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# HARMONIC VOLTAGES AND CURRENTS IN Y- AND DELTA-CONNECTED TRANSFORMERS

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ABSTRACT OF PAPER

The paper reviews the conditions under which triple harmonic voltages and currents are produced in Y-and delta-connected transformers. These voltages are produced by hysteresis in the core. In a single-phase transformer, increase of series resistance tends to suppress the current harmonic and produce the voltage harmonic. In three-phase transformers, a Y connection suppresses the current harmonic and allows the full flux and voltage harmonics to appear. Delta connection provides a closed path for the current harmonic, and suppresses the triple voltage.

A case is cited where a Y-connected auto-transformer was used to step up from 6600 to 12,000 volts at a substation. The neutral was not grounded, and trouble resulted due to partial resonance at triple frequency between line capacity and transformer reactance. The paper shows that, although not generally recognized, a triple component can exist in the line-to-line e.m.f. wave of a three-phase system. This is possible in a case where a two-to-three-phase transformation is used, and when the e.m.f. wave of the two-phase generator contains a triple harmonic. Vector diagrams and curves are given illustrating this possible effect.

IN CONSIDERING the relative advantages and disadvantages of Y and delta connections of transformer windings, it is necessary to pay some attention to the production of harmonic voltages and currents occuring in such windings due to hysteresis in the core. This has been treated by several since the present writer first drew attention to the effect,[1] but it may not be out of place in the present discussion to review briefly the conditions under which such harmonics become noticeable, and to point out a further possible case which, so far as the writer is aware, has escaped notice.

If we take the case of a transformer winding connected to an a-c. source of supply, we find that though the e.m.f. wave may be sine-shaped, the current wave necessary to produce the sine flux wave contains harmonics, notably a third and a fifth, which are produced by hysteresis and by the variation of permeability of the iron. These current harmonics may be regarded as in-

1. See the *Electrician* for 10th November 1905 and 5th January 1906.

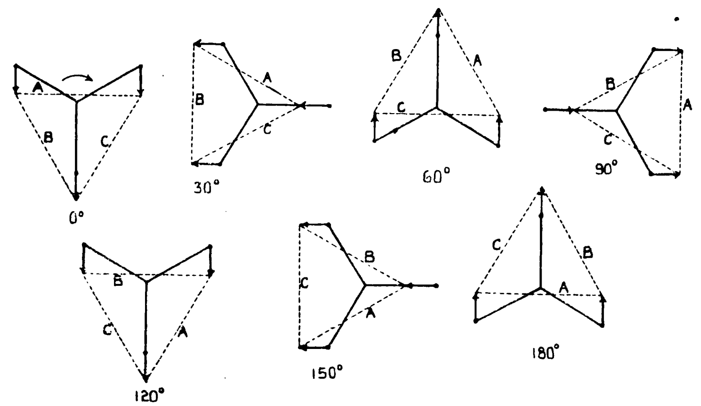
duced in the circuit by harmonic flux variations arising in the core. In such a case, however, the current harmonics produced "wipe out" almost completely the flux harmonic, because the resistance of the circuit is usually low. Hence the value of flux harmonic actually existing is only that sufficient to induce a voltage in the coil equal to $i \times r$, where $i$ is the harmonic current, and $r$ the resistance of the winding. By increasing the resistance, however, the flux and voltage harmonics become very noticeable, and a distorted wave form appears across the coil terminals. (Incidentally, this has an important bearing upon the use of series resistance for varying the voltage of an insulation—testing transformer, which sometimes results in the production of a high peak voltage. One remedy is to shunt a resistance across the transformer terminals, which allows of the use of a lower series resistance and therefore, reduced distortion). Without increasing the resistance, however, there is another way by which the current harmonic may be suppressed, viz., by the use of a star-connection of circuits, using the term "star" in its general senses implying $n$ circuits for an $n$-phase system. A little consideration shows that the $n$th harmonic of e.m.f. induced by the iron is directed in each circuit simultaneously towards or away from the neutral point, and that therefore no current of this frequency can flow. Hence, with a three-phase Y connection, the triple component of the magnetizing current is eliminated, and the full triple frequency flux variation occurs, producing a distorted wave form of high peak value, across each winding. With a Y-Y connection of transformers, particularly with single-phase transformers or shell type three-phase transformers, the full flux variation occurs. In the case of one set of windings, either primary or secondary, being delta connected, the flux variation is almost wholly suppressed by the circulation of the triple-frequency current in the closed triangle. From this point of view, therefore, a delta connection on either primary or secondary is desirable, as (1) It avoids the extra insulation stress due to the higher peak voltage, and (2) In the case of a transmission line with grounded neutral, it prevents triple-frequency capacity current in the ground wire, which, in the case of long transmission and high voltage, may reach a high value. In core-type three-phase transformers, the triple-frequency voltage is not marked, as the magnetic circuit of the flux harmonic is a partly open one, the path being up the three limbs in parallel and back through the air or tank. Oscillograms showing the conditions obtained in various cases were presented in an interesting paper by Mr. J. J. Frank, before the A. I. E. E. in May 1910. A case where trouble occurred which could only be ascribed to the above effects was brought to the writer's notice some time ago. The conditions were as follows:

From the power house, a five-mile 6600-volt line ran to a substation, where the voltage was raised to 12,000 by two Y-connected auto-transformers of 600 kv-a. capacity, with insulated neutrals. The 12,000-volt line supplied a substation five miles away, where step-down transformers were installed. Trouble was experienced due to the gap arresters on the 12,000-volt line continually sparking to ground. Also two breakdowns on terminal leads occurred, and there were other general indications of high voltage to ground, although the line to line voltage was correct. Voltage readings between the three lines and the auto-transformer neutral, on the 12,000-volt side, showed that with normal phase voltage on the transformer and the 12,000-volt line disconnected, the pressure between each 12,000 volt terminal and neutral was 17 per cent greater than the phase voltage $\div \sqrt{3}$. Then the 12,000-volt line was switched on, the voltage to earth rose further to 33 per cent above the normal value. Grounding the neutral of the auto-transformers cured the trouble, and reduced the voltage between the 12,000-volt lines and the ground to normal. It should be noted that the neutral of the generators was grounded through a low resistance at the power house. A ready explanation of these effects is afforded when the triple-voltage harmonics induced in the transformers are taken into account. With 12,000-volt lines disconnected, there is a 17 per cent increase of voltage to ground due to triple harmonic. This is aggravated, however, when the 12,000-volt line is connected, due to partial resonance, the capacity being in series with the high reactance of the auto-transformer winding. In this case the circuit is completed through the grounded generator neutral and the 6600-volt line. The effect of grounding is to connect the neutrals of generator and auto-transformer, and allow the triple component of current to be supplied, thus eliminating the voltage harmonic.

It has sometimes been stated[2] that no triple harmonic can exist in the e.m.f. between the lines of a three-phase system. It appears to the writer, however, that, though not often met with

2. See Steinmetz, Paper before N. E. L. A., June 1907., also Rhodes TRANS. A.I.E.E., 1910.

in practise, such an effect is quite possible. Consider the case of a two-phase generator supplying a three-phase line through Scott-connected transformers, and suppose that the generator




FIGS. 1-7—DIAGRAMS SHOWING TRIPLE HARMONIC IN THREE-PHASE SYSTEM AS PRODUCED BY THREE-PHASE GENERATOR OR BY Y-CONNECTED TRANSFORMERS—DOTTED TRIANGLE SHOWS LINE-TO-LINE E. M. FS.

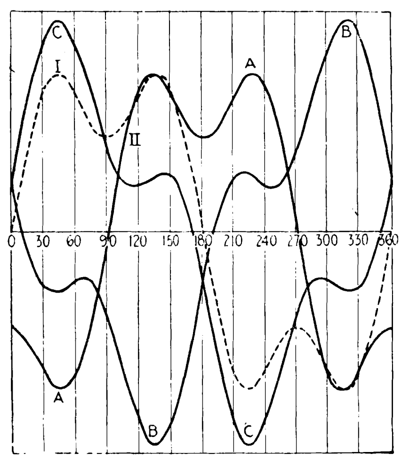
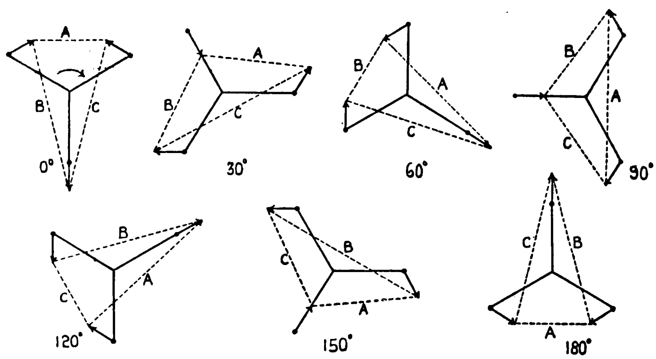



FIG. 8—WAVE SHAPES OF E. M. FS. BETWEEN LINES OF THREE-PHASE SYSTEM, SHOWING TRIPLE HARMONIC TRANSFORMED FROM TWO-PHASE SYSTEM.

Curves I and II two-phase e.m.fs. Curves A, B and C three-phase e.m.fs.
See Figs. 9-15 for vector diagram.

e.m.f. waves contain triple harmonics. As the main waves differ by 90 deg. in phase, the harmonics in the two phases will differ 3 × 90 deg. or 270 deg. in phase, *i.e.*, they will be in quadrature. Hence we have a two-phase supply at triple frequency impressed upon the primary, and this is transformed to a three-phase e.m.f. at triple frequency which appears between the lines as a superposed harmonic on the fundamental three-phase e.m.f. It is a curious fact that this effect cannot be obtained by direct three-phase generation, but, apparently, only by transformation from a two-phase supply. The accompanying diagrams will make clear the difference between the two cases, viz., (1) Triple harmonic due to three-phase generator, or Y connection of transformers. Harmonic appears only between line and neutral, and not between lines. Figs. 1 to 7 show the vectors at progressively varying phases, during one-third of a cycle.

(2) Triple harmonic impressed on system by two- to three-phase transformation. Harmonic appears both between lines,




FIGS. 9–15—VECTOR DIAGRAMS SHOWING TRIPLE HARMONIC TRANSFORMED INTO THREE-PHASE SYSTEM FROM TWO-PHASE SYSTEM. (SEE FIG. 8 FOR WAVE SHAPES.)

and between each line and neutral. Fig. 8 shows the three-phase waves produced between lines, assuming particular values for the amplitude and phase of the triple harmonic in the two-phase e.m.f. Curves I and II are the assumed two-phase waves, having exaggerated harmonics. $A$, $B$, and $C$ are the resulting three-phase line voltages. Note that these waves are dissimilar.

Figs. 9 to 15 give the vector diagrams corresponding to Fig. 8. The dotted line $A$, representing one of the three-phase line voltages, also represents by its projections on vertical and horizontal the two-phase voltages I and II respectively.

The writer is not aware of any previous reference to this possible effect, and it would be interesting to hear if such has been observed on any line employing two- to three-phase transformation.