Д/з 3.

1) а) $\lim\limits_{n\to\infty} \frac{(23-2n^2)(3n+17)^2}{4n^6+n-1} = \left(\frac{\infty}{\infty}\right) = \frac{-2\cdot 3^2}{4} = -\frac{9}{2}$

б) $\lim\limits_{n\to\infty} \frac{(97-2n)^3}{2n(3n^2+15)+8n} = \left(\frac{\infty}{\infty}\right) = \frac{(-2)^3}{2\cdot 3} = -\frac{4}{3}$

в) $\lim\limits_{n\to\infty} \frac{2n^3+13n(n+18)}{(27-n)(2n+19)^2} = \left(\frac{\infty}{\infty}\right) = \frac{2}{-1\cdot 2^2} = -\frac{1}{2}$

г) $\lim\limits_{n\to\infty} \left(\sqrt{n^2+1}-n\right) = (\infty-\infty) =$

$\lim\limits_{n\to\infty} \left(\sqrt{n^2+1}-n\right)\cdot\frac{\sqrt{n^2+1}+n}{\sqrt{n^2+1}+n} = \lim\limits_{n\to\infty}\frac{n^2+1-n^2}{\sqrt{n^2+1}+n} =$

$= \lim\limits_{n\to\infty}\frac{1}{\sqrt{n^2+1}+n} = 0$

g) $\lim_{n\to\infty} \frac{(-4)^n + 5\cdot 7^n}{(-4)^{n-1} + 7^{n+2}} = \left(\frac{\infty}{\infty}\right) =$

$\lim_{n\to\infty} = \frac{(-4)^n + 5\cdot 7^n}{\left(-\frac{1}{4}\right)\cdot(-4)^n + 49\cdot 7^n} =$

$\lim_{n\to\infty} \frac{7^n\cdot\left(\left(-\frac{4}{7}\right)^n + 5\right)}{7^n\cdot\left(\left(-\frac{1}{4}\right)\cdot\left(-\frac{4}{7}\right)^n + 49\right)} = \frac{5}{49}$

2) $1 = \frac{1}{2} + \frac{1}{2}$

$\frac{1}{2} = \frac{1}{2\cdot 3} + \frac{1}{3} \Rightarrow 1 = \frac{1}{2} + \frac{1}{6} + \frac{1}{3}$

4) $a_n = \frac{\sin 1}{2} + \frac{\sin 2}{2^2} + \frac{\sin 3}{2^3} + \dots \frac{\sin n}{2^n}$

$a_{n+k} = \frac{\sin 1}{2} + \frac{\sin 2}{2^2} \dots \frac{\sin(n+k)}{2^{n+k}}$

$|a_n - a_{n+k}| = \left|\frac{\sin(n+1)}{2^{n+1}} + \frac{\sin(n+2)}{2^{n+2}} \dots + \frac{\sin(n+k)}{2^{n+k}}\right| \leq$

$\leq \frac{|\sin(n+1)|}{2^{n+1}} + \frac{|\sin(n+2)|}{2^{n+2}} \dots \frac{|\sin(n+k)|}{2^{n+k}} \leq$

$\leq \frac{1}{2^{n+1}} + \frac{1}{2^{n+2}} + \dots + \frac{1}{2^{n+k}} = \frac{1}{2^n}\left(\frac{1}{2} + \frac{1}{4} \dots \frac{1}{2^k}\right) =$

$= \frac{1}{2}\cdot\frac{\frac{1}{2}\left(1 - \frac{1}{2^k}\right)}{1 - \frac{1}{2}} = \frac{1}{2^n}\left(1 - \frac{1}{2^k}\right) < \frac{1}{2^n} < \frac{1}{2^{N(\varepsilon)}} = \varepsilon$

$\varepsilon = 10^{-7}$

$N(10^{-7}) = -\log_2 10^{-7} = 7 \cdot \log_2 10 \approx 23.25$

В кач-ве предела помпено взять 24 и далее.