MAT221

Discrete Mathematics – Fall 2017

Final Exam – Form A2

Time: 70 Minutes

Instructions:

- **The exam consists of 5 problems on 4 pages. Most problems are subdivided into sections like 1(a), 1(b), etc. The last problem is 5b. Make sure your exam is complete before you begin.**
- **Show all work in detail or your answer will not receive any credit. All answers without supporting work receive ZERO credit.**
- **Write neatly and box all answers. Write Question ID to your answer sheet.**
- **Include appropriate units on all questions that apply. When drawing graphs, make sure to clearly label axes, scale, and curves.**
- **Do not use your own scratch paper. You may ask for scratch paper at the front desk (or from your instructor if the exam is conducted in class).**
- **Turn off your handy phone. Leave all electronic devices in your backpack, and leave your backpack at the front of the room.**
- **No calculators with QWERTY keyboards or ones like the Casio FX-2, TI-89 or TI-92 that do symbolic algebra may be used.**
- **Add your class, student ID, name, signature and submit this form together with your answer sheet.**

Honor Statement:

By signing below you confirm that you have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the Faculty of Information Technology (FIT) and your instructor. Furthermore, you agree not to discuss this exam with anyone until the exam testing period is over. In addition, your calculator's program memory and menus may be checked at any time and cleared by any testing center proctor or FIT's instructor.

Class	Student ID	Student Name	Signature

Multiple Choice Questions (3 points)

1. (0.3 points) An ci~~rcuit in a~~ graph G is a simple circuit containing every edge of G. An path in G is a si~~mple pat~~h containing every edge of G.
   (a) Hamilton
   (b) Euler
   (c) closed
   (d) open

2. (0.3 points) Let G be a simple graph. A a ________ tree of G is a subgraph of G that is tree containing every vertex of G.
   (a) spanning.
   (b) rooted.
   (c) balance.
   (d) simple.

3. (0.3 points) The simple graphs $G_1 = (V_1,E_1)$ and $G_2 = (V_2,E_2)$ are ________ if there exists a one-to-one and onto function f from V_1 to V_2 with the property that a and b are adjacent in G_1 if and only if $f(a)$ and $f(b)$ are adjacent in $G_2, \forall a,b \in V_1$.

   (a) equivalent (b) isomorphic (c) open

4. (0.3 points) Two vertices u and v in an undirected graph G are called ________ in G if u and v are endpoints of an edge e of G. Such an edge e is called incident with the vertices u and v and e is said to connect u and v.

   (a) adjacent (c) an ordered pair
   (b) incident (d) a pair

5. (0.3 points) An undirected graph has an ________ number of vertices of ________ degree.

   (a) even, odd (c) even, even
   (b) odd, even (d) odd, odd

6. (0.3 points) Suppose you need to prove that four statements, numbered 1, 2, 3, 4 are equivalent. Which of the following lists of implications will show that the four statements are equivalent?

(a) $1 \to 2, 3 \to 1, 3 \to 2, 4 \to 2, 3 \to 4, 4 \to 3$.

(b) $1 \to 2, 1 \to 3, 2 \to 4, 3 \to 4, 4 \to 1$.x

(c) $1 \to 2, 2 \to 1, 3 \to 4, 4 \to 3$.x

(d) $1 \to 3, 2 \to 1, 3 \to 2, 4 \to 1, 4 \to 2, 4 \to 3$.

7. (0.3 points) How many nonzero entries does the matrix representing the relation R on $A = \{1,2,3,...,100\}$ consisting of the first 100 positive integers have if R is $\{(a,b)|a = b + 1\}$.

(a) 4950 (b) 99 (c) 9900 (d) 100

8. (0.3 points) Let R be a set, with two operations addition $(a,b \mapsto a+b)$ and multiplication $(a,b \mapsto a \times b)$ are defined where $a,b \in R$. $(R,+,\times)$ will be a ring if the following holds:

(a) Closure, associate law, identity element, inverse element.

(b) Closure, associate law, commutative law, distributive law, additive identity element, additive inverse element.

(c) Closure, associative law, commutative law, distributive law, additive identity element, additive inverse element, multiplicative identity element, multiplicative inverse element.

(d) Closure, associative law, commutative law, distributive law, multiplicative identity element, multiplicative inverse element.

9. (0.3 points) Which of the following multiplication tables (Figure 1) defined on the set $G = \{a,b,c,d\}$ form a group? Support your answer in each case.

∘	a	b	c	d
a	a	c	d	a
b	b	b	c	d
c	c	d	a	b
d	d	a	b	c

(a)

∘	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d	d	c	b	a

(b)

∘	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

(c)

∘	a	b	c	d
a	a	b	c	d
b	b	a	c	d
c	c	b	a	d
d	d	d	b	c

(d)

Figure 1: Multiplication tables on the set G

10. (0.3 points) A relation R on a set A is called ________ if whenever $(a,b) \in R$ and $(b,c) \in R$, then $(a,c) \in R$, for all $a,b,c \in A$.

(a) transitive

(b) reflexive

(c) symmetric

(d) anti-symmetric

Short Answer Questions (7 points)

1. (1 point) Alice set up a secret sharing scheme where she want to distribute five shares to five persons such that any three or more persons can figure out the secret, but two or fewer persons cannot. She uses a primitive polynomial $P(x)$ in $GF(7)$ (where $P(0) = s$ is the secret) with the shares are $P(1) = 6$, $P(2) = 5$, $P(3) = 5$, $P(4) = 6$, and $P(5) = 1$.

   (a) Find $P(x)$ and the secret key s.

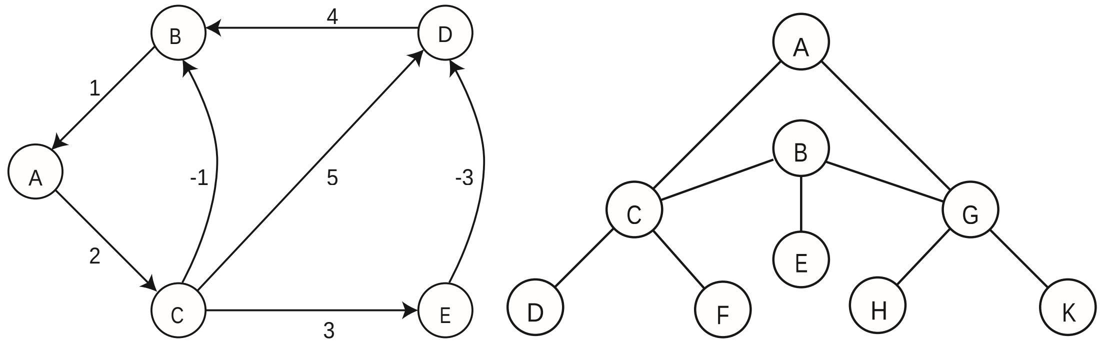
   (b) Prove that $P_{145}(x) \Leftrightarrow P_{523}(x)$. $P_{abc}(x)$ is the polynomial created by the combination of the shares of the person a^{th}, b^{th}, and c^{th}.

2. (2 point) Five data packages, {1,0,5,3,5}, are received through a transmission line. Given that one package has been changed. If the data is generated from a primitive polynomial over $GF(7)$, then find and correct the error package? <u>*Note:*</u> *Student must use the inverse matrix to solve the problem*.

$$A^{-1} = \begin{pmatrix} 11/10 & -3/5 & -19/20 & -7/10 & 23/20 \\ -3/2 & 1 & 5/4 & 1/2 & -5/4 \\ 3/2 & -1 & -7/4 & -1/2 & 7/4 \\ 19/10 & -2/5 & -11/20 & -3/10 & 7/20 \\ -2 & 1 & 2 & 1 & -2 \end{pmatrix}.$$

3. (1 point) Find the shortest path from A to all other vertices in the Figure 2(a) by using Bellman-Ford's algorithm.

4. (1 point) Using Breadth-First-Search's algorithm, show visiting process and the final level graph with starting vertex is A in the Figure 2(b).



(a)Bellman-FordAlgorithm (b)BFSAlgorithm

Figure 2: Graphs

5. (2 points) Given a data source of 6 characters A, B, C, D, E, F with probability of appearance as Table 1:

A	B	C	D	E	F
0.21	0.16	0.19	0.30	0.09	0.05

Table 1: Probabilities

(a) (1 point) Encode the source and find the codewords for those characters. Show the final Huffman tree.

(b) (1 point) If the length of the source is 1000, calculate C and S by the following equations

$$\text{Compression Ratio} : C = \frac{\text{Uncomressed Size}}{\text{Comressed Size}} \quad (1)$$

$$\text{Space Saving} : S = 1 - \frac{\text{Comressed Size}}{\text{Uncomressed Size}} \quad (2)$$