

Задача 33.

| 0. | угол | sin | cos | tg |
|----|------|--------------|--------------|--------------|
| | 30° | 1/2 | $\sqrt{3}/2$ | $\sqrt{3}/3$ |
| | 45° | $\sqrt{2}/2$ | $\sqrt{2}/2$ | 1 |
| | 60° | $\sqrt{3}/2$ | 1/2 | $\sqrt{3}$ |
| | 90° | 1 | 0 | — |
| | 180° | 0 | -1 | 0 |

2. Линейное преобр. на плоскости

$$X = a_{11}x + a_{12}y + a_{13}$$

$$Y = a_{21}x + a_{22}y + a_{23}$$

Ортонормированные оси:

$$a_{11}^2 + a_{21}^2 = 1 \quad a_{12}^2 + a_{22}^2 = 1 \quad a_{11}a_{12} + a_{21}a_{22} = 0$$

$$A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$$

Пусть $M_1(x_1, y_1)$ и $M_2(x_2, y_2)$ — произвольные точки в $M_1'(x'_1, y'_1)$ и $M_2'(x'_2, y'_2) \Rightarrow M_1, M_2 = M_1', M_2'$

$$|M_1' M_2'|^2 = (x'_2 - x'_1)^2 + (y'_2 - y'_1)^2, \text{ где:}$$

$$x'_2 = a_{11}x_2 + a_{12}y_2 + a_{13}$$

$$x'_1 = a_{11}x_1 + a_{12}y_1 + a_{13}$$

$$y'_2 = a_{21}x_2 + a_{22}y_2 + a_{23}$$

$$y'_1 = a_{21}x_1 + a_{22}y_1 + a_{23}$$

$$\begin{aligned}
 & (a_{11}x_2 + a_{12}y_2 - a_{11}x_1 - a_{12}y_1)^2 + (a_{21}x_2 + a_{22}y_2 - a_{21}x_1 - a_{22}y_1)^2 = \\
 & (a_{11}(x_2 - x_1) + a_{12}(y_2 - y_1))^2 + (a_{21}(x_2 - x_1) + a_{22}(y_2 - y_1))^2 = \\
 & a_{11}^2(x_2 - x_1)^2 + a_{12}^2(y_2 - y_1)^2 + 2a_{11}a_{12}(x_2 - x_1)(y_2 - y_1) + \\
 & a_{21}^2(x_2 - x_1)^2 + a_{22}^2(y_2 - y_1)^2 + 2a_{21}a_{22}(x_2 - x_1)(y_2 - y_1) = \\
 & (a_{11}^2 + a_{21}^2)(x_2 - x_1)^2 + (a_{12}^2 + a_{22}^2)(y_2 - y_1)^2 + 2(a_{11}a_{12} + a_{21}a_{22})(x_2 - x_1)(y_2 - y_1) = \\
 & (x_2 - x_1)^2 + (y_2 - y_1)^2 = |M_1 M_2|^2 \\
 & |M_1' M_2'|^2 = |M_1 M_2|^2
 \end{aligned}$$

4. a) $y = x^2 - 1$

$$\exp(x) + x(1-y) = 1 \Rightarrow y = \frac{\exp(x) + x - 1}{x}$$

б) $y = x^2 - 1$

$$\exp(x) + x(1-y) > 1$$

$$y = x^2 - 1$$

$$y = -\frac{1}{x} + 2$$

В нрор.