COMP 3008 Homework #1

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1/15/2021

1.Write the following statement in symbolic form: "Every odd integer larger than 2 can be written as a sum of two primes"

If a number is an integer and an odd number larger than 2, then it can be written as a sum of two prime numbers

$\forall n \in Z, n= 2k+1 \wedge n>2 \rightarrow n= Z' + Z'$

(**Z** is the set of integers)

(**Z'** represents the set of **prime** integers)

2. Negate this sentence: "If a novel is longer than 300 pages, then someone in the class will not read it."

p= novel > 300 pages

q= someone in class will read it

$\neg$ q= someone in class will not read it

Current statement "p $\rightarrow\neg$ q"

p	q	$\neg$ q	p $\rightarrow$ q	p $\rightarrow\neg$ q	**$\neg$(p $\rightarrow\neg$ q)**	**p$\wedge$ q**
T	T	F	T	F	**T**	**T**
T	F	T	F	T	**F**	**F**
F	T	F	T	T	**F**	**F**
F	F	T	T	T	**F**	**F**

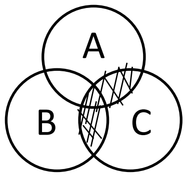
*bold columns are logically equivalent

Negation of above statement: **"A novel is greater than 300 pages AND someone in class will read it"**

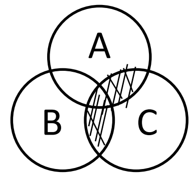
3. Use set-builder notation to give a possible definition of the set {3,10,29,66,127,...}

$S=\{ x= k^3+2 : k \in Z^+\}$

4. If A, B, and C are sets, determine if the statement $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ is necessarily true.



$(A \cup B) \cap C$



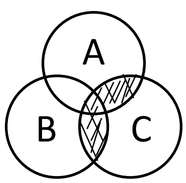
$(A \cap C) \cup (B \cap C)$

A	B	C	A∪B	**(A∪B) ∩ C**	(A ∩ C)	(B ∩ C)	**(A ∩ C) ∪ (B ∩ C)**
F	F	F	F	**F**	F	F	**F**
F	F	T	F	**F**	F	F	**F**
F	T	F	T	**F**	F	F	**F**
F	T	T	T	**T**	F	T	**T**
T	F	F	T	**F**	F	F	**F**
T	F	T	T	**T**	T	F	**T**
T	T	F	T	**F**	F	F	**F**
T	T	T	T	**T**	T	T	**T**

*bold columns are logically equivalent

$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ are logically equivalent given the truth table above.

5. Use set operations to define the set depicted in this Venn diagram.



$(A \cap C) \cup (B \cap C)$

A	B	C	**(A ∩ C) ∪ (B ∩ C)**
F	F	F	**F**
F	F	T	**F**
F	T	F	**F**
F	T	T	**T**
T	F	F	**F**
T	F	T	**T**
T	T	F	**F**
T	T	T	**T**

6. Negate the statement $\forall x \in S, (A(x) \vee B(x))$.

$\neg(\forall x \in S, (A(x) \vee B(x)) = \exists x$

$\neg(p \vee q) = \neg p \wedge \neg q$ (DeMorgan's Law)

$\exists x \in S, \neg A(x) \wedge \neg B(x)$

7. Show that the sum of two odd numbers is even.

Definition: an integer n is odd if n=2k+1 for some integer k

Proof: Let m, n be two odd integers

By definition, there exists integers a, b such that $m=2a+1$, $n=2b+1$

$m+n= (2a+1)+ (2b+1)$

$m+n= 2a+2b+2$

$m+n= 2(a+b+1)$

since $a+b \in Z$, the sum of m+n is even ■

8. Find the error in the proof below. If the statement is true, then provide a correct proof; otherwise exhibit a counter-example.

Show that the product of two odd numbers has a remainder of 1 when you divide it by 4. "Proof": Given two numbers a and b, since they are both odd, $a=2m+1$ and $b=2m+1$ for some integer m. Then $a*b= (2m+1)(2m+1)= 4m^2+4m+1= 4(m^2+m) +1$, so it has a remainder of 1 when divided by 4.

Error in the proof: "m" is given as a single variable and the proof calls for 2 unique variables (odd numbers).

This proof is not true, given the following counterexample:

Counterexample:

let a = 3 and b=1, both odd numbers by definition

$ab=3*1=3$

$ab/4=$ ¾, this example does not have a reminder of 1 when divided by 4

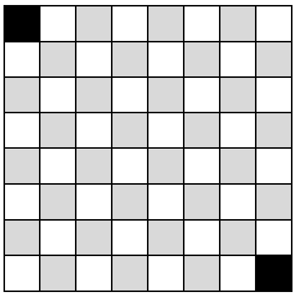
9. Suppose you have an unlimited number of 7¢ and 4¢ postage stamps. What are all the possible values of stamps you can create?

All possible values of stamps that can be created are represented by this set:

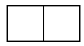
{{7¢}, {4¢}, {4¢, 7¢}, {Ø}}

10. Prove or disprove that an 8x8 board with opposite corners removed can be tiled with 1x2 (and 2x1) pieces.

Board:



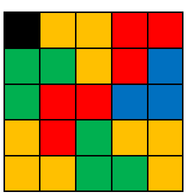
Tile shape:



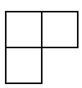
Color the 8x8 board as shown. The black squares represent the removed tiles. Notice that any 2x1 tile will cover 1 gray and 1 white square. If tiling were possible, there would have to be the same number of gray and white squares. Since there are more white than gray squares in this configuration, then no tiling is possible.

11. Prove or disprove that a 5x5 board with one corner removed can be tiled with pieces consisting of three squares in an L-shape.

Board:



Tile Shape:



Yes, a 5x5 board with one corner removed can be tiled with pieces consisting of three squares in an L-shape, as shown in picture.