MAT221

Discrete Mathematics – Fall 2017

Final Exam – Form A3

Time: 70 Minutes

Instructions:

- **The exam consists of 5 problems on 5 pages. Most problems are subdivided into sections like 1(a), 1(b), etc. The last problem is 5b. Make sure your exam is complete before you begin.**
- **Show all work in detail or your answer will not receive any credit. All answers without supporting work receive ZERO credit.**
- **Write neatly and box all answers. Write Question ID to your answer sheet.**
- **Include appropriate units on all questions that apply. When drawing graphs, make sure to clearly label axes, scale, and curves.**
- **Do not use your own scratch paper. You may ask for scratch paper at the front desk (or from your instructor if the exam is conducted in class).**
- **Turn off your handy phone. Leave all electronic devices in your backpack, and leave your backpack at the front of the room.**
- **No calculators with QWERTY keyboards or ones like the Casio FX-2, TI-89 or TI-92 that do symbolic algebra may be used.**
- **Add your class, student ID, name, signature and submit this form together with your answer sheet.**

Honor Statement:

By signing below you confirm that you have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the Faculty of Information Technology (FIT) and your instructor. Furthermore, you agree not to discuss this exam with anyone until the exam testing period is over. In addition, your calculator's program memory and menus may be checked at any time and cleared by any testing center proctor or FIT's instructor.

Class	Student ID	Student Name	Signature

Multiple Choice Questions (3 points)

1. (0.3 points) Suppose you are examining a conjecture of the form $\forall x(P(x) \wedge Q(x))$. To show that the conjecture is false, you MUST show which of the following?

   (a) There is a value x_1 such that $P(x_1)$ is false and a value x_2 such that $Q(x_2)$ is false.

   (b) There is a value x either $P(x)$ is false or $Q(x)$ is false.

   (c) For every choice of x, $P(x)$ and $Q(x)$ are both false.

   (d) For every choice of x, either $P(x)$ is false or $Q(x)$ is false.

2. (0.3 points) Suppose that $P(n)$ is the statement "$n + 1 = n + 2$". What is wrong with following "proof" that the statement $P(n)$ is true for all nonnegative integer n.

   You assume that $P(k)$ is true for some positive integer k, that is, that $k+1 = k+2$. Then you add 1 to both sides of this equation to obtain $k + 2 = k + 3$; therefore $P(k + 1)$ is true. By the principle of mathematical induction $P(n)$ is true for all nonnegative integers n.

   (a) There is nothing wrong with this proof.

   (b) The proof is incorrect because the statement used in the inductive hypothesis is incorrect.

   (c) The proof is incorrect because there is no basic step.

   (d) The proof is incorrect because you cannot add one to both sides of the equation in the inductive step.

3. (0.3 points) In a graph with directed edges the ________ of a vertex v is the number of edges with v as their terminal vertex.

   (a) in-degree, (b) out-degree, (c) $deg^+(v)$

4. (0.3 points) Which of the following is the negation of the statement "I will watch TV or read a book, but not both"?

   (a) I will watch TV and read a book.

   (b) I will neither watch TV nor read a book.

   (c) Either I will watch TV and read a book, or I will do neither.

5. (0.3 points) Let G be a graph in which there are two designated vertices, one the *source* of all flow, and the other the *sink*, or recipient of all flow. At every other vertex, the amount of flow into the vertex equals the amount of flow out of the vertex. The flows are limited by weights, or ,~~on the~~ edges. The edges may be undirected or directed.

(a) quantity. (b) volume (c) capacities

6. (0.3 points) Let T be an ordered rooted tree with root r. If T consists only of r, then r is thetr~~aversal o~~f T. Otherwise, suppose that $T_1, T_2, \ldots, T_n$ are the subtrees at r from left to right. The traversal begins by traversing T_1 in , then T_2 in , ..., then T_n ~~in~~ , and ends~~ by visiting~~ r.

(a) pre-order (b) in-order (c) post-order

7. (0.3 points) How many edges does a full binary tree with 1000 internal vertices have?

(a) 1000 (b) 2000 (c) 3000 (d) 4000

8. (0.3 points) Choose the best answer for the definition of Euler path in a graph G.

(a) An Euler path in a graph G is a simple path containing every edge of G.

(b) An Euler path in a graph G is a simple circuit that passes through every vertex of G.

(c) An Euler path in a graph G is a weak connected graph.

(d) Both (a) and (c) are true.

9. (0.3 points) Consider the following rule with domain **N**(the set of natural numbers {0,1,2,3,...}):

$$f(n) = \frac{2}{n + 0.5}$$

Which one of the following statement is correct?

(a) This rule does not define a function with codomain **R** (the set of real numbers).

(b) This is a function with codomain **R** because for each $n \in$ **N**, $f(n)$ is a single real number.

(c) This is not a function with codomain **R** because $f(1) = \frac{2}{1.5}$ is not an element of **N**.

(d) This is not a function with codomain **R** because $f(-2) = -\frac{2}{1.5}$ is not an element of **N**.

(e) This is a function because there are no values $n \in \mathbf{N}$ such that $f(n) = 0$.

10. (0.3 points) A graph with $V = \{1,2,3,4\}$ is described by $G = \left(a_{(1,2)}, b_{(1,2)}, c_{(1,4)}, d_{(2,3)}, e_{(3,4)}, f_{(3,4)}\right)$.

It has weights on its edges given by

$$\lambda = \begin{pmatrix} a & b & c & d & e & f \\ 3 & 2 & 1 & 2 & 4 & 2 \end{pmatrix}$$

How many minimum spanning trees does it have?

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

Short Answer Questions (7 points)

1. (1 point) Bob set up a secret sharing scheme where he want to distribute five shares to five persons such that any three or more persons can figure out the secret, but two or fewer persons cannot. He uses a primitive polynomial $P(x)$ in $GF(7)$ (where $P(0) = s$ is the secret) with the shares are $P(1) = 2$, $P(2) = 5$, $P(3) = 0$, $P(4) = 1$, and $P(5) = 1$.

   (a) Find $P(x)$ and the secret key s.

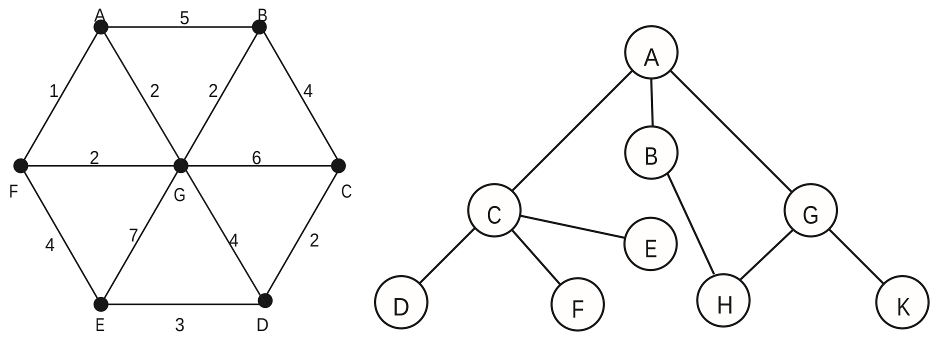
   (b) Prove that $P_{431}(x) \Leftrightarrow P_{254}(x)$. $P_{abc}(x)$ is the polynomial created by the combination of the shares of the person a^{th}, b^{th}, and c^{th}.

2. (2 points) Five data packages, {1,1,0,6,4}, are received through a transmission line. Given that one package has been changed. If the data is generated from a primitive polynomial over $GF(7)$, then find and correct the error package? <u>Note:</u> *Student must use the inverse matrix to solve the problem*.

$$A^{-1} = \begin{pmatrix} 1/2 & -3/10 & -7/20 & -2/5 & 11/20 \\ 1/6 & 1/6 & -5/12 & -1/3 & 5/12 \\ -3/2 & 1/2 & 5/4 & 1 & -5/4 \\ 7/6 & -1/30 & 11/60 & 1/15 & -23/60 \\ 2/3 & -1/3 & -2/3 & -1/3 & 2/3 \end{pmatrix}.$$

3. (1 point) Find a minimum spanning tree (MST) for the given weighted graph provided in Figure 1(a) by using Prim's algorithm.

4. (1 point) Using Breadth-First-Search's algorithm, show visiting process and the final level graph with starting vertex is A in the Figure 1(b).



(a)PrimAlgorithm (b)BFSAlgorithm

Figure 1: Graphs

5. (2 points) Given a data source of 6 characters A, B, C, D, E, F with probability of appearance as Table 1:

A	B	C	D	E	F
0.11	0.29	0.22	0.15	0.05	0.18

Table 1: Probabilities

(a) (1 point) Encode the source and find the codewords for those characters. Show the Huffman tree.

(b) (1 point) Calculate the average length of the codewords ($\overline{L}$) and the entropy (H) of the source by the following equations

$$\overline{L} = \sum_{i=1}^{m} L_i \times p_i; \;\; H = \sum_{i=1}^{m} p_i \times \log_2\left(\frac{1}{p_i}\right)$$

where m is number of different characters, L_i and p_i are length and probability of the ith codeword.