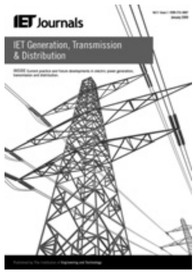
www.ietdl.org

Published in IET Generation, Transmission & Distribution
Received on 12th October 2012
Revised on 30th January 2013
Accepted on 10th February 2013
doi: 10.1049/iet-gtd.2012.0724



ISSN 1751-8687

Turn-to-earth fault modelling of power transformer based on symmetrical components

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Abstract: In this paper, a new approach to simulate transformer turn to earth fault, by using symmetrical components approach, has been presented. In the suggested novel modeling technique, the transformer turn to earth fault is simulated as a new healthy transformer with new short circuit (SC) impedance. This SC impedance is considered as the SC impedance of the transformer with the fraction part of the transformer winding through which the turn to earth fault occurs. The SC impedance of the reminder part of the transformer winding is added to the transmission line impedance or load impedance as external impedance. Hence, by using this suggested modelling technique, the system of turn to earth fault transformer is represented with a new system of a new healthy transformer, and the turn to earth fault is represented as an external line to ground fault. The values of the turn to earth fault current from this novel modelling technique (using the symmetrical components approach) were compared with those obtained by Electromagnetic Transient Program/Alternative Transient Program (EMTP/ATP). Finally, a suggestion method to estimate general information about the SC impedance of a three-phase three-winding transformer at different percentages of turn to earth fault is discussed.

1 Introduction

The power transformer is one of the most important equipment of the electric power system. The development of modern power systems has been reflected in advances in the transformer design. This was resulted in a wide range of transformers with sizes ranging from a few kVA to several hundred MVA being available for use in a wide variety of applications. Different faults can occur inside the transformer and at the electrical system where the transformer is connected. Transformer faults can be divided into two classes, internal and external faults [1].

Analysis of modern transformer failures, which occurred over 20 years, showed that inter-turn/internal faults are between 70 and 80% of transformer faults [2].

This type of fault occurs suddenly and usually requires fast action by the protective relay to disconnect the transformer from the electric power system.

Modelling of transformer winding faults requires accurate representation of the transformer elements. The development of an accurate transformer model is being to be very complex because of the large number of core designs and due to the fact that several transformer parameters are both non-linear and frequency dependent.

Physical attributes, whose behavior may need to be correctly represented, are core and coil configurations, self- and mutual inductances between coils, leakage fluxes, skin effect and proximity effect in coils, magnetic core saturation, hysteresis and eddy current losses in core and capacitive effect [3].

Transformer models for network simulation can be divided into four main categories as follows [4]:

1. Steady-state models: in which the core representation is not critical and can usually be neglected in load flow, short-circuit (SC) and any steady-state calculations [5].
2. Models based on matrix and circuit representation: where, the iron core behaviour can be linearised, however simulation errors occur when the core is driven in the saturation area. This approach is used in BCTRAN (a routine generating impedence matrix of the transformer) component in EMTP/ATP. To improve the core representation, excitation can be extracted from the main circuit and an additional non-linear circuit can be externally attached at the model terminal [6].
3. Topologically correct models: which are based on transformer geometry and duality theorem. The unified magnetic equivalent circuit model in power systems computer aided design/electromagnetic transients including direct current (PSCAD/EMTDC) is one of the first models offered in a simulation package to take advantage of this approach. Another model that uses the geometry and duality approach is the hybrid transformer model recently implemented in Electromagnetic Transient Program (EMTP/ATP) under the name XFMR, which is a routine used to model the transformer and supported three sources of data: design parameters, test report and typical values. In these transformer models each individual limb of the magnetic circuit is represented and contributed to the magnetisation characteristic. This approach can very accurately represent

any type of core but it requires a slightly larger set of data [7–9].
4. Models based on finite element methods: such type of modelling technique can be very accurate but it has the disadvantages to be valid only for a specific unit and requires huge computing resources. This modelling technique still retains several approximations [10].

As mentioned before, there are many methods that can simulate the transformer under internal faults; each of them needs some experience and information data about the transformer itself at each percentage of faulted winding. That of course is considered consuming of time and tired the researchers.

In addition, the novel suggested technique, in this paper, saves the time and does not need any experience that required by the previous methods. The only required is the basic of the fault calculation by using the symmetrical components approach, and the value of the healthy transformer SC impedance. Hence, it is easily to use this technique to calculate the fault current at any percent of faulted winding.

The matrix representation of transformer is used in this paper, to simulate turn-to-earth faulted transformer. Also, the values of the SC impedance of the faulted transformer winding at different percentages (b) of faulted winding are calculated from the SC test. These values of the SC impedance are used to calculate the fault current by using the suggested technique based on the symmetrical components approach.

2 Matrix representation of the transformer

Matrix representation of the power transformer is an important step towards realisation of transformer winding fault in the EMTP/ATP. The BCTRAN routine computes two matrices [$\boldsymbol{R}$] and [$\boldsymbol{L}$], which represent the transformer based on the data of the excitation and SC tests for positive and zero sequences.

2.1 Healthy transformer model

The transformer model could be built by using a lot of components, that are, BCTRAN, saturated transformer, coupled inductance etc. BCTRAN is used in this paper because of the available data of the transformers. The inputs data to the BCTRAN consist of MVA, voltage, winding connection, grounding, impedance, losses, frequency, SC and open-circuit tests etc., which can be easily taken from the transformer factory acceptance tests. BCTRAN can carry out model for any power transformer, two and three winding, single and three phases, star (wye) and delta winding and autotransformer.

In order to explain the concept, a single phase two-winding transformer, shown in Fig. 1, will be considered, which can be described by the following steady-state phasor equations [1, 3, 11, 12]

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{1}$$

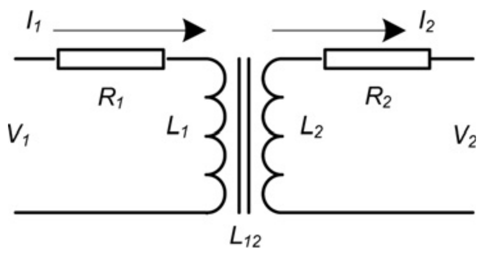

Fig. 1 *Single phase transformer*

According to the differential equation of electromagnetic transient analysis, (1) can be written as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{2}$$

Where [$\boldsymbol{R}$] is the resistance matrix (real part of impedance matrix [$\boldsymbol{Z}$]) and [$\boldsymbol{L}$] is the inductance matrix (imaginary part of impedance matrix [$\boldsymbol{Z}$]) divided by ω. This model is accommodated by BCTRAN routine in the EMTP/ATP. Extension of (1) to three-phase transformer is conceptually easy. Each winding in (1) consists of a single coil, but it consists of three coils for the three phases, as shown in Fig. 2*a*. This means that each element of impedance matrix [$\boldsymbol{Z}$] becomes a 2×2 matrix as follows

$$\begin{bmatrix} Z_S & Z_M \\ Z_M & Z_S \end{bmatrix} \tag{3}$$

where Z_S is the self-impedance of each phase and Z_M is the mutual impedance among any two phases. Replacing each element of [$\boldsymbol{Z}$] by the 3×3 sub-matrix of (3), the three-phase matrix is obtained. Hence, the [$\boldsymbol{R}$] and [$\boldsymbol{L}$] matrices of the three-phase three-winding will be 9×9.

2.2 Turn-to-earth faulted transformer

To model a fault between a coil turns and the earth, the corresponding winding is divided into two sub-coils. Thus the [$\boldsymbol{R}$] and [$\boldsymbol{L}$] matrices are modified, where some of their

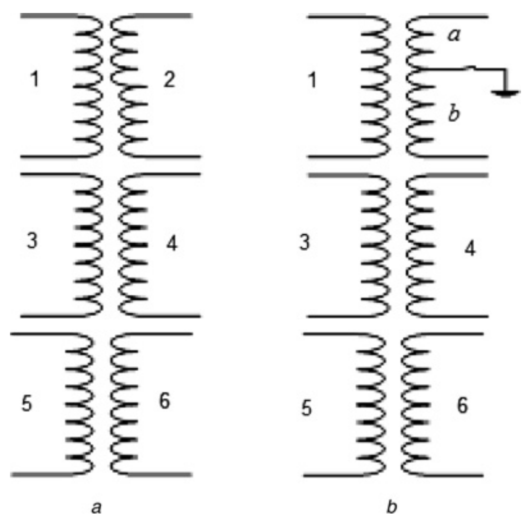

Fig. 2 *Three-phase two-winding transformer*
a Healthy transformer
b Transformer with turn-to-earth fault

elements are computed by the BCTRAN for the healthy transformer and the other elements are computed from mathematical equations modelling faults in the windings. Fig. 2*b* shows the simulation of the transformer with turn-to-earth fault condition. There are two methods for calculating the new impedance matrix to represent that transformer. In the first method, the self and mutual impedances of the transformer are calculated directly [13–16]. This method has limitation because it needs the very detail design data of the transformer and sometimes it is hard to find these data. In the second method, the leakage inductance is calculated by using leakage factor [11, 12, 17, 18]. This method is used in this paper as it does not require special tests and utilises the same test results requested by the BCTRAN module.

The winding with turn-to-earth fault is simulated by creating internal node in the equivalent matrix generated by the BCTRAN subroutine, by splitting the faulted winding into two sub-windings (sub-coils *a* and *b*, with percentages $a + b = 1$) in case of turn-to-earth fault, as shown in Fig. 2*b*.

As shown in Fig 2*b*, each sub-coil (*a* or *b*) will has its own resistance, self-inductance and mutual inductance with the other sub-coil and all remaining coils of the transformer. This leads to a formation of a new 10×10 [$\boldsymbol{R}$] and [$\boldsymbol{L}$] matrices [18]. Hence, the updated matrices would be as follows

$$\boldsymbol{R} = \begin{bmatrix} R_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & R_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & R_b & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & R_5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & R_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_9 \end{bmatrix} \tag{4}$$

$$\boldsymbol{L} = \begin{bmatrix} L_{11} & L_{1a} & L_{1b} & L_{13} & L_{14} & L_{15} & L_{16} & L_{17} & L_{18} & L_{19} \\ L_{a1} & L_a & L_{ab} & L_{a3} & L_{a4} & L_{a5} & L_{a6} & L_{a7} & L_{a8} & L_{a9} \\ L_{b1} & L_{ba} & L_b & L_{b3} & L_{b4} & L_{b5} & L_{b6} & L_{b7} & L_{b8} & L_{b9} \\ L_{31} & L_{3a} & L_{3b} & L_{33} & L_{34} & L_{35} & L_{36} & L_{37} & L_{38} & L_{39} \\ L_{41} & L_{4a} & L_{4b} & L_{43} & L_{44} & L_{45} & L_{46} & L_{47} & L_{48} & L_{49} \\ L_{51} & L_{5a} & L_{5b} & L_{53} & L_{54} & L_{55} & L_{56} & L_{57} & L_{58} & L_{59} \\ L_{61} & L_{6a} & L_{6b} & L_{63} & L_{64} & L_{65} & L_{66} & L_{67} & L_{68} & L_{69} \\ L_{71} & L_{7a} & L_{7b} & L_{73} & L_{74} & L_{75} & L_{76} & L_{77} & L_{78} & L_{79} \\ L_{81} & L_{8a} & L_{8b} & L_{83} & L_{84} & L_{85} & L_{86} & L_{87} & L_{88} & L_{89} \\ L_{91} & L_{9a} & L_{9b} & L_{93} & L_{94} & L_{95} & L_{96} & L_{97} & L_{98} & L_{99} \end{bmatrix} \tag{5}$$

The first step, in this method, is to determine the self and mutual inductances of the faulty sub-coils. That can be achieved according to three rules: consistency, leakage and proportionality.

2.2.1 Consistency: If coils *a* and *b* are supplied in series by current *I* without any fault between them, the same results must be obtained by using the 10×10 matrix as that obtained by using the 9×9 matrix.

$$\begin{aligned} \varphi_a &= (L_a + L_{ab})I \\ \varphi_b &= (L_b + L_{ab})I \end{aligned} \tag{6}$$

In this way

$$\begin{aligned} \varphi_a + \varphi_b &= (L_a + 2L_{ab} + L_b)I \\ \varphi_2 &= L_{22}I \end{aligned} \tag{7}$$

where $\varphi_a + \varphi_b = \varphi_2$ is the magnetic flux through this loop.

These relations lead to the well-known expression of L_{22}, for any two inductances in series

$$L_a + 2L_{ab} + L_b = L_{22} \tag{8}$$

where L_a, L_{ab} and L_b, are elements of the 10×10 matrix, while L_{22} is an element of the 9×9 matrix.

2.2.2 Leakage: Taking into account a leakage factor between coils *a* and *b* is essential, since the fault current will largely depends on the leakage. The leakage factor is [18]

$$\sigma_{ab} = 1 - \frac{L_{ab}^2}{L_a L_b} \tag{9}$$

2.2.3 Proportionality: To determine the three unknowns L_a, L_{ab} and L_b, another equation should be added to relations (8) and (9), as follow

$$\frac{L_a}{L_b} = \left(\frac{n_a}{n_b}\right)^2 \tag{10}$$

Consider

$$k = \frac{n_a}{n_b} \tag{11}$$

where n_a and n_b are the number of turns in coils *a* and *b*, respectively.

The 10×10 matrix [$\boldsymbol{R}$] will be determined with the help of the relations in [12, 17, 18] as follows

$$\begin{aligned} R_a &= aR_2 \\ R_b &= bR_2 \end{aligned} \tag{12}$$

where R_2 is the element of the 9×9 matrix.

The 10×10 matrix [$\boldsymbol{L}$] will be determined with the help of the relations in [12, 17] as follows

$$L_a = \frac{L_{22}}{(1/k^2) + \left(2\sqrt{1 - \sigma_{ab}}/(k)\right) + 1} \tag{13}$$

$$L_b = \frac{L_{22}}{k^2 + 2k\sqrt{1 - \sigma_{ab}} + 1} \tag{14}$$

$$L_{ab} = \frac{L_{22}\sqrt{1 - \sigma_{ab}}}{(k + (1/k)) + 2\sqrt{1 - \sigma_{ab}}} \tag{15}$$

If the considered coil *i* is wound on the same leg as *a* and *b* (the faulted coil) and if $a > b$, in these conditions

$$L_{ai} = L_{2i}\sqrt{\varepsilon}\sqrt{\frac{L_a}{L_{22}}}\sqrt{1 + \left(\frac{1 - \varepsilon}{\varepsilon}\right)\left(\frac{L_{22}L_{ii}}{L_{2i}^2}\right)} \tag{16}$$

and

$$L_{bi} = L_{2i} - L_{ai} \tag{17}$$

where L_{22}, L_{2i} and L_{ii} are elements of the (9×9) $[\boldsymbol{L}]$ matrix computed by BCTRAN. If coil i is wound on the different leg than that of a and b then

$$L_{ai} = \frac{k}{1+k} L_{2i} \tag{18}$$

$$L_{bi} = \frac{1}{1+k} L_{2i} \tag{19}$$

$$\frac{L_{ai}}{L_{bi}} = \frac{a}{b} = k \tag{20}$$

$$\varepsilon = \frac{\sigma_{ai}}{\sigma_{2i}} \tag{21}$$

where σ_{ai} is the leakage factor between coil a and winding i, and σ_{2i} is the leakage factor between winding 2 and winding i. Unknown ratio of two leakage factor (ε) is assumed to be 1 as recommended by Kezunovic *et al.* [19]. The leakage factor σ_{ab} proposed by Darwish *et al.* [12] is equal to

$$\sigma_{ab} = a\sigma_{12} \tag{22}$$

where σ_{12} is leakage factor between primary and secondary winding of the healthy transformer and could be calculated by

$$\sigma_{12} = 1 - \frac{L_{12}^2}{L_{11} L_{22}} \tag{23}$$

3 EMTP/ATP simulation of turn-to-earth fault

The modelled transformers in this paper are 40 MVA, 220/70/11.5 kV three-phase three-winding YNyn0d11 transformer, and 300 MVA, 230/109.8/50 kV three-phase three-winding YNyn0d11 transformer. Their details are presented separately in the appendix.

The electrical system with three-phase three-winding transformer, under internal turn-to-earth fault conditions at different percentages of the winding of the low-voltage side, is built in the EMTP/ATP network as shown in Fig. 3. Under internal turn-to-earth fault conditions at different percentages of the winding, the new 10×10 impedance matrix of the transformer is calculated by using the principles of consistency, leakage and proportionality and is written as source data in the library. This library will react as BCTRAN routine for the faulty transformer. The internal turn-to-earth fault, at different percentages of the winding, is applied by closing the switch F between the node of the fault point and the ground, as shown in Fig. 3. Then, the simulated fault current of turn-to-earth fault of the transformer winding can be obtained.

4 Results and discussion

4.1 EMTP/ATP simulation

The electrical system with faulty transformer is built in the EMTP/ATP network as shown in Fig. 3. The magnitude of the fault current on a transformer winding depends on source impedance, neutral grounding impedance, transformer leakage reactance, fault voltage and winding connection [19]. The simulated turn-to-earth fault currents waveform at different percentage of the faulty winding and 30 Ω earthed resistance of phase A of 40 MVA-220/70/11.5 kV transformer is shown in Fig. 4. From Fig. 4, it is noticed that the winding turn-to-earth fault current depends on the percentages (b) of the faulty winding. Since the fault voltage is directly proportional to this percent, the turn-to-earth fault current increases as the percent of faulted turns from the neutral point increases.

Fig. 5 shows the variation of the turn-to-earth fault current, at 70% faulted winding, with the variation of the earthed resistance. It is seen that, the turn-to-earth fault current decreases with the increase of the earthed resistance.

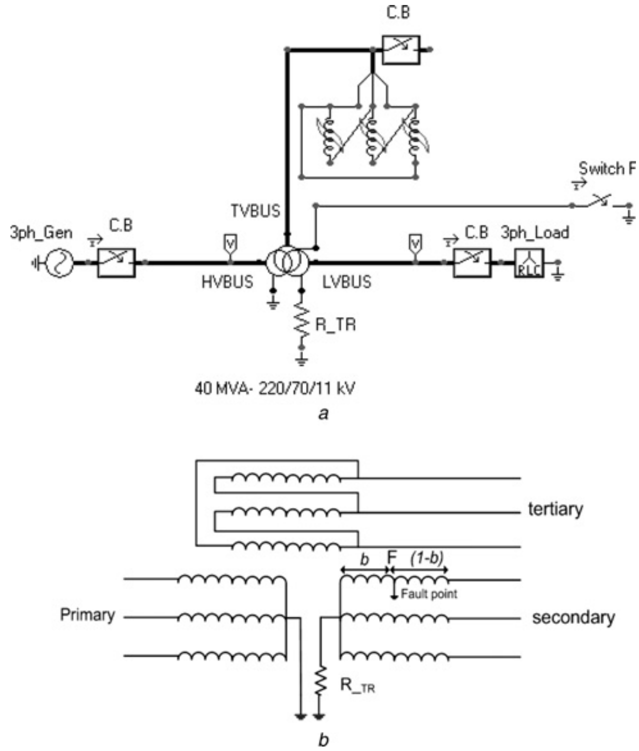

Fig. 3 *Presentation of the transformer under turn-to-earth fault*
a EMTP/ATP simulation
b Electric circuit

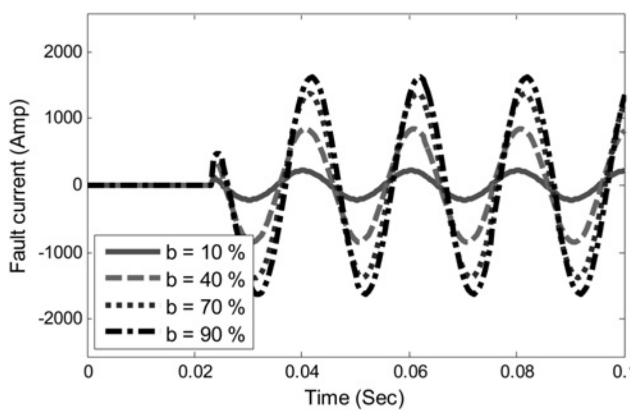

Fig. 4 *Simulated turn-to-earth fault currents waveform at different percentage of faulty winding of phase A of the secondary side of 40 MVA-220/70/11.5 kV transformer*

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doi: 10.1049/iet-gtd.2012.0724

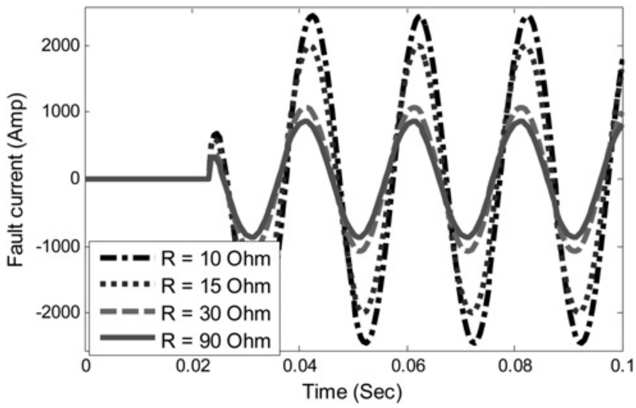

Fig. 5 *Simulated turn-to-earth fault currents waveform at different earthed resistance of faulty winding of phase A of the secondary side of 40 MVA-220/70/11.5 kV transformer*

4.2 Three-winding transformer SC test

For a three-phase three-winding transformer that has wye-connected primary and secondary windings, with their neutrals grounded, and a delta-connected tertiary winding, the zero-sequence SC test between the primary and secondary windings will not only has the secondary winding shorted but the tertiary winding as well, since a closed delta connection provides a SC path for zero-sequence currents.

With the well-known equivalent star circuit of Fig. 6*a*, the three test values (the equivalent SC reactance of primary and secondary windings, $X_{HL}^{\text{closed}\Delta}$; the equivalent SC reactance of primary and tertiary windings, X_{HL}; and the equivalent SC reactance of secondary and tertiary windings, X_{LT}) supplied by the manufacturer are [6]

$$X_{HL}^{\text{closed}\Delta} = X_H + \frac{X_L X_T}{X_L + X_T} \tag{24}$$

$$X_{HL} = X_H + X_T \tag{25}$$

$$X_{LT} = X_L + X_T \tag{26}$$

which can be solved to calculate X_H, X_L and X_T as

$$X_{\rm H} = X_{\rm HT} - \sqrt{X_{\rm LT}X_{\rm HT} - X_{\rm LT}X_{\rm HL}^{\text{closed}\Delta}} \tag{27}$$

$$X_{\rm L} = X_{\rm LT} - X_{\rm HT} + X_{\rm H} \tag{28}$$

$$X_{\rm T} = X_{\rm HT} - X_{\rm H} \tag{29}$$

when the fault occurs in the secondary side, $X_{\rm L}$ is replaced by $X_{{\rm L}f}$ as shown in Fig. 6*b*. Also, $X_{\rm HL}^{\text{closed}\Delta}$ is replaced by $X_{\rm F}$ which equals

$$X_{{\rm HL},f} = X_{\rm H} + \frac{X_{{\rm L}f} X_{\rm T}}{X_{{\rm L}f} + X_{\rm T}}$$

$X_{\rm LT}$ is replaced by $X_{{\rm LT},f}$ which equals $X_{\rm L} + X_{\rm T}$; and $X_{\rm HT}$ is remained the same as that of the healthy transformer.

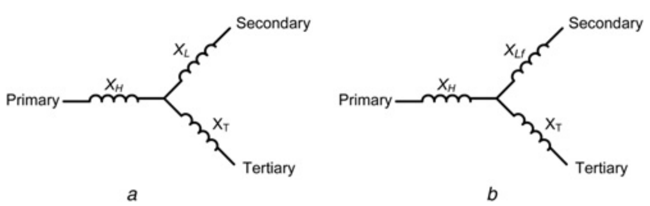

Fig. 6 *Equivalent star circuit for zero-sequence SC tests of a three-winding transformer*

a Healthy transformer
b Transformer with turn-to-earth fault

4.3 Estimation of turn-to-earth faulty transformer SC impedance

The SC impedance of any transformer is related to the leakage impedance of that transformer [20]. There are many parameters that affect the value of the leakage inductance of any transformer, such as: the arrangement of the windings, the shape of the core, the thickness of copper foil and the thickness of insulator layer, the permeability of copper foil and the permeability of insulator, the number of turns and the length and the width of conductor [21–23].

The SC impedance $X_{\rm F}$, of the transformer under internal turn-to-earth fault at different percentage (b) of the winding of the low-voltage side, can be calculated with the help of Fig. 7, as follow

The supply is connected to the primary winding and the tertiary winding is left open. The two healthy phases of the secondary winding are short circuited with the point of faulted winding of the third phase. In each step the selector arrow is changed from zero (neutral point) to 100% (total length of winding), and X_F, is calculated from the following relation

$$X_f(\text{pu}) = 100\,\frac{V_{\rm p,L}\text{MVA}}{\sqrt{3}I_{\rm p,L}(\text{kV})^2} \tag{30}$$

Where $V_{\rm p,L}$ is the primary line voltage and $I_{\rm p,L}$ is the primary line current of faulty phase.

Also, $X_{\rm R}$, which is the SC impedance of the remaining part of the turn-to-earth faulty winding, is calculated as follow:

If the fault occurs at percentage b%, then $X_{\rm R} = X_{\rm F}$ at percentage (100–b)%.

Fig. 8 shows the values of SC impedance $X_{\rm R}$ and $X_{\rm F}$ of the three-phase three-winding transformer (40 MVA, 220/70/11.5 kV, YNyn0d11) at different percentages of faulted winding.

The normalised values of the SC impedance $X_{\rm F}$ (to the value of the SC impedance of 100% of faulted winding) are general values of that type of transformers that can describe any three-phase three-winding transformer at any percentages of faulted winding.

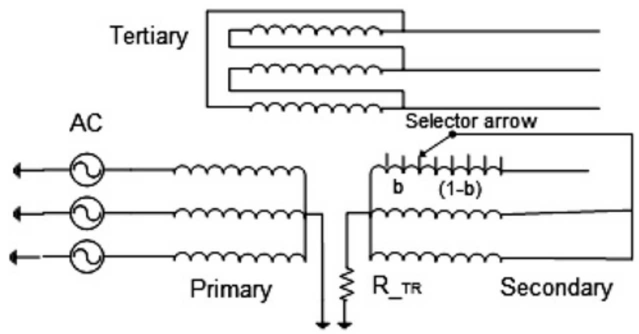

Fig. 7 *Circuit for calculating X_F*

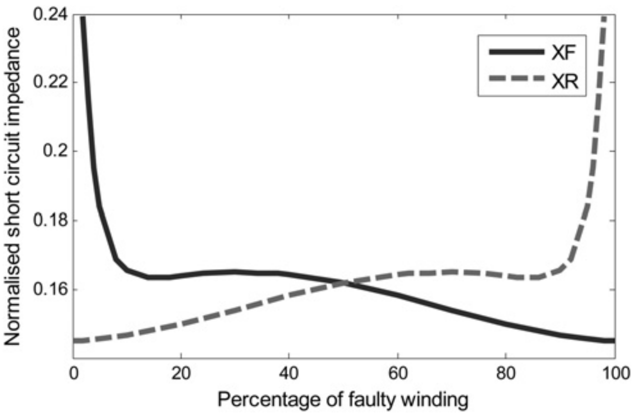

Fig. 8 *SC impedance of the three-phase three-winding transformer at different percentages of faulty winding*

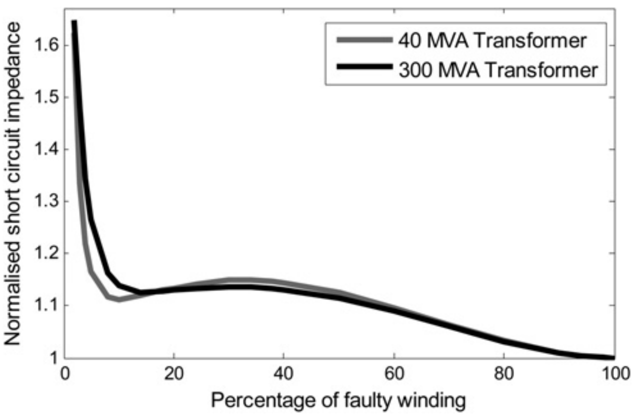

Fig. 9 *Normalised values of the SC impedance at different percentages of faulty winding*

Fig. 9 shows the comparison between the normalised values of the SC impedance X_F of the two different three-phase three-winding transformers (40 MVA, 220/70/11.5 kV, YNyn0d11 and 300 MVA, 230/109.8/50 kV, YNyn0d11) at different percentages of faulted winding. It is noticed that the results of the two transformers are in a good agreement with each other. This means that, these normalised values of the SC impedance at different percentages of faulted winding are general values that can be used to estimate the value of the SC impedance of any three-phase three-winding (YYD) transformer, if the value of the SC impedance at 100% of faulted winding is obtained.

The value of the SC impedance at 100% of winding for any transformer is obtained from the data of the SC test.

Hence, the value of X_F, which represents the new healthy transformer as well as the value of X_R, which represents the value of the remaining part of the SC impedance, are easily estimated, for any transformer (YYD), by using the normalised data of Fig. 9. The only required to estimate these values, are the data of the SC test of that transformer.

5 Simulation by symmetrical components

In this novel modelling technique, the turn-to-earth faulted transformer is simulated as a new healthy transformer with new SC impedance equals X_F. The SC impedance of the remaining part of the turn-to-earth faulted winding, X_R, is represented as an external impedance, which can be added to the transmission line impedance or external load impedance. Hence, by using this suggested modelling technique, the system of the faulted transformer is represented with a new system of a new healthy transformer, and the turn-to-earth fault is represented as an external line-to-ground fault, as shown in Fig. 10. Hence, the fault current can be easily calculated by using the symmetrical components.

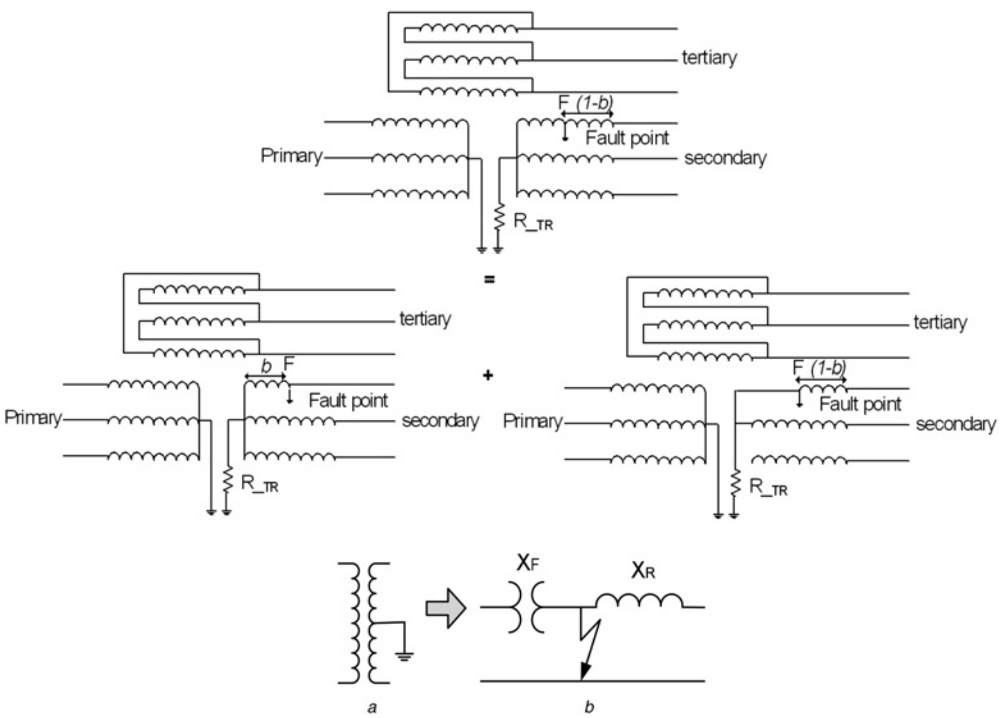

Fig. 10 *Representation of the turn-to-earth faulty transformer*

a Faulty winding
b Faulty transformer representation

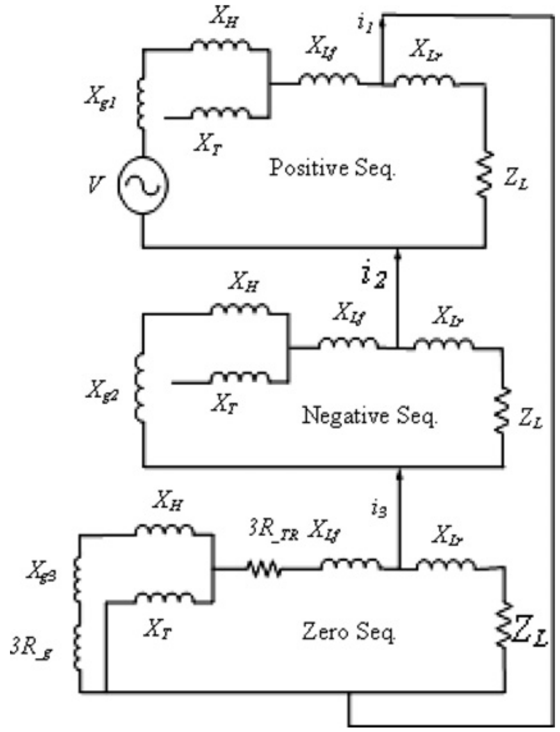

Fig. 11 *Positive, negative and zero sequences representation of a three-phase three-winding power transformer under turn-to-earth fault condition*

In the symmetrical components simulation method the base impedance and base current are calculated as follows

$$Z_{\text{base},1} = \frac{(b \times \text{kV}_\text{s})^2}{\text{MVA}}$$

$$Z_{\text{base},2} = \frac{(\text{kV}_\text{p})^2}{\text{MVA}}$$

$$I_\text{base} = \frac{\text{MVA}}{\sqrt{3} \times b \times \text{kV}}$$

where MVA is the rating power of the transformer, b is the percentage of the faulty winding from neutral point, V_s and V_p is secondary and primary voltages, respectively.

The zero I_{ao}, positive I_{a1} and negative I_{a2} sequence currents are calculated as follows [24]

$$I_{a0} = I_{a1} = I_{a2} = \frac{V_\text{pu}}{Z_0 + Z_1 + Z_2} \quad (34)$$

where Z_0, Z_1 and Z_2 are zero, positive and negative sequence impedances, respectively, which are calculated from the following relations [24]

$$V_\text{pu} = 1 \quad (35)$$

$$Z_1 = (\text{j}X_{g1} + \text{j}X_{F1})//(\text{j}X_{lr,1} + \text{j}Z_\text{L}) \quad (36)$$

$$Z_2 = (\text{j}X_{g2} + \text{j}X_{F2})//(\text{j}X_{lr,2} + \text{j}Z_\text{L}) \quad (37)$$

$$Z_0 = \left[\left((3R_{ng} + \text{j}X_{g0} + \text{j}X_\text{H})//\text{j}X_\text{T}\right) + 3R_{ntr} + \text{j}X_{Lf}\right]// (\text{j}X_{lr,0} + Z_\text{L}) \quad (38)$$

Also, X_{F1} and X_{F2} can be defined for three-phase three-winding transformer as

$$X_{F1} = X_{F2} = X_\text{H} + \frac{X_{Lf}X_\text{T}}{X_{Lf} + X_\text{T}} \quad \text{and}$$

$$X_{lr,1} = X_{lr,2} = X_{lr,0} = X_{Lr}$$

as given in Fig. 11, which shows the representation of the three-phase three-winding transformer system by positive, negative and zero sequence components, where the tertiary winding is unloaded, and the low-voltage winding has turn-to-earth fault condition.

Where X_{lf} is the SC impedances of turn-to-earth faulted winding, and X_{lr} is the SC impedances of the remaining part, $R_{\_\text{TR}}$ is the resistances of the transformer neutral point, X_{g0}, X_{g1} and X_{g2} are the impedances of zero, positive and negative sequence of circuit supply, $R_{\_g}$ is the resistances of the neutral point of the supply in the circuits, X_H and X_T are the SC impedances of high voltage winding and tertiary winding, respectively, and Z_L is the load impedance.

Finally, the fault current I_f is calculated as follow [24]

$$I_\text{f}(\text{pu}) = 3I_{a0} \quad (39)$$

$$I_\text{f}(\text{kA}) = I_\text{f}(\text{pu}) \times I_\text{base} \quad (40)$$

Table 1 shows the comparison between simulated fault currents obtained by EMTP/ATP program and symmetrical components method for three-phase three-winding turn-to-earth faulted transformers. From Table 1, it is seen that the results of the two methods are in an acceptable agreement with each other.

Table 1 Comparison between simulated fault currents, which are obtained by EMTP/ATP program and symmetrical components method, for three-phase three-winding faulty transformer

Percentage of faulty winding, %	40 MVA, three-phase three-winding transformer			300 MVA, three-phase three-winding transformer		
	Simulated fault currents in Ampere	Calculated fault currents in Ampere	Percentage of error, %	Simulated fault currents in Ampere	Calculated fault currents in Ampere	Percentage of error, %
10	217.8	221.3	1.61	552	554.6	0.47
40	851	876.7	3.02	2200	2216.5	0.75
70	1377.8	1430	3.79	3810	3848.6	1.013
90	1620	1684	3.95	4830	4882.7	1.091

6 Conclusions

The new suggested technique represents the transformer with tern to earth fault by a new healthy transformer with new SC impedance and the turn-to-earth fault is represented as an external line-to-ground fault. The results of the novel technique (symmetrical components method) are compared with those obtained by using EMTP/ATP program. The comparison shows that the results of the two methods are in an acceptable agreement with each other. The slightly error between them is because of the approximation that was used in the calculation of the values of the SC impedance, depending on the principles of consistency, leakage and proportionality.

Also, this paper presents a suggested method, to give general information about the SC impedance of any three-phase three-winding (YYD) transformer at different percentages of turn-to-earth fault.

By using these normalised values of SC impedance, the symmetrical components method is seem to be an efficient method compared with EMTP/ATP method. Where, the calculation of the turn-to-earth fault current by using EMTP/ATP program, at different percentages of turn-to-earth faulted winding, is time consuming.

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8 Appendix

See Table 2.

Table 2 Modelled transformers details

(1) Transformer: 40 MVA, three-phase three-winding transformer's data	
power rating	40/40/15 MVA
voltage rating	220/70/11.5 kV
frequency	50 Hz
three-legged core type	
vector group	YNyn0d11
open test data	
excitation losses	30.45 kW
excitation voltage	11.5 kV
excitation current	5.44 A
SC test data	
ZSC HL = 0.1449 pu based on 40 MVA	
SC losses =	203.48 kW
ZSC HT = 0.0761 pu based on 15 MVA	
SC losses =	46.02 kW
ZSC LT = 0.01920 pu based on 15 MVA	
SC losses =	37.41 kW
(2) Transformer: 300 MVA, three-phase three-winding transformer's data	
power rating	300/300/76 MVA
voltage rating	230/109.8/50 kV
frequency	50 Hz
three-legged core type	
vector group	YNyn0d11
open test data	
excitation losses	135.75 kW
excitation current	0.428% based on 300 MVA
SC test data	
ZSC HL = .0874 pu based on 300 MVA	
SC losses	903.56 kW
ZSC HT = 0.0868 pu based on 76 MVA	
SC losses	85.738 kW
ZSC LT = 0.0531 pu based on 76 MVA	
SC losses	98.86 kW

IET Gener. Transm. Distrib., 2013, Vol. 7, Iss. 7, pp. 709–716
doi: 10.1049/iet-gtd.2012.0724