COMP 3008 Homework #5 Krista Miller

2/16/2021

__Part 1__

1. Consider the following process. Starting with 1 pile of n stones, and the value x= 0, successively split a pile into two smaller piles until there are n piles with 1 stone each. Whenever a pile is split into smaller piles of size r and s, say, the value rs is added to x. For example, if you start with a pile of 4 stones, and split it into two piles of 2 stones, x=0 is incremented to x= 2(2)=4. Then each of these piles is split into two piles of 1 stone each. The result is to increment x= 4+1(1)+1(1)=6, the final value of x. Prove that for any positive integer n and any sequence of splits, x=n(n-1)/2.

   __Proof (by induction):__

   $$\sum_{x=0}^{n} x = \frac{n(n-1)}{2}$$

   __Base case:__

   For n=2 stones,

   1·1 stones= 1 ; $\frac{2(2-1)}{2} = \frac{2}{2}$=1, so the base case holds.

   __Inductive step:__

   Assume x= $\frac{n(n-1)}{2}$ is correct for all values of n

   x= $\frac{k(k-1)}{2} = \sum_{n=0}^{k-1} n$

   Notice that we want to show that this holds for all k+1: (assistance with this step during office hours with Jurgen 2/12/2021):

   $$\sum_{x=0}^{k+1} x = \frac{(k+1)((k+1)-1)}{2}$$

   $$\sum_{x=0}^{k} x + k + 1 = \frac{(k+1)k}{2}$$

   k+ $\frac{k(k-1)}{2} = \frac{k(k+1)}{2}$

   $\frac{2k}{2} + \frac{k^2-k}{2} = \frac{k^2+k}{2}$

   $\frac{k^2-k+2k}{2} = \frac{k^2+k}{2}$

   $$\frac{k^2+k}{2} = \frac{k^2+k}{2}$$

   This holds for k+1 cases, so by induction, any positive integer n and any sequence of splits, x=n(n-1)/2∎

The Fibonacci sequence is defined recursively as $a_1=a_2=1$, and $a_n=a_n-1+a_n-2$ for integers $n>2$. Prove that the formula: $b_n=\frac{1}{\sqrt{5}}(\frac{1+\sqrt{5}}{2})^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$ is an explicit formula for the Fibonacci sequence.

$a_n=a_{n-1}+a_{n-2}$ $\quad a_1=a_2=1$

$b_n=\frac{1}{\sqrt{5}}(\frac{1+\sqrt{5}}{2})^n - \frac{1}{\sqrt{5}}(\frac{1-\sqrt{5}}{2})^n$

$a_{n-1}+a_{n-2}=\frac{1}{\sqrt{5}}(\frac{1+\sqrt{5}}{2})^n - \frac{1}{\sqrt{5}}(\frac{1-\sqrt{5}}{2})^n$

Proof:

Base case: $a_3=2$

$a_2+a_1= 1+1=2$

$\frac{1}{\sqrt{5}}(\frac{1+\sqrt{5}}{2})^3 - \frac{1}{\sqrt{5}}(\frac{1-\sqrt{5}}{2})^3=2$

Inductive step: Assume this relation is true for all values of n from 3 to k-1. (Algebra for this step done with Jurgen in office hours 2/12/21).

$a_{k-1}\equiv a_{k-2}+a_{k-3}=\frac{1}{\sqrt{5}}(\frac{1+\sqrt{5}}{2})^{k-1} - \frac{1}{\sqrt{5}}(\frac{1-\sqrt{5}}{2})^{k-1}$

$a_{k-1}+a_{k-2} = \frac{1}{\sqrt{5}}(\frac{1+\sqrt{5}}{2})^k - \frac{1}{\sqrt{5}}(\frac{1-\sqrt{5}}{2})^k$

$$(\frac{1+\sqrt{5}}{2})^{k-1} - (\frac{1-\sqrt{5}}{2})^{k-1} + (\frac{1+\sqrt{5}}{2})^{k-2} - \left(\frac{1-\sqrt{5}}{2}\right)^{k-2} = (\frac{1+\sqrt{5}}{2})^k - \left(\frac{1-\sqrt{5}}{2}\right)^k$$

$$(\frac{1+\sqrt{5}}{2})^{k-1} + (\frac{1+\sqrt{5}}{2})^{k-2} - (\left(\frac{1-\sqrt{5}}{2}\right)^{k-1} + \left(\frac{1-\sqrt{5}}{2}\right)^{k-2}) = (\frac{1+\sqrt{5}}{2})^k - \left(\frac{1-\sqrt{5}}{2}\right)^k$$

$$\left(\frac{1+\sqrt{5}}{2}\right)^k (\left(\frac{2}{1+\sqrt{5}} + \left(\frac{4}{(1+\sqrt{5})^2}\right)\right) - ((\left(\frac{1-\sqrt{5}}{2}\right)^k (\left(\frac{2}{1-\sqrt{5}} + \left(\frac{4}{(1-\sqrt{5})^2}\right)\right) = (\frac{1+\sqrt{5}}{2})^k - \left(\frac{1-\sqrt{5}}{2}\right)^k$$

$$\left(\frac{1+\sqrt{5}}{2}\right)^k (\left(\frac{2}{1+\sqrt{5}} + \left(\frac{4}{(6+2\sqrt{5})}\right)\right) - ((\left(\frac{1-\sqrt{5}}{2}\right)^k (\left(\frac{2}{1-\sqrt{5}} + \left(\frac{4}{(6-2\sqrt{5})}\right)\right) = (\frac{1+\sqrt{5}}{2})^k - \left(\frac{1-\sqrt{5}}{2}\right)^k$$

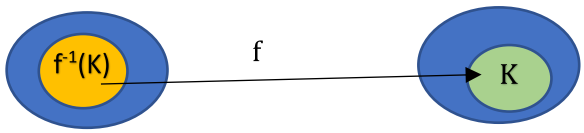
$$\left(\frac{1+\sqrt{5}}{2}\right)^k (\left(\frac{2(1+\sqrt{5})+4}{6+2\sqrt{5}}\right) - ((\left(\frac{1-\sqrt{5}}{2}\right)^k (\left(\frac{2(1-\sqrt{5})+4}{6-2\sqrt{5}}\right) = (\frac{1+\sqrt{5}}{2})^k - \left(\frac{1-\sqrt{5}}{2}\right)^k$$

$$\left(\frac{1+\sqrt{5}}{2}\right)^k (\left(\frac{6+2\sqrt{5}}{6+2\sqrt{5}}\right) - (\left(\frac{1-\sqrt{5}}{2}\right)^k (\left(\frac{6-2\sqrt{5}}{6-2\sqrt{5}}\right) = (\frac{1+\sqrt{5}}{2})^k - \left(\frac{1-\sqrt{5}}{2}\right)^k$$

$$\left(\frac{1+\sqrt{5}}{2}\right)^k (1) - \left(\frac{1-\sqrt{5}}{2}\right)^k (1) = (\frac{1+\sqrt{5}}{2})^k - \left(\frac{1-\sqrt{5}}{2}\right)^k$$

Therefore, $b_n=\frac{1}{\sqrt{5}}(\frac{1+\sqrt{5}}{2})^n - \frac{1}{\sqrt{5}}(\frac{1-\sqrt{5}}{2})^n$ is an explicit formula for the Fibonacci sequence ■

2. Given a function: X↦Y, suppose that B1 and B2 are both subsets of Y. If the following statement is true, prove it, and if it is not true, give a counter-example:
$f^{-1}(B_1 \cup B_2) \subseteq f^{-1}(B_1) \cup f^{-1}(B_2)$
idea: K is a subset of Y, $f^{-1}(k) = \{x \in X \mid f(x) \in k\}$
X= domain Y= codomain

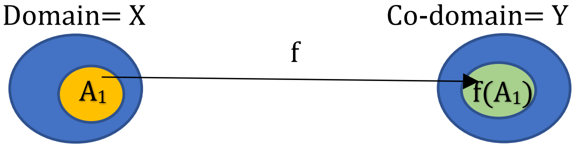


So, if x is an element of $f^{-1}(k)$, f(x) lives in k.

Proof: Claim $f^{-1}(B_1 \cup B_2) \subseteq f^{-1}(B_1) \cup f^{-1}(B_2)$
Note x is an element of $f^{-1}(B_1 \cup B_2)$ if and only if f(x) is an element of $(B_1 \cup B_2)$, which is the definition of pre-image. f(x) is an element of B_1 or f(x) is an element of B_2. x is an element of $f^{-1}(B_1)$ or x is an element of $f^{-1}(B_2)$. x is an element of $f^{-1}(B_1) \cup f^{-1}(B_2)$. Therefore, the inverse image of $(B_1 \cup B_2)$ is the inverse image of B_1 union with the inverse image of B_2 ■

3. Given a function X↦Y, suppose that A1 and A2 are both subsets of X. If the following statement is true, prove it, and if it is not true, give a counter-example:
$f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$
$f(A) = \{y \in Y \mid y = f(x) \text{ for some } x \in A\}$

Domain= X Co-domain= Y



Proof: claim $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$
Let y be an element of $f(A_1 \cup A_2)$. This means there exists an element x which is inside the union of A_1 and A_2 such that f(x)=y. Since x is in $A_1 \cup A_2$, then by the definition of union, then x is an element of A_1 or x is an element of A_2. So, y= f(x) is an element of $f(A_1)$ or y=f(x) is an element of $f(A_2)$. Thus, $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$.
Now suppose y is an element of $f(A_1) \cup f(A_2)$. So, by the definition of union, y is an element of $f(A_1)$ or y is an element of $f(A_2)$. Assume that y is an element of $f(A_1)$. This means that there exists some x in A_1 such that y=f(x). Now A is a subset of $A_1 \cup A_2$, so x is an element of $A_1 \cup A_2$, so y= f(x) is an element of the direct image of $f(A_1 \cup A_2)$. This shows that $f(A_1) \cup f(A_2)$ is contained in $f(A_1 \cup A_2)$. Therefore, $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$ ■

4. Determine if the function f: $\mathbb{R} \setminus \{1\} \rightarrow \mathbb{R} \setminus \{2\}$ defined by $f(x) = \frac{2x+1}{x-1}$ is bijective. Prove or disprove.
Proof: In order to prove a bijection, we must show that the function is an injection and that it is also a surjection.
Definitions: A function f is injective if f(a)=f(b), then a=b.
A function f is surjective if for every element b in B, there exists some element a in A, such that f(a)=b.

Injective proof: Suppose f(a)=f(b) for some a ,b, where a and b cannot be 1.

$\frac{2a+1}{a-1} = \frac{2b+1}{b-1}$

$2ab + b - 2a = 2ba - 2b + a$

$b - 2a = -2b + a$

$b + 2b = a + 2a$

$3b = 3a$

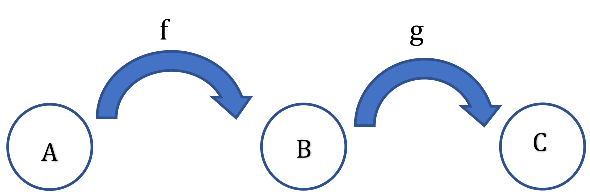
$b = a$

This shows that f is injective.

Surjective proof: Take any b from B in the set of real numbers (excluding 2). Then, a=$\frac{b+1}{b-2}$ is an element of ℝ. Then, f(a)= $\frac{2a+1}{a-1} = \frac{2(\frac{b+1}{b-1})+1}{(\frac{b+1}{b-1})-1} = \frac{3b+1}{2}$

f(a)≠b, therefore this function is not surjective. Because this function does not satisfy surjective definition, the function is not bijective ■

5. Prove that if f: A→B and g: B→C are both surjective functions, then g∘f: A→C is also surjective.
   Def: f: X→Y is surjective if ∀y∈Y ∃x∈X such that f(x)=y.
   Proof:
   Suppose c is some ∈ C. Since g is surjective, by definition of surjective, there exists some b ∈ B such that g(b)=c. Since f is surjective, by definition of surjective, there exists some a ∈ A such that f(a)=b.
   Then, (g∘f)(a)=g(f(a))= g(b)=c. Since c ∈ C is arbitrary, this holds for all c in C. Therefore, (g∘f) → C is surjective ■



6. Suppose that you have three coins: two fair coins, for which the probability of "H" is 0.5 and the probability of "T" is 0.5; and a biased coin, for which the probability of "H" is 0.8 and the probability of "T" is 0.2. You choose a coin in such a way that each coin is equally likely to be chosen. You flip the coin three times, getting "H" all three times. What is the probability that the coin is the biased coin?

Probability	Coin 1 "fair"	Coin 2 "fair"	Coin 3 "biased"
Heads	0.5	0.5	0.8
Tails	0.5	0.5	0.2

b(k;n,p)=$\binom{n}{k} p^k (1-p)^{(n-k)}$

k= number of successes, n= number of trials, p= probability of success

Probability for HHH for fair coin 1: (3; 3, 0.5)= $\binom{3}{3}0.5^3\,(0.5)^0$= 0.125

Probability for HHH for fair coin 2: (3; 3, 0.5)= $\binom{3}{3}0.5^3\,(0.5)^0$= 0.125

Probability for HHH for biased coin 3: (3; 3, 0.8)= $\binom{3}{3}0.8^3\,(0.2)^0$= 0.512

P(HHH)= $\frac{1}{3}P_1 + \frac{1}{3}P_2 + \frac{1}{3}P_3$= 0.125+0.125+0.512= 0.762

P(biased coin3|HHH)= $\frac{\text{P(biased coin3)}}{P(HHH)} = \frac{0.512}{0.762}$= **0.6719**

7. Suppose that a medical test run on 372 people resulted in 38 positive results. Of those, 22 people were eventually confirmed to have the illness. Among the people who tested negative, 3 were eventually diagnosed through other means, and the rest were healthy. Find the sensitivity of the test, the specificity of the test, and the positive and negative predictive values. The positive predictive value is the probability that a person is ill given that they tested positive, and the negative predictive value is the probability they are healthy if they tested negative.

	Has Condition?		
	Yes	No	Total
Tests Positive	22	16	38
Tests Negative	3	331	334
Total	25	347	372

Sensitivity= $\frac{P(positive\ test\ |has\ condition)}{P(has\ condition)} = \frac{22}{25}$

Specificity= $\frac{P(negative\ test\ |does\ not\ have\ condition)}{P(does\ not\ have\ condition)} = \frac{331}{347}$

Positive predictive value= $\frac{True\ Positive}{True\ Positive+False\ Positive} = \frac{22}{38}$

Negative predictive value= $\frac{True\ Negative}{True\ Negative+False\ Negative} = \frac{331}{334}$

8. If A and B are independent events, then are A and B^c necessarily independent events? Prove, or find a counter-example.

<u>Proof:</u>

By definition, if A and B are independent events, then P(A∩B)= P(A)P(B). In other words, P(A|B) = P(A).

We need to show that this implies P(A∩B^c)= P(A)P(B^c).

$$\begin{aligned} P(B^C\cap A) &= P(A)\text{- }P(B\cap A)\\ &=P(A)\text{-}P(B)P(A)\\ &=P(A)(1\text{-}P(B)\\ &=P(A)P(B^c) = P(A)P(B^c)\text{, therefore if A and B are}\end{aligned}$$

independent events, then A and B^c are also independent events ■

Part 2:

1. Let Y be a random variable distributed as shown in the table below. First confirm it is a probability distribution; then find E[Y], E[Y^2], and Var[Y].

y	1	2	3	4
y^2	1	4	9	16
p(y)	0.4	0.3	0.2	0.1

For Y to be a probability distribution, the sum of probabilities of all possible outcomes must be equal to 1. **0.4+0.3+0.2+0.1= 1, so this is a probability distribution**.

E[Y]= $\mu = \sum_{n=1}^{4} yp(y)$= 1(0.4)+2(0.3)+3(0.2)+4(0.1)= **2**

E[Y^2]= $\sum_{n=1}^{4} y^2p(y)$=1(0.4)+4(0.3)+9(0.2)+16(0.1)= **5**

Var[Y]= E[Y^2]-μ^2= 5-(2^2)= **1**

2. Find the mean and the variance for the sum of the number of spots rolled on two fair six-sided dice.

Sample Space:

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of number of spots:

2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12

Probability distribution:

n	2	3	4	5	6	7	8	9	10	11	12
n^2	4	9	16	25	36	49	64	81	100	121	144
p(n)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

E[Y]= $\sum_{n=2}^{12} yp(y)$

=2(1/36)+3(2/36)+4(3/36)+5(4/36)+6(5/36)+7(6/36)+8(5/36)+9(4/36)+10(3/36)+11(2/36)+12(1/36)

E[Y]= **μ= 7**

E[Y^2]= $\sum_{n=2}^{12} y^2p(y)$=

4(1/36)+9(2/36)+16(3/36)+25(4/36)+36(5/36)+49(6/36)+64(5/36)+81(4/36)+100(3/36)+121(2/36)+144(1/36)

≈ 54.83

Var[Y]= E[Y^2]-μ^2= 54.83-(7^2) ≈**5.83**

3. Find the mean and variance for the minimum of the number of spots rolled on two fair six-sided dice.

Sample Space:

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Minimum number of spots:

1	1	1	1	1	1
1	2	2	2	2	2
1	2	3	3	3	3
1	2	3	4	4	4
1	2	3	4	5	5
1	2	3	4	5	6

Probability distribution:

n	1	2	3	4	5	6
n^2	1	4	9	16	25	36
p(n)	11/36	9/36	7/36	5/36	3/36	1/36

E[Y]= $n\sum_{n=1}^{6} yp(y)$

= 1(11/36)+2(9/36)+3(7/36)+4(5/36)+5(3/36)+6(1/36)

E[Y]= **μ =91/36 ≈ 2.528**

$E[Y^2] = \sum_{n=1}^{6} y^2 p(y)$ = 1(11/36)+4(9/36)+9(7/36)+16(5/36)+25(3/36)+36(1/36)

$E[Y^2] \approx 8.361$

$Var[Y] = E[Y^2] - \mu^2 = 8.361 - (2.528^2) \approx$ **1.970**

4. Suppose a patient who has a medical condition takes a diagnostic test, which has a sensitivity of 80%.

   Sensitivity means that the test will report positive 80% of the time for a true positive test.

   a. If the test is given repeatedly n times, what is the probability that the patient tests positive for the first time on the nth test?

      n-1 times fail: p(n^{th} is success)= $(0.2)^{(n-1)} \cdot (0.8)^1$

      $$= \left(\frac{1}{5}\right)^{(n-1)} \cdot \frac{4}{5}$$

      $$= \frac{1}{5^{n-1}} \cdot \frac{4}{5}$$

      $$= \boldsymbol{\frac{4}{5^n}}$$

   b. What is the probability that the patient tests positive at least once in those n tests?

      P(one or more successes)= 1-P(no successes)= $\boldsymbol{1 - \frac{1}{5^n}}$

   c. What is the expected number of times the patient would have to repeat the test before seeing a positive test result?

      Mean of geometric distribution= $\frac{1}{p}$

      $$\mathbf{E[X]} = \frac{1}{\left(\frac{4}{5}\right)} = \boldsymbol{\frac{5}{4}}$$

5. Suppose we randomly sample customers leaving a grocery store until we find one who as purchased our product and agrees to a survey. Our first successful survey happens on the

second customer stopped 16% of the time. Can you determine the average number of customers we must stop before our first successful survey?
Geometric distribution
$P(X=n) = p(1-p)^{n-1}$
$P(X=2)= p(1-p)^{2-1}= 0.16$
$=p(1-p)= \frac{4}{25}$
$=p-p^2= \frac{4}{25}$
$25p^2-25p-4=0$
$p=0.8$

Average number of customers we must stop before our first successful survey: $\mu=\frac{1}{p}= \frac{1}{0.8}=\frac{\mathbf{5}}{\mathbf{4}}$

6. Basketball players in the NBA average about 75% success in free throws. If there are 23 free-throw attempts in a game, what is the probability that more than 20 are good?
Probability of success= 0.75
Probability of failure= 0.25
We are interested in the random variable X which counts as the number of successes 21, 22, or 23 are in 23 trials, or the cumulative probability $P(X>20)$.
$b(k;n,p)=(\genfrac{}{}{0pt}{}{n}{k})p^k(1-p)^{(n-k)}$
k= number of successes, n= number of trials, p= probability of success
probability that 21 are good: $b(k;n,p)=(\genfrac{}{}{0pt}{}{23}{21})0.75^{21}(1-0.75)^{(23-21)}=$ **0.037**
probability that 22 are good: $b(k;n,p)=(\genfrac{}{}{0pt}{}{23}{22})0.75^{22}(1-0.75)^{(23-22)}=$ **0.010**
probability that 23 are good: $b(k;n,p)=(\genfrac{}{}{0pt}{}{23}{23})0.75^{23}(1-0.75)^{(23-23)}=$ **0.001**
total: 0.048= 4.8%

7. Find a formula for the mean and variance of a uniform random variable on the integers from a to b.
$P(x_a \leq X \leq x_b)=\sum_{i=a}^{b} f(x) = \sum_{i=a}^{b} \frac{1}{n}$
Mean= $E[X]= \sum_n xf(x) = \sum_a^b \left(\frac{x}{n}\right)=\frac{\sum_a^n x}{n} = \frac{\boldsymbol{(b+a)}}{\mathbf{2}}$
$E[Y^2]= \sum_y y^2 p(y) = \sum_{n=a}^{b} n^2 \left(\frac{a}{b}\right) = \frac{a}{b}(\frac{b(b+1)(2b+1)}{6})= \frac{a(b+1)(2b+1)}{6}$
Variance= $Var[Y]= E[Y^2]-(E[Y])^2= \frac{a(b+1)(2b+1)}{6} - [\frac{(b+a)}{2}]^2= \frac{\boldsymbol{(b-a+1)^2-1}}{\mathbf{12}}$

8. Use summations to find the formula for the variance of a binomial random variable with parameters n and p. (I was unable to solve this independently, and used this resource to help me from jbstatistics: https://www.youtube.com/watch?v=8fqkQRjcR1M)
Expected value of a binomial random variable: $E[X]= \sum_{k=0}^{n} k \cdot \frac{n!}{k!(n-k)!} p^k(1-p)^{(n-k)}$
$=\sum_{k=1}^{n} \frac{n!}{(k-1)!(n-k)!} p^k(1-p)^{(n-k)}$
$=\sum_{k=0}^{n-1} \frac{n!}{k!(n-1-k)!} p^{k+1}(1-p)^{(n-k-1)}$

$=np\sum_{k=0}^{n-1}\frac{(n-1)!}{k!(n-1-k)!}\ p^k(1-p)^{(n-k-1)}$

=np(1)

E[X]=np

Var[X]= E[(X-μ)^2]

$=\sum_x (x-\mu)^2 p(x)$

E[(X-μ)^2]= E(X^2)-[E(X)]^2, we know that E[X]= np, so the goal is to find E(X^2).

E(X^2)= $\sum_{x=0}^{n} x^2 \frac{n!}{x!(n-x)!} p^x (1-p)^{(n-x)}$

E[X(X-1)]= $\sum_{x=0}^{n} x(x-1)\frac{n!}{x!(n-x)!} p^x (1-p)^{(n-x)}$

E(X^2-X)= $\sum_{x=2}^{n} x(x-1)\frac{n!}{x!(n-x)!} p^x (1-p)^{(n-x)}$ *terms x=0 and x=1 are both 0

x! can be replaced with x(x-1)(x-2)!

E(X^2-X)= $\sum_{x=2}^{n}\frac{n!}{(x-2)!(n-x)!} p^x (1-p)^{(n-x)}$

E(X^2-X)=n(n-1)p^2 $\sum_{x=2}^{n}\frac{(n-2)!}{(x-2)!(n-x)!} p^{(x-2)} (1-p)^{(n-x)}$

E(X^2-X)=n(n-1)p^2 $\sum_{x=2}^{n}\frac{(n-2)!}{(x-2)!((n-2)-(x-2))!} p^{(x-2)} (1-p)^{(n-2)-(x-2)}$ *let m=n-2 and y=x-2

E(X^2-X)=n(n-1)p^2 $\boldsymbol{\sum_{y=0}^{m}\frac{m!}{y!(m-y)!} p^{(y)} (1-p)^{(m-y)}}$ ***bolded part of expression evaluates to 1

because (p+(1-p))^m= 1^m=1

E(X^2)-E(X)= n(n-1)p^2

We know that E(X)= np

Var(X)= E(X^2)-[E(X)]^2

=n(n-1)p^2 +np –(np)^2

=np[(n-1)p+1-np]

=np(np-p+1-np)

Var(X)=np(1-p)