

MAT221

Discrete Mathematics – Fall 2017

Final Exam – Form A4

Time: 70 Minutes

Instructions:

- The exam consists of 5 problems on 4 pages. Most problems are subdivided into sections like 1(a), 1(b), etc. The last problem is 5b. Make sure your exam is complete before you begin.
- Show all work in detail or your answer will not receive any credit. All answers without supporting work receive ZERO credit.
- Write neatly and box all answers. Write Question ID to your answer sheet.
- Include appropriate units on all questions that apply. When drawing graphs, make sure to clearly label axes, scale, and curves.
- Do not use your own scratch paper. You may ask for scratch paper at the front desk (or from your instructor if the exam is conducted in class).
- Turn off your handy phone. Leave all electronic devices in your backpack, and leave your backpack at the front of the room.
- No calculators with QWERTY keyboards or ones like the Casio FX-2, TI-89 or TI-92 that do symbolic algebra may be used.
- Add your class, student ID, name, signature and submit this form together with your answer sheet.

Honor Statement:

By signing below you confirm that you have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the Faculty of Information Technology (FIT) and your instructor. Furthermore, you agree not to discuss this exam with anyone until the exam testing period is over. In addition, your calculator's program memory and menus may be checked at any time and cleared by any testing center proctor or FIT's instructor.

Class

Student ID

Student Name

Signature

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Multiple Choice Questions (3 points)

1. (0.3 points) The implication $q \rightarrow \neg p$ is true for all possible assignments of truth values to p and q except for which assignment?
(a) **p true, q true.** (b) p true, q false. (c) p false, q true. (d) p false, q false.
2. (0.3 points) In a(n) _____ cipher, the same key is used by both the sender and receiver.
(a) **symmetric-key** (c) either (a) or (b)
(b) asymmetric-key (d) neither (a) nor (b)
3. (0.3 points) Suppose $f: \mathbf{R} \rightarrow \mathbf{R}$ has the rule $f(x) = \left\lfloor \frac{x}{2} \right\rfloor$ and suppose $g: \mathbf{R} \rightarrow \mathbf{R}$ has the rule $g(x) = \left\lfloor \frac{5 - 2x}{3} \right\rfloor$. Find $(f \circ g)(5)$.
(a) 1. (b) -1 (c) 0. (d) **none of these.**
4. (0.3 points) Let G be a simple graph. A spanning tree of G is a subgraph of G that is a tree containing _____ vertex of G .
(a) **every** (c) one
(b) some (d) only
5. (0.3 points) A relation R on a set A is called _____ if $(a, a) \in R$ for every element $a \in A$.
(a) **reflexive** (c) transitive
(b) symmetric (d) anti-symmetric
6. (0.3 points) Given $n + 1$ data pairs, a unique polynomial of degree _____ passes through the $n + 1$ data points.
(a) $n + 1$ (c) **n or less**
(b) n (d) $n + 1$ or less

(a)

7. (0.3 points) A graph with $V = \{1,2,3,4\}$ is described by $G = (a_{(1,2)}, b_{(1,2)}, c_{(1,4)}, d_{(2,3)}, e_{(3,4)}, f_{(3,4)})$.

How many Hamiltonian cycles does it have?

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- 1 (b) 2 (c) 4 (d) 16 (e) 32

8. (0.3 points) Let R be a set, with two operations addition $(a,b \mapsto a+b)$ and multiplication $(a,b \mapsto a \times b)$ are defined where $a,b \in R$. $(R,+, \times)$ will be a ring if the following holds:

(a) Closure, associate law, identity element, inverse element.

(b) Closure, associate law, commutative law, distributive law, additive identity element, additive inverse element.

(c) Closure, associative law, commutative law, distributive law, additive identity element, additive inverse element, multiplicative identity element, multiplicative inverse element.

(d) Closure, associative law, commutative law, distributive law, multiplicative identity element, multiplicative inverse element.

9. (0.3 points) Which of the following multiplication tables (Figure 1) defined on the set $G = \{a,b,c,d\}$ form a group? Support your answer in each case.

\circ	a	b	c	d
a	a	c	d	a
b	b	b	c	d
c	c	d	a	b
d	d	a	b	c

(a)

\circ	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d	d	c	b	a

(b)

\circ	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

(c)

\circ	a	b	c	d
a	a	b	c	d
b	b	a	c	d
c	c	b	a	d
d	d	d	b	c

(d)

Figure 1: Multiplication tables on the set G

10. (0.3 points) Determine whether the relations represented by the matrix A is reflexive, irreflexive, symmetric, anti-symmetric, and/or transitive.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(a) Reflexive, symmetric, transitive

(b) Symmetric, transitive

(c) Anti-symmetric, transitive

(d) Transitive

Short Answer Questions (7 points)

1. (1 point) Alice set up a secret sharing scheme where she want to distribute five shares to five persons such that any three or more persons can figure out the secret, but two or fewer persons cannot. She uses a primitive polynomial $P(x)$ in $GF(7)$ (where $P(0) = s$ is the secret) with the shares are $P(1) = 1$, $P(2) = 0$, $P(3) = 5$, $P(4) = 2$, and $P(5) = 5$.

(a)

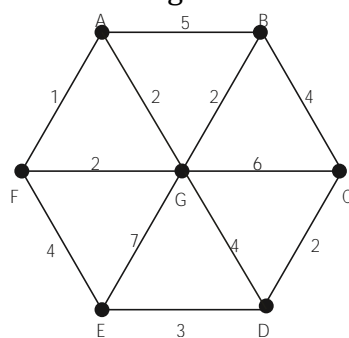
Find $P(x)$ and the secret key s .

(b) Prove that $P_{342}(x) \Leftrightarrow P_{315}(x)$. $P_{abc}(x)$ is the polynomial created by the combination of the shares of the person a^{th} , b^{th} , and c^{th} .

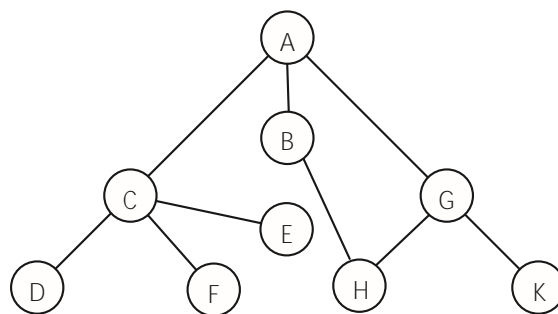
2. (2 points) Five data packages, $\{6,0,5,6,1\}$, are received through a transmission line. Given that one package has been changed. If the data is generated from a primitive polynomial over $GF(7)$, then find and correct the error package? *Note: Student must use the inverse matrix to solve the problem.*

$$A^{-1} = \begin{bmatrix} -1/10 & 0 & 1/4 & -1/10 & -1/20 \\ 3/2 & 9/4 & 1 & -9/4 & 5/2 \\ -1/2 & -3/4 & 0 & -9/4 & 1/2 \\ 5/2 & -4 & -31/4 & -39/10 & 3/4 \\ 1/2 & 1/2 & 1 & 1/2 & 151/20 \end{bmatrix}$$

3. (1 point) Find a minimum spanning tree (MST) for the given weighted graph provided in Figure 2(a) by using Kruskal's algorithm.
4. (1 point) Using Depth-First-Search's algorithm, show visiting process and the final level graph with starting vertex is A in the Figure 2(b).



(a)KruskalAlgorithm



(b)DFSAlgorithm

Figure 2: Graphs

(a)

5. (2 points) Given a data source of 6 characters A, B, C, D, E, F with probability of appearance as Table 1:

A	B	C	D	E	F
0.23	0.25	0.17	0.09	0.21	0.05

Table 1: Probabilities

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(1 point) Encode the source and find the codewords for those characters. Show the Huffman tree.

- (b) (1 point) Calculate the average length of the codewords (\bar{L}) and the entropy (H) of the source by the following equations

$$\bar{L} = \sum_{i=1}^m L_i \times p_i; \quad H = - \sum_{i=1}^m p_i \times \log_2 \left(\frac{1}{p_i} \right)$$

where m is number of different characters, L_i and p_i are length and probability of the i th codeword.