e-11
```

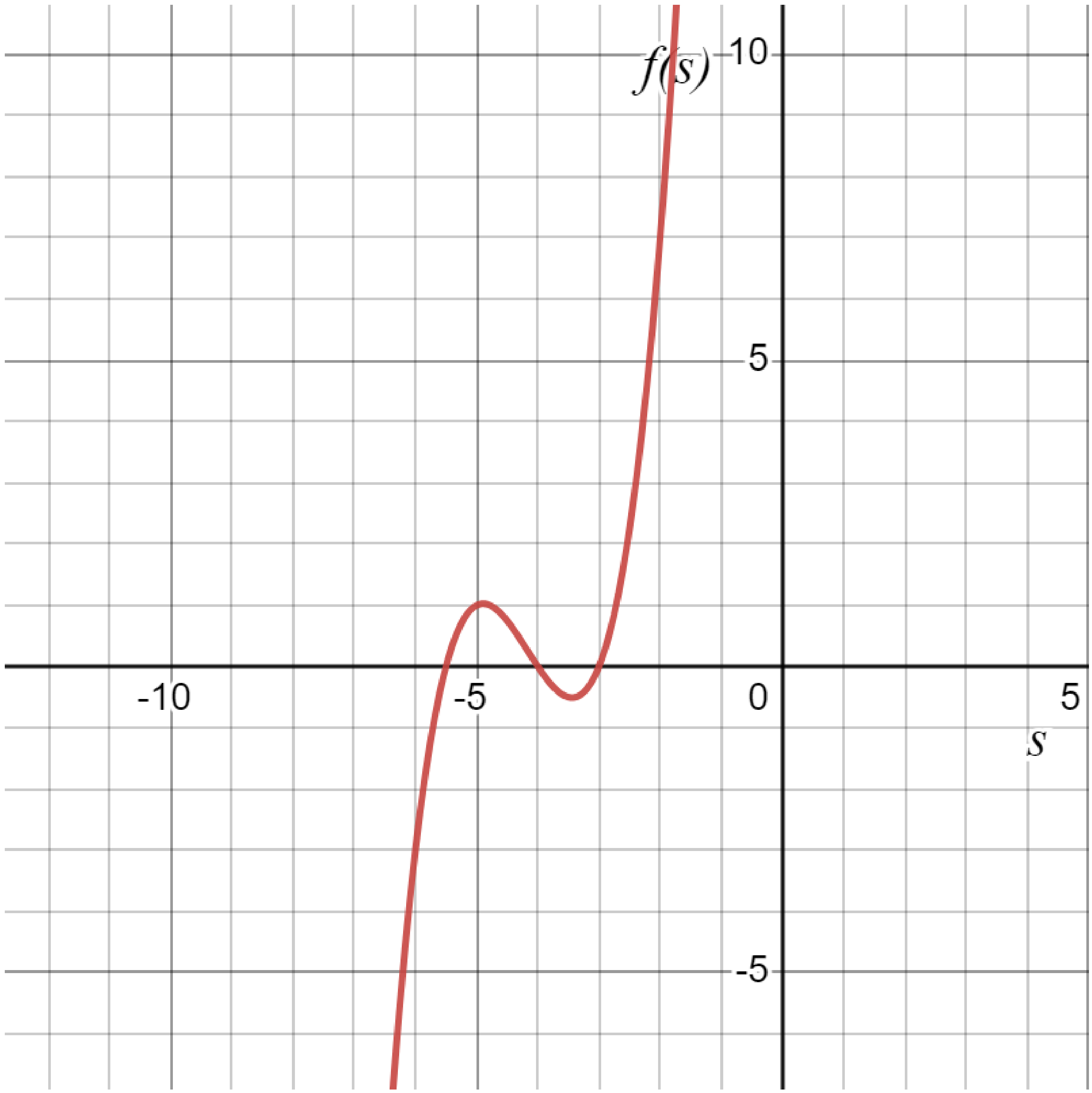
a. last iteration's values for x0, x1, x2, f(x0), f(x1), f(x2), A, B, C, x3, f(x3) and approximate relative error:

```
-4.000 -4.000 -4.000 0.0006128 6.455e-07 2.672e-12  0.49945 -1.500 2.672e-12 -4.000 2.842e-14 4.452e-11
```

b. values returned by the MullerMethod function:

```
> csn_root
$f
function(s) s^3 + 12.5*s^2 + 50.5*s + 66
<bytecode: 0x000001f702a52488>

$given_x0
[1] -5

$given_x1
[1] -4.7

$given_x2
[1] -4.4

$x3
[1] -4

$iterations
[1] 6

$ea
[1] 4.452e-11
```

c. Graph: C(s) = s^3 + 12.5*s^2 + 50.5*s + 66 (using desmos)



[2] Finding the root for $N(s)$

N(s) = s^4 + 19*s^3 + 122*s^2 + 296*s + 192

Function call:

```
> nsn_root = MullerMethod(nsn, x0, x1, x2, macheps, max, TRUE)
     x0    x1    x2     f(x0)     f(x1)     f(x2)       A      B         C    x3     f(x3)  Error(%)
1 -5.00 -4.70 -4.40 12.000000 1.111e+01 7.834e+00 -13.270 -14.91 7.834e+00 -4.01 2.372e-01 9.728e+00
2 -4.70 -4.40 -4.01 11.111100 7.834e+00 2.372e-01 -12.389 -24.31 2.372e-01 -4.00 5.134e-03 2.428e-01
3 -4.40 -4.01 -4.00  7.833600 2.372e-01 5.134e-03 -11.066 -24.01 5.134e-03 -4.00 2.199e-06 5.347e-03
4 -4.01 -4.00 -4.00  0.237213 5.134e-03 2.199e-06 -10.030 -24.00 2.199e-06 -4.00 6.821e-13 2.290e-06
5 -4.00 -4.00 -4.00  0.005134 2.199e-06 6.821e-13  -9.996 -24.00 6.821e-13 -4.00 0.000e+00 7.105e-13
```

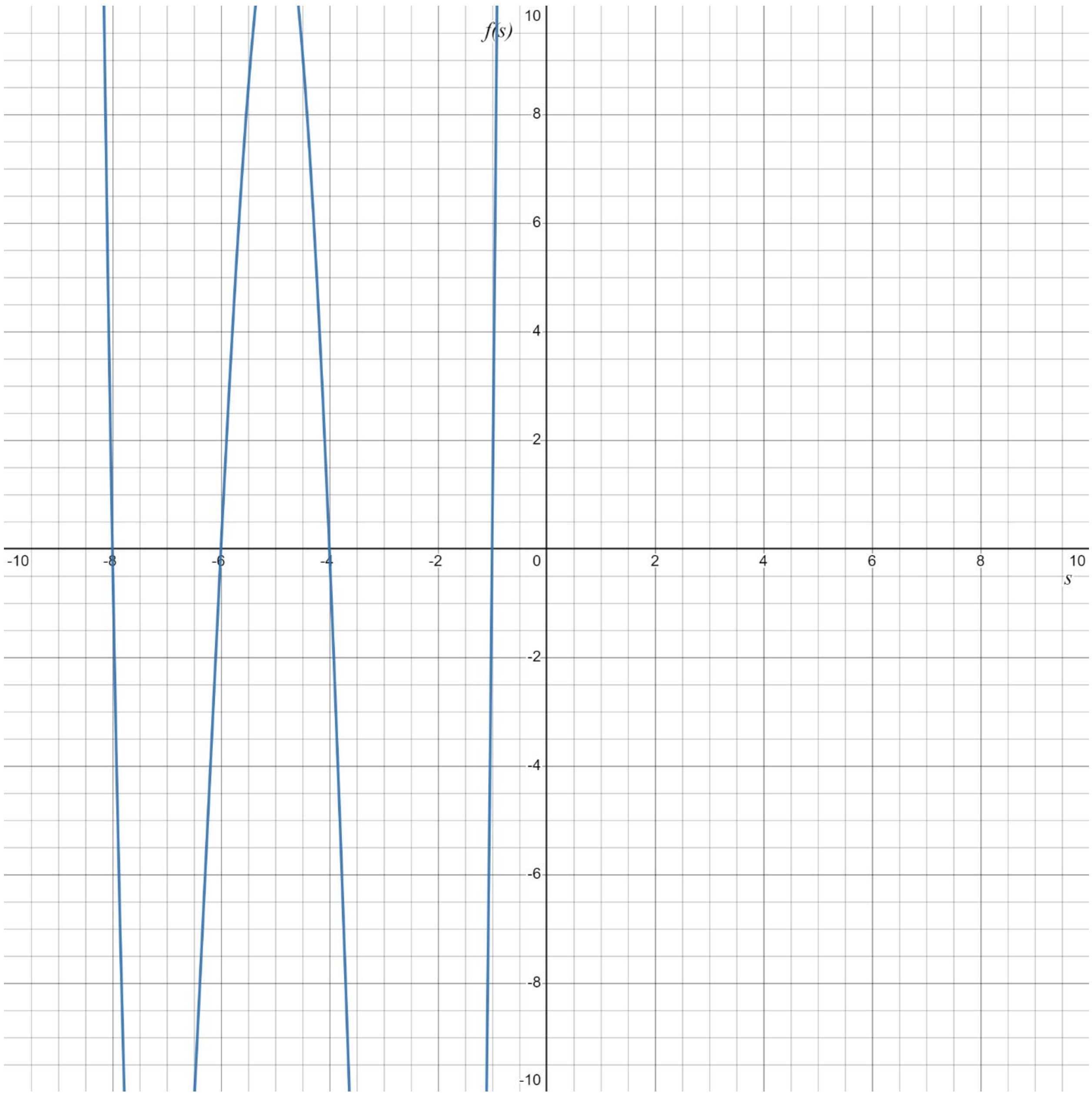
a. last iteration's values for x0, x1, x2, f(x0), f(x1), f(x2), A, B, C, x3, f(x3) and approximate relative error:

```
-4.00 -4.00 -4.00  0.005134 2.199e-06 6.821e-13  -9.996 -24.00 6.821e-13 -4.00 0.000e+00 7.105e-13
```

b. values returned by the MullerMethod function:

```
> nsn_root
$f
function(s) s^4 + 19*s^3 + 122*s^2 + 296*s + 192
<bytecode: 0x000001f70276e4d0>

$given_x0
[1] -5

$given_x1
[1] -4.7

$given_x2
[1] -4.4

$x3
[1] -4

$iterations
[1] 5

$ea
[1] 7.105e-13
```

c. Graph of N(s) = s^4 + 19*s^3 + 122*s^2 + 296*s + 192 (using desmos)



Conclusion:

The computed roots for $C(s)$ and $N(s)$ using the Muller's method are -4 and -4, respectively.