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CMSC 150 -B3L

November 23, 2023

Exercise 08

Part 2: Word Problem

Problem 1: False-Position Method

You are designing a spherical tank (Fig. 1) to hold water for a small village in a developing country. The volume of liquid it can hold can be computed as:

$$V = \pi h^2 \frac{[3R - h]}{3}$$

where V = volume (m^3), h = depth of water in tank (m), and R = the tank radius (m).

If $R = 4\ m$, to what depth must the tank be filled so that it holds 40 m^3 ? Use the false-position method to determine your answer. Employ initial guesses of 0 and R.

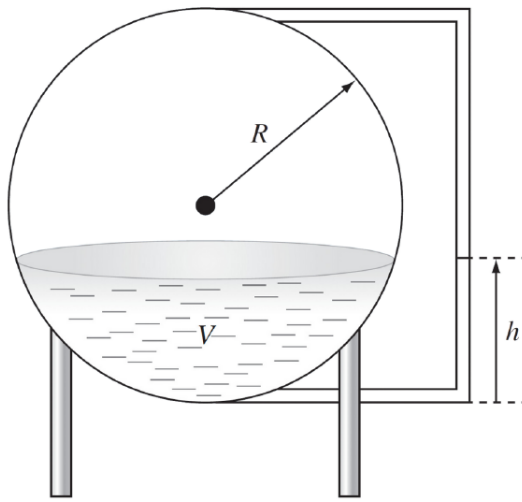

Fig. 1

Legend:

R = radius
h = depth/height
V = volume

Given formula for the volume of liquid the spherical tank can hold:

$$V = \pi h^2 \frac{3R-h}{3}$$

Given:

R = 4m
V = 40 m^3
h = ?

Asked: Find the depth (h) of the spherical tank given that R=4m and V=40 m^3.

Solution:

To compute for h, we can use the false position method to find the root of a certain equation with one unknown variable (h).

We can obtain the desired equation by playing with the formula $V = \pi h^2 \frac{3R-h}{3}$ such that one of its sides would only contain 0.

$$V = \pi h^2 \frac{3R-h}{3}$$

$$0 = \pi h^2 \frac{3R-h}{3} - V$$

We can then plugin the given values for R and V in the derived equation.

$$0 = \pi h^2 \frac{3(4)-h}{3} - 40$$

Now that we're only left with one unknown variable h in the derived equation, we can now use the false position method (we made in our program) to solve for its value/root (approximate).

Setting up some values/variables for the computation:

```
# -----------------------------------------------------------------------------
# problem 1
# Given:
V = 40
R = 4
# V = pi*(h^2) * ( (3*R -h)/3 )
# 0 = pi*(h^2) * ( (3*R -h)/3 ) - V
problem1_function = function (h) pi*(h^2) * ( (3*R -h)/3 ) - V
# Employ initial guesses of 0 and R.
a = 0
b = R
problem1 = FalsePositionMethod(problem1_function, 0, 4, macheps, max_no_iterations)
print(problem1)
```

Results of the process per iteration:

	a	b	f(a)	f(b)	c	f(c)	Error(%)
1	0.000	4	-40.000000	94.04	1.194	-2.388e+01	1.000e+02
2	1.194	4	-23.876105	94.04	1.762	-6.718e+00	3.225e+01
3	1.762	4	-6.718126	94.04	1.911	-1.412e+00	7.808e+00
4	1.911	4	-1.412443	94.04	1.942	-2.762e-01	1.592e+00
5	1.942	4	-0.276235	94.04	1.948	-5.323e-02	3.094e-01
6	1.948	4	-0.053229	94.04	1.949	-1.023e-02	5.955e-02
7	1.949	4	-0.010228	94.04	1.949	-1.964e-03	1.144e-02
8	1.949	4	-0.001964	94.04	1.949	-3.771e-04	2.197e-03

Return values:

```
$f
function (h) pi*(h^2) * ( (3*R -h)/3 ) - V
<bytecode: 0x000001b45c5466a8>

$given_a
[1] 0

$given_b
[1] 4

$c
[1] 1.949

$iterations
[1] 8

$ea
[1] 0.002197
```

Final (approximate) value of h after 8 iterations: 1.949
(approximate error with machine epsilon 1×10^{-2} is 0.002197 %)

Interpretation:

Given that the radius of the spherical tank is 4m, the tank must be filled to a depth of about 1.949 m so that it holds 40 m^3 of water.

Problem 2: Secant Method

Determine the real root of $\mathrm{f}(x) = -26 + 85x - 91x^2 + 44x^3 + 8x^4 + x^5$. Use the secant method and employ initial guesses of $x_l = 0.5$ and $x_u = 1.0$.

Given:

$\mathrm{f}(x) = -26 + 85x - 91x^2 + 44x^3 + 8x^4 + x^5$
$x_0 = 0.5$
$x_1 = 1.0$

Asked:

Find the real root of the given mathematical function.

Solution:

Use the secant method (we made in our program) to solve for the root of the given function $\mathrm{f}(x) = -26 + 85x - 91x^2 + 44x^3 + 8x^4 + x^5$.

Setting up some values/variables for the computation:

```
# -----------------------------------------------------------------------------
# problem 2
problem2_function = function (x) -26 + 85*x -91*(x^2) + 44*(x^3) + 8*(x^4) + (x^5)
x0 = 0.5
x1 = 1.0
problem2 = SecantMethod(problem2_function, x0, x1, macheps, max_no_iterations, TRUE)
print(problem2)
```

Results of the process per iteration:

	x0	x1	f(x0)	f(x1)	x	f(x)	Error(%)
1	0.5000	1.0000	-0.21875	2.100e+01	0.5052	-5.765e-02	1.000e+02
2	1.0000	0.5052	21.00000	-5.765e-02	0.5065	-1.541e-02	2.675e-01
3	0.5052	0.5065	-0.05765	-1.541e-02	0.5070	-9.665e-06	9.745e-02
4	0.5065	0.5070	-0.01541	-9.665e-06	0.5070	-1.605e-09	6.117e-05

Return values:

```
$f
function (x) -26 + 85*x -91*(x^2) + 44*(x^3) + 8*(x^4) + (x^5)
<bytecode: 0x000001b466e750b8>

$given_x0
[1] 0.5

$given_x1
[1] 1

$x
[1] 0.507

$iterations
[1] 4

$ea
[1] 6.117e-05
```

Final (approximate) value of x after 8 iterations: 1.949
(approximate error with machine epsilon 1 X 10^{-2} is 0.00006117 %)

Interpretation:

The real root of $f(x) = -26 + 85x - 91x^2 + 44x^3 + 8x^4 + x^5$ is about 0.507.