MAT221
Discrete Mathematics – Fall 2017
Final Exam – Form A1

Time: 70 Minutes

Instructions:

- **The exam consists of 5 problems on 4 pages. Most problems are subdivided into sections like 1(a), 1(b), etc. The last problem is 5b. Make sure your exam is complete before you begin.**
- **Show all work in detail or your answer will not receive any credit. All answers without supporting work receive ZERO credit.**
- **Write neatly and box all answers. Write Question ID to your answer sheet.**
- **Include appropriate units on all questions that apply. When drawing graphs, make sure to clearly label axes, scale, and curves.**
- **Do not use your own scratch paper. You may ask for scratch paper at the front desk (or from your instructor if the exam is conducted in class).**
- **Turn off your handy phone. Leave all electronic devices in your backpack, and leave your backpack at the front of the room.**
- **No calculators with QWERTY keyboards or ones like the Casio FX-2, TI-89 or TI-92 that do symbolic algebra may be used.**
- **Add your class, student ID, name, signature and submit this form together with your answer sheet.**

Honor Statement:
By signing below you confirm that you have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the Faculty of Information Technology (FIT) and your instructor. Furthermore, you agree not to discuss this exam with anyone until the exam testing period is over. In addition, your calculator's program memory and menus may be checked at any time and cleared by any testing center proctor or FIT's instructor.

Class	Student ID	Student Name	Signature

Multiple Choice Questions (3 points)

1. (0.3 points) Let $G = (V,E)$ be any bipartite graph with $V = V_1 \cup V_2$. A subset of edges M contained in E is called a ________ matching if every vertex in V is contained in exactly one edge of M.

   (a) perfect. (c) correct.

   (b) exact. (d) simple.

2. (0.3 points) The ________ of a vertex in an undirected graph is the number of edges incident with it, except that a loop at a vertex contributes twice to the ________ of that vertex.

   (a) degree (b) order (c) rank

3. (0.3 points) The maximum flow from vertex s to vertex t 6= s in a directed graph G with capacities on its edges is less than or equal to the capacity of any ________ $(A,V(G)-A)$ having $s \in A$ and t /∈ A.

   (a) cut. (c) balance set.

   (b) subset. (d) balance graph.

4. (0.3 points) A clique in a graph G is a ________ of G that is a complete graph.

   (a) planar graph (c) subgraph.

   (b) multigraph. (d) pseudograph.

5. (0.3 points) A ________ is a sequence of edges that begins at a vertex of a graph and travels from vertex to vertex along edges of the graph.

   (a) trail (c) walk.

   (b) path. (d) circuit.

6. (0.3 points) ________ paths and circuits can be used to solve practical problems. For example, many applications ask for a path or circuit that visits each road intersection in a city, each place pipelines intersect in a utility grid, or each node in a

communications network exactly once. Finding a ________ path or circuit in the appropriate graph model can solve such problems. The famous traveling salesperson problem or TSP (also known in older literature as the traveling salesman problem) asks for the shortest route a traveling salesperson should take to visit a set of cities. This problem reduces to finding a ________ circuit in a complete graph such that the total weight of its edges is as small as possible.

(a) Hamilton. (c) simple.

(b) Euler. (d) complex.

7. (0.3 points) A ________ is a connected undirected graph with no simple circuits.

(a) trail. (c) path.

(b) tree. (d) binary tree.

8. (0.3 points) Let T be an ordered rooted tree with root r. If T consists only of r, then r is the ________ traversal of T. Otherwise, suppose that $T_1, T_2, \dots, T_n$ are the subtrees at r from left to right. The ________ traversal begins by traversing T_1 in , th~~en visiting~~g r. It continues by traversing T_2 in , then T_3 in , ~~..., and~~ finally ________ T_n in ________.

(a) pre-order (b) in-order (c) post-order

9. (0.3 points) Let G be a simple graph. A ________ of G is a subgraph of G that is a tree containing every vertex of G.

(a) Steiner tree (c) spanning tree

(b) Red-black tree (d) balanced tree

10. (0.3 points) A relation on a set A is called an equivalence relation if it is ________.

(a) symmetric,transitive (c) anti-symmetric

(b) reflexive, symmetric, and transitive (d) Transitive

Short Answer Questions (7 points)

1. (1 point) Bob set up a secret sharing scheme where he want to distribute five shares to five persons such that any three or more persons can figure out the secret, but two or fewer persons cannot. He uses a primitive polynomial $P(x)$ in $GF(7)$ (where $P(0) = s$ is the secret) with the shares are $P(1) = 1$, $P(2) = 1$, $P(3) = 5$, $P(4) = 6$, and $P(5) = 4$.

   (a) Find $P(x)$ and the secret key s.

   (b) Prove that $P_{123}(x) \Leftrightarrow P_{345}(x)$. $P_{abc}(x)$ is the polynomial created by the combination of the shares of the person a^{th}, b^{th}, and c^{th}.

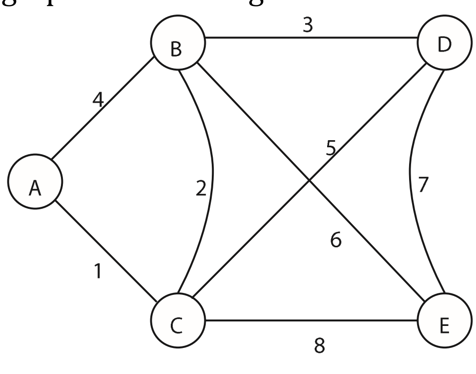
2. (2 point) Five data packages, {2,2,0,1,1}, are received through a transmission line. Given that one package has been changed. If the data is generated from a primitive

polynomial over $GF(7)$, then find and correct the error package? <u>*Note:*</u> *Student must use the inverse matrix to solve the problem*.

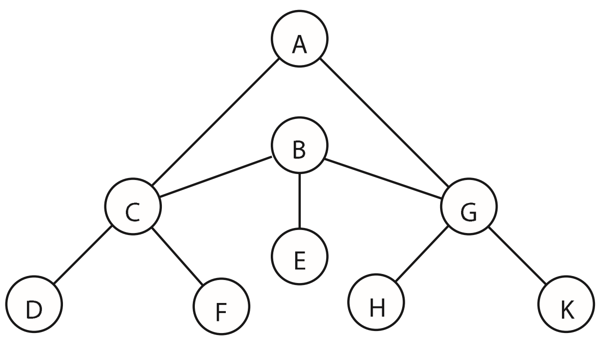
$$A^{-1} = \begin{pmatrix} 2/15 & -7/60 & 1/60 & -13/60 & 11/60 \\ 1/3 & 5/12 & 1/12 & -1/12 & -1/12 \\ 0 & -1/4 & -1/4 & 1/4 & 1/4 \\ -2/15 & 37/60 & 89/60 & 43/60 & -101/60 \\ 2/3 & -1/3 & -2/3 & -1/3 & 2/3 \end{pmatrix}.$$

3. (1 point) Find the shortest path from A to all other vertices in the Figure 1(a) by using Dijkstra's algorithm.

4. (1 point) Using Depth-First-Search's algorithm, show visiting process and the final level graph with starting vertex is A in the Figure 1(b).



(a)Dijkstra'sAlgorithm



(b)DFSAlgorithm

Figure 1: Graphs

5. (2 point) Given a data source of 6 characters A, B, C, D, E, F with probability of appearance as Table 1:

A	B	C	D	E	F
0.18	0.23	0.30	0.17	0.09	0.03

Table 1: Probabilities

   (a) (1 point) Encode the source and find the codewords for those characters. Show the final Huffman tree.

   (b) (1 point) If the length of the source is 1000, calculate C and S by the following equations

$$\text{Compression Ratio} : C = \frac{\text{Uncomressed Size}}{\text{Comressed Size}} \tag{1}$$

$$\text{Space Saving} : S = 1 - \frac{\text{Comressed Size}}{\text{Uncomressed Size}} \tag{2}$$

<u>Note:</u> Uncompressed size of a string is the number of bits used to encode that string as an ASCII array.