Appendix to "Fiscal DSGE model for Latvia" published in Baltic Journal of Economics

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A Calibration & Estimation of the Model's Non-Fiscal Part

Table A.1: **Calibrated parameters, non-fiscal part**

Parameter	Value	Description
		Core block
α	0.4000	Capital share in production of intermediate goods
β	0.9995	Time discount factor, optimizing households
ω_c	0.4500	Import share in consumption goods
ω_i	0.6500	Import share in investment goods
ω_x	0.3300	Import share in export goods
$\tilde{\phi}_a$	0.0100	Elasticity of domestic risk premium to net foreign assets position
μ_z	1.0031	Steady-state quarterly growth rate of neutral technology
μ_ψ	1.0000	Steady-state growth rate of investment technology
$\bar{\pi}$	1.0043	Steady-state gross inflation target
λ_d	1.3000	Price mark-up for domestic good
$\lambda_{m,c}$	1.0500	Price mark-up for imported consumption good
$\lambda_{m,i}$	1.0500	Price mark-up for imported investment good
$\lambda_{m,x}$	1.0500	Price mark-up for imported exports good
λ_x	1.0500	Price mark-up for exported good
ν^j	0.5000	Working capital fraction, all goods
θ^d	0.5000	Technology diffusion parameter (common)
		Financial frictions block
$F(\bar{\omega})$	0.0200	Steady state bankruptcy rate
$100W_e/y$	0.1000	Transfers to entrepreneurs
		Labour market block
L	0.8744	Steady-state fraction of employment (1 - unemployment rate)
Q	0.7000	Vacancy filling rate
bshare	0.1000	Unemployment benefit share of gross wage
hshare, %	0.2000	Hiring fixed costs as a share of GDP
vshare, %	0.0000	Vacancy posting cost as a share of GDP
ρ	0.7000	Exogenous survival rate of a match
σ	0.7392	Unemployment share in matching technology

Notes: This table contains the key calibrated parameters and steady states of the core block of the model.

Table A.2: **Targeted steady states and selected implied parameters**

Parameter	Description	Value
	Targeted steady states	
$p_x x\tilde{\varphi}/y$	Exports to gross output	0.6100
$p_i i/y$	Investment to gross output	0.2200
$n/(p_{k'}k)$	Firm net worth to capital	0.7000
	Implied parameters	
δ	Capital quarterly depreciation rate	0.0490
$\tilde{\varphi}$	Real exchange rate	0.9079
γ	Entrepreneurial survival rate	0.9473
σ_m	Level parameter in matching function	0.6823
f	Job finding rate	0.6762
v	Vacancy rate	0.4286
$p_c c/y$	Private consumption to gross output	0.6477

Notes: This table contains the key targeted steady states in the model and selected implied parameters.

Table A.3: **Estimated parameters and shocks, non-fiscal part**

Parameter	Description	Prior $\mathcal{D}$,**Mean**,St.d.	Posterior **Mode**,St.d.
ξ_d	Calvo parameter, domestic good	β,**0.75**,0.075	**0.688**,0.032
ξ_x	Calvo parameter, exports	β,**0.75**,0.075	**0.877**,0.014
$\xi_{m,c}$	Calvo parameter, imports for consumption	β,**0.75**,0.075	**0.947**,0.014
$\xi_{m,i}$	Calvo parameter, imports for investment	β,**0.75**,0.075	**0.578**,0.023
$\xi_{m,x}$	Calvo parameter, imports for exports	β,**0.66**,0.10	**0.819**,0.022
κ_d	Inflation indexation, domestic good	β,**0.50**,0.15	**0.212**,0.060
κ_x	Inflation indexation, exports	β,**0.05**,0.02	**0.062**,0.012
$\kappa_{m,c}$	Inflation indexation, imports for consumption	β,**0.50**,0.15	**0.436**,0.054
$\kappa_{m,i}$	Inflation indexation, imports for investment	β,**0.50**,0.15	**0.293**,0.068
$\kappa_{m,x}$	Inflation indexation, imports for exports	β,**0.05**,0.02	**0.042**,0.009
b	Habit in consumption	β,**0.65**,0.15	**0.607**,0.050
$0.1S''$	Investment adjustment costs	Γ,**0.40**,0.15	**0.069**,0.013
σ_a	Variable capital utilization	Γ,**0.30**,0.1	**0.369**,0.035
η_x	Elasticity of substitution, domestic good & imports for exports	$\underline{\Gamma}$,**1.50**,0.25	**1.821**,0.139
η_c	Elasticity of substitution, domestic good & imports for consumption	$\underline{\Gamma}$,**1.50**,0.25	**1.854**,0.118
η_i	Elasticity of substitution, domestic good & imports for investment	$\underline{\Gamma}$,**1.50**,0.25	**1.059**,0.050
η_f	Elasticity of substitution, exports & foreign good	$\underline{\Gamma}$,**1.50**,0.25	**1.010**,0.038
μ	Monitoring cost	β,**0.35**,0.075	**0.557**,0.035
λ_r	Share of restricted households	β,**0.5**,0.075	**0.318**,0.030
woo	Worker outside option parameter	β,**0.40**,0.075	**0.557**,0.038
α^u	Power parameter in outside option equation	β,**0.90**,0.075	**0.862**,0.029
ρ_ϵ	Persistence, stationary technology	β,**0.85**,0.075	**0.891**,0.028
ρ_Υ	Persistence, marginal efficiency of private investment	β,**0.85**,0.075	**0.567**,0.035
ρ_{ζ^c}	Persistence, consumption preference shock	β,**0.85**,0.075	**0.817**,0.037
$\rho_{\tilde{\phi}}$	Persistence, domestic risk premium	β,**0.85**,0.075	**0.962**,0.019
ρ_g	Persistence, government spending	β,**0.85**,0.075	**0.841**,0.020
ρ_γ	Persistence, entrepreneurial wealth	β,**0.85**,0.075	**0.731**,0.038
ρ_{ϵ^x}	Persistence, exports stationary technology	β,**0.85**,0.075	**0.916**,0.026
ρ_{bu}	Persistence, outside option	β,**0.80**,0.075	**0.934**,0.013
	Shock standard deviations		
$10\sigma_\epsilon$	Stationary technology	Γ^{-1},**0.10**,inf	**0.097**,0.010
σ_Υ	Marginal efficiency of private investment	Γ^{-1},**0.15**,inf	**0.099**,0.010
σ_{ζ^c}	Consumption preference shock	Γ^{-1},**0.15**,inf	**0.063**,0.007
$100\sigma_{\tilde{\phi}}$	Domestic risk premium	Γ^{-1},**0.40**,inf	**0.108**,0.010
σ_{τ^d}	Mark-up, domestic good	Γ^{-1},**0.10**,inf	**0.106**,0.029
σ_{τ^x}	Markup, exports	Γ^{-1},**0.40**,inf	**0.183**,0.060
$\sigma_{\tau^{m,c}}$	Mark-up, imports for consumption	Γ^{-1},**0.50**,inf	**0.729**,0.434
$\sigma_{\tau^{m,i}}$	Mark-up, imports for investment	Γ^{-1},**0.50**,inf	**0.657**,0.095
$\sigma_{\tau^{m,x}}$	Mark-up, imports for exports	Γ^{-1},**0.50**,inf	**0.250**,0.105
$10\sigma_\gamma$	Entrepreneurial wealth	Γ^{-1},**0.15**,inf	**0.271**,0.027
$10\sigma_{\epsilon^x}$	Exports stationary technology	Γ^{-1},**0.15**,inf	**0.173**,0.028
$10\sigma_{bu}$	Outside option	Γ^{-1},**0.10**,inf	**0.190**,0.049
	Model fit statistic		
$\mathcal{L}$	Log marginal likelihood (Laplace approximation)		-4419.0

Notes: This table contains the estimated parameters for the core block of the model. Note that truncated priors, denoted by $\underline{\Gamma}$, with no mass below 1.01 have been used for the elasticity parameters η_j, $j = \{x, c, i, f\}$.

Table A.4: **Estimated foreign SVAR parameters**

Parameter	Description	Prior Distr.	Prior **Mean**	Prior St.d.	Posterior **Mode**	Posterior St.d.
ρ_{μ_z}	Persistence, unit-root technology	β	**0.60**	0.075	**0.707**	0.057
a_{11}	Persistence, euro area output	β	**0.90**	0.075	**0.994**	0.014
a_{22}	Persistence, euro area inflation	β	**0.30**	0.075	**0.310**	0.063
a_{33}	Persistence, euro area interest rate	β	**0.70**	0.075	**0.668**	0.060
a_{66}	Persistence, competitors price inflation	β	**0.30**	0.075	**0.304**	0.059
a_{77}	Persistence, foreign demand	β	**0.85**	0.075	**0.941**	0.023
ρ_s	Persistence, nominal effective exchange rate	β	**0.20**	0.075	**0.155**	0.029
a_{12}	Foreign SVAR parameter	N	**0.00**	0.10	**0.012**	0.082
a_{13}	Foreign SVAR parameter	N	**-0.10**	0.05	**-0.059**	0.037
a_{21}	Foreign SVAR parameter	N	**0.10**	0.10	**0.071**	0.032
a_{23}	Foreign SVAR parameter	N	**-0.10**	0.05	**-0.097**	0.049
a_{24}	Foreign SVAR parameter	N	**0.00**	0.10	**0.024**	0.064
a_{26}	Foreign SVAR parameter	N	**0.00**	0.05	**0.010**	0.014
a_{31}	Foreign SVAR parameter	N	**0.00**	0.05	**0.106**	0.019
a_{32}	Foreign SVAR parameter	N	**0.10**	0.005	**0.099**	0.005
a_{34}	Foreign SVAR parameter	N	**0.00**	0.005	**0.002**	0.005
a_{61}	Foreign SVAR parameter	N	**0.00**	0.05	**-0.009**	0.047
a_{62}	Foreign SVAR parameter	N	**0.00**	0.05	**0.007**	0.052
a_{67}	Foreign SVAR parameter	N	**0.00**	0.05	**-0.023**	0.022
a_{71}	Foreign SVAR parameter	N	**0.00**	0.05	**0.001**	0.048
a_{76}	Foreign SVAR parameter	N	**0.20**	0.05	**0.246**	0.047
c_{17}	Foreign SVAR parameter	N	**0.10**	0.10	**0.096**	0.017
c_{21}	Foreign SVAR parameter	N	**0.05**	0.05	**0.018**	0.049
c_{31}	Foreign SVAR parameter	N	**0.00**	0.05	**0.111**	0.033
c_{32}	Foreign SVAR parameter	N	**0.00**	0.05	**0.063**	0.022
c_{24}	Foreign SVAR parameter	N	**-0.05**	0.05	**-0.067**	0.045
c_{34}	Foreign SVAR parameter	N	**0.00**	0.005	**0.001**	0.005
c_{67}	Foreign SVAR parameter	N	**0.20**	0.05	**-0.023**	0.022
	Shock standard deviations					
$100\sigma_{\mu_z}$	Unit root technology	Γ^{-1}	**0.25**	inf	**0.223**	0.038
$100\sigma_{y^{ea}}$	Euro area GDP	Γ^{-1}	**0.20**	inf	**0.256**	0.031
$1000\sigma_{\pi^{ea}}$	Euro area inflation	Γ^{-1}	**0.25**	inf	**0.256**	0.019
$100\sigma_{R^{ea}}$	Euro area interest rate	Γ^{-1}	**0.05**	inf	**0.048**	0.005
$10\sigma_{dem^*}$	Foreign demand	Γ^{-1}	**0.20**	inf	**0.239**	0.017
$10\sigma_{\pi^{compx}}$	Competitors price on exports side	Γ^{-1}	**0.15**	inf	**0.160**	0.013
$10\sigma_s$	Nominal effective exchange rate	Γ^{-1}	**0.15**	inf	**0.154**	0.013

Notes: This table contains the estimated parameters for the foreign economy. Informative priors are used to generate plausible impulse response functions.

B Detailed Description of the Model

Section 2 just outlined the new fiscal elements of the model in detail and just contained a brief description of the rest of the model. In this appendix, we describe the full model, including the new fiscal elements. Appendix C gives further details and describes how to normalize the model. We describe the model below by proceeding first with describing the households, then outlining the production sectors (domestic homogeneous good, final consumption goods, final investment goods, export goods, and imports), the labour market friction and the financial friction in the model, next delving into the foreign economy block, the fiscal sector, and monetary policy, before concluding with the aggregate resource constraint and the description of the current account.

B.1 Households

There are two types of households in our model: optimizing households and restricted households, who just consume their available income and do not solve any optimization problems. The optimizing households have access to a number of financial assets. They choose their holdings of these assets optimally. The optimizing households have mass $(1-\lambda_r)$ and the restricted households have mass λ_r with $\lambda_r \in [0,1]$. Therefore, aggregate consumption is given by the following aggregate (C_t) of optimizing households' private consumption $(C_{o,t})$ and restricted households' private consumption $(C_{r,t})$:

$$C_t = \lambda_r C_{r,t} + (1-\lambda_r)C_{o,t}. \tag{B.1}$$

B.1.1 Optimizing households

The preferences of optimizing households over their consumption bundle $\widetilde{C}_t$, defined as an aggregate over private consumption $C_{o,t}$ and public consumption $G_{c,t}$, are given by

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\zeta_t^c\log(\widetilde{C}_{o,t}-b\widetilde{C}_{o,t-1})\right)\right], \tag{B.2}$$

where ζ_t^c denotes a consumption preference shock, b is the consumption habit parameter. The household owns the economy's stock of physical capital. It determines the rate at which the capital stock is accumulated and the rate at which it is utilized. The household also owns the stock of net foreign assets and determines its rate of accumulation. The consumption bundle is defined as follows

$$\widetilde{C}_{o,t} = \left(\alpha_c^{\frac{1}{\nu_c}}(C_{o,t})^{\frac{\nu_c-1}{\nu_c}} + (1-\alpha_c)^{\frac{1}{\nu_c}}(G_{c,t})^{\frac{\nu_c-1}{\nu_c}}\right)^{\frac{\nu_c}{\nu_c-1}}, \tag{B.3}$$

where the elasticity of substitution between private and public consumption is denoted by ν_c and the weight on private consumption is given by α_c.

The optimizing household faces the following period-by-period real budget constraint:

$$\begin{aligned}(1+\tau_t^c)p_t^c C_{o,t} &+ \frac{P_t^i}{P_t}I_t + B_{g,t+1} + \frac{S_t A_{t+1}^*}{P_t\Phi_t R_t^*} + T_{ls,t} + \Gamma_{g,t} \\ &= (1-\tau_t^y-\tau_{w,t}^w)\frac{W_t L_t}{P_t} + \frac{\tau_t^k\bar{K}_t}{P_t}[P_t^i a(u_t) + \delta P_t^i - u_t r_t^k] + (1-\tau^b)D_t^f + D_{b,t}(1-L_t) \\ &\quad + TR_{o,t} + B_{g,t}\Phi_{g,t-1}R_{t-1}^* + \left(1-\tau^b+\frac{\tau^b\pi_t}{R_{t-1}^*\Phi_{t-1}}\frac{S_{t-1}}{S_t}\right)\frac{S_t A_t^*}{P_{t-1}} + \Xi_t^B + \Xi_t^{A^*}\end{aligned} \tag{B.4}$$

where P_t^c and P_t^i are the prices of a unit of the private consumption good $C_{o,t}$ and the investment good I_t, respectively. The variable W_t denotes the gross wage rate for the labour services provided to firms, L_t. The expression $D_{b,t}(1-L_t)$ denotes the unemployment benefits received by the optimizing households. The variable R_t^k is the rental rate for the capital services rented to firms, K_t. Moreover, D_t^f are the dividends paid by the firms owned by the household and R_t^* is the rate of return on the domestic government bond and on the internationally traded foreign bond. The holdings of these bonds are denoted by $B_{g,t}$ and A_t^*, respectively. The household needs to pay quadratic adjustment costs $\Gamma_{g,t}$ for holdings of the domestic government bond in excess of a certain level. The internationally traded foreign bond is denominated in foreign currency, thus its value depends on the nominal exchange rate S_t. The fiscal authority taxes the household's gross income to finance the government expenditure. The variable τ_t^c denotes the consumption tax rate that is applied to the household's consumption purchases; τ_t^y, τ_t^k, and τ^b are the tax rates levied on household's wage income, capital income, and dividend/interest income. We assume that the depreciated physical capital $\delta P_t^i \bar{K}_t$ and utilization costs $P_t^i a(u_t)$ are exempt from taxation. Furthermore, $\tau_{w,t}^w$ is the social security contribution paid by the household, $T_{ls,t}$ are lump-sum taxes, and $TR_{o,t}$ are lump-sum transfers.

The effective return on the risk-free domestic bonds depends on a financial intermediation premium $\Phi_{g,t}$. Similarly, when taking a position in the international bond market, the household encounters an external financial intermediation premium Φ_t. This implies that, in the deterministic steady state, households have no incentive to hold foreign bonds and the private sector's net foreign asset position is zero. The incurred intermediation premia are rebated in the form of lump-sum payments, $\Xi_{i,t}^B$ and $\Xi_{i,t}^{B^*}$.

Finally, the physical capital stock owned by the household evolves according to the following capital accumulation process. It takes into account investment adjustment costs as introduced by Christiano et al. (2005):[3]

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + \Upsilon_t \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right) I_t, \tag{B.5}$$

where δ denotes the depreciation rate of the private capital stock, the term inside the large parentheses is the adjustment cost function in investment. Furthermore, Υ_t denotes the marginal efficiency of investment shock that affects how investment is transformed into capital.[4]

Define the real rate of return on a period t investment in a unit of physical capital R_{t+1}^k as follows:

$$R_{t+1}^k = \frac{(1-\tau_t^k)\left[u_{t+1} r_{t+1}^k - \frac{P_{t+1}^i}{P_{t+1}} a(u_{t+1})\right] P_{t+1} + (1-\delta) P_{t+1} P_{k',t+1} + \tau_t^k \delta P_t P_{k',t}}{P_t P_{k',t}}, \tag{B.6}$$

where τ_t^k is the capital tax rate, r_{t+1}^k is the nominal rental rate of capital which is scaled by P_t, $a(u_t)$ is the capital utilization function with functional form given in Equation (C.4), $P_{k',t}$ denotes the price of a unit of newly installed physical capital which operates in period $t+1$. This price is expressed in units of the homogeneous good, so that $P_t P_{k',t}$ is the domestic currency price of physical capital. The price of a unity of the final investment good is denoted by $P_{i,t}$. The numerator in the expression for R_{t+1}^k represents the period $t+1$ payoff from a unit additional physical capital. The expression in square brackets captures the idea that maintenance expenses associated with the operation of capital are deductible from taxes. The last expression in the numerator expresses the idea that physical depreciation is deductible at historical cost.

Household consumption and investment decisions. The first order condition for private consumption is

$$\left(\frac{\zeta_t^c}{\widetilde{C}_{o,t} - b\widetilde{C}_{o,t-1}} - \beta b \mathbb{E}_t\left[\frac{\zeta_{t+1}^c}{\widetilde{C}_{o,t+1} - b\widetilde{C}_{o,t}}\right]\right) \alpha_c^{\frac{1}{\nu_c}} C_{o,t}^{-\frac{1}{\nu_c}} \widetilde{C}_{o,t}^{\frac{1}{\nu_c}} - \upsilon_t P_t^c (1+\tau_t^c) = 0, \tag{B.7}$$

where τ_t^c is the consumption tax rate, P_t^c the price of a unit of the consumption good, and υ_t is the Lagrange multiplier attached to the household's budget constraint.

By differentiating the Lagrangian representation of the household's problem with respect to I_t, the investment first order condition is

$$\mathbb{E}_t\left[\mathrm{M}_{t,t+1}^o P_{k',t+1} \Upsilon_{t+1} \tilde{S}'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] = P_t^i - P_{k',t}\Upsilon_t\left[1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right) - \tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right], \tag{B.8}$$

[3] See Appendix C for the functional form of the investment adjustment costs, $\tilde{S}$.

[4] This is the shock whose importance is emphasized by Justiniano et al. (2011).

where $\mathbb{M}^o_{t,t+1} := \beta\upsilon_{t+1}/\upsilon_t$ is the stochastic discount factor of optimizing households.

Equation (B.8) can be thought of as reflecting that the household builds and sells physical capital, or it can be interpreted as the FOC of many identical competitive firms that build capital (note that each has a state variable in the form of lagged investment).

Financial assets. The household does the domestic economy's saving. Period t saving occurs by the acquisition of net foreign assets, A^*_{t+1}, and a domestic government bond that we will discuss in Section B.8 of this appendix.

The date t first order condition associated with the asset A^*_{t+1} that pays R^*_t in terms of foreign currency is

$$1 = \mathbb{E}_t\left[\mathbb{M}^o_{t,t+1}\left[\frac{S_{t+1}}{S_t}R^*_t\Phi_t - \tau^b\left(\frac{S_{t+1}}{S_t}R^*_t\Phi_t - \pi_{t+1}\right)\right]\right], \tag{B.9}$$

where τ^b is the tax rate on the real interest rate on bond income.[5] Recall that S_t is the domestic currency price of a unit foreign currency. The left side of this expression is the cost of acquiring a unit of the foreign asset. The currency cost is S_t and this is converted into utility terms by multiplying by the multiplier on the household's budget constraint, υ_t. The term in square brackets is the after-tax payoff of the foreign asset in domestic currency units. The period $t+1$ pre-tax interest payoff on A^*_{t+1} is $S_{t+1}R^*_t\Phi_t$. Here, R^*_t is the foreign nominal rate of interest, which is risk free in foreign currency units. The term Φ_t represents a relative risk adjustment of the foreign asset return, so that a unit of the foreign asset acquired in t pays off $R^*_t\Phi_t$ units of foreign currency in $t+1$. The determination of Φ_t is discussed below. The remaining term in brackets pertains to the impact of taxation on returns on foreign assets.[6]

The risk adjustment term has the following form:

$$\Phi_t = \Phi\left(\frac{S_tA^*_{t+1}}{P_tz^+_t}, R^*_t - R_t, \tilde{\phi}_t\right) = \exp\left(-\tilde{\phi}_a\left(\frac{S_tA^*_{t+1}}{P_tz^+_t} - \frac{SA^*}{Pz^+}\right) - \tilde{\phi}_s(R^*_t - R_t - (R^* - R)) + \tilde{\phi}_t\right), \tag{B.10}$$

where $\tilde{\phi}_t$ is a mean zero country risk premium shock, and $\tilde{\phi}_a$ and $\tilde{\phi}_s$ are positive parameters.[7]

B.1.2 Restricted households

The restricted households do not optimize over their consumption, and just consume their disposable income each period. Their period-by-period real budget constraint is therefore given by:

$$(1+\tau^c_t)p^c_tC_{r,t} = (1-\tau^y_t - \tau^w_{w,t})\frac{W_tL_t}{P_t} + D_{b,t}(1-L_t) + \mathrm{TR}_{r,t}, \tag{B.11}$$

where $C_{r,t}$ is amount of restricted households' consumption, $D_{b,t}$ are the unemployment benefits, $\mathrm{TR}_{r,t}$ the amount of government transfers directed to restricted households, τ^y_t is the payroll tax rate, $\tau^w_{w,t}$ is the social security contribution rate for workers, W_t is the wage, and L_t is the total labour supply by households.

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[5] A consequence of this treatment of the taxation on domestic bonds is that the real after-tax return on bonds in the steady state is invariant to steady-state inflation π.

[6] If we ignore the term after the minus sign in parentheses, then the taxation is applied to the whole nominal payoff on the bond, including principal. The term after the minus sign is designed to ensure that the principal is deducted from taxes. The principal is expressed in nominal terms and is set so that the real value at $t+1$ coincides with the real value of the currency used to purchase the asset in period t. Recall that S_t is the period t domestic currency cost of a unit (in terms of foreign currency) of foreign assets. So the period t real cost of the asset is S_t/P_t. The domestic currency value in period $t+1$ of this real quantity is $P_{t+1}S_t/P_t$.

[7] The dependence of Φ_t on a_t ensures that there is a unique steady state value of a_t that is independent of the initial net foreign assets and the capital stock of the economy. The dependence of Φ_t on the relative level of the interest rate, $R^*_t - R_t$, is designed to allow the model to reproduce two types of observations. The first concerns observations related to uncovered interest parity. The second concerns the hump-shaped response of output to a domestic monetary policy shock. The particular calibration sets $\tilde{\phi}_s = 0$ to ensure the nominal interest rate peg regime.

Similarly to the preferences of optimizing households, the preferences of restricted households over their consumption bundle $\widetilde{C}_{r,t}$, defined as an aggregate over private consumption $C_{r,t}$ and public consumption $G_{c,t}$, are given by

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\zeta_t^c\log(\widetilde{C}_{r,t}-b\widetilde{C}_{r,t-1})\right)\right]. \tag{B.12}$$

The consumption bundle is defined as follows

$$\widetilde{C}_{r,t}=\left(\alpha_c^{\frac{1}{\nu_c}}\left(C_{r,t}\right)^{\frac{\nu_c-1}{\nu_c}}+\left(1-\alpha_c\right)^{\frac{1}{\nu_c}}\left(G_{c,t}\right)^{\frac{\nu_c-1}{\nu_c}}\right)^{\frac{\nu_c}{\nu_c-1}}. \tag{B.13}$$

B.2 Production of the domestic homogeneous good

B.2.1 Final domestic good

A homogeneous domestic good, Y_t, is produced by competitive, identical firms using

$$Y_t=\left[\int_0^1 Y_{j,t}^{1/\lambda_d}dj\right]^{\lambda_d},\ 1\leq\lambda_d<\infty, \tag{B.14}$$

and taking the price of output, P_t, and the price of intermediate goods, $P_{i,t}$, as given. Here, $Y_{i,t}$ denotes intermediate goods and $1/\lambda_d$ their degree of substitutability. The representative firm chooses the amount of intermediate goods $Y_{i,t}$ to maximize profits

$$P_tY_t-\int_0^1 P_{i,t}Y_{i,t}di \tag{B.15}$$

subject to the production function (B.14). The firm's first order condition for producer i^{th}'s demand for intermediate goods is

$$Y_{i,t}=\left(P_t/P_{i,t}\right)^{\frac{\lambda_d}{\lambda_d-1}}Y_t. \tag{B.16}$$

B.2.2 Retailer

The intermediate good j in Equation (B.14) is produced by a retailer using the following production function:

$$Y_{j,t}=\left(z_tH_{j,t}\right)^{1-\alpha}\epsilon_t\widetilde{K}_{j,t}^{\alpha}-z_t^{+}\phi, \tag{B.17}$$

where $\widetilde{K}_{j,t}$ denotes a bundle of private and public capital services rented by the j-th retailer, $\log\epsilon_t$ is a stationary neutral technology shock, and ϕ denotes a fixed production cost. The object z_t^+ in Equation (B.17) is defined as[8]

$$z_t^+=\Psi_t^{\frac{\alpha}{1-\alpha}}z_t,$$

where $\log z_t$ is a technology shock whose first difference has a positive mean. The economy has two sources of growth: the positive drift in $\log z_t$ and a positive drift in $\log\Psi_t$, where Ψ_t is an investment-specific technology shock. The bundle of private and public capital services is given by

$$\widetilde{K}_{j,t}=\left(\alpha_k^{\frac{1}{\nu_k}}\left(K_{j,t}\right)^{\frac{\nu_k-1}{\nu_k}}+\left(1-\alpha_k\right)^{\frac{1}{\nu_k}}\left(K_{g,j,t}\right)^{\frac{\nu_k-1}{\nu_k}}\right)^{\frac{\nu_k}{\nu_k-1}}, \tag{B.18}$$

where $K_{j,t}$ denotes private capital services and $K_{g,j,t}$ public capital services, rented by the j-th retailer. The parameters α_k and ν_k denote the weight of private capital and the elasticity of substitution between private and public capital services, respectively.

[8] The details regarding the scaling of variables are collected in Appendix C.

$H_{j,t}$ is the quantity of specialized labour inputs purchased by the j-th retailer. This good is purchased in competitive markets at price P_t^H from a wholesaler.

The retailer must borrow a fraction ν^f of $P_t^H H_{j,t}$ at the gross nominal interest rate, R_t, so that the costs of purchasing one unit of the specialized labour input is denoted by

$$P_t^H R_t^f,$$

with

$$R_t^f = \nu^f R_t + 1 - \nu^f. \tag{B.19}$$

The retailer repays the loan at the end of period t after receiving sales revenues.

The firm's marginal cost is denoted by MC_t:

$$MC_t = \frac{\tau_t^d}{\epsilon_t}\left(\frac{1}{1-\alpha}\right)^{1-\frac{\alpha\nu_k}{\nu_k-1}}\left(\frac{1}{\alpha}\right)^{\frac{\alpha\nu_k}{\nu_k-1}}\left(\frac{1}{\alpha_k}\right)^{\frac{\alpha}{\nu_k-1}}\left(r_t^k P_t\right)^{\frac{\alpha\nu_k}{\nu_k-1}}\left(P_t^H R_t^f\right)^{1-\frac{\alpha\nu_k}{\nu_k-1}}, \tag{B.20}$$

where r_t^k is the nominal rental rate of capital scaled by P_t. Also, τ_t^d is a tax-like shock which affects marginal cost but does not appear in the production function.[9]

Productive efficiency dictates that marginal cost is equal to the cost of producing another unit using the specialized labour input, which implies:

$$MC_t = \frac{\tau_t^d P_t^H R_t^f}{\epsilon_t (1-\alpha) z_t^{1-\alpha}\left(\frac{\widetilde{K}_{j,t}}{H_{j,t}}\right)^{\alpha}}. \tag{B.21}$$

The retailer is a monopolist in the product market and is competitive in factor markets. The j-th retailer sets its price, $P_{j,t}$ subject to the demand curve (B.16) and Calvo-type price frictions. With probability ξ_d the intermediate good firm cannot reoptimize its price, in which case,

$$P_{j,t} = \tilde{\pi}_{d,t} P_{j,t-1}, \quad \tilde{\pi}_{d,t} := (\pi_{t-1})^{\kappa_d}(\bar{\pi}_t^c)^{1-\kappa_d-\varkappa_d}(\breve{\pi})^{\varkappa_d},$$

where κ_d, $\varkappa_d$, $\kappa_d + \varkappa_d \in (0,1)$ are parameters, π_{t-1} is the lagged inflation rate and $\bar{\pi}_t^c$ is the central bank's (implicit) target inflation rate. Also, $\breve{\pi}$ is a scalar which allowing to capture, among other things, the case in which non-optimizing firms either do not change price at all (i.e., $\breve{\pi} = \varkappa_d = 1$) or that they index only to the steady state inflation rate (i.e., $\breve{\pi} = \bar{\pi}$, $\varkappa_d = 1$). Note that there is a price dispersion in steady state if $\varkappa_d > 0$ and if $\breve{\pi}$ is different from the steady state value of π.

With probability $1-\xi_d$ the firm can change the price. The problem of the j-th domestic intermediate good producer which has the opportunity to change price is to maximize discounted profits:

$$\mathbb{E}_t\left[\sum_{j=0}^{\infty}\beta^j \upsilon_{t+j}\left\{P_{j,t+j}Y_{i,t+j} - MC_{t+j}Y_{i,t+j}\right\}\right], \tag{B.22}$$

subject to the requirement that production equals demand. In the above expression, υ_t is the multiplier on the household's nominal budget constraint. It measures the marginal value to the household of one unit of profits in terms of currency. In states of nature when the firm can reoptimize price, it does so to maximize its discounted profits subject to the price setting frictions and to the requirement that it satisfies demand given by

$$\left(\frac{P_t}{P_{j,t}}\right)^{\frac{\lambda_d}{\lambda_d-1}} Y_t = Y_{j,t}. \tag{B.23}$$

The equilibrium conditions associated with the price setting problem and their derivation are reported in Appendix C.

The domestic homogeneous good is allocated among alternative uses as follows:

$$Y_t = Z_t + G_{c,t}^d + C_t^d + G_{i,t}^d + I_t^d + \int_0^1 X_{j,t}^d dj + D_t, \tag{B.24}$$

[9] In the linearized version of the model in which there are no price and wage distortions in the steady state, τ_t^d is isomorphic to a disturbance in λ_d, i.e., a markup shock.

where Z_t is wasteful government expenditure, $G_{c,t}^d$ denotes the amount of domestic homogeneous goods used (together with foreign public consumption goods) to produce final public consumption goods, C_t^d denotes the amount of domestic homogeneous goods used (together with foreign consumption goods) to produce final private consumption goods, $G_{i,t}^d$ is the amount of domestic homogeneous goods used (in combination with imported foreign public investment goods) to produce final public investment goods, I_t^d is the amount of domestic homogeneous goods used (in combination with imported foreign investment goods) to produce final private investment goods. Furthermore, the integral in Equation (B.24) denotes domestic resources allocated to exports. Finally, the variable D_t incorporates monitoring costs due to financial frictions and the costs of holding public debt. The determination of consumption, investment, and export demand is discussed below.

B.3 Production of final private consumption, final public consumption, private investment, public investment, and export goods

This section outlines how the five different final goods – private consumption goods, public consumption goods, private investment goods, public investment goods, and export goods – are produced in our small open economy.

B.3.1 Private consumption goods

Final private consumption goods are purchased by households. These goods are produced by a representative competitive firm using the following linear homogeneous technology:

$$C_t = \left[(1-\omega_c)^{\frac{1}{\eta_c}} \left(C_t^d\right)^{\frac{\eta_c-1}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} \left(C_t^m\right)^{\frac{\eta_c-1}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c-1}}. \tag{B.25}$$

Strictly speaking, one would have to take care of existing entrepreneurs' consumption (see Section B.6 for details on the modeling of entrepreneurs and financial frictions). The net worth of these entrepreneurs is $(1-\gamma_t)V_t$ and it is assumed that a fraction $1-\Theta$ of it is taxed and transferred in lump-sum form to households, while the complementary fraction Θ is consumed by the existing entrepreneurs. This consumption can be taken into account by subtracting

$$\Theta \frac{1-\gamma_t}{\gamma_t}(N_{t+1} - W_t^e)$$

from the right side of Equation (B.25). In practice we do not make this adjustment because we assume Θ is sufficiently small so that the adjustment is negligible.

The representative firm takes the price of final private consumption goods output, P_t^c, as given. Final private consumption goods output is produced using two inputs. The first, C_t^d, is a one-for-one transformation of the homogeneous domestic good and therefore has price P_t. The second input, C_t^m, is the homogeneous composite of specialized consumption import goods discussed in the next subsection. The price of C_t^m is $P_t^{m,c}$. The representative firm takes the input prices P_t and $P_t^{m,c}$ as given. Profit maximization leads to the following demand for the intermediate inputs:

$$C_t^d = (1-\omega_c)\left(\frac{P_t^c}{P_t}\right)^{\eta_c} C_t \tag{B.26}$$

$$C_t^m = \omega_c \left(\frac{P_t^c}{P_t^{m,c}}\right)^{\eta_c} C_t, \tag{B.27}$$

The price of C_t is related to the price of the inputs by

$$P_t^c = \left[(1-\omega_c)(P_t)^{1-\eta_c} + \omega_c (P_t^{m,c})^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}}. \tag{B.28}$$

The rate of inflation of the private consumption good is

$$\pi_t^c = \frac{P_t^c}{P_{t-1}^c} = \left[\frac{(1-\omega_c)(P_t)^{1-\eta_c} + \omega_c (P_t^{m,c})^{1-\eta_c}}{(1-\omega_c)(P_{t-1})^{1-\eta_c} + \omega_c (P_{t-1}^{m,c})^{1-\eta_c}}\right]^{\frac{1}{1-\eta_c}}. \tag{B.29}$$

B.3.2 Public consumption goods

Final public consumption goods are purchased by the government. Analogous to the production of final private consumption goods, these goods are produced by a representative competitive firm using the following linear homogeneous technology:

$$G_{c,t} = \left[(1-\omega_{g,c})^{\frac{1}{\eta_{g,c}}} \left(G^d_{c,t} \right)^{\frac{\eta_{g,c}-1}{\eta_{g,c}}} + \omega_{g,c}^{\frac{1}{\eta_{g,c}}} \left(G^m_{c,t} \right)^{\frac{\eta_{g,c}-1}{\eta_{g,c}}} \right]^{\frac{\eta_{g,c}}{\eta_{g,c}-1}}. \tag{B.30}$$

The representative firm takes the price of final public consumption goods output, $P_t^{g,c}$, as given. Final public consumption goods output is produced using two inputs. The first, $G^d_{c,t}$, is a one-for-one transformation of the homogeneous domestic good and therefore has price P_t. The second input, $G^m_{c,t}$, is the homogeneous composite of specialized consumption import goods discussed in the next subsection. The price of $G^m_{c,t}$ is $P_t^{g,m,c}$. The representative firm takes the input prices P_t and $P_t^{g,m,c}$ as given. Profit maximization leads to the following demand for the intermediate inputs:

$$G^d_{c,t} = (1-\omega_{g,c}) \left(\frac{P_t^{g,c}}{P_t} \right)^{\eta_{g,c}} G_{c,t} \tag{B.31}$$

$$G^m_{c,t} = \omega_{g,c} \left(\frac{P_t^{g,c}}{P_t^{g,m,c}} \right)^{\eta_{g,c}} G_{c,t}, \tag{B.32}$$

The price of $G_{c,t}$ is related to the price of the inputs by

$$P_t^{g,c} = \left[(1-\omega_{g,c})(P_t)^{1-\eta_{g,c}} + \omega_{g,c}(P_t^{g,m,c})^{1-\eta_{g,c}} \right]^{\frac{1}{1-\eta_{g,c}}}. \tag{B.33}$$

The rate of inflation of the public consumption good is

$$\pi_t^{g,c} = \frac{P_t^{g,c}}{P_{t-1}^{g,c}} = \left[\frac{(1-\omega_{g,c})(P_t)^{1-\eta_{g,c}} + \omega_{g,c}(P_t^{g,m,c})^{1-\eta_{g,c}}}{(1-\omega_{g,c})(P_{t-1})^{1-\eta_{g,c}} + \omega_{g,c}(P_{t-1}^{g,m,c})^{1-\eta_{g,c}}} \right]^{\frac{1}{1-\eta_{g,c}}}. \tag{B.34}$$

B.3.3 Private investment goods

Private investment goods are produced by a representative competitive firm using the following technology:

$$I_t + a(u_t)\bar{K}_t = \Psi_t \left[(1-\omega_i)^{\frac{1}{\eta_i}} \left(I^d_t \right)^{\frac{\eta_i-1}{\eta_i}} + \omega_i^{\frac{1}{\eta_i}} \left(I^m_t \right)^{\frac{\eta_i-1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i-1}},$$

where investment is defined as the sum of private investment goods I_t used in the accumulation of physical capital plus private investment goods used in capital maintenance, $a(u_t)\bar{K}_t$. The maintenance is discussed below. See Appendix C for the functional form of $a(u_t)$. The quantity u_t denotes the utilization rate of capital, with capital services being defined by

$$K_t = u_t \bar{K}_t.$$

In order to accommodate the possibility that the price of investment goods relative to the price of consumption goods declines over time, it is assumed that the investment-specific technology shock Ψ_t is a unit root process with a potentially positive drift. As in the consumption good sector, the representative private investment goods producer takes all relevant prices as given. Profit maximization leads to the following demand for the intermediate inputs:

$$I^d_t = (1-\omega_i) \left(\frac{P_t^i}{P_t} \right)^{\eta_i} \left(I_t + a(u_t)\bar{K}_t \right) \tag{B.35}$$

$$I^m_t = \omega_i \left(\frac{P_t^i}{P_t^{m,i}} \right)^{\eta_i} \left(I_t + a(u_t)\bar{K}_t \right). \tag{B.36}$$

The price of I_t is related to the price of the inputs by

$$P_t^i = \left[(1-\omega_i)(P_t)^{\eta_i} + \omega_i (P_t^{m,i})^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}}. \tag{B.37}$$

The rate of inflation of the private investment good is

$$\pi_t^i = \left[\frac{(1-\omega_i)(P_t)^{\eta_i} + \omega_i \left(P_t^{m,i}\right)^{1-\eta_i}}{(1-\omega_i)(P_{t-1})^{\eta_i} + \omega_i \left(P_{t-1}^{m,i}\right)^{1-\eta_i}}\right]^{\frac{1}{1-\eta_i}}. \tag{B.38}$$

B.3.4 Public investment goods

Public capital $K_{g,t}$ accumulates according to the following two equations, where public investment $G_{i,t}$ obeys a condition that incorporates a two-period lag in building public capital from public investment (instead of a one-period lag in building private capital):

$$K_{g,t+1} = (1-\delta_g)K_{g,t} + A_{g,i,t-1}, \tag{B.39}$$

$$G_{i,t} = b_0 A_{g,i,t} + b_1 A_{g,i,t-1}. \tag{B.40}$$

The capital depreciation rate is denoted by δ_g and the parameters b_0 and b_1 control how much needs to be invested over the two years until public capital is fully built and ready to use. If we set $b_0 = 1$ and $b_1 = 0$, the full amount of public investment needs to be put down at time $t-1$ so that public capital is ready to use at time $t+1$.

In contrast to private capital, public capital is not subject to an endogenous utilization rate and is thus always fully utilized in the production of domestic intermediate goods. Moreover, there is no investment-specific technology shock present in the accumulation equation of public capital.

Public investment $G_{i,t}$ is a CES aggregate of the domestic good $G_{i,t}^d$ and the imported good $G_{i,t}^m$:

$$G_{i,t} = \left((1-\omega_{g,i})^{\frac{1}{\eta_{g,i}}} \left(G_{i,t}^d\right)^{\frac{\eta_{g,i}-1}{\eta_{g,i}}} + (\omega_{g,i})^{\frac{1}{\eta_{g,i}}} \left(G_{i,t}^m\right)^{\frac{\eta_{g,i}-1}{\eta_{g,i}}}\right)^{\frac{\eta_{g,i}}{\eta_{g,i}-1}}, \tag{B.41}$$

where the fraction of imports is given by the constant $\omega_{g,i}$ and the elasticity of substitution between the domestic good and the imported good is denoted by $\eta_{g,i}$.

The representative public investment goods producer takes all relevant prices as given. Profit maximization leads to the following demand for the intermediate inputs:

$$G_{i,t}^d = (1-\omega_{g,i}) \left(\frac{P_t^{g,i}}{P_t}\right)^{\eta_{g,i}} G_{i,t} \tag{B.42}$$

$$G_{i,t}^m = \omega_{g,i} \left(\frac{P_t^i}{P_t^{g,m,i}}\right)^{\eta_{g,i}} G_{i,t}. \tag{B.43}$$

The price of $G_{i,t}$ is related to the price of the inputs by

$$P_t^{g,i} = \left[(1-\omega_{g,i})(P_t)^{\eta_{g,i}} + \omega_{g,i} (P_t^{g,m,i})^{1-\eta_{g,i}}\right]^{\frac{1}{1-\eta_{g,i}}}. \tag{B.44}$$

The rate of inflation of the public investment good is

$$\pi_t^{g,i} = \left[\frac{(1-\omega_{g,i})(P_t)^{\eta_{g,i}} + \omega_{g,i} \left(P_t^{m,i}\right)^{1-\eta_{g,i}}}{(1-\omega_{g,i})(P_{t-1})^{\eta_{g,i}} + \omega_{g,i} \left(P_{t-1}^{m,i}\right)^{1-\eta_{g,i}}}\right]^{\frac{1}{1-\eta_{g,i}}}. \tag{B.45}$$

B.3.5 Export goods

Export activities involve Calvo price setting frictions and therefore require the presence of market power. A Dixit-Stiglitz aggregator is used to introduce a range of specialized goods. This allows there to be market power without the counterfactual implication that there is a small number of firms in the export sector. Thus, exports involve a continuum of exporters, each of which is a monopolist which produces a specialized export good. Each monopolist produces the export good using a homogeneous domestically produced good and a homogeneous good derived from imports. The specialized export goods are sold to foreign competitive retailers which create a homogeneous good that is sold to foreign citizens.

There is total demand by foreigners for domestic exports, which takes on the following form:

$$X_t = \left(\frac{P_t^x}{P_t^*}\right)^{-\eta_f} Y_t^f \epsilon_t^x, \tag{B.46}$$

where Y_t^f is foreign demand, P_t^* is the foreign currency price of foreign homogeneous goods, P_t^x is an index of export prices defined below, and ϵ_t^x is a stationary exports-specific technology shock. The goods X_t are produced by a representative competitive foreign retailer firm using specialized inputs as follows:

$$X_t = \left[\int_0^1 X_{i,t}^{\frac{1}{\lambda_x}} di\right]^{\lambda_x}, \tag{B.47}$$

where $X_{i,t}$, $i \in (0,1)$ are specialized intermediate goods for export good production. The retailer that produces X_t takes its output price P_t^x and its input prices $P_{i,t}^x$ as given. Optimization leads to the following demand for specialized exports:

$$X_{i,t} = \left(\frac{P_{i,t}^x}{P_t^x}\right)^{\frac{-\lambda_x}{\lambda_x-1}} X_t. \tag{B.48}$$

Combining Equations (B.47) and (B.48) yields:

$$P_t^x = \left[\int_0^1 \left(P_{i,t}^x\right)^{\frac{1}{1-\lambda_x}} di\right]^{1-\lambda_x}.$$

The i-th specialized export is produced by a monopolist using the following technology:

$$X_{i,t} = \left[\omega_x^{\frac{1}{\eta_x}} \left(X_{i,t}^m\right)^{\frac{\eta_x-1}{\eta_x}} + (1-\omega_x)^{\frac{1}{\eta_x}} \left(X_{i,t}^d\right)^{\frac{\eta_x-1}{\eta_x}}\right]^{\frac{\eta_x}{\eta_x-1}},$$

where $X_{i,t}^m$ and $X_{i,t}^d$ are the i-th exporter's use of the imported and domestically produced goods, respectively. The marginal cost associated with the CES production function is derived from the multiplier associated with the Lagrangian representation of the cost minimization problem:

$$C = \min \tau_t^x \left[P_t^{m,x} R_t^x X_{i,t}^m + P_t R_t^x X_{i,t}^d\right] + \lambda \left\{X_{i,t} - \left[\omega_x^{\frac{1}{\eta_x}} \left(X_{i,t}^m\right)^{\frac{\eta_x-1}{\eta_x}} + (1-\omega_x)^{\frac{1}{\eta_x}} \left(X_{i,t}^d\right)^{\frac{\eta_x-1}{\eta_x}}\right]^{\frac{\eta_x}{\eta_x-1}}\right\},$$

where $P_t^{m,x}$ is the price of the homogeneous import good and P_t is the price of the homogeneous domestic good. Using the first order conditions of this problem and the production function, the real marginal cost is derived as

$$MC_t^x = \frac{\lambda P_t}{S_t P_t^x} = \frac{\tau_t^x R_t^x}{q_t P_t^c P_t^x} \left[\omega_x \left(P_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)(P_t)^{1-\eta_x}\right]^{\frac{1}{1-\eta_x}}, \tag{B.49}$$

where

$$R_t^x = \nu^x R_t + 1 - \nu^x, \tag{B.50}$$

$$\frac{S_t P_t^x}{P_t} = \frac{S_t P_t^f}{P_t^c} \frac{P_t^c}{P_t} \frac{P_t^x}{P_t^f} = q_t \frac{P_t^c}{P_t} \frac{P_t^x}{P_t^f}, \tag{B.51}$$

and q_t denotes the real exchange rate defined as

$$q_t = \frac{S_t P_t^f}{P_t^c}. \tag{B.52}$$

From the solution to the same problem, the demand for domestic inputs for export production is

$$X_{i,t}^d = (1-\omega_x)\left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x} X_{i,t}. \tag{B.53}$$

The quantity of the domestic homogeneous good used by specialized exporters is

$$\int_0^1 X_{i,t}^d di,$$

which, in terms of aggregates, is [by plugging Equation (B.53) into this integral, see also Appendix C]

$$X_t^d = \int_0^1 X_{i,t}^d di = \left[\omega_x (P_t^{m,x})^{1-\eta_x} + (1-\omega_x)(P_t)^{1-\eta_x}\right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x)(\mathring{p}_t^x)^{\frac{-\lambda_x}{\lambda_x-1}} (P_t^x)^{-\eta_f} Y_t^f, \tag{B.54}$$

where $\mathring{P}_t^x$ is a measure of the price dispersion and is defined in Appendix C. Using a similar derivation as for X_t^d,

$$X_t^m = \omega_x \left(\frac{\left[\omega_x (P_t^{m,x})^{1-\eta_x} + (1-\omega_x)(P_t)^{1-\eta_x}\right]^{\frac{1}{1-\eta_x}}}{P_t^{m,x}}\right)^{\eta_x} (\mathring{p}_t^x)^{\frac{-\lambda_x}{\lambda_x-1}} (P_t^x)^{-\eta_f} Y_t^f. \tag{B.55}$$

The i-th export good firm [$i \in (0,1)$] takes Equation (B.48) as its demand curve. This producer sets prices subject to a Calvo sticky-price mechanism. With probability ξ_x, the i-th export good firm cannot reoptimize its price, in which case it updates the price as follows:

$$P_{i,t}^x = \tilde{\pi}_t^x P_{i,t-1}^x, \quad \tilde{\pi}_t^x = (\pi_{t-1}^x)^{\kappa_x} (\pi^x)^{1-\kappa_x-\varkappa_x} (\breve{\pi})^{\varkappa_x}, \tag{B.56}$$

where κ_x, $\varkappa_x$, $\kappa_x + \varkappa_x \in (0,1)$.

The equilibrium conditions associated with price setting by exporters that do get to reoptimize their prices are analogous to the ones derived for the domestic intermediate good producers and are reported in Appendix C. The pricing frictions in exports help the model to produce a hump-shape in the response of output to a domestic monetary shock.

B.4 Imports

In the case of imports, specialized domestic importers purchase a homogeneous foreign good which they turn into a specialized input and sell to domestic retailers. There are five types of domestic retailers. One uses the specialized import goods to create the homogeneous good used as an input into the production of specialized exports. Another uses the specialized import goods to create an input used in the production of private investment goods. Similarly, there are specialized domestic importers who create an input used in the production of public investment goods. The fourth type uses specialized imports to produce a homogeneous input used in the production of private consumption goods. Finally, the fifth type produces a homogeneous input used in the production of public consumption goods.

Imported goods are combined with domestic inputs before being passed on to final domestic users. There are pricing frictions. In all cases it is assumed that prices are set in the currency of the buyer ('pricing to market').[10]

Foreign firms sell a homogeneous good to domestic importers. The importers convert the homogeneous good into a specialized input ('brand name it') and supply that input monopolistically to

[10] Pricing frictions in imports help the model account for the evidence that exchange rate shocks take time to pass into domestic prices.

domestic retailers. Importers are subject to Calvo price setting frictions. There are five types of importing firms: (i) one produces goods used to produce an intermediate good for the production of private consumption, (ii) one produces goods used to produce an intermediate good for the production of public consumption, (iii) one produces goods used to produce an intermediate good for the production of private investment, (iv) one produces goods used to produce an intermediate good for the production of public investment, and (v) one produces goods used to produce an intermediate good for the production of exports.

Consider the first type first (intermediate goods for production of private consumption). The production function of the domestic retailer of imported consumption goods is

$$C_t^m = \left[\int_0^1 (C_{i,t}^m)^{\frac{1}{\lambda_{m,c}}} di\right]^{\lambda_{m,c}},$$

where $C_{i,t}^m$ is the output of the i-th specialized producer and C_t^m is an intermediate good used in the production of private consumption goods. Let $P_t^{m,c}$ denote the price index of C_t^m and let $P_{i,t}^{m,c}$ denote the price of the i-th intermediate input. The domestic retailer is competitive and takes $P_t^{m,c}$ and $P_{i,t}^{m,c}$ as given. The demand curve for specialized inputs is given by the domestic retailer's first order necessary condition for profit maximization:

$$C_{i,t}^m = C_t^m \left(\frac{P_t^{m,c}}{P_{i,t}^{m,c}}\right)^{\frac{\lambda_{m,c}}{\lambda_{m,c}-1}}.$$

We now turn to the producer of $C_{i,t}^m$ who takes the previous equation as a demand curve. This producer buys the homogeneous foreign good and converts it one-for-one into the domestic differentiated good, $C_{i,t}^m$. The intermediate good producer's marginal cost is

$$\tau_t^{m,c} S_t P_t^f R_t^{\nu,*}, \tag{B.57}$$

where

$$R_t^{\nu,*} = \nu^* R_t^* + 1 - \nu^*, \tag{B.58}$$

and where R_t^* is the foreign nominal rate of interest.[11]

As in the homogeneous domestic good sector, $\tau_t^{m,c}$ is a tax-like shock which affects marginal costs but does not appear in a production function.[12]

The total value of imports accounted for by the private consumption sector is

$$S_t P_t^f R_t^{\nu,*} C_t^m (\mathring{p}_t^{m,c})^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}}, \quad \mathring{p}_t^{m,c} = \frac{P_{i,t}^{m,c}}{P_t^{m,c}},$$

where $\mathring{p}_t^{m,c}$ is a measure of price dispersion in the differentiated good, $C_{i,t}^m$.

Next, consider the second type of firms (producing intermediate goods for production of public consumption). The production function of the domestic retailer of imported consumption goods is

$$G_{c,t}^m = \left[\int_0^1 (G_{c,i,t}^m)^{\frac{1}{\lambda_{g,m,c}}} di\right]^{\lambda_{g,m,c}},$$

where $G_{c,i,t}^m$ is the output of the i-th specialized producer and $G_{c,t}^m$ is an intermediate good used in the production of public consumption goods. Let $P_t^{g,m,c}$ denote the price index of $G_{c,t}^m$ and let $P_{i,t}^{g,m,c}$

[11] The notion here is that the intermediate good firm must pay the inputs with foreign currency and because they have no resources themselves at the beginning of the period, they must borrow those resources if they are to buy the foreign inputs needed to produce $C_{i,t}^m$. The financing need is in the foreign currency, so the loan is taken in that currency. There is no risk to this working capital loan because all shocks are realized at the beginning of the period and so there is no uncertainty within the duration of the loan about the realization of prices and exchange rates.

[12] In the linearization of a version of the model in which there are no price and wage distortions in the steady state, $\tau_t^{m,c}$ is isomorphic to a markup shock.

denote the price of the i-th intermediate input. The domestic retailer is competitive and takes $P_t^{g,m,c}$ and $P_{i,t}^{g,m,c}$ as given. The demand curve for specialized inputs is given by the domestic retailer's first order necessary condition for profit maximization:

$$G_{c,i,t}^m = G_{c,t}^m \left(\frac{P_t^{g,m,c}}{P_{i,t}^{g,m,c}} \right)^{\frac{\lambda_{g,m,c}}{\lambda_{g,m,c}-1}}.$$

We now turn to the producer of $G_{c,i,t}^m$ who takes the previous equation as a demand curve. This producer buys the homogeneous foreign good and converts it one-for-one into the domestic differentiated good, $G_{c,i,t}^m$. The intermediate good producer's marginal cost is

$$\tau_t^{g,m,c} S_t P_t^f R_t^{\nu,*}, \tag{B.59}$$

where

$$R_t^{\nu,*} = \nu^* R_t^* + 1 - \nu^*, \tag{B.60}$$

and where R_t^* is the foreign nominal rate of interest. The variable $\tau_t^{g,m,c}$ is a tax-like shock which affects marginal costs but does not appear in a production function.

The total value of imports accounted for by the public consumption sector is

$$S_t P_t^f R_t^{\nu,*} G_{c,t}^m (\mathring{p}_t^{g,m,c})^{\frac{\lambda_{g,m,c}}{1-\lambda_{g,m,c}}}, \quad \mathring{p}_t^{g,m,c} = \frac{P_{i,t}^{g,m,c}}{P_t^{g,m,c}}$$

where $\mathring{p}_t^{g,m,c}$ is a measure of price dispersion in the differentiated good, $G_{c,i,t}^m$.

Now consider the third type of firms (private investment goods). The production function for the domestic retailer of imported private investment goods, I_t^m, is

$$I_t^m = \left[\int_0^1 (I_{i,t}^m)^{\frac{1}{\lambda_{m,i}}} di \right]^{\lambda_{m,i}}.$$

The retailer of imported private investment goods is competitive and takes output prices $P_t^{m,i}$ and input prices $P_{i,t}^{m,i}$ as given.

The producer of the i-th intermediate input in the above production function buys the homogeneous foreign good and converts it one-for-one into the differentiated good, $I_{i,t}^m$. The marginal cost of $I_{i,t}^m$ is the analogue of Equation (B.57):

$$\tau_t^{m,i} S_t P_t^f R_t^{\nu,*},$$

which implies the importing firm's cost is P_t^* (before borrowing costs, exchange rate conversion, and markup shocks), which is the same cost for the specialized inputs used to produce C_t^m.

The total value of imports associated with the production of private investment goods is analogous to what was obtained for the private consumption good sector:

$$S_t P_t^* R_t^{\nu,*} I_t^m (\mathring{p}_t^{m,i})^{\frac{\lambda_{m,i}}{1-\lambda_{m,i}}}, \quad \mathring{p}_t^{m,i} = \frac{P_{i,t}^{m,i}}{P_t^{m,i}}. \tag{B.61}$$

The fourth type of firms (public investment goods) is considered in what follows. The production function for the domestic retailer of imported public investment goods, $G_{i,t}^m$, is

$$G_{i,t}^m = \left[\int_0^1 (G_{i,j,t}^m)^{\frac{1}{\lambda_{g,m,i}}} dj \right]^{\lambda_{g,m,i}}.$$

The retailer of imported public investment goods is competitive and takes output prices $P_t^{g,m,i}$ and input prices $P_{j,t}^{g,m,i}$ as given.

The producer of the i-th intermediate input in the above production function buys the homogeneous foreign good and converts it one-for-one into the differentiated good, $G_{i,j,t}^m$. The marginal cost of $G_{i,j,t}^m$ is the analogue of Equation Equation (B.59):

$$\tau_t^{g,m,i} S_t P_t^f R_t^{\nu,*},$$

which implies the importing firm's cost is P_t^* (before borrowing costs, exchange rate conversion, and markup shocks), which is the same cost for the specialized inputs used to produce $G_{c,t}^m$.

The total value of imports associated with the production of public investment goods is analogous to what was obtained for the public consumption good sector:

$$S_t P_t^* R_t^{\nu,*} G_{i,t}^m (\mathring{p}_t^{g,m,i})^{\frac{\lambda_{g,m,i}}{1-\lambda_{g,m,i}}}, \quad \mathring{p}_t^{g,m,i} = \frac{P_{j,t}^{g,m,i}}{P_t^{g,m,i}}. \tag{B.62}$$

Finally, consider the fifth type of firms producing intermediate goods for exports. The production function of the domestic retailer of imported goods used in the production of the input X_t^m for the production of export goods is

$$X_t^m = \left[\int_0^1 (X_{i,t}^m)^{\frac{1}{\lambda_{m,x}}} di\right]^{\lambda_{m,x}}.$$

The imported good retailer is competitive and takes output prices $P_t^{m,x}$ and input prices $P_{i,t}^{m,x}$ as given. The producer of the specialized input $X_{i,t}^m$ has marginal cost

$$\tau_t^{m,x} S_t P_t^f R_t^{\nu,*}.$$

The total value of imports associated with the production of X_t^m is

$$S_t P_t^* R_t^{\nu,*} X_t^m (\mathring{p}_t^{m,x})^{\frac{\lambda_{m,x}}{1-\lambda_{m,x}}}, \quad \mathring{p}_t^{m,x} = \frac{P_{i,t}^{m,x}}{P_t^{m,x}}. \tag{B.63}$$

Each of the above five types of intermediate good firm is subject to Calvo price-setting frictions. With probability $1-\xi_{m,j}$, the j-th type of firm can reoptimize its price and with probability $\xi_{m,j}$ it updates its price according to

$$P_{i,t}^{m,j} = \tilde{\pi}_t^{m,j} P_{i,t-1}^{m,j}, \quad \tilde{\pi}_t^{m,j} := (\pi_{t-1}^{m,j})^{\kappa_{m,j}} (\bar{\pi}_t^c)^{1-\kappa_{m,j}-\varkappa_{m,j}} \check{\pi}^{\varkappa_{m,j}}, \quad j = c, i, x. \tag{B.64}$$

The equilibrium conditions associated with price setting by importers are analogous to the ones derived for domestic intermediate good producers and are reported in Appendix C.

B.5 Wholesalers and the labour market

The law of motion for aggregate employment, L_t, is given by:

$$L_t = (\rho_{t-1} + \chi_t) L_{t-1}. \tag{B.65}$$

Here, ρ_{t-1} is the probability that a given firm/worker match continues from one period to the next, and its law of motion is given by $\log \rho_t = (1-\rho_\rho)\log\rho + \rho_\rho \log \rho_{t-1} + \epsilon_{\rho,t}$. Note that ρ_{t-1} is predetermined in Equation (B.65).[13] So, $\rho_{t-1}L_{t-1}$ denotes the number of workers that were attached to firms in period $t-1$ and remain attached at the start of period t. Also, $\chi_t L_{t-1}$ denotes the number of new firm/worker meetings at the start of period t. The variable χ_t denotes the hiring rate which is implicitly determined in equilibrium. In equilibrium, meetings always result in employment; that is, the number of new matches m_t is

$$m_t = \chi_t L_{t-1}, \tag{B.66}$$

where, according to Equation (B.65), workers are engaged in production as soon as they are hired.

The number of workers searching for work at the start of period t is the sum of the number of unemployed workers in period $t-1$, $1 - L_{t-1}$, and the number of workers that separate from firms at

[13] Contrary to the assumption of a fixed separation rate in Christiano et al. (2016), here this is a simple way of modeling the time-varying separation rate that allows to match this variable to the data. It is reasonable to assume that the decision on separation takes some time and thus it is made before the current period's shocks are realized.

the end of period $t-1$, $(1-\rho_{t-1})L_{t-1}$. The probability f_t that a searching worker meets a firm is given by:

$$f_t = \frac{\chi_t L_{t-1}}{1-\rho_{t-1}L_{t-1}}. \tag{B.67}$$

Wholesaler firms produce the specialized labour input using labour which has a fixed marginal productivity of unity. A wholesaler firm that wishes to meet a worker in period t must post a vacancy at cost K_t^v, expressed in units of the homogeneous domestic good. The vacancy is filled with probability Q_t:

$$Q_t = \chi_t / v_t, \tag{B.68}$$

where v_t is the vacancy rate, so that the total number of vacancies posted by firms at the start of period t is:

$$v_t^{tot} = v_t L_{t-1}. \tag{B.69}$$

In case the vacancy is filled, the firm must pay a fixed real cost, K_t^v, before bargaining with the newly-matched worker. Let J_t denote the value of a worker to the firm expressed in units of the homogeneous domestic good:

$$J_t = \Theta_t^p - W_t^p. \tag{B.70}$$

Here, Θ_t^p denotes the expected present value of the nominal price of the specialized labour input over the duration of the worker/firm match. Also, W_t^p denotes the expected present value of the nominal labour cost paid by the firm. The recursive form is as follows:

$$\Theta_t^p = P_t^H + \rho_t \mathbb{E}_t[\mathbb{M}_{t,t+1}^o \Theta_{t+1}^p], \tag{B.71}$$

where $\mathbb{M}_{t,t+1}^o$ is the stochastic discount factor of the optimizing households which firms and workers view as an exogenous stochastic process (the stochastic discount factor of optimizing households is used here, as these households own the firms in the economy). Similarly, we have:

$$W_t^p = (1+\tau_{e,t}^w)W_t + \rho_t \mathbb{E}_t[\mathbb{M}_{t,t+1}^o W_{t+1}^p]. \tag{B.72}$$

Here, $\tau_{e,t}^w$ denotes the employer's social security contribution rate. Free entry by wholesalers implies that, in equilibrium, the expected benefit of a vacancy equals the cost:

$$Q_t\left(J_t - K_t^h\right) = K_t^v, \tag{B.73}$$

where K_t^h is the hiring cost a firm must pay and K_t^v is the firm's vacancy posting cost. The present value of (net) wage received by the worker is:

$$W_{p,t} = (1-\tau_t^y - \tau_{w,t}^w)W_t + \rho_t \mathbb{E}_t[((1-\lambda_r)\mathbb{M}_{t+1}^o + \lambda_r \mathbb{M}_{t+1}^r)W_{p,t+1}], \tag{B.74}$$

where τ_t^y denotes the labour income tax rate and $\tau_{w,t}^w$ stands for the employee's social security contribution rate. Moreover, $\mathbb{M}_{t+1}^r = \beta v_{t+1}^r / v_t^r$ is the stochastic discount factor of restricted households and λ_r $(1-\lambda_r)$ the share of restricted (optimizing) households in the economy. It enters the present value of work to a worker:

$$V_t = W_{p,t} + A_t^w, \tag{B.75}$$

where A_t^w is the present value of worker pay-off after separation,

$$\begin{aligned} A_t^w = {} & (1-\rho_t)\,\mathbb{E}_t\Big[((1-\lambda_r)\mathbb{M}_{t+1}^o + \lambda_r \mathbb{M}_{t+1}^r)\left[f_{t+1}V_{t+1} + (1-f_{t+1})\,U_{t+1}\right]\Big] \\ & + \rho_t \mathbb{E}_t[((1-\lambda_r)\mathbb{M}_{t+1}^o + \lambda_r \mathbb{M}_{t+1}^r)A_{t+1}^w]. \end{aligned} \tag{B.76}$$

Here, U_t denotes the value of being unemployed,

$$U_t = B_t^u + \tilde{U}_t, \tag{B.77}$$

and $\tilde{U}_t$ denotes the continuation value of unemployment:

$$\tilde{U}_t := \mathbb{E}_t\left[((1-\lambda_r)\mathbb{M}_{t+1}^o + \lambda_r\mathbb{M}_{t+1}^r)\left[f_{t+1}V_{t+1} + (1-f_{t+1})\,U_{t+1}\right]\right]. \tag{B.78}$$

We deviate from the literature by assuming that B_t^u is endogenous and pro-cyclical, in particular, linked to the labour market conditions. The practical reason of doing so is that the pro-cyclical worker outside option is a simple mechanism generating pro-cyclical worker bargaining power, thus the much needed pro-cyclical wages. Although we fully acknowledge that this mechanism is not built from micro-foundations and that the true reasons for wage pro-cyclicality may be different (thus more work on this mechanism is justified), it can be reasoned that labour market conditions affect the outside option of a worker. When the labour market is slack – as in a downturn – a worker faces less options of finding a job elsewhere, thus her bargaining power is lower compared to normal or boom times, hence she might accept a lower wage at the current employer. And, vice versa, as the labour market tightens, firms compete for workers more, which is reflected in a higher bargaining power of a worker via her higher outside option; as a result, there is a pressure on labour costs. This mechanism assumes relatively frequent and de-centralized wage re-negotiation, which may suit Latvia, as labour unions in Latvia are scarce.

This mechanism is formalized via linking the worker outside option $B_t^u = b_t^u P_t z_t^+$ to the labour market tightness[14]:

$$b_t^u = (b_{t-1}^u)^{\rho_{b^u}}(b^u)^{1-\rho_{b^u}}\left(\frac{1-L}{1-L_{t-1}}\right)^{(1-\rho_{b^u})\alpha^u}\exp(\epsilon_{b^u,t}), \tag{B.79}$$

where the term in the large parentheses is the measure of labour market tightness, that is, the unemployment rate relative to the non-accelerating inflation rate of unemployment (NAIRU), the latter being for simplicity assumed constant and equal to the steady state of unemployment rate. The parameter α^u controls for the sensitivity of the worker bargaining power to the labour market conditions, the persistence parameter ρ_{b^u} allows for persistence in the worker outside option. This expression adds a new shock $\epsilon_{b^u,t}$ to the model, which we interpret as a wage cost-push shock. Note that in the resulting wage setting mechanism, there is no additionally-imposed wage frictions or indexation. Also note that it is straightforward to switch back to the standard setup by calibrating $\alpha^u = 0$.

The vacancy filling rate, Q_t, and the job finding rate for workers, f_t, are assumed to be related to labour market tightness, Γ_t, as follows:

$$f_t = \sigma_m\Gamma_t^{1-\sigma}, \quad Q_t = \sigma_m\Gamma_t^{-\sigma}, \quad \sigma_m > 0, \quad 0 < \sigma < 1,$$

where

$$\Gamma_t = \frac{v_t L_{t-1}}{1-\rho_{t-1}L_{t-1}}. \tag{B.80}$$

In other words, the job matching function is a weighted product of the total number of job-seekers, $1-\rho_{t-1}L_{t-1}$, and the total number of vacancies, $v_t L_{t-1}$:

$$m_t = \sigma_m(1-\rho_{t-1}L_{t-1})^{\sigma}(v_t L_{t-1})^{1-\sigma}. \tag{B.81}$$

[14] In the codes, the expression is entered in its log form

$$\log b_t^u = \rho_{b^u}\log b_{t-1}^u + (1-\rho_{b^u})\log b^u + (1-\rho_{b^u})\alpha^u\left(\log(1-L) - \log(1-L_{t-1})\right) + \epsilon_{b^u,t}.$$

The bargained gross wage W_t is determined by maximizing the Nash product with respect to wage:

$$\max_{W_t} \left\{ (V_t - U_t)^\eta J_t^{1-\eta} \right\}, \tag{B.82}$$

yielding the following first-order condition:

$$J_t = \frac{1 + \tau_{e,t}^w}{1 - \tau_t^y - \tau_{w,t}^w} \frac{1-\eta}{\eta} (V_t - U_t). \tag{B.83}$$

Thus, the presence of labour taxes affects the bargained outcome by altering the shares of surplus going to a worker and a firm.

Market clearing of specialized labour inputs requires

$$\int_0^1 H_{i,t} di = L_t. \tag{B.84}$$

B.6 Financial friction

A number of activities in the model require financing. Producers of specialized inputs must borrow working capital within the period. The management of capital involves financing because the construction of capital requires a substantial initial outlay of resources, while the return from capital comes in over time as a flow. Without financial frictions, financing requirements only slightly affect the allocations. This is because none of the messy realities of actual financial markets are present. There is no asymmetric information between borrower and lender, there is no risk to lenders. In the case of capital accumulation, the borrower and lender are actually the same household who puts up finances and later reaps the rewards. When real-world financial frictions are introduced into the model, then intermediation becomes distorted by the presence of balance sheet constraints and other factors.

This subsection assumes that the accumulation and management of capital involves frictions following Bernanke et al. (1999). It is assumed that working capital loans are frictionless.

Recall that households deposit money with banks, and that the interest rate that households receive is nominally non state-contingent. This gives rise to potentially interesting wealth effects of the sort emphasized by Fisher (1933). The banks then lend funds to entrepreneurs using a standard nominal debt contract which is optimal given the asymmetric information. The amount that banks are willing to lend to an entrepreneur under the standard debt contract is a function of the entrepreneurial net worth. This is how balance sheet constraints enter the model. When a shock occurs that reduces the value of the entrepreneurial assets, this cuts into their ability to borrow. As a result, they acquire less capital and this translates into a reduction in investment and ultimately into a slowdown in the economy.

Although individual entrepreneurs are risky, banks themselves are not. It is supposed that banks lend to a sufficiently diverse group of entrepreneurs that the uncertainty that exists in individual entrepreneurial loans washes out across all loans. The net worth of entrepreneurs is empirically measured by using a stock market index.

Entrepreneurs all have different histories, as they experience different idiosyncratic shocks. Thus, in general, solving for the aggregate variables would require also solving for the distribution of entrepreneurs according to their characteristics and for the law of motion for that distribution. However, as emphasized in Bernanke et al. (1999), the right functional form assumption have been made in the model to guarantee the result that the aggregate variables associated with entrepreneurs are not a function of distributions. The loan contract specifies that all entrepreneurs, regardless of their net worth, receive the same interest rate. Also, the loan amount received by an entrepreneur is proportional to his level of net worth. These characteristics are enough to guarantee the aggregation result. The financial frictions bring a net increase of two equations over the equations in the baseline model. In addition, they introduce two new endogenous variables, one related to the interest rate paid by entrepreneurs and the other to their net worth. The financial frictions also allow to introduce two new shocks. A formal discussion of the model follows.

B.6.1 The individual entrepreneur

At the end of period t each entrepreneur has a level of net worth, N_{t+1}. The entrepreneur's net worth, N_{t+1}, constitutes his state at this time, and nothing else about his history is relevant. There are many entrepreneurs for each level of net worth and for each level of net worth there is a competitive bank with free entry that offers a loan contract. The contract is defined by a loan amount and by an interest rate, both of which are derived as the solution to a particular optimization problem.

Consider a type of entrepreneur with particular level of net worth, N_{t+1}. The entrepreneur combines this net worth with a bank loan, B_{t+1}, to purchase new installed physical capital, $\bar{K}_{t+1}$, from the household (or, equivalently, one could assume capital producers are present in the economy who manage the capital stock). The loan the entrepreneur requires for this is

$$B_{t+1} = P_t P_{k',t} \bar{K}_{t+1} - N_{t+1}. \tag{B.85}$$

The entrepreneur is required to pay the gross interest rate Z_{t+1} on the bank loan at the end of period $t+1$, if it is feasible to do so. After purchasing capital, the entrepreneur experiences an idiosyncratic productivity shock which converts the purchased capital, $\bar{K}_{t+1}$, into $\bar{K}_{t+1}\omega$, where ω is a unit mean, log-normally and independently distributed random variable across entrepreneurs with $V(\log \omega) = \sigma_t^2$. The t subscript indicates that σ_t is itself the realization of a random variable. This allows to consider the effects of an increase in the riskiness of individual entrepreneurs and we call σ_t the shock to idiosyncratic uncertainty. Denote the cumulative distribution function of ω by $F(\omega;\sigma)$ and its partial derivatives by $F_\omega(\omega;\sigma)$ and $F_\sigma(\omega;\sigma)$.

After observing the period $t+1$ shocks, the entrepreneur sets the utilization rate, u_{t+1}, of capital and rents out capital in competitive markets at the nominal rental rate, $P_{t+1}r_{t+1}^k$. In choosing the capital utilization rate, the entrepreneur takes into account that operating one unit of physical capital at rate u_{t+1} requires $a(u_{t+1})$ of domestically produced investment goods for maintenance expenditures, where a is defined in Appendix C. The first order condition associated with capital utilization is given by:[15]

$$r_t^k = \frac{P_t^i a'(u_t)}{P_t}. \tag{B.86}$$

The entrepreneur then sells the undepreciated part of physical capital back to the households. Per unit of physical capital purchased, the entrepreneur who draws idiosyncratic productivity ω earns a return (after taxes) of $R_{t+1}^k\omega$, where R_{t+1}^k is defined in Equation (B.6). Because the mean of ω across entrepreneurs is unity, the average return across all entrepreneurs is R_{t+1}^k.

After entrepreneurs sell their capital, they settle their bank loans. At this point the resources available to an entrepreneur who has purchased $\bar{K}_{t+1}$ units of physical capital in period t and who experiences an idiosyncratic productivity shock ω are $P_t P_{k',t} R_{t+1}^k \omega \bar{K}_{t+1}$. There is a cutoff value of ω, $\bar{\omega}_{t+1}$, such that the entrepreneur has just enough resources to pay interest:

$$\bar{\omega}_{t+1} R_{t+1}^k P_t P_{k',t} \bar{K}_{t+1} = Z_{t+1} B_{t+1}. \tag{B.87}$$

Entrepreneurs with $\omega < \bar{\omega}_{t+1}$ are bankrupt and turn over all their resources,

$$R_{t+1}^k \omega P_t P_{k',t} \bar{K}_{t+1},$$

which is less than $Z_{t+1}B_{t+1}$, to the bank. In this case, the bank monitors the entrepreneur at the cost

$$\mu R_{t+1}^k \omega P_t P_{k',t} \bar{K}_{t+1},$$

where $\mu \geq 0$ is a parameter.

Banks obtain the funds loaned in the period t to entrepreneurs by issuing deposits to households at gross nominal rate of interest R_t. The subscript on R_t indicates that the payoff to households in $t+1$ is not contingent on the period $t+1$ uncertainty. There is no risk in household bank deposits.

There is competition and free entry among banks and banks participate in no financial arrangements other than the liabilities issued to households and the loans issued to entrepreneurs. It follows that the

[15] The tax rate on capital income does not enter here because of the deductibility of maintenance costs.

bank's cash flow in each state of period $t+1$ is zero for each loan amount.[16] For loans in the amount B_{t+1}, the bank receives gross interest $Z_{t+1}B_{t+1}$ from the fraction $1-F(\bar{\omega}_{t+1};\sigma_t)$ of entrepreneurs who are not bankrupt. The bank takes all the resources possessed by bankrupt entrepreneurs, net of monitoring costs. Thus, the state-by-state zero profit condition is

$$[1-F(\bar{\omega}_{t+1};\sigma_t)]Z_{t+1}B_{t+1}+(1-\mu)\int_0^{\bar{\omega}_{t+1}}\omega dF(\omega;\sigma_t)R^k_{t+1}P_tP_{k',t}\bar{K}_{t+1}=R_tB_{t+1},$$

or, after making use of Equation (B.87) and rearranging,

$$[\Gamma(\bar{\omega}_{t+1};\sigma_t)-\mu G(\bar{\omega}_{t+1};\sigma_t)]\frac{R^k_{t+1}}{R_t}\varrho_t=\varrho_t-1, \tag{B.88}$$

where

$$\begin{aligned} G(\bar{\omega}_{t+1};\sigma_t) &= \int_0^{\bar{\omega}_{t+1}}\omega dF(\omega;\sigma_t) \\ \Gamma(\bar{\omega}_{t+1};\sigma_t) &= \bar{\omega}_{t+1}[1-F(\bar{\omega}_{t+1};\sigma_t)]+G(\bar{\omega}_{t+1};\sigma_t) \\ \varrho_t &= \frac{P_tP_{k',t}\bar{K}_{t+1}}{N_{t+1}}. \end{aligned}$$

The expression $\Gamma(\bar{\omega}_{t+1};\sigma_t)-\mu G(\bar{\omega}_{t+1};\sigma_t)$ is the share of revenues earned by entrepreneurs that borrow B_{t+1} which goes to banks. Note that $\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)=1-F(\bar{\omega}_{t+1};\sigma_t)>0$ and $G_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)=\bar{\omega}_{t+1}F_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)>0$. Therefore, the share of entrepreneurial revenues accruing to banks is non-monotone with respect to $\bar{\omega}_{t+1}$.[17]

The optimal contract is derived in Appendix C. ϱ_t and $\bar{\omega}_{t+1}$ are the same for all entrepreneurs regardless of their net worth. This result for the leverage ratio ϱ_t implies that

$$\frac{B_{t+1}}{N_{t+1}}=\varrho_t-1,$$

i.e., an entrepreneur's loan amount is proportional to his net worth. Rewriting Equations (B.85) and (B.87), the rate of interest paid by the entrepreneur is

$$Z_{t+1}=\frac{\bar{\omega}_{t+1}R^k_{t+1}}{1-\frac{N_{t+1}}{P_tP_{k',t}\bar{K}_{t+1}}}=\frac{\bar{\omega}_{t+1}R^k_{t+1}}{1-\frac{1}{\varrho_t}}, \tag{B.89}$$

which also is the same for all entrepreneurs regardless of their net worth.

B.6.2 Aggregation across entrepreneurs and the external financing premium

The law of motion for the net worth on an individual entrepreneur is

$$V_t=R^k_tP_{t-1}P_{k',t-1}K_t-\Gamma(\bar{\omega}_t;\sigma_{t-1})R^k_tP_{t-1}P_{k',t-1}K_t.$$

[16] Absence of state contingent securities markets guarantee that cash flow is non-negative. Free entry guarantees that ex ante profits are zero. Given that each state of nature receives positive probability, the two assumptions imply the state-by-state zero profit condition.

[17] Bernanke et al. (1999) argue that the expression on the left of (B.88) has an inverted 'U' shape, achieving a maximum value at $\bar{\omega}_{t+1}=\omega^*$. The expression is increasing for $\bar{\omega}_{t+1}<\omega^*$ and decreasing for $\bar{\omega}_{t+1}>\omega^*$. Thus, for any given value of the leverage ratio, ϱ_t, and R^k_{t+1}/R_t, there are either no values of $\bar{\omega}_{t+1}$ or two that satisfy Equation (B.88). The value of $\bar{\omega}_{t+1}$ realized in equilibrium must be the one on the left side of inverted 'U' shape. This is because, according to Equation (B.87), the lower value of $\bar{\omega}_{t+1}$ corresponds to a lower interest rate for entrepreneurs which yields them higher welfare. The equilibrium contract is the one that maximizes entrepreneurial welfare subject to the zero profit condition on banks. This reasoning leads to the conclusion that $\bar{\omega}_{t+1}$ falls with a period $t+1$ shock that drives R^k_{t+1} up. The fraction of entrepreneurs that experience bankruptcy is $F(\bar{\omega}_{t+1};\sigma_t)$, so it follows that a shock which drives up R^k_{t+1} has a negative contemporaneous impact on the bankruptcy rate. According to Equation (B.6), shocks that drive R^k_{t+1} up include anything which raises the value of physical capital and/or the rental rate of capital.

Each entrepreneur faces an identical and independent probability $1-\gamma_t$ of being selected to exit the economy. With the probability γ_t each entrepreneur remains. Because the selection is random, the net worth of the entrepreneurs who survive is $\gamma_t \bar{V}_t$. A fraction $1-\gamma_t$ of new entrepreneurs arrive. Entrepreneurs who survive or who are new arrivals receive a transfer W_t^e. This ensures that all entrepreneurs, whether new arrivals or survivors that experienced bankruptcy, have sufficient funds to obtain at least some amount of loans. The average net worth across all entrepreneurs after the W_t^e transfers have been made and exits and entry have occurred, is $\bar{N}_{t+1} = \gamma_t \bar{V}_t + W_t^e$, or

$$\bar{N}_{t+1} = \gamma_t \left\{ R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \left[R_{t-1} + \frac{\mu \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t}{P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t} \right] \right.$$
$$\left. \times \left(P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t \right) \right\} + W_t^e. \tag{B.90}$$

where upper bar over a letter denotes its aggregate average value. Because of its direct effect on entrepreneurial net worth, γ_t is referred to as the shock to net worth. For a derivation of the aggregation across entrepreneurs, see Appendix C.

We now turn to the external financing premium for entrepreneurs. The cost to the entrepreneur of internal funds (i.e., his own net worth) is the interest rate R_t which he loses by applying it to capital rather than buying a risk-free domestic asset. The average payment by all entrepreneurs to the bank is the entire object in square brackets in Equation (B.90). So, the term involving μ represents the excess of external funds over the internal cost of funds. As a result, this is one measure of the financing premium in the model. Another is $Z_{t+1} - R_t$, the excess of the interest rate paid by entrepreneurs who are not bankrupt over the risk-free rate. This paper calls this the interest rate spread.

B.7 Foreign economy

The adaptation of the DSGE model for forecasting purposes over the last years has altered the specification of the foreign block to include such observables as foreign demand (measured as the weighted average imports of the trading partners), the competitors' prices, as well as the nominal effective exchange rate.

The representation of foreign variables takes into account the assumption that the euro area output, Y_t^*, is affected by disturbances to z_t^+, just as domestic variables are. In particular,

$$\ln(Y_t^*) = \ln(y_t^*) + \ln(z_t^+) = \ln(y_t^*) + \ln(z_t) + \frac{\alpha}{1-\alpha} \ln(\psi_t),$$

where $\ln(y_t^*)$ is assumed to be a stationary process.

The first five rows below represent the euro area (EA) output, EA inflation, EA interest rate, the neutral unit-root technological process, and the (suspended in the benchmark model) investment-specific unit-root technological process. The sixth and seventh rows reflect competitors' prices and foreign demand, while the last row is the log-difference of the nominal effective exchange rate, modeled as an AR(1) process.

$$
\begin{pmatrix}
\ln\left(\frac{y_t^*}{y^*}\right) \\
\pi_t^* - \pi^* \\
R_t^* - R^* \\
\ln\left(\frac{\mu_{z,t}}{\mu_z}\right) \\
\ln\left(\frac{\mu_{\psi,t}}{\mu_\psi}\right) \\
\pi_t^f - \pi^f \\
\ln\left(\frac{y_t^f}{y^f}\right) \\
\ln\left(s_t\right)
\end{pmatrix}
=
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & 0 & 0 & 0 & 0 & 0 \\
a_{21} & a_{22} & a_{23} & a_{24} & \frac{a_{24}\alpha}{1-\alpha} & a_{26} & 0 & 0 \\
a_{31} & a_{32} & a_{33} & a_{34} & \frac{a_{34}\alpha}{1-\alpha} & 0 & 0 & 0 \\
0 & 0 & 0 & \rho_{\mu_z} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \rho_{\mu_\psi} & 0 & 0 & 0 \\
a_{61} & a_{62} & 0 & 0 & 0 & a_{66} & a_{67} & 0 \\
a_{71} & 0 & 0 & 0 & 0 & a_{76} & a_{77} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_s
\end{bmatrix}
\begin{pmatrix}
\ln\left(\frac{y_{t-1}^*}{y^*}\right) \\
\pi_{t-1}^* - \pi^* \\
R_{t-1}^* - R^* \\
\ln\left(\frac{\mu_{z,t-1}}{\mu_z}\right) \\
\ln\left(\frac{\mu_{\psi,t-1}}{\mu_\psi}\right) \\
\pi_{t-1}^f - \pi^f \\
\ln\left(\frac{y_{t-1}^f}{y^f}\right) \\
\ln\left(s_{t-1}\right)
\end{pmatrix}
$$

$$
+
\begin{bmatrix}
\sigma_{y^*} & 0 & 0 & 0 & 0 & 0 & c_{17} & 0 \\
c_{21} & \sigma_{\pi^*} & 0 & c_{24} & \frac{c_{24}\alpha}{1-\alpha} & 0 & 0 & 0 \\
c_{31} & c_{32} & \sigma_{R^*} & c_{34} & \frac{c_{34}\alpha}{1-\alpha} & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_{\mu_z} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma_{\mu_\psi} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma_{\pi^f} & c_{67} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \sigma_{y^f} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_s
\end{bmatrix}
\begin{pmatrix}
\varepsilon_{y^*,t} \\
\varepsilon_{\pi^*,t} \\
\varepsilon_{R^*,t} \\
\varepsilon_{\mu_z,t} \\
\varepsilon_{\mu_\psi,t} \\
\varepsilon_{\pi^f,t} \\
\varepsilon_{y^f,t} \\
\varepsilon_{s,t}
\end{pmatrix},
\tag{B.91}
$$

where ε_t's are mean zero, unit variance, Gaussian i.i.d. processes uncorrelated with each other. Note that $\mu_{z,t} = z_t/z_{t-1}$ and $\mu_{\psi,t} = \psi_t/\psi_{t-1}$.

B.8 Fiscal sector

Total public expenditure is a constant fraction η_g of GDP in the steady state. The total public expenditure consists of public consumption expenditure, public investment expenditure, total government transfers to households, unemployment benefits expenditure, and interest payments on public debt. The residual part is wasteful public expenditure Z_t. Wasteful public expenditure is given by the sum of a constant and an exogenous wasteful spending shock, $Z_t = \bar{Z} + \varepsilon_{g,t}$. This implies the following breakdown of total public expenditure:

$$
G_t = G_{c,t}^{exp} + G_{i,t}^{exp} + TR_t + D_{b,t}(1 - L_t) + \ln(R_{g,t-1})/\pi_t \cdot D_{g,t-1} + Z_t. \tag{B.92}
$$

The debt interest rate $R_{g,t}$ between time t and $t+1$ equals the foreign interest rate (i.e. Euribor) plus a government-specific risk premium:

$$
R_{g,t} = \Phi_{g,t} R_t^*, \tag{B.93}
$$

$$
\Phi_{g,t} = \ln(R) - \ln(R^*) + \tilde{\phi}_{g,r}\left(\frac{D_{g,t}}{Y_t + Y_{t-1} + Y_{t-2} + Y_{t-3}} - \overline{dgy}\right) + \varepsilon_{g,t}^{rp}. \tag{B.94}
$$

Above, R_t^* is the foreign nominal (risk-free) interest rate, R_t is the domestic nominal (risk-free) interest rate, $\varepsilon_{g,t}^{rp}$ is a public debt risk premium shock, and $\tilde{\phi}_{g,r}$ is a parameter controlling the sensitivity of the risk premium to deviations of the public debt to GDP ratio from target. Specifically, $\tilde{\phi}_{g,r} > 0$ allows to model a risk premium on public debt depending on the annual public debt to GDP ratio of the domestic economy.

On the revenue side, the government levies distortionary taxes on consumption τ_t^c, capital income τ_t^k, labour income τ_t^y, as well as SSC of employers $\tau_{e,t}^w$ and employees $\tau_{w,t}^w$. The total real tax revenue

T_t from distortionary and lump-sum taxes $T_{ls,t}$ is given by:[18]

$$T_t = \tau_t^c p_t^c(\lambda_r C_{r,t} + (1-\lambda_r)C_{o,t}) + (\tau_t^y + \tau_{e,t}^w + \tau_{w,t}^w)\frac{W_t L_t}{P_t} + \frac{\tau_t^k \bar{K}_t}{P_t}[u_t r_t^k - P_t^i a(u_t) - \delta P_t^i] + T_{ls,t}. \tag{B.95}$$

The government budget constraint is given by:

$$G_t = \text{Deficit}_{g,t} + T_t, \tag{B.96}$$

where $\text{Deficit}_{g,t}$ is the fiscal deficit at time t. Real public debt is governed by the following law of motion:

$$D_{g,t} = D_{g,t-1}/\pi_t + \text{Deficit}_{g,t}. \tag{B.97}$$

The steady-state or target level of public debt is denoted by $\overline{dgy}$. Only a fraction ω_h of public debt is held by domestic households in the steady state, the rest is held abroad. The variable $B_{g,t}$ denotes domestic public debt holdings, and the variable $B_{g,t}^*$ denotes foreign public debt holdings between time $t-1$ and t. In particular, domestic households can hold a constant share of the total public debt $\overline{B}_{g,t+1} = \omega_h D_{g,t}$ freely, and need to pay quadratic adjustment costs $\Gamma_{g,t} = 0.5\gamma_g(B_{g,t+1} - \overline{B}_{g,t+1})^2/\overline{B}_{g,t+1}$ at time t for deviating from this level. Therefore, public debt holdings evolve as follows:

$$D_{g,t} = B_{g,t+1} + B_{g,t+1}^*, \qquad B_g = \omega_h D_g. \tag{B.98}$$

The optimizing household's first order condition with respect to public debt holdings is equal to:

$$B_{g,t+1} = \left(1 + \gamma_g^{-1}\left(\mathbb{E}_t\left[\mathbb{M}_{t,t+1}^o \Phi_{g,t} R_t^* - 1\right]\right)\right)\overline{B}_{g,t+1} \tag{B.99}$$

In the steady state, total public expenditure is a constant fraction of output:

$$G = \eta_g Y. \tag{B.100}$$

Dynamically, total public expenditure changes due to adjustments in the revenue or expenditure elements of the government budget.

These adjustments in the revenue and expenditure elements are controlled by the following eight fiscal rules:[19]

$$\begin{aligned}
\ln(x_t) = {} & (1-\rho_x)\ln(x) + \rho_x \ln(x_{t-1}) + (1-\rho_x)\phi_{x,d}\left(\ln\left(\frac{D_{g,t}}{Y_t + Y_{t-1} + Y_{t-2} + Y_{t-3}}\right) - \ln(\overline{dgy})\right) \\
& + (1-\rho_x)\phi_{x,dd}\left(\ln\left(\frac{D_{g,t}}{Y_t + Y_{t-1} + Y_{t-2} + Y_{t-3}}\right) - \ln\left(\frac{D_{g,t-1}}{Y_{t-1} + Y_{t-2} + Y_{t-3} + Y_{t-4}}\right)\right) \\
& + (1-\rho_x)\phi_{x,y}(\ln(y_t) - \ln(y)) + (1-\psi_x)\varepsilon_{x,t} + \psi_x \varepsilon_{x,t-1},
\end{aligned} \tag{B.101}$$

where x_t is an element of the set $\mathbb{X} = \{g_{i,t}^{exp} = G_{i,t}^{exp}/z_t^+; g_{c,t}^{exp} = G_{c,t}^{exp}/z_t^+; \text{tr}_t = \text{TR}_t/z_t^+; \tau_t^c; \tau_t^y; \tau_{e,t}^w; \tau_{w,t}^w; t_{ls,t}\}$.[20]

Therefore, the government adjusts the taxes or expenditures by reacting to the deviation of the debt-to-GDP ratio from the steady state (parameter $\phi_{x,d}$), to the growth rate of the debt-to-GDP ratio (parameter $\phi_{x,dd}$), and to the output gap (parameter $\phi_{x,y}$). Moreover, the government allows for the smoothing of tax rates and expenditures which is controlled by the parameter ρ_x. Finally, anticipated ($\varepsilon_{x,t-1}$) or unanticipated ($\varepsilon_{x,t}$) fiscal shocks may occur. The size of anticipated shocks relative to unanticipated shocks is controlled by the parameter ψ_x.

The capital income tax follows a simple AR(1) process:

$$\ln(\tau_t^k) = (1-\rho_{\tau,k})\ln(\tau^k) + \rho_{\tau,k}\ln(\tau_{t-1}^k) + \varepsilon_{\tau,k,t}. \tag{B.102}$$

[18] Lump-sum taxes are levied on optimizing households. If $T_{ls,t}$ is negative, these are actually transfers.

[19] Plus, optionally, by exogenous changes to wasteful government spending (suspended in estimation).

[20] Lower-case variables denote normalized quantities (e.g., $g_{c,t}^{exp}$, tr_t, or y_t), i.e. divided by the growth variable z_t^+.

B.8.1 Public capital accumulation and public investment expenditure

The amount of public capital accumulates according to the following equations, where public investment $G_{i,t}$ obeys a condition that incorporates a two-period lag in building public capital from public investment (instead of a one-period lag in building private capital):

$$K_{g,t+1} = (1-\delta_g)K_{g,t} + A_{g,i,t-1}, \quad G_{i,t} = b_0 A_{g,i,t} + b_1 A_{g,i,t-1}. \tag{B.103}$$

Public investment expenditure in the deterministic steady state is an exogenously specified fraction τ_i^g of total public expenditure, and public investment expenditure is split into expenditure directed towards the domestic good and the imported public investment good (with price $p_t^{m,g,i}$) as follows:

$$G_i^{exp} = \tau_i^g G, \quad G_{i,t}^{exp} = G_{i,t}^d + p_t^{m,g,i} G_{i,t}^m. \tag{B.104}$$

B.8.2 Public consumption expenditure

Public consumption expenditure $G_{c,t}^{exp}$ in the deterministic steady state is calibrated to be a fraction τ_c^g of total public expenditure. This expenditure is split into domestic goods purchase of public goods $G_{c,t}^d$ and imported goods purchase of public goods $p_t^{m,g,c} G_{c,t}^m$ as follows:

$$G_c^{exp} = \tau_c^g G, \quad G_{c,t}^{exp} = G_{c,t}^d + p_t^{m,g,c} G_{c,t}^m. \tag{B.105}$$

B.8.3 Government transfers

Government transfers (less unemployment benefits) TR_t to both optimizing and restricted households are present the model. These transfers are such that in the steady state they are an exogenously given percentage τ_{tr}^g of total public expenditure:

$$\mathrm{TR} = \tau_{tr}^g G. \tag{B.106}$$

Dynamically, transfers follow a fiscal rule specified in the main text. These transfers are split into transfers to optimizing and restricted households. We assume that there is a share λ_r of restricted households so that the remaining share $1-\lambda_r$ is the mass of optimizing households. Total transfers are split according to the following distribution rule:

$$\mathrm{TR}_t = \lambda_r \cdot \mathrm{TR}_{r,t} + (1-\lambda_r)\mathrm{TR}_{o,t}, \quad \tau_r^{tr} \cdot \mathrm{TR}_{o,t} = (1-\tau_r^{tr})\mathrm{TR}_{r,t}, \tag{B.107}$$

where $\mathrm{TR}_{r,t}$ denotes the transfers to restricted households and $\mathrm{TR}_{o,t}$ the transfers to optimizing households. The constant τ_r^{tr} determines the fraction of transfers to restricted households.

B.8.4 Unemployment benefits

The steady state level of unemployment benefits per unemployed $D_{b,t}$ is calibrated to be a share of steady-state gross wage, $D_b = \mathrm{bshare} \cdot W$. Dynamically, unemployment benefits per unemployed are endogenous and form a constant share of the worker outside option, $D_{b,t} = \mathrm{bshare}/woo \cdot B_t^u/P_t$, where woo is the steady-state share of worker outside option in the gross wage.

B.9 Monetary policy

Monetary policy is conducted according to a hard peg of the domestic nominal interest rate to the foreign nominal interest rate. Only a risk-adjustment term drives differences between the domestic and the foreign nominal interest rate. The risk-adjustment term is defined in Equation (B.10). This implies

$$R_t = \Phi_t R_t^*. \tag{B.108}$$

B.10 Aggregate resource constraint and current account

The fact that there is potentially steady state price dispersion in prices complicates the expression for the domestic homogeneous good, Y_t, in terms of aggregate factors of production. The relationship derived in Appendix C can be expressed as

$$Y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\epsilon_t \widetilde{K}_t^{\alpha} L_t^{1-\alpha} - \phi z_t^{+} \right], \tag{B.109}$$

where $\mathring{p}_t$ denotes the degree of price dispersion in the intermediate domestic good.

Resource constraint for domestic homogeneous output. Above we defined real, scaled output in terms of aggregate factors of production. It is convenient to also have an expression that exhibits the uses of domestic homogeneous output. Using Equation (B.54) implies

$$Y_t - D_t = Z_t + G_{c,t}^d + C_t^d + G_{i,t}^d + I_t^d + \left[\omega_x \left(P_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)(P_t)^{1-\eta_x} \right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x)(\mathring{p}_t^{x})^{\frac{-\lambda_x}{\lambda_x - 1}} (P_t^x)^{-\eta_f} Y_t^f,$$

where monitoring costs due to financial frictions plus the costs of holding public debt are given by

$$D_t = \mu G(\bar{\omega}_t; \sigma_{t-1}) R_t^k P_{k',t-1} \bar{K}_t + \left(\frac{K_t^v}{Q_t} + K_t^h \right) \chi_t L_{t-1} + \frac{\gamma_g (B_{g,t+1} - \overline{B}_{g,t+1})^2}{2\overline{B}_{g,t+1}}.$$

When GDP is matched to the data, capital utilization costs are subtracted from Y_t (see Appendix C):

$$GDP_t = Y_t - D_t - (1-\omega_i)(P_t^i)^{\eta_i} \left(a(u_t) \bar{K}_t \right).$$

Trade balance. Expenses on imports, new purchases of net foreign assets, A_{t+1}^*, and expenses on foreign-held debt must equal income from exports, from previously purchased net foreign assets, and from new government borrowing from abroad:

$$S_t A_{t+1}^* + B_{g,t}^* R_{g,t-1} + expenses\, on\, imports_t = receipts\, from\, exports_t + R_{t-1}^* \Phi_{t-1} S_t A_t^* + B_{g,t+1}^*,$$

where $B_{g,t}^*$ is borrowing from abroad. Expenses on imports correspond to the purchases of specialized importers for the consumption, investment, and export sectors:[21]

$$\begin{aligned} expenses\, on\, imports_t = S_t P_t^* R_t^{\nu,*} \Bigg(& C_t^m (\mathring{p}_t^{m,c})^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}} + G_{c,t}^m (\mathring{p}_t^{g,m,c})^{\frac{\lambda_{g,m,c}}{1-\lambda_{g,m,c}}} + I_t^m (\mathring{p}_t^{m,i})^{\frac{\lambda_{m,i}}{1-\lambda_{m,i}}} \\ & + G_{i,t}^m (\mathring{p}_t^{g,m,i})^{\frac{\lambda_{g,m,i}}{1-\lambda_{g,m,i}}} + X_t^m (\mathring{p}_t^{m,x})^{\frac{\lambda_{m,x}}{1-\lambda_{m,x}}} \Bigg). \end{aligned}$$

The current account can be written as follows, using Equation (B.51):

$$\begin{aligned} & S_t A_{t+1}^* + B_{g,t}^* R_{g,t-1} + S_t P_t^* R_t^{\nu,*} \Bigg(C_t^m (\mathring{p}_t^{m,c})^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}} + G_{c,t}^m (\mathring{p}_t^{g,m,c})^{\frac{\lambda_{g,m,c}}{1-\lambda_{g,m,c}}} + I_t^m (\mathring{p}_t^{m,i})^{\frac{\lambda_{m,i}}{1-\lambda_{m,i}}} \\ & + G_{i,t}^m (\mathring{p}_t^{g,m,i})^{\frac{\lambda_{g,m,i}}{1-\lambda_{g,m,i}}} + X_t^m (\mathring{p}_t^{m,x})^{\frac{\lambda_{m,x}}{1-\lambda_{m,x}}} \Bigg) = S_t P_t^c P_t^x X_t + R_{t-1}^* \Phi_{t-1} S_t A_t^* + B_{g,t+1}^*. \end{aligned} \tag{B.110}$$

This completes the description of the model. Additional equilibrium conditions and the normalized versions of the equilibrium conditions are summarized in Appendix C.

[21] Note the presence of the price distortion terms here. To understand these terms, recall that, e.g., C_t^m is produced as a linear homogeneous function of specialized imported goods. Because the specialized importers only buy foreign goods, it is their total expenditures that interests us here. When the imports are distributed evenly across differentiated goods, then the total quantity of those imports is C_t^m, and the value of imports associated with domestic production of consumption goods is $S_t P_t^* R_t^{\nu,*} C_t^m$. When there are price distortion among imported intermediate goods then the sum of the homogeneous import goods is higher for any given value of C_t^m.

C Model Normalization and Further Details

C.1 Scaling of variables

We adopt the following scaling of variables. The neutral shock to technology is z_t and its growth rate is $\mu_{z,t}$:

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}.$$

The variable Ψ_t is an investment-specific shock to technology and it is convenient to define the following combination of investment-specific and neutral technology:

$$\begin{aligned} z_t^+ &= \Psi_t^{\frac{\alpha}{1-\alpha}} z_t, \\ \mu_{z^+,t} &= \mu_{\Psi,t}^{\frac{\alpha}{1-\alpha}} \mu_{z,t}. \end{aligned} \tag{C.1}$$

Private capital $\bar{K}_t$, private investment I_t, public capital $K_{g,t}$, and public investment $G_{i,t}$ are scaled by $z_t^+\Psi_t$. Public investment and public capital are scaled by z_t^+. Foreign and domestic inputs into the production of I_t (we denote these by I_t^d and I_t^m, respectively) and $G_{i,t}$ (we denote these by $G_{i,t}^d$ and $G_{i,t}^m$) are scaled by z_t^+. Private consumption goods (C_t^m are imported intermediate private consumption goods, C_t^d are domestically produced intermediate private consumption goods, and C_t are final private consumption goods) and public consumption goods ($G_{c,t}^m$ are imported intermediate public consumption goods, $G_{c,t}^d$ are domestically produced intermediate public consumption goods, and $G_{c,t}$ are final public consumption goods) are scaled by z_t^+. Government expenditure G_t, the real wage W_t/P_t, and real foreign assets A_{t+1}^*/P_t are scaled by z_t^+. Exports (X_t^m are imported intermediate goods for use in producing exports and X_t are final export goods) are scaled by z_t^+. Also, υ_t is the shadow value in utility terms to the household of domestic currency and $\upsilon_t P_t$ is the shadow value of one unit of the homogeneous domestic good. The latter must be multiplied by z_t^+ to induce stationarity. The variable $\tilde{P}_t$ is the within-sector relative price of a good. Thus,

$$\begin{aligned}
k_{t+1} &= \frac{K_{t+1}}{z_t^+\Psi_t},\ \bar{k}_{t+1} = \frac{\bar{K}_{t+1}}{z_t^+\Psi_t},\ k_{g,t+1} = \frac{K_{g,t+1}}{z_t^+\Psi_t},\ i_t^d = \frac{I_t^d}{z_t^+},\ i_t = \frac{I_t}{z_t^+\Psi_t},\ i_m^t = \frac{I_t^m}{z_t^+},\ g_{i,t}^d = \frac{G_{i,t}^d}{z_t^+}, \\
g_{i,t} &= \frac{G_{i,t}}{z_t^+\Psi_t},\ g_{i,t}^m = \frac{G_{i,t}^m}{z_t^+},\ c_t^m = \frac{C_t^m}{z_t^+},\ c_t^d = \frac{C_t^d}{z_t^+},\ c_t = \frac{C_t}{z_t^+},\ g_{c,t}^m = \frac{G_{c,t}^m}{z_t^+},\ g_{c,t}^d = \frac{G_{c,t}^d}{z_t^+},\ g_{c,t} = \frac{G_{c,t}}{z_t^+}, \\
g_t &= \frac{G_t}{z_t^+},\ \bar{w}_t = \frac{W_t}{z_t^+P_t},\ a_t := \frac{S_tA_{t+1}^*}{z_t^+P_t},\ x_t^d = \frac{X_t^d}{z_t^+},\ x_t^m = \frac{X_t^m}{z_t^+},\ x_t = \frac{X_t}{z_t^+},\ \psi_{z^+,t} = \upsilon_tP_tz_t^+, \\
y_t &= \tilde{y}_t = \frac{Y_t}{z_t^+},\ \vartheta_t = \frac{P_t^h}{P_tz_t^+},\ \tilde{p}_t = \frac{\tilde{P}_t}{P_t},\ \widetilde{k}_t = \frac{\widetilde{K}_t}{z_t^+\Psi_t},\ n_{t+1} = \frac{\bar{N}_{t+1}}{P_tz_t^+},\ w_t^e = \frac{W_t^e}{P_tz_t^+},\ \kappa_t^v = \frac{K_t^v}{P_tz_t^+},\ \kappa_t^h = \frac{K_t^h}{P_tz_t^+}, \\
c_{r,t} &= \frac{C_{r,t}}{z_t^+},\ c_{o,t} = \frac{C_{o,t}}{z_t^+},\ \widetilde{c}_{o,t} = \frac{\widetilde{C}_{o,t}}{z_t^+},\ \widetilde{c}_{r,t} = \frac{\widetilde{C}_{r,t}}{z_t^+},\ \xi_t^B = \frac{\Xi_t^B}{z_t^+},\ \xi_t^{A^*} = \frac{\Xi_t^{A^*}}{z_t^+},\ \gamma_{g,t} = \frac{\Gamma_{g,t}}{z_t^+},\ d_t^f = \frac{D_t^f}{z_t^+}, \\
\vartheta_t^p &= \frac{\Theta_t^p}{P_tz_t^+},\ \bar{j}_t = \frac{J_t}{P_tz_t^+},\ \bar{w}_t^p = \frac{W_t^p}{P_tz_t^+},\ w_{p,t} = \frac{W_{p,t}}{P_tz_t^+},\ \bar{v}_t = \frac{V_t}{P_tz_t^+},\ \bar{u}_t = \frac{U_t}{P_tz_t^+},\ \tilde{u}_t = \frac{\tilde{U}_t}{P_tz_t^+},\ a_t^w = \frac{A_t^w}{P_tz_t^+}.
\end{aligned}$$

We define the scaled date t price of new installed physical capital for the start of period $t+1$ as $p_{k',t}$ and we define the scaled real rental rate of capital as $\bar{r}_t^k$:

$$p_{k',t} = \frac{\Psi_tP_{k',t}}{P_t}, \quad \bar{r}_t^k = \Psi_tr_t^k,$$

where $P_{k',t}$ is in units of the domestic homogeneous good. The nominal exchange rate is denoted by S_t and its growth rate is s_t:

$$s_t = \frac{S_t}{S_{t-1}}.$$

Utilized normalized capital is given by:

$$k_t = \bar{k}_t u_t. \tag{C.2}$$

We define the following inflation rates:

$$\pi_t = \frac{P_t}{P_{t-1}},\ \pi_t^c = \frac{P_t^c}{P_{t-1}^c},\ \pi_t^{g,c} = \frac{P_t^{g,c}}{P_{t-1}^{g,c}},\ \pi_t^* = \frac{P_t^*}{P_{t-1}^*},\ \pi_t^f = \frac{P_t^f}{P_{t-1}^f},\ \pi_t^i = \frac{P_t^i}{P_{t-1}^i},\ \pi_t^{g,i} = \frac{P_t^{g,i}}{P_{t-1}^{g,i}},\ \pi_t^x = \frac{P_t^x}{P_{t-1}^x}.$$

Here, P_t is the price of a domestic homogeneous output good, P_t^c is the price of the domestic final private consumption good (i.e., the 'consumer price index'), $P_t^{g,c}$ is the price of the domestic final public consumption good, P_t^* is the price of a foreign homogeneous good, P_t^i is the price of the domestic final private investment good, $P_t^{g,i}$ is the price of the domestic final public investment good, and P_t^x is the price (in foreign currency units) of a final export good. Moreover, we define the following inflation rates for imported goods:

$$\pi_t^{m,c} = \frac{P_t^{m,c}}{P_{t-1}^{m,c}},\ \pi_t^{g,m,c} = \frac{P_t^{g,m,c}}{P_{t-1}^{g,m,c}},\ \pi_t^{m,i} = \frac{P_t^{m,i}}{P_{t-1}^{m,i}},\ \pi_t^{g,m,i} = \frac{P_t^{g,m,i}}{P_{t-1}^{g,m,i}},\ \pi_t^{m,x} = \frac{P_t^{m,x}}{P_{t-1}^{m,x}}.$$

With one exception, we define a lower case price as the corresponding uppercase price divided by the price of the homogeneous good. When the price is denominated in domestic currency units, we divide by the price of the domestic homogeneous good (P_t). When the price is denominated in foreign currency units we divide the price by P_t^*, the price of the foreign homogeneous good. The exceptional case has to do with handling of the prices of private investment goods P_t^i and public investment goods $P_t^{g,i}$. These grow at a rate potentially slower than P_t, and we therefore scale them by P_t/Ψ_t. Thus,

$$\begin{aligned} p_t^{m,x} &= \frac{P_t^{m,x}}{P_t},\ p_t^{m,c} = \frac{P_t^{m,c}}{P_t},\ p_t^{g,m,c} = \frac{P_t^{g,m,c}}{P_t},\ p_t^{m,i} = \frac{P_t^{m,i}}{P_t},\ p_t^{g,m,i} = \frac{P_t^{g,m,i}}{P_t}, \\ p_t^x &= \frac{P_t^x}{P_t^f},\ p_t^c = \frac{P_t^c}{P_t},\ p_t^{g,c} = \frac{P_t^{g,c}}{P_t},\ p_t^i = \frac{\Psi_t P_t^i}{P_t},\ p_t^{g,i} = \frac{\Psi_t P_t^{g,i}}{P_t}. \end{aligned} \tag{C.3}$$

Fiscal sector variables, which have not been mentioned yet, are normalized as follows:

$$\begin{aligned} t_t &= \frac{T_t}{z_t^+},\ z_t = \frac{Z_t}{z_t^+},\ d_{b,t} = \frac{D_{b,t}}{z_t^+},\ \mathrm{tr}_t = \frac{\mathrm{TR}_t}{z_t^+},\ \mathrm{tr}_{r,t} = \frac{\mathrm{TR}_{r,t}}{z_t^+},\ \mathrm{tr}_{o,t} = \frac{\mathrm{TR}_{o,t}}{z_t^+},\ d_{g,t} = \frac{D_{g,t}}{z_t^+}, \\ \mathrm{deficit}_t &= \frac{\mathrm{Deficit}_t}{z_t^+},\ b_{g,t+1} = \frac{B_{g,t+1}}{z_t^+},\ b_{g,t+1}^* = \frac{B_{g,t+1}^*}{z_t^+},\ g_{i,t}^{exp} = \frac{G_{i,t}^{exp}}{z_t^+},\ g_{c,t}^{exp} = \frac{G_{c,t}^{exp}}{z_t^+},\ a_{g,i,t} = \frac{A_{g,i,t}}{z_t^+\Psi_t}. \end{aligned}$$

C.2 Functional forms

We adopt the following functional form for capital utilization a:

$$a(u) = 0.5\sigma_b\sigma_a u^2 + \sigma_b(1-\sigma_a)u + \sigma_b((\sigma_a/2) - 1), \tag{C.4}$$

where σ_a and σ_b are parameters. The functional form of investment adjustment costs as well as its derivatives are

$$\begin{aligned} \tilde{S}(x) &= \frac{1}{2}\left\{\exp\left[\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right] + \exp\left[-\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right] - 2\right\} \\ &= 0,\ x = \mu_{z^+}\mu_\Psi, \end{aligned} \tag{C.5}$$

$$\begin{aligned} \tilde{S}'(x) &= \frac{1}{2}\sqrt{\tilde{S}''}\left\{\exp\left[\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right] - \exp\left[-\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right]\right\} \\ &= 0,\ x = \mu_{z^+}\mu_\Psi, \end{aligned} \tag{C.6}$$

$$\begin{aligned} \tilde{S}''(x) &= \frac{1}{2}\tilde{S}''\left\{\exp\left[\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right] + \exp\left[-\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right]\right\} \\ &= \tilde{S}'',\ x = \mu_{z^+}\mu_\Psi. \end{aligned}$$

C.3 Normalized model

C.3.1 First order conditions in optimizing household problem

The optimizing households' budget constraint in scaled terms is given by:

$$\begin{aligned}(1+\tau_t^c)p_t^c c_{o,t} + p_t^i i_t + b_{g,t+1} + \frac{a_t}{\Phi_t R_t^*} + t_{ls,t} + \gamma_{g,t} \\ = (1-\tau_t^y-\tau_{w,t}^w)\bar{w}_t L_t + \tau_t^k \bar{k}_t[p_t^i a(u_t) + \delta p_t^i - u_t \bar{r}_t^k] + (1-\tau^b)d_t^f + d_{b,t}(1-L_t) \\ + tr_{o,t} + b_{g,t}\Phi_{g,t-1}R_{t-1}^* + \left(1-\tau^b+\frac{\tau^b \pi_t}{R_{t-1}^*\Phi_{t-1}}\right)a_{t-1} + \xi_t^B + \xi_t^{A^*}\end{aligned} \tag{C.7}$$

The first order condition for consumption in scaled terms is given by:

$$\left(\frac{\zeta_t^c}{\widetilde{c}_{o,t} - b\widetilde{c}_{o,t-1}\frac{1}{\mu_{z^+,t}}} - \beta b \mathbb{E}_t\left[\frac{\zeta_{t+1}^c}{\widetilde{c}_{o,t+1}\mu_{z^+,t+1} - b\widetilde{c}_{o,t}}\right]\right)\alpha_c^{\frac{1}{\nu_c}} c_{o,t}^{-\frac{1}{\nu_c}} \widetilde{c}_{o,t}^{\frac{1}{\nu_c}} - \psi_{z^+,t}p_t^c(1+\tau^c) = 0. \tag{C.8}$$

where $\psi_{z^+,t}$ is the marginal utility of optimizing households:

$$\psi_{z^+,t} = \upsilon_t P_t z_t^+.$$

Let us define for later the marginal utility of restricted households:

$$\psi_{z^+,t}^r = \upsilon_t^r P_t z_t^+,$$

which satisfies the following restricted households'first order condition :

$$\left(\frac{\zeta_t^c}{\widetilde{c}_{r,t} - b\widetilde{c}_{r,t-1}\frac{1}{\mu_{z^+,t}}} - \beta b \mathbb{E}_t\left[\frac{\zeta_{t+1}^c}{\widetilde{c}_{r,t+1}\mu_{z^+,t+1} - b\widetilde{c}_{r,t}}\right]\right)\alpha_c^{\frac{1}{\nu_c}} c_{r,t}^{-\frac{1}{\nu_c}} \widetilde{c}_{r,t}^{\frac{1}{\nu_c}} - \psi_{z^+,t}^r p_t^c(1+\tau^c) = 0. \tag{C.9}$$

The investment first order condition is in scaled terms

$$\begin{aligned}-\psi_{z^+,t}p_t^i + \psi_{z^+,t}p_{k',t}\Upsilon_t\left[1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right) - \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right] \\ + \beta\psi_{z^+,t+1}p_{k',t+1}\Upsilon_{t+1}\tilde{S}'\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1}i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2 \mu_{\Psi,t+1}\mu_{z^+,t+1} = 0.\end{aligned}$$

The first order condition associated with capital utilization is in scaled terms:

$$\bar{r}_t^k = p_t^i a'(u_t). \tag{C.10}$$

The first order condition with respect foreign asset holdings in scaled terms is given by:

$$\psi_{z^+,t} = \mathbb{E}_t\left[\frac{\beta\psi_{z^+,t+1}}{\pi_{t+1}\mu_{z^+,t+1}}\left(s_{t+1}R_t^*\Phi_t - \tau^b(s_{t+1}R_t^*\Phi_t - \pi_{t+1})\right)\right], \tag{C.11}$$

where $s_t = S_t/S_{t-1}$. The risk adjustment Φ_t, using scaled variables, has the following form:

$$\Phi_t = \Phi\left(a_t, R_t^* - R_t, \tilde{\phi}_t\right) = \exp\left(-\tilde{\phi}_a(a_t-\bar{a}) - \tilde{\phi}_s(R_t^* - R_t - (R^* - R)) + \tilde{\phi}_t\right), \tag{C.12}$$

where

$$a_t = \frac{S_t A_{t+1}^*}{P_t z_t^+}.$$

The optimizing household's first order condition with respect to public debt holdings, in scaled terms, is equal to:

$$b_{g,t+1} = \left(1+\gamma_g^{-1}\left(\mathbb{E}_t\left[\frac{\beta\psi_{z^+,t+1}}{\psi_{z^+,t}}\frac{\Phi_{g,t}R_t^*}{\pi_{t+1}\mu_{z^+,t+1}} - 1\right]\right)\right)\bar{b}_{g,t+1}, \quad \bar{b}_{g,t+1} = \omega_h d_{g,t}. \tag{C.13}$$

C.3.2 Scaling of households' budget constraints and aggregation across households

The restricted households' budget constraint in scaled terms is given by:

$$(1+\tau_t^c)p_t^c c_{r,t} = (1-\tau_t^y-\tau_{w,t}^w)\bar{w}_t L_t + d_{b,t}(1-L_t) + \mathrm{tr}_{r,t}. \tag{C.14}$$

The aggregate level of consumption across the two household types is given by:

$$c_t = \lambda_r c_{r,t} + (1-\lambda_r)c_{o,t}. \tag{C.15}$$

Moreover, the consumption bundles for both household types are defined as follows, using scaled variables:

$$\widetilde{c}_{o,t} = \left(\alpha_c^{\frac{1}{\nu_c}}(c_{o,t})^{\frac{\nu_c-1}{\nu_c}} + (1-\alpha_c)^{\frac{1}{\nu_c}}(g_{c,t})^{\frac{\nu_c-1}{\nu_c}}\right)^{\frac{\nu_c}{\nu_c-1}}, \tag{C.16}$$

$$\widetilde{c}_{r,t} = \left(\alpha_c^{\frac{1}{\nu_c}}(c_{r,t})^{\frac{\nu_c-1}{\nu_c}} + (1-\alpha_c)^{\frac{1}{\nu_c}}(g_{c,t})^{\frac{\nu_c-1}{\nu_c}}\right)^{\frac{\nu_c}{\nu_c-1}}. \tag{C.17}$$

C.3.3 First order conditions for domestic homogeneous goods price setting

The firm's marginal cost, divided by the price of the homogeneous good is denoted by mc_t and given by the following expression, using scaled variables:

$$mc_t = \frac{\tau_t^d}{\epsilon_t}\left(\frac{1}{1-\alpha}\right)^{1-\frac{\alpha\nu_k}{\nu_k-1}}\left(\frac{1}{\alpha}\right)^{\frac{\alpha\nu_k}{\nu_k-1}}\left(\frac{1}{\alpha_k}\right)^{\frac{\alpha}{\nu_k-1}}\left(\bar{r}_t^k\right)^{\frac{\alpha\nu_k}{\nu_k-1}}\left(\vartheta_t R_t^f\right)^{1-\frac{\alpha\nu_k}{\nu_k-1}}.$$

Rewriting Equation (B.21) in scaled variables yields:

$$mc_t = \frac{\tau_t^d \vartheta_t R_t^f}{\epsilon_t(1-\alpha)\left(\frac{\widetilde{k}_{i,t}}{\mu_{z^+,t}\mu_{\Psi,t}H_{i,t}}\right)^{\alpha}} \tag{C.18}$$

Substituting Equations (B.23) into (B.22) and rearranging,

$$\mathbb{E}_t\left[\sum_{j=0}^{\infty}\beta^j \upsilon_{t+j}P_{t+j}Y_{t+j}\left\{\left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{1-\frac{\lambda_d}{\lambda_d-1}} - mc_{t+j}\left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{\frac{-\lambda_d}{\lambda_d-1}}\right\}\right],$$

or,

$$\mathbb{E}_t\left[\sum_{j=0}^{\infty}\beta^j \upsilon_{t+j}P_{t+j}Y_{t+j}\left\{(X_{t,j}\tilde{p}_t)^{1-\frac{\lambda_d}{\lambda_d-1}} - mc_{t+j}(X_{t,j}\tilde{p}_t)^{\frac{-\lambda_d}{\lambda_d-1}}\right\}\right],$$

where

$$\frac{P_{i,t+j}}{P_{t+j}} = X_{t,j}\tilde{p}_t, \quad X_{t,j} := \begin{cases} \frac{\tilde{\pi}_{d,t+j}\cdots\tilde{\pi}_{d,t+1}}{\pi_{t+j}\cdots\pi_{t+1}}, & j>0 \\ 1, & j=0 \end{cases}.$$

The i-th firm maximizes profits by choice over the within-sector relative price $\tilde{p}_t$. The fact that this variable does not have an index i reflects that all firms that have the opportunity to reoptimize in period t solve the same problem, and hence have the same solution. Differentiating its profit function, multiplying the result by $\tilde{p}_t^{\frac{\lambda_d}{\lambda_d-1}+1}$, rearranging, and scaling, yields

$$\mathbb{E}_t\left[\sum_{j=0}^{\infty}(\beta\xi_d)^j A_{t+j}^x[\tilde{p}_t X_{t,j} - \lambda_d mc_{t+j}]\right] = 0,$$

where A^x_{t+j} is exogenous from the point of view of the firm:

$$A^x_{t+j} = \psi_{z^+,t+j}\tilde{y}_{t+j}X_{t,j}.$$

After rearranging the optimizing intermediate good firm's first order condition for prices, we obtain

$$\tilde{p}^d_t = \frac{E_t\sum_{j=0}^{\infty}(\beta\xi_d)^j A^x_{t+j}\lambda_d mc_{t+j}}{E_t\sum_{j=0}^{\infty}(\beta\xi_d)^j A^x_{t+j}X_{t,j}} = \frac{K^d_t}{F^d_t},$$

where

$$K^d_t := \mathbb{E}_t\left[\sum_{j=0}^{\infty}(\beta\xi_d)^j A^x_{t+j}\lambda_d mc_{t+j}\right],$$
$$F^d_t := \mathbb{E}_t\left[\sum_{j=0}^{\infty}(\beta\xi_d)^j A^x_{t+j}X_{t,j}\right].$$

These objects have the following convenient recursive representations

$$\mathbb{E}_t\left[\psi_{z^+,t}\tilde{y}_t + \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_d}}\beta\xi_d F^d_{t+1} - F^d_t\right] = 0$$
$$\mathbb{E}_t\left[\lambda_d\psi_{z^+,t}\tilde{y}_t mc_t + \beta\xi_d\left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_d}{1-\lambda_d}} K^d_{t+1} - K^d_t\right] = 0.$$

Turning to the aggregate price index:

$$\begin{aligned} P_t &= \left[\int_0^1 P_{it}^{\frac{1}{1-\lambda_d}}di\right]^{1-\lambda_d} \\ &= \left[(1-\xi_p)\tilde{P}_t^{\frac{1}{1-\lambda_d}} + \xi_p(\tilde{\pi}_{d,t}P_{t-1})^{\frac{1}{1-\lambda_d}}\right]^{1-\lambda_d} \end{aligned} \tag{C.19}$$

After dividing by P_t and rearranging

$$\frac{1-\xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d} = (\tilde{p}^d_t)^{\frac{1}{1-\lambda_d}}. \tag{C.20}$$

In sum, the equilibrium conditions associated with price setting for producers of the domestic homogeneous good are

$$0 = \mathbb{E}_t\left[\psi_{z^+,t}y_t + \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_d}}\beta\xi_d F^d_{t+1} - F^d_t\right], \tag{C.21}$$

$$0 = \mathbb{E}_t\left[\lambda_d\psi_{z^+,t}y_t mc_t + \beta\xi_d\left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_d}{1-\lambda_d}} K^d_{t+1} - K^d_t\right], \tag{C.22}$$

$$\mathring{p}_t = \left[(1-\xi_d)\left(\frac{1-\xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d}\right)^{\lambda_d} + \xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\mathring{p}_{t-1}\right)^{\frac{\lambda_d}{1-\lambda_d}}\right]^{\frac{1-\lambda_d}{\lambda_d}}, \tag{C.23}$$

$$\frac{K^d_t}{F^d_t} = \left[\frac{1-\xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d}\right]^{1-\lambda_d}, \tag{C.24}$$

$$\tilde{\pi}_{d,t} := (\pi_{t-1})^{\kappa_d}(\bar{\pi}^c_t)^{1-\kappa_d-\varkappa_d}(\breve{\pi})^{\varkappa_d}. \tag{C.25}$$

After linearizing around the steady state and setting $\varkappa_d = 0$, the aforementioned equations reduce to

$$\begin{aligned}\hat{\pi} - \hat{\bar{\pi}}_t^c =& \frac{\beta}{1+\kappa_d\beta}\mathbb{E}_t[\hat{\pi}_{t+1} - \hat{\bar{\pi}}_{t+1}^c] + \frac{\kappa_d}{1+\kappa_d\beta}(\hat{\pi}_{t-1} - \hat{\bar{\pi}}_t^c) \\ & - \frac{\kappa_d\beta(1-\rho_\pi)}{1+\kappa_d\beta}\hat{\bar{\pi}}_t^c \\ & + \frac{1}{1+\kappa_d\beta}\frac{(1-\beta\xi_d)(1-\xi_d)}{\xi_d}\widehat{mc}_t,\end{aligned}$$

where a hat indicates log-deviation from steady state.

C.3.4 Export demand

Scaling Equations (B.46), (B.54), and (B.55) yields

$$x_t = \epsilon_t^x (p_t^x)^{-\eta_f} y_t^f, \tag{C.26}$$

$$x_t^d = \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x)\right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x)(\mathring{p}_t^x)^{\frac{-\lambda_x}{\lambda_x - 1}} (p_t^x)^{-\eta_f} y_t^f, \tag{C.27}$$

$$x_t^m = \omega_x \left(\frac{\left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}}{p_t^{m,x}} \right)^{\eta_x} (\mathring{p}_t^x)^{\frac{-\lambda_x}{\lambda_x - 1}} (p_t^x)^{-\eta_f} y_t^f. \tag{C.28}$$

C.3.5 FOCs for export goods price setting

The real marginal cost in terms of stationary variables, mc_t^x, is derived as

$$mc_t^x = \frac{\lambda}{S_t P_t^x} = \frac{\tau_t^x R_t^x}{q_t p_t^c p_t^x} \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}, \tag{C.29}$$

$$0 = \mathbb{E}_t \left[\psi_{z^+,t} q_t p_t^c p_t^x x_t + \left(\frac{\tilde{\pi}_{t+1}^x}{\pi_{t+1}^x}\right)^{\frac{1}{1-\lambda_x}} \beta\xi_x F_{x,t+1} - F_{x,t} \right], \tag{C.30}$$

$$0 = \mathbb{E}_t \left[\lambda_x \psi_{z^+,t} q_t p_t^c p_t^x x_t mc_t^x + \beta\xi_x \left(\frac{\tilde{\pi}_{t+1}^x}{\pi_{t+1}^x}\right)^{\frac{\lambda_x}{1-\lambda_x}} K_{x,t+1} - K_{x,t} \right], \tag{C.31}$$

$$\mathring{p}_t^x = \left[(1-\xi_x)\left(\frac{1-\xi_x \left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\right)\frac{1}{1-\lambda_x}}{1-\xi_x} \right)^{\lambda_x} + \xi_x \left(\frac{\tilde{\pi}_t^x}{\pi_t^x} \mathring{p}_{t-1}^x \right)^{\frac{\lambda_x}{1-\lambda_x}} \right]^{\frac{1-\lambda_x}{\lambda_x}}, \tag{C.32}$$

$$\frac{K_{x,t}}{F_{x,t}} = \left[\frac{1-\xi_x \left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\right)^{\frac{1}{1-\lambda_x}}}{1-\xi_x} \right]^{1-\lambda_x}. \tag{C.33}$$

When linearized around steady state and $\varkappa_{m,j} = 0$, Equations (C.30)-(C.33) reduce to

$$\begin{aligned}\hat{\pi}_t^x =& \frac{\beta}{1+\kappa_x\beta}\mathbb{E}_t[\hat{\pi}_{t+1}^x] + \frac{\kappa_x}{1+\kappa_x\beta}\hat{\pi}_{t-1}^x \\ & + \frac{1}{1+\kappa_x\beta}\frac{(1-\beta\xi_x)(1-\xi_x)}{\xi_x}\widehat{mc}_t^x\end{aligned}$$

where a hat over a variable indicates log-deviation from steady state.

C.3.6 Demand for domestic inputs in export production

Integrating Equation (B.53) yields

$$\begin{aligned}\int_0^1 X_{i,t}^d di &= \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x} (1-\omega_x) \int_0^1 X_{i,t} di \\ &= \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x} (1-\omega_x) X_t \frac{\int_0^1 (P_{i,t}^x)^{\frac{-\lambda_x}{\lambda_x-1}} di}{(P_t^x)^{\frac{-\lambda_x}{\lambda_x-1}}}. \end{aligned} \tag{C.34}$$

Define $\mathring{P}_t^x$, a linear homogeneous function of $P_{i,t}^x$, as follows:

$$\mathring{P}_t^x = \left[\int_0^1 (P_{i,t}^x)^{\frac{-\lambda_x}{\lambda_x-1}} di\right]^{\frac{\lambda_x-1}{-\lambda_x}}.$$

Then

$$\left(\mathring{P}_t^x\right)^{\frac{-\lambda_x}{\lambda_x-1}} = \int_0^1 (P_{i,t}^x)^{\frac{-\lambda_x}{\lambda_x-1}} di,$$

and

$$\int_0^1 X_{i,t}^d di = \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x} (1-\omega_x) X_t (\mathring{p}_t^x)^{\frac{-\lambda_x}{\lambda_x-1}}, \tag{C.35}$$

where

$$\mathring{p}_t^x := \frac{P_{i,t}^x}{P_t^x},$$

and the law of motion of $\mathring{p}_t^x$ is given in Equation (C.32). We now simplify Equation (C.35). Rewriting the second equality in (B.49) yields

$$\frac{\lambda}{P_t \tau_t^x R_t^x} = \frac{S_t P_t^x}{P_t q_t p_t^c p_t^x} \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}},$$

or

$$\frac{\lambda}{P_t \tau_t^x R_t^x} = \frac{S_t P_t^x}{P_t \frac{S_t P_t^*}{P_t^c} \frac{P_t^c}{P_t} \frac{P_t^x}{P_t^*}} \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}},$$

or

$$\frac{\lambda}{P_t \tau_t^x R_t^x} = \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}.$$

Substituting into Equation (C.35),

$$X_t^d = \int_0^1 X_{i,t}^d di = \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x)\right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) (\mathring{p}_t^x)^{\frac{-\lambda_x}{\lambda_x-1}} (p_t^x)^{-\eta_x} Y_t^f.$$

C.3.7 Demand for imported inputs in export production

Scaling Equation (B.55) yields

$$x_t^m = \omega_x \left(\frac{\left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}}{p_t^{m,x}}\right)^{\eta_x} (\mathring{p}_t^x)^{\frac{-\lambda_x}{\lambda_x-1}} (p_t^x)^{-\eta_f} y_t^f. \tag{C.36}$$

C.3.8 Value of imports of the intermediate private consumption goods producers

It is of interest to have a measure of the value of imports of the intermediate consumption good producers:

$$S_t P_t^f R_t^{\nu,*} \int_0^1 C_{i,t}^m di.$$

In order to relate this to C_t^m, substitute the demand curve into the previous expression:

$$\begin{aligned} S_t P_t^f R_t^{\nu,*} \int_0^1 C_t^m \left(\frac{P_t^{m,c}}{P_{i,t}^{m,c}} \right)^{\frac{\lambda_{m,c}}{\lambda_{m,c}-1}} di &= S_t P_t^f R_t^{\nu,*} C_t^m (P_t^{m,c})^{\frac{\lambda_{m,c}}{\lambda_{m,c}-1}} \int_0^1 (P_{i,t}^{m,c})^{\frac{-\lambda_{m,c}}{\lambda_{m,c}-1}} di \\ &= S_t P_t^f R_t^{\nu,*} C_t^m \left(\frac{\mathring{P}_t^{m,c}}{P_t^{m,c}} \right)^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}}, \end{aligned}$$

where

$$\mathring{P}_t^{m,c} = \left[\int_0^1 (P_{i,t}^{m,c})^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}} \right]^{\frac{1-\lambda_{m,c}}{\lambda_{m,c}}}.$$

Thus, the total value of imports accounted for by the consumption sector is

$$S_t P_t^f R_t^{\nu,*} C_t^m (\mathring{p}_t^{m,c})^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}}, \tag{C.37}$$

where

$$\mathring{p}_t^{m,c} = \frac{P_{i,t}^{m,c}}{P_t^{m,c}}.$$

The derivation for the value of imports used by the public consumption, private investment, public investment, and export production sectors are analogous.

C.3.9 Marginal costs of importers

Real marginal cost is defined as

$$mc_t^{m,j} = \tau_t^{m,j} \frac{S_t P_t^f}{P_t^{m,j}} R_t^{\nu,*} = \tau_t^{m,j} \frac{S_t P_t^f P_t^c P_t}{P_t^c P_t^{m,j} P_t} R_t^{\nu,*} = \tau_t^{m,j} \frac{q_t p_t^c}{p_t^{m,j}} R_t^{\nu,*}, \tag{C.38}$$

for $j = c, i, x$. Similarly,

$$mc_t^{g,m,j} = \tau_t^{g,m,j} \frac{S_t P_t^f}{P_t^{g,m,j}} R_t^{\nu,*} = \tau_t^{g,m,j} \frac{S_t P_t^f P_t^c P_t}{P_t^c P_t^{g,m,j} P_t} R_t^{\nu,*} = \tau_t^{g,m,j} \frac{q_t p_t^c}{p_t^{g,m,j}} R_t^{\nu,*}, \tag{C.39}$$

for $j = c, i$.

C.3.10 Demand equations and inflation rates for intermediate inputs

Scaling the demand equations for intermediate inputs and intermediate input inflation rates yields:

$$c_t^d = (1-\omega_c)(p_t^c)^{\eta_c} c_t, \tag{C.40}$$

$$c_t^m = \omega_c \left(\frac{p_t^c}{p_t^{m,c}}\right)^{\eta_c} c_t, \tag{C.41}$$

$$p_t^c = \left[(1-\omega_c) + \omega_c (p_t^{m,c})^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}}, \tag{C.42}$$

$$\pi_t^c = \pi_t \left[\frac{(1-\omega_c) + \omega_c (p_t^{m,c})^{1-\eta_c}}{(1-\omega_c) + \omega_c (p_{t-1}^{m,c})^{1-\eta_c}}\right]^{\frac{1}{1-\eta_c}}, \tag{C.43}$$

$$g_{c,t}^d = (1-\omega_{g,c})(p_t^c)^{\eta_{g,c}} g_{c,t}, \tag{C.44}$$

$$g_{c,t}^m = \omega_{g,c} \left(\frac{p_t^{g,c}}{p_t^{g,m,c}}\right)^{\eta_c} g_{c,t}, \tag{C.45}$$

$$p_t^{g,c} = \left[(1-\omega_{g,c}) + \omega_{g,c} (p_t^{g,m,c})^{1-\eta_{g,c}}\right]^{\frac{1}{1-\eta_{g,c}}}, \tag{C.46}$$

$$\pi_t^{g,c} = \pi_t \left[\frac{(1-\omega_{g,c}) + \omega_{g,c} (p_t^{g,m,c})^{1-\eta_{g,c}}}{(1-\omega_{g,c}) + \omega_{g,c} (p_{t-1}^{g,m,c})^{1-\eta_{g,c}}}\right]^{\frac{1}{1-\eta_{g,c}}}, \tag{C.47}$$

$$i_t^d = \left(p_t^i\right)^{\eta_i} \left(i_t + a(u_t)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right)(1-\omega_i), \tag{C.48}$$

$$i_t^m = \omega_i \left(\frac{p_t^i}{p_t^{m,i}}\right)^{\eta_i} \left(i_t + a(u_t)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right), \tag{C.49}$$

$$p_t^i = \left[(1-\omega_i) + \omega_i (p_t^{m,i})^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}}, \tag{C.50}$$

$$\pi_t^i = \frac{\pi_t}{\mu_{\Psi,t}} \left[\frac{(1-\omega_i) + \omega_i \left(p_t^{m,i}\right)^{1-\eta_i}}{(1-\omega_i) + \omega_i \left(p_{t-1}^{m,i}\right)^{1-\eta_i}}\right]^{\frac{1}{1-\eta_i}}, \tag{C.51}$$

$$g_{i,t}^d = \left(p_t^{g,i}\right)^{\eta_{g,i}} g_{i,t}(1-\omega_{g,i}), \tag{C.52}$$

$$g_{i,t}^m = \omega_{g,i} \left(\frac{p_t^{g,i}}{p_t^{g,m,i}}\right)^{\eta_{g,i}} g_{i,t}, \tag{C.53}$$

$$p_t^{g,i} = \left[(1-\omega_{g,i}) + \omega_{g,i} (p_t^{g,m,i})^{1-\eta_{g,i}}\right]^{\frac{1}{1-\eta_{g,i}}}, \tag{C.54}$$

$$\pi_t^{g,i} = \frac{\pi_t}{\mu_{\Psi,t}} \left[\frac{(1-\omega_{g,i}) + \omega_{g,i} \left(p_t^{g,m,i}\right)^{1-\eta_{g,i}}}{(1-\omega_{g,i}) + \omega_{g,i} \left(p_{t-1}^{g,m,i}\right)^{1-\eta_{g,i}}}\right]^{\frac{1}{1-\eta_{g,i}}}. \tag{C.55}$$

C.3.11 FOCs for import goods price setting

$$0 = \mathbb{E}_t\left[\psi_{z^+,t} p_t^{m,j} \Xi_t^j + \left(\frac{\tilde{\pi}_{t+1}^{m,j}}{\pi_{t+1}^{m,j}}\right)^{\frac{1}{1-\lambda_{m,j}}} \beta \xi_{m,j} F_{m,j,t+1} - F_{m,j,t}\right], \tag{C.56}$$

$$0 = \mathbb{E}_t\left[\lambda_{m,j} \psi_{z^+,t} p_t^{m,j} mc_t^{m,j} \Xi_t^j + \beta \xi_{m,j} \left(\frac{\tilde{\pi}_{t+1}^{m,j}}{\pi_{t+1}^{m,j}}\right)^{\frac{\lambda_{m,j}}{1-\lambda_{m,j}}} K_{m,j,t+1} - K_{m,j,t}\right], \tag{C.57}$$

$$\mathring{p}_t^{m,j} = \left[(1-\xi_{m,j}) \left(\frac{1-\xi_{m,j}\left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}}\right)^{\frac{1}{1-\lambda_{m,j}}}}{1-\xi_{m,j}}\right)^{\lambda_{m,j}} + \xi_{m,j}\left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}} \mathring{p}_{t-1}^{m,j}\right)^{\frac{\lambda_{m,j}}{1-\lambda_{m,j}}}\right]^{\frac{1-\lambda_{m,j}}{\lambda_{m,j}}}, \tag{C.58}$$

$$\frac{K_{m,j,t}}{F_{m,j,t}} = \left[\frac{1-\xi_{m,j}\left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}}\right)^{\frac{1}{1-\lambda_{m,j}}}}{1-\xi_{m,j}}\right]^{1-\lambda_{m,j}}, \tag{C.59}$$

for $j = c, i, x$, where

$$\Xi_t^j = \begin{cases} c_t^m & j = c, \\ x_t^m & j = x, \\ i_t^m & j = i. \end{cases}$$

When linearized around steady state and $\varkappa_{m,j} = 0$, the Equations (C.56)–(C.59) reduce to

$$\begin{aligned} \hat{\pi}_t^{m,j} - \hat{\bar{\pi}}_t^c =& \frac{\beta}{1+\kappa_{m,j}\beta} \mathbb{E}_t[\hat{\pi}_{t+1}^{m,j} - \hat{\bar{\pi}}_{t+1}^c] + \frac{\kappa_{m,j}}{1+\kappa_{m,j}\beta}\left(\hat{\pi}_{t-1}^{m,j} - \hat{\bar{\pi}}_t^c\right) \\ &- \frac{\kappa_{m,j}\beta(1-\rho_\pi)}{1+\kappa_{m,j}\beta} \hat{\bar{\pi}}_t^c \\ &+ \frac{1}{1+\kappa_{m,j}\beta} \frac{(1-\beta\xi_{m,j})(1-\xi_{m,j})}{\xi_{m,j}} \widehat{mc}_t^{m,j}. \end{aligned}$$

Similarly, we have the following normalized conditions for the FOCs related to public consumption and investment goods:

$$0 = \mathbb{E}_t\left[\psi_{z^+,t} p_t^{g,m,j} \Xi_t^j + \left(\frac{\tilde{\pi}_{t+1}^{g,m,j}}{\pi_{t+1}^{m,j}}\right)^{\frac{1}{1-\lambda_{g,m,j}}} \beta \xi_{m,j} F_{g,m,j,t+1} - F_{g,m,j,t}\right], \tag{C.60}$$

$$0 = \mathbb{E}_t\left[\lambda_{g,m,j} \psi_{z^+,t} p_t^{g,m,j} mc_t^{g,m,j} \Xi_t^j + \beta \xi_{m,j} \left(\frac{\tilde{\pi}_{t+1}^{g,m,j}}{\pi_{t+1}^{g,m,j}}\right)^{\frac{\lambda_{g,m,j}}{1-\lambda_{g,m,j}}} K_{g,m,j,t+1} - K_{g,m,j,t}\right], \tag{C.61}$$

$$\mathring{p}_t^{g,m,j} = \left[(1-\xi_{g,m,j}) \left(\frac{1-\xi_{g,m,j}\left(\frac{\tilde{\pi}_t^{g,m,j}}{\pi_t^{g,m,j}}\right)^{\frac{1}{1-\lambda_{g,m,j}}}}{1-\xi_{g,m,j}}\right)^{\lambda_{g,m,j}} + \xi_{g,m,j}\left(\frac{\tilde{\pi}_t^{g,m,j}}{\pi_t^{g,m,j}} \mathring{p}_{t-1}^{g,m,j}\right)^{\frac{\lambda_{g,m,j}}{1-\lambda_{g,m,j}}}\right]^{\frac{1-\lambda_{g,m,j}}{\lambda_{g,m,j}}}, \tag{C.62}$$

$$\frac{K_{g,m,j,t}}{F_{g,m,j,t}} = \left[\frac{1-\xi_{g,m,j}\left(\frac{\tilde{\pi}_t^{g,m,j}}{\pi_t^{g,m,j}}\right)^{\frac{1}{1-\lambda_{g,m,j}}}}{1-\xi_{g,m,j}}\right]^{1-\lambda_{g,m,j}}, \tag{C.63}$$

for $j = c, i$, where

$$\Xi_t^j = \begin{cases} g_{c,t}^m & j = c, \\ g_{i,t}^m & j = i. \end{cases}$$

C.3.12 Wage setting conditions

Scaling the value of a worker to a firm expressed in units of the homogeneous domestic good and the corresponding value definitions yields:

$$\bar{j}_t = \vartheta_t^p - \bar{w}_t^p, \tag{C.64}$$
$$\vartheta_t^p = \vartheta_t + \rho_t \mathbb{E}_t[\beta \psi_{z^+,t+1}/\psi_{z^+,t} \cdot \vartheta_{t+1}^p], \tag{C.65}$$
$$\bar{w}_t^p = (1+\tau_{e,t}^w)\bar{w}_t + \rho_t \mathbb{E}_t[\beta \psi_{z^+,t+1}/\psi_{z^+,t} \cdot \bar{w}_{t+1}^p], \tag{C.66}$$
$$w_{p,t} = (1-\tau_t^y - \tau_{w,t}^w)\bar{w}_t + \rho_t \mathbb{E}_t[((1-\lambda_r)\beta \psi_{z^+,t+1}/\psi_{z^+,t} + \lambda_r \beta \psi_{z^+,t+1}^r/\psi_{z^+,t}^r) \cdot w_{p,t+1}] \tag{C.67}$$

Free entry by wholesalers is determined in scaled form by the following equation:

$$Q_t\left(\bar{j}_t - \kappa_t^h\right) = \kappa_t^v \tag{C.68}$$

The value functions related to supplying labour in scaled form are given by:

$$\bar{v}_t = w_{p,t} + a_t^w, \tag{C.69}$$
$$\begin{aligned} a_t^w = {} & (1-\rho_t)\,\mathbb{E}_t\left[\left((1-\lambda_r)\beta \psi_{z^+,t+1}/\psi_{z^+,t} + \lambda_r \beta \psi_{z^+,t+1}^r/\psi_{z^+,t}^r\right) \cdot [f_{t+1}\bar{v}_{t+1} + (1-f_{t+1})\,\bar{u}_{t+1}]\right] \\ & + \rho_t \mathbb{E}_t[((1-\lambda_r)\beta \psi_{z^+,t+1}/\psi_{z^+,t} + \lambda_r \beta \psi_{z^+,t+1}^r/\psi_{z^+,t}^r) \cdot a_{t+1}^w], \end{aligned} \tag{C.70}$$
$$\bar{u}_t = b_t^u + \tilde{u}_t, \tag{C.71}$$
$$\tilde{u}_t = \mathbb{E}_t\left[\left((1-\lambda_r)\beta \psi_{z^+,t+1}/\psi_{z^+,t} + \lambda_r \beta \psi_{z^+,t+1}^r/\psi_{z^+,t}^r\right) \cdot [f_{t+1}\bar{v}_{t+1} + (1-f_{t+1})\,\bar{u}_{t+1}]\right]. \tag{C.72}$$

The unemployment benefits are given in scaled form as follows

$$b_t^u = \frac{B_t^u}{z_t^+ P_t}. \tag{C.73}$$

The bargaining first-order condition is given by

$$\bar{j}_t = \frac{1+\tau_{e,t}^w}{1-\tau_t^y-\tau_{w,t}^w}\frac{1-\eta}{\eta}(\bar{v}_t - \bar{u}_t). \tag{C.74}$$

C.3.13 Scaling laws of motion of private and public capital, capital return, and capital bundle

Using Equation (B.5), the law of motion of private capital in scaled terms is

$$\bar{k}_{t+1} = \frac{1-\delta}{\mu_{z^+,t}\mu_{\Psi,t}}\bar{k}_t + \Upsilon_t\left(1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right)\right) i_t. \tag{C.75}$$

Expressing R_t^k in scaled terms gives:

$$R_{t+1}^k = \frac{\pi_{t+1}}{\mu_{\Psi,t+1}} \frac{(1-\tau^k)\left[u_{t+1}\bar{r}_{t+1}^k - p_{t+1}^i a(u_{t+1})\right] + (1-\delta)p_{k',t+1} + \tau^k \delta \frac{\mu_{\Psi,t+1}}{\pi_{t+1}} p_{k',t}}{p_{k',t}}. \tag{C.76}$$

The law of motion of public capital in scaled terms is given by

$$k_{g,t+1} = (1-\delta_g)\frac{k_{g,t}}{\mu_{\psi,t}\mu_{z^+,t}} + \frac{a_{g,i,t-1}}{\mu_{\psi,t}\mu_{z^+,t}}, \qquad g_{i,t} = b_0 a_{g,i,t} + b_1 \frac{a_{g,i,t-1}}{\mu_{\psi,t}\mu_{z^+,t}}. \tag{C.77}$$

The capital bundle in scaled terms is equal to:

$$\widetilde{k}_t = \left(\alpha_k^{\frac{1}{\nu_k}}(k_t)^{\frac{\nu_k-1}{\nu_k}} + (1-\alpha_k)^{\frac{1}{\nu_k}}(k_{g,t})^{\frac{\nu_k-1}{\nu_k}}\right)^{\frac{\nu_k}{\nu_k-1}}. \tag{C.78}$$

C.3.14 Output and aggregate factors of production

Below we derive a relationship between total output of the domestic homogeneous good, Y_t, and aggregate factors of production. Consider the unweighted average of the intermediate goods:

$$\begin{aligned} Y_t^{sum} &= \int_0^1 Y_{i,t} di \\ &= \int_0^1 \left[(z_t H_{i,t})^{1-\alpha} \epsilon_t \widetilde{K}_{i,t}^{\alpha} - z_t^+ \phi \right] di \\ &= \int_0^1 \left[z_t^{1-\alpha} \epsilon_t \left(\frac{\widetilde{K}_{i,t}}{H_{i,t}} \right)^{\alpha} H_{i,t} - z_t^+ \phi \right] di \\ &= z_t^{1-\alpha} \epsilon_t \left(\frac{\widetilde{K}_t}{L_t} \right)^{\alpha} \int_0^1 H_{i,t} di - z_t^+ \phi, \end{aligned}$$

where K_t is the economy-wide average stock of capital services and L_t is the economy-wide average of labour. The last expression exploits the fact that all intermediate good firms confront the same factor prices, and so they adopt the same capital services to labour ratio. This follows from cost minimization, and holds for all firms, regardless whether or not they have an opportunity to reoptimize. Then, using the labour market clearing condition

$$Y_t^{sum} = z_t^{1-\alpha} \epsilon_t \widetilde{K}_t^{\alpha} L_t^{1-\alpha} - z_t^+ \phi.$$

Recall that the demand for $Y_{j,t}$ is

$$\left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_d}{\lambda_d - 1}} = \frac{Y_{i,t}}{Y_t},$$

so that

$$\mathring{Y}_t := \int_0^1 Y_{i,t} di = \int_0^1 Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_d}{\lambda_d - 1}} di = Y_t P_t^{\frac{\lambda_d}{\lambda_d - 1}} \left(\mathring{P}_t \right)^{\frac{\lambda_d}{1 - \lambda_d}},$$

where

$$\mathring{P}_t = \left[\int_0^1 P_{i,t}^{\frac{\lambda_d}{1-\lambda_d}} di \right]^{\frac{1-\lambda_d}{\lambda_d}}. \tag{C.79}$$

Dividing by P_t,

$$\mathring{p}_t = \left[\int_0^1 \left(\frac{P_{it}}{P_t} \right)^{\frac{\lambda_d}{1-\lambda_d}} di \right]^{\frac{1-\lambda_d}{\lambda_d}},$$

or,

$$\mathring{p}_t = \left[(1 - \xi_p) \left(\frac{1 - \xi_p \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \right)^{\frac{1}{1-\lambda_d}}}{1 - \xi_p} \right)^{\lambda_d} + \xi_p \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \mathring{p}_{t-1} \right)^{\frac{\lambda_d}{1-\lambda_d}} \right]^{\frac{1-\lambda_d}{\lambda_d}}. \tag{C.80}$$

The preceding equations imply

$$Y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \mathring{Y}_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[z_t^{1-\alpha} \epsilon_t \widetilde{K}_t^{\alpha} L_t^{1-\alpha} - z_t^+ \phi \right],$$

or, after scaling by z_t^+,

$$y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\epsilon_t \left(\frac{1}{\mu_{\Psi,t} \mu_{z^+,t}} \widetilde{k}_t \right)^{\alpha} L_t^{1-\alpha} - \phi \right].$$

C.3.15 Restrictions across inflation rates

We now consider the restrictions across inflation rates implied by the relative price formulas. In terms of the expressions in Equation (C.3), there are the restrictions implied by $P_t^{m,j}/P_{t-1}^{m,j}$, $j = x, c, i$, by $P_t^{g,m,j}/P_{t-1}^{g,m,j}$, $j = c, i$, and p_t^x. The restrictions implied by the other two relative prices in Equation (C.3), p_t^i and p_t^c, have already been used in Equations (B.38) and (C.75), respectively. Finally, we also use the restriction across inflation rates implied by q_t/q_{t-1} and Equation (B.52). Thus,

$$\frac{p_t^{m,x}}{p_{t-1}^{m,x}} = \frac{\pi_t^{m,x}}{\pi_t}, \tag{C.81}$$

$$\frac{p_t^{m,c}}{p_{t-1}^{m,c}} = \frac{\pi_t^{m,c}}{\pi_t}, \tag{C.82}$$

$$\frac{p_t^{m,i}}{p_{t-1}^{m,i}} = \frac{\pi_t^{m,i}}{\pi_t}, \tag{C.83}$$

$$\frac{p_t^{x}}{p_{t-1}^{x}} = \frac{\pi_t^{x}}{\pi_t^f}, \tag{C.84}$$

$$\frac{q_t}{q_{t-1}} = \frac{s_t \pi_t^f}{\pi_t^c}, \tag{C.85}$$

$$\frac{p_t^{g,m,c}}{p_{t-1}^{g,m,c}} = \frac{\pi_t^{g,m,c}}{\pi_t}, \tag{C.86}$$

$$\frac{p_t^{g,m,i}}{p_{t-1}^{g,m,i}} = \frac{\pi_t^{g,m,i}}{\pi_t}. \tag{C.87}$$

C.3.16 Scaling of fiscal sector equations

The government budget constraint is given by, in scaled terms:

$$g_t = \text{deficit}_{g,t} + t_t, \tag{C.88}$$

Real public debt is governed by the following law of motion, using scaled variables:

$$d_{g,t} = \frac{d_{g,t-1}}{\mu_{z^+,t}\pi_t} + \text{deficit}_{g,t}. \tag{C.89}$$

The total expenditure of the government in scaled terms is given by:

$$g_t = g_{c,t}^{exp} + g_{i,t}^{exp} + \text{tr}_t + d_{b,t}(1 - L_t) + \frac{\ln(R_{g,t-1})}{\pi_t} \cdot \frac{d_{g,t-1}}{\mu_{z^+,t}} + z_t. \tag{C.90}$$

The total tax revenues, using scaled variables, become:

$$\begin{aligned} t_t = {} & \tau_t^c p_t^c(\lambda_r c_{r,t} + (1-\lambda_r)c_{o,t}) + (\tau_t^y + \tau_{e,t}^w + \tau_{w,t}^w)L_t\bar{w}_t \\ & + \tau_t^k \bar{k}_t((u_t\bar{r}_t^k - p_t^i a(u_t))/(\mu_{\psi,t}\mu_{z+,t}) - \delta p_t^i/(\pi_t\mu_{z^+,t})) + t_{ls,t}. \end{aligned} \tag{C.91}$$

Public debt holdings evolve as follows:

$$d_{g,t} = b_{g,t+1} + b_{g,t+1}^*. \tag{C.92}$$

The split of total transfers in scaled terms is given by:

$$\text{tr}_t = \lambda_r \cdot \text{tr}_{r,t} + (1-\lambda_r)\text{tr}_{o,t}, \qquad \tau_r^{tr} \cdot \text{tr}_{o,t} = (1-\tau_r^{tr})\text{tr}_{r,t}. \tag{C.93}$$

C.3.17 Scaling of aggregate resource constraint and current account

The resource constraint in scaled terms is

$$\begin{aligned} y_t - d_t = z_t + g^d_{c,t} + g^d_{i,t} + (1-\omega_c)(p^c_t)^{\eta_c} c_t + \left(p^i_t\right)^{\eta_i} \left(i_t + a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}} \right) (1-\omega_i) \\ + \left[\omega_x (p^{m,x}_t)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x)(\mathring{p}^x_t)^{\frac{-\lambda_x}{\lambda_x - 1}} (p^x_t)^{-\eta_x} y^f_t, \end{aligned} \quad \text{(C.94)}$$

where

$$d_t = \frac{\mu G(\bar{\omega}_t; \sigma_{t-1}) R^k_t p_{k',t-1} \bar{k}_t}{\pi_t \mu_{z^+,t}} + \left(\frac{\kappa^v_t}{Q_t} + \kappa^h_t \right) \chi_t L_{t-1} + \frac{\gamma_g (b_{g,t+1} - \bar{b}_{g,t+1})^2}{2\bar{b}_{g,t+1}}.$$

When GDP is matched to the data, the following scaled equation emerges:

$$gdp_t = y_t - d_t - (p^i_t)^{\eta_i} \left(a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}} \right) (1-\omega_i).$$

The current account can be written as follows in scaled form:

$$\begin{aligned} a_t + b^*_{g,t} R_{g,t-1} + q_t p^c_t R^{\nu,*}_t \left(c^m_t (\mathring{p}^{m,c}_t)^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}} + i^m_t (\mathring{p}^{m,i}_t)^{\frac{\lambda_{m,i}}{1-\lambda_{m,i}}} + x^m_t (\mathring{p}^{m,x}_t)^{\frac{\lambda_{m,x}}{1-\lambda_{m,x}}} \right) \\ = q_t p^c_t p^x_t x_t + R^*_{t-1} \Phi_{t-1} s_t \frac{a_{t-1}}{\pi_t \mu_{z^+,t}} + b^*_{g,t+1}, \end{aligned} \quad \text{(C.95)}$$

where $a_t = S_t A^*_{t+1} / (P_t z^+_t)$.

C.3.18 Equilibrium conditions for financial frictions

Derivation of optimal contract. As noted in the text, it is supposed that the equilibrium debt contract maximizes entrepreneurial welfare subject to the zero profit condition on banks and the specified required return on household bank liabilities. The date t debt contract specifies a level of debt B_{t+1} and a state $t+1$-contingent rate of interest, Z_{t+1}. We suppose that entrepreneurial welfare corresponds to the entrepreneur's expected wealth at the end of the contract. It is convenient to express welfare as a ratio to the amount the entrepreneur could receive by depositing his net worth in a bank:

$$\begin{aligned} \frac{\mathbb{E}_t \left[\int_{\bar{\omega}_{t+1}}^{\infty} \left(R^k_{t+1} \omega P_t P_{k',t} \bar{K}_{t+1} - Z_{t+1} B_{t+1} \right) dF(\omega; \sigma_t) \right]}{R_t N_{t+1}} &= \frac{\mathbb{E}_t \left[\int_{\bar{\omega_{t+1}}}^{\infty} (\omega - \bar{\omega}_{t+1}) dF(\omega;\sigma_t) R^k_{t+1} P_t P_{k',t} \bar{K}_{t+1} \right]}{R_t N_{t+1}} \\ &= \mathbb{E}_t \left[[1 - \Gamma(\bar{\omega}_{t+1}; \sigma_t)] \frac{R^k_{t+1}}{R_t} \right] \varrho_t, \end{aligned}$$

after making use of Equations (B.85), (B.87), and

$$1 = \int_0^{\infty} \omega dF(\omega; \sigma_t) = \int_{\bar{\omega}_{t+1}}^{\infty} \omega dF(\omega;\sigma_t) + G(\bar{\omega}_{t+1}; \sigma_t).$$

We can equivalently characterize the contract by a state-$t+1$ contingent set of values for $\bar{\omega}_{t+1}$ and a value of ϱ_t. The equilibrium contract is the one involving $\bar{\omega}_{t+1}$ and ϱ_t which maximizes entrepreneurial welfare (relative to $R_t N_{t+1}$) subject to the bank zero profits condition. The Lagrangian representation of this problem is

$$\max_{\varrho_t, \{\bar{\omega}_{t+1}\}} \mathbb{E}_t \left[[1 - \Gamma(\bar{\omega}_{t+1}; \sigma_t)] \frac{R^k_{t+1}}{R_t} \varrho_t + \lambda_{t+1} \left([\Gamma(\bar{\omega}_{t+1}; \sigma_t) - \mu G(\bar{\omega}_{t+1}; \sigma_t)] \frac{R^k_{t+1}}{R_t} \varrho_t - \varrho_t + 1 \right) \right],$$

where λ_{t+1} is the Lagrange multiplier which is defined for each period $t+1$ state of nature. The FOCs for this problem are:

$$\mathbb{E}_t\left[[1-\Gamma(\bar{\omega}_{t+1};\sigma_t)]\frac{R^k_{t+1}}{R_t}+\lambda_{t+1}\left([\Gamma(\bar{\omega}_{t+1};\sigma_t)-\mu G(\bar{\omega}_{t+1};\sigma_t)]\frac{R^k_{t+1}}{R_t}-1\right)\right]=0,$$

$$-\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)\frac{R^k_{t+1}}{R_t}+\lambda_{t+1}[\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)-\mu G_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)]\frac{R^k_{t+1}}{R_t}=0,$$

$$[\Gamma(\bar{\omega}_{t+1};\sigma_t)-\mu G(\bar{\omega}_{t+1};\sigma_t)]\frac{R^k_{t+1}}{R_t}\varrho_t-\varrho_t+1=0,$$

where the absence of λ_{t+1} from the complementary slackness condition reflects that it is assumed that $\lambda_{t+1}>0$ in each period $t+1$ state of nature. Substituting out for λ_{t+1} from the second equation into the first, the FOCs reduce to

$$\mathbb{E}_t\begin{bmatrix}[1-\Gamma(\bar{\omega}_{t+1};\sigma_{t+1})]\frac{R^k_{t+1}}{R_t}+\frac{\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)}{\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)-\mu G_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)}\times \\ \left([\Gamma(\bar{\omega}_{t+1};\sigma_t)-\mu G(\bar{\omega}_{t+1};\sigma_t)]\frac{R^k_{t+1}}{R_t}-1\right)\end{bmatrix}=0 \tag{C.96}$$

$$[\Gamma(\bar{\omega}_{t+1};\sigma_t)-\mu G(\bar{\omega}_{t+1};\sigma_t)]\frac{R^k_{t+1}}{R_t}\varrho_t-\varrho_t+1=0 \tag{C.97}$$

for $t=0,1,2,\ldots,\infty$ and for $t=-1,0,1,2,\ldots$ respectively.

Since N_{t+1} does not appear in the last two equations, ϱ_t and $\bar{\omega}_{t+1}$ are the same for all entrepreneurs regardless of their net worth.

Derivation of aggregation of across entrepreneurs. Let $f(N_{t+1})$ denote the density of entrepreneurs with net worth N_{t+1}. Then, aggregate average net worth $\bar{N}_{t+1}$ is given by

$$\bar{N}_{t+1}=\int_{N_{t+1}}N_{t+1}f(N_{t+1})dN_{t+1}.$$

We now derive the law of motion of $\bar{N}_{t+1}$. Consider the set of entrepreneurs who in period $t-1$ had net worth N. Their net worth after they have settled with the bank in period t is denoted V^N_t, where

$$V^N_t=R^k_tP_{t-1}P_{k',t-1}\bar{K}^N_t-\Gamma(\bar{\omega}_t;\sigma_{t-1})R^k_tP_{t-1}P_{k',t-1}\bar{K}^N_t, \tag{C.98}$$

where $\bar{K}^N_t$ is the amount of physical capital that entrepreneurs with net worth N_t acquired in period $t-1$. Clearing in the market for capital requires:

$$\bar{K}_t=\int_{N_t}\bar{K}^N_tf(N_t)dN_t.$$

Multiplying Equation (C.98) by $f(N_t)$ and integrating over all entrepreneurs yields

$$V_t=R^k_tP_{t-1}P_{k',t-1}\bar{K}_t-\Gamma(\bar{\omega}_t;\sigma_{t-1})R^k_tP_{t-1}P_{k',t-1}\bar{K}_t.$$

Writing this out fully:

$$\begin{aligned}V_t&=R^k_tP_{t-1}P_{k',t-1}\bar{K}_t-\left\{[1-F(\bar{\omega}_t;\sigma_{t-1})]\bar{\omega}_t+\int_0^{\bar{\omega}_t}\omega dF(\omega;\sigma_{t-1})\right\}R^k_tP_{t-1}P_{k',t-1}\bar{K}_t\\&=R^k_tP_{t-1}P_{k',t-1}\bar{K}_t\\&-\left\{[1-F(\bar{\omega}_t;\sigma_{t-1})]\bar{\omega}_t+(1-\mu)\int_0^{\bar{\omega}_t}\omega dF(\omega;\sigma_{t-1})+\mu\int_0^{\bar{\omega}_t}\omega dF(\omega;\sigma_{t-1})\right\}R^k_tP_{t-1}P_{k',t-1}\bar{K}_t.\end{aligned}$$

Note that the first two terms in braces correspond to the net revenues of the bank, which must equal $R_{t-1}(P_{t-1}P_{k',t-1}\bar{K}_t-\bar{N}_t)$. Therefore, we obtain

$$V_t=R^k_tP_{t-1}P_{k',t-1}\bar{K}_t-\left\{R_{t-1}+\frac{\mu\int_0^{\bar{\omega}_t}\omega dF(\omega;\sigma_{t-1})R^k_tP_{t-1}P_{k',t-1}\bar{K}_t}{P_{t-1}P_{k',t-1}\bar{K}_t-\bar{N}_t}\right\}(P_{t-1}P_{k',t-1}\bar{K}_t-\bar{N}_t),$$

which implies Equation (B.90) in the main text.

C.3.19 Scaling of equilibrium conditions related to the financial friction in the model

Dividing both sides of Equation (B.90) by $P_t z_t^+$, we obtain the scaled law of motion for net worth:

$$n_{t+1} = \frac{\gamma_t}{\pi_t \mu_{z^+,t}} [R_t^k p_{k',t-1} \bar{k}_t - R_{t-1}(p_{k',t-1} \bar{k}_t - n_t) - \mu G(\bar{\omega}_t; \sigma_{t-1}) R_t^k p_{k',t-1} \bar{k}_t] + w_t^e \tag{C.99}$$

for $t = 0, 1, 2, \ldots$. Equation (C.99) has a simple intuitive interpretation. The first object in square brackets is the average gross return across all entrepreneurs in period t. The two negative terms correspond to what the entrepreneurs pay to the bank, including the interest paid by non-bankrupt entrepreneurs and the resources turned over to the bank by the bankrupt entrepreneurs. Since the bank makes zero profits, the payment to the bank by entrepreneurs must equal bank costs. The term involving R_{t-1} represents the cost of funds loaned to entrepreneurs by the bank, and the term involving μ represents the bank's total expenditures on monitoring costs.

The zero profit condition on banks, Equation (C.97), can be expressed in terms of the scaled variables as

$$\Gamma(\bar{\omega}_{t+1}; \sigma_t) - \mu G(\bar{\omega}_{t+1}; \sigma_t) = \frac{R_t}{R_{t+1}^k} \left(1 - \frac{n_{t+1}}{p_{k',t} \bar{k}_{t+1}} \right) \tag{C.100}$$

for $t = -1, 0, 1, 2, \ldots$.

D Measurement equations

Below are the measurement equations linking the model variables to the data. Where present, $\varepsilon_{i,t}^{me}$ denotes a measurement error for variable i.
Nominal deposit interest rate:

$$R_{D,t}^{data} = 400(R_t - 1) \tag{D.1}$$

Nominal government bond yield:

$$R_{L,t}^{data} = 400(R_{g,t} - 1) \tag{D.2}$$

Foreign (euro area) nominal interest rate:

$$R_t^{*,data} = 400(R_t^* - 1) \tag{D.3}$$

GDP deflator inflation:

$$\pi_t^{d,data} = 400 \log \pi_t - 400 \log \pi + \varepsilon_{\pi,t}^{me} \tag{D.4}$$

CPI inflation (after consumption tax):

$$\pi_t^{c,data} = 400 \log \pi_t^{c,\tau_c} - 400 \log \pi^c + \varepsilon_{\pi^c,t}^{me} \tag{D.5}$$

where

$$\pi_t^{c,\tau_c} = \pi_t^c \frac{1 + \tau_t^c}{1 + \tau_{t-1}^c} \tag{D.6}$$

Investment deflator inflation:

$$\pi_t^{i,data} = 400 \log \pi_t^i - 400 \log \pi^i + \varepsilon_{\pi^i,t}^{me} \tag{D.7}$$

Foreign (euro area) CPI inflation:

$$\pi_t^{*,data} = 400 \log \pi_t^* \tag{D.8}$$

Competitors' export price inflation:

$$\pi_t^{f,data} = 400 \log \pi_t^f \tag{D.9}$$

GDP quarter-on-quarter growth:

$$\Delta \log y_t^{data} = 100(\log \mu_{z^+,t} + \Delta \log y_t^{gdp} - \log \mu_{z^+}) + \varepsilon_{y,t}^{me} \tag{D.10}$$

where

$$\begin{aligned} y_t^{gdp} = y_t - p_t^i a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}} - \frac{\mu G(\bar{\omega}_t; \sigma_{t-1}) R_t^k p_{k',t-1} \bar{k}_t}{\pi_t \mu_{z^+,t}} \\ - \left(n_{\kappa_v,t}\kappa_v Q_t^{-1} + n_{\kappa_h,t}\kappa_h\right) \chi_t L_{t-1} - \Gamma_{g,t}/z_t^+ \end{aligned} \tag{D.11}$$

is the measured GDP without capital utilization costs, entrepreneur monitoring costs, vacancy posting and recruiting costs, and bond adjustment costs.
Foreign (euro area) GDP quarter-on-quarter growth:

$$\Delta \log y_t^{*,data} = 100(\log \mu_{z^+,t} + \Delta \log y_t^* - \log \mu_{z^+}) \tag{D.12}$$

Foreign demand (trading partners' imports) quarter-on-quarter growth:

$$\Delta \log y_t^{f,data} = 100(\log \mu_{z^+,t} + \Delta \log y_t^f - \log \mu_{z^+}) \tag{D.13}$$

Nominal effective exchange rate quarter-on-quarter growth:

$$\log s_t^{data} = 100 \log s_t + \varepsilon_{s,t}^{me} \tag{D.14}$$

Private consumption quarter-on-quarter growth:

$$\Delta \log c_t^{data} = 100(\log \mu_{z^+,t} + \Delta \log c_t - \log \mu_{z^+}) + \varepsilon_{c,t}^{me} \tag{D.15}$$

(Total) investment quarter-on-quarter growth:

$$\Delta \log i_t^{data} = 100[\log \mu_{z^+,t} + \log \mu_{\psi,t} + \Delta \log i_t^{tot}] - 100(\log \mu_{z^+} + \log \mu_{\psi}) + \varepsilon_{i,t}^{me} \tag{D.16}$$

where

$$i_t^{tot} = i_t + g_{i,t} \tag{D.17}$$

is total real investment.
Exports quarter-on-quarter growth:

$$\Delta \log x_t^{data} = 100(\log \mu_{z^+,t} + \Delta \log x_t - \log \mu_{z^+}) + \varepsilon_{x,t}^{me} \tag{D.18}$$

Imports quarter-on-quarter growth:

$$\Delta \log imp_t^{data} = 100(\log \mu_{z^+,t} + \Delta \log imp_t - \log \mu_{z^+}) + \varepsilon_{imp,t}^{me} \tag{D.19}$$

where

$$imp_t = \begin{pmatrix} c_t^m (\mathring{p}_t^{m,c})^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}} \\ +i_t^m (\mathring{p}_t^{m,i})^{\frac{\lambda_{m,i}}{1-\lambda_{m,i}}} \\ +x_t^m (\mathring{p}_t^{m,x})^{\frac{\lambda_{m,x}}{1-\lambda_{m,x}}} \\ +g_{c,t}^m (\mathring{p}_t^{m,g,c})^{\frac{\lambda_{m,g,c}}{1-\lambda_{m,g,c}}} \\ +g_{i,t}^m (\mathring{p}_t^{m,g,i})^{\frac{\lambda_{m,g,i}}{1-\lambda_{m,g,i}}} \end{pmatrix} \tag{D.20}$$

is total imports, and where the last two rows represent imports for government consumption and government investment.
Real labour cost quarter-on-quarter growth, deflated by after-tax CPI[22]:

$$\Delta \log w_t^{lc,data} = 100(\log \mu_{z^+,t} + \Delta \log w_t^{lc} - \log \mu_{z^+} + \log \pi_t - \log \pi_t^{c,\tau_c}) + \varepsilon_{lc,t}^{me} \tag{D.21}$$

where

$$w_t^{lc} = (1 + \tau_{e,t}^w)\bar{w}_t \tag{D.22}$$

Unemployment rate quarter-on-quarter growth:

$$\Delta \log Unemp_t^{data} = 100\Delta \log(1 - L_t) + \varepsilon_{Unemp,t}^{me} \tag{D.23}$$

Net worth as measured by real house price index, quarter-on-quarter growth:

$$\Delta \log n_t^{data} = 100(\log \mu_{z^+,t} + \Delta \log n_t - \log \mu_{z^+}) + \varepsilon_{n,t}^{me} \tag{D.24}$$

Nominal interest rate spread between lending and risk-free rate:

$$Spread_t^{data} = 400(spread_t - spread) + \varepsilon_{Spread,t}^{me} \tag{D.25}$$

[22] The default model variable is deflated by GDP deflator but its data counterpart is considerably noisier than that deflated by after-tax CPI; thus, here we transform the model variable in the measurement equation such that the deflator is after-tax CPI, consistent with the data.

where

$$spread_t = Z_{t+1} - R_t = \frac{\bar{\omega}_{t+1} R^k_{t+1}}{1 - \frac{n_{t+1}}{p_{k',t} \bar{k}_{t+1}}} - R_t \tag{D.26}$$

Government consumption quarter-on-quarter growth:

$$\Delta \log g^{data}_{c,t} = 100(\log \mu_{z^+,t} + \Delta \log g_{c,t} - \log \mu_{z^+}) + \varepsilon^{me}_{g,c,t} \tag{D.27}$$

Government investment quarter-on-quarter growth:

$$\Delta \log g^{data}_{i,t} = 100[\log \mu_{z^+,t} + \log \mu_{\psi,t} + \Delta \log g_{i,t}] - 100(\log \mu_{z^+} + \log \mu_{\psi}) + \varepsilon^{me}_{g,i,t} \tag{D.28}$$

Government debt-to-GDP ratio:

$$debt2GDP^{data}_t = \frac{D_{g,t}}{Y_t + Y_{t-1} + Y_{t-2} + Y_{t-3}} \tag{D.29}$$

Government transfers quarter-on-quarter growth:

$$\Delta \log tr^{data}_t = 100(\log \mu_{z^+,t} + \Delta \log tr_t - \log \mu_{z^+}) + \varepsilon^{me}_{tr,t} \tag{D.30}$$

Effective consumption tax rate:

$$\tau^{c,data}_t = \tau^c_t + \varepsilon^{me}_{\tau^c,t} \tag{D.31}$$

Effective labour income tax rate:

$$\tau^{y,data}_t = \tau^y_t + \varepsilon^{me}_{\tau^y,t} \tag{D.32}$$

Effective rate of social security contribution by employer:

$$\tau^{w,data}_{e,t} = \tau^w_{e,t} + \varepsilon^{me}_{\tau^w_e,t} \tag{D.33}$$

Effective rate of social security contribution by employee:

$$\tau^{w,data}_{w,t} = \tau^w_{w,t} + \varepsilon^{me}_{\tau^w_w,t} \tag{D.34}$$

E First-Order (Marginal Cost) Conditions

E.1 Model without public capital

Intermediate goods firms minimize total costs subject to the production function. Therefore, the optimization problem we solve here is:

$$\min_{\{H_t,K_t\}} \left\{TC_t = \tau_t^d P_t^H R_t^f H_t + \tau_t^d r_t^k P_t K_t + z_t^+ \phi\right\}, \tag{E.1}$$

subject to:

$$Y_t = (z_t H_t)^{1-\alpha} \varepsilon_t K_t^{\alpha} - z_t^+ \phi. \tag{E.2}$$

The Lagrange multiplier is denoted by MC_t as it equals the nominal marginal costs function in equilibrium. The first order conditions are:

$$K_t : \tau_t^d r_t^k P_t = \mathrm{MC}_t (z_t H_t)^{1-\alpha} \varepsilon_t \alpha K_t^{\alpha-1}, \tag{E.3}$$

$$H_t : \tau_t^d P_t^H R_t^f = \mathrm{MC}_t (1-\alpha) z_t^{1-\alpha} H_t^{-\alpha} \varepsilon_t K_t^{\alpha}. \tag{E.4}$$

These conditions become using stationary (normalized) variables:

$$\tau_t^d \bar{r}_t^k = \mathrm{mc}_t \alpha \varepsilon_t H_t^{1-\alpha} \left(\frac{k_t}{\mu_{z+,t}\mu_{\psi,t}}\right)^{\alpha-1}, \tag{E.5}$$

$$\tau_t^d \vartheta_t R_t^f = \mathrm{mc}_t (1-\alpha) \varepsilon_t H_t^{-\alpha} \left(\frac{k_t}{\mu_{z+,t}\mu_{\psi,t}}\right)^{\alpha}. \tag{E.6}$$

These normalized conditions can also be written as follows:

$$\mathrm{mc}_t = \frac{\tau_t^d \bar{r}_t^k}{\alpha \varepsilon_t \left(\frac{k_t}{\mu_{z+,t}\mu_{\psi,t} H_t}\right)^{\alpha-1}}, \tag{E.7}$$

$$\mathrm{mc}_t = \frac{\tau_t^d \vartheta_t R_t^f}{(1-\alpha) \varepsilon_t \left(\frac{k_t}{\mu_{z+,t}\mu_{\psi,t} H_t}\right)^{\alpha}}. \tag{E.8}$$

Equating Equations (E.7) and (E.8) and substituting the following resulting equation,

$$\frac{k_t}{\mu_{z+,t}\mu_{\psi,t}} = \frac{\vartheta_t R_t^f H_t}{\bar{r}_t^k} \frac{\alpha}{1-\alpha}, \tag{E.9}$$

into Equation (E.7) leads to the following expression after some simple algebra:

$$\mathrm{mc}_t = \frac{\tau_t^d}{\varepsilon_t} \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\bar{r}_t^k\right)^{\alpha} \left(\vartheta_t R_t^f\right)^{1-\alpha}. \tag{E.10}$$

E.2 Model with public capital in CES bundle

Intermediate goods firms minimize total costs subject to the production function. Therefore, the optimization problem we solve here is:

$$\min_{\{H_t,K_t\}} \left\{TC_t = \tau_t^d P_t^H R_t^f H_t + \tau_t^d r_t^k P_t K_t + z_t^+ \phi\right\}, \tag{E.11}$$

subject to:

$$Y_t = (z_t H_t)^{1-\alpha} \varepsilon_t \widetilde{K}_t^{\alpha} - z_t^+ \phi, \tag{E.12}$$

where:

$$\widetilde{K}_t = \left(\alpha_k^{\frac{1}{\nu_k}} (K_t)^{\frac{\nu_k-1}{\nu_k}} + (1-\alpha_k)^{\frac{1}{\nu_k}} (K_{g,t})^{\frac{\nu_k-1}{\nu_k}}\right)^{\frac{\nu_k}{\nu_k-1}}. \tag{E.13}$$

The Lagrange multiplier is denoted by MC_t as it equals the nominal marginal costs function in equilibrium. The first order conditions are:

$$K_t : \tau_t^d r_t^k P_t = \mathrm{MC}_t \left(z_t H_t\right)^{1-\alpha} \varepsilon_t \alpha \alpha_k^{1/\nu_k} \widetilde{K}_t^{\alpha+1/\nu_k-1} K_t^{-1/\nu_k}, \tag{E.14}$$

$$H_t : \tau_t^d P_t^H R_t^f = \mathrm{MC}_t (1-\alpha) z_t^{1-\alpha} H_t^{-\alpha} \varepsilon_t \widetilde{K}_t^{\alpha}. \tag{E.15}$$

These conditions become using stationary (normalized) variables:

$$\tau_t^d \bar{r}_t^k = \mathrm{mc}_t \alpha \alpha_k^{1/\nu_k} \varepsilon_t H_t^{1-\alpha} \left(\frac{\widetilde{k}_t}{\mu_{z+,t}\mu_{\psi,t}}\right)^{\alpha+1/\nu_k-1} \left(\frac{k_t}{\mu_{z+,t}\mu_{\psi,t}}\right)^{-1/\nu_k}, \tag{E.16}$$

$$\tau_t^d \vartheta_t R_t^f = \mathrm{mc}_t (1-\alpha) \varepsilon_t H_t^{-\alpha} \left(\frac{\widetilde{k}_t}{\mu_{z+,t}\mu_{\psi,t}}\right)^{\alpha}. \tag{E.17}$$

These normalized conditions can also be written as follows:

$$\mathrm{mc}_t = \frac{\tau_t^d \bar{r}_t^k}{\alpha \alpha_k^{1/\nu_k} \varepsilon_t \left(\frac{\widetilde{k}_t}{\mu_{z+,t}\mu_{\psi,t} L_t}\right)^{\alpha+1/\nu_k-1} \left(\frac{k_t}{\mu_{z+,t}\mu_{\psi,t} H_t}\right)^{-1/\nu_k}}, \tag{E.18}$$

$$\mathrm{mc}_t = \frac{\tau_t^d \vartheta_t R_t^f}{(1-\alpha)\varepsilon_t \left(\frac{\widetilde{k}_t}{\mu_{z+,t}\mu_{\psi,t} H_t}\right)^{\alpha}}. \tag{E.19}$$

Equating Equations (E.18) and (E.19) and substituting the following resulting equation:

$$\frac{k_t}{\mu_{z+,t}\mu_{\psi,t}} = \left(\frac{\bar{r}_t^k}{R_t^f \vartheta_t H_t}\right)^{-\nu_k} \left(\frac{1-\alpha}{\alpha}\right)^{-\nu_k} \alpha_k \left(\frac{\widetilde{k}_t}{\mu_{z+,t}\mu_{\psi,t}}\right)^{1-\nu_k}, \tag{E.20}$$

into Equation (E.18) leads to the following expression after some simple algebra:

$$\mathrm{mc}_t = \frac{\tau_t^d}{\varepsilon_t} \left(\frac{\widetilde{k}_t}{\mu_{z+,t}\mu_{\psi,t} H_t}\right)^{-\alpha} \frac{\vartheta_t R_t^f}{1-\alpha}. \tag{E.21}$$

Alternatively, equating Equations (E.18) and (E.19) and substituting the following resulting equation:

$$\frac{\widetilde{k}_t}{\mu_{z+,t}\mu_{\psi,t}} = \left(\frac{\bar{r}_t^k}{R_t^f \vartheta_t H_t}\right)^{\nu_k/(1-\nu_k)} \left(\frac{1-\alpha}{\alpha}\right)^{\nu_k/(1-\nu_k)} \alpha_k^{\nu_k-1} \left(\frac{k_t}{\mu_{z+,t}\mu_{\psi,t}}\right)^{1/(1-\nu_k)}, \tag{E.22}$$

into Equation (E.21) leads to the following expression after some not so simple algebra:

$$\mathrm{mc}_t = \frac{\tau_t^d}{\varepsilon_t} \left(\vartheta_t R_t^f\right)^{\frac{\alpha\nu_k}{1-\nu_k}+1} \left(\bar{r}_t^k\right)^{\frac{\alpha\nu_k}{\nu_k-1}} (\alpha)^{\frac{\alpha\nu_k}{1-\nu_k}} (\alpha_k)^{\frac{\alpha}{1-\nu_k}} (1-\alpha)^{\frac{\alpha\nu_k}{\nu_k-1}-1} \left(\frac{k_t}{\mu_{z+,t}\mu_{\psi,t} H_t}\right)^{\frac{\alpha}{\nu_k-1}}. \tag{E.23}$$

E.3 Model with public capital as an additional production factor

Intermediate goods firms minimize total costs subject to the production function. Therefore, the optimization problem we solve here is:

$$\min_{\{H_t, K_t\}} \left\{ TC_t = \tau_t^d P_t{}^{,H} R_t^f H_t + \tau_t^d r_t^k P_t K_t + z_t^+ \phi \right\}, \tag{E.24}$$

subject to:

$$Y_t = z_t^{1-\alpha-\alpha_k} H_t^{1-\alpha} \varepsilon_t K_t^{\alpha} K_{G,t}^{\alpha_k} - z_t^+ \phi. \tag{E.25}$$

The Lagrange multiplier is denoted by MC_t as it equals the nominal marginal costs function in equilibrium. The first order conditions are:

$$K_t : \tau_t^d r_t^k P_t = \mathrm{MC}_t z_t^{1-\alpha-\alpha_k} H_t^{1-\alpha} \varepsilon_t \alpha K_t^{\alpha-1} (K_{G,t})^{\alpha_k}, \tag{E.26}$$

$$H_t : \tau_t^d P_t^H R_t^f = \mathrm{MC}_t (1-\alpha) z_t^{1-\alpha-\alpha_k} H_t^{-\alpha} \varepsilon_t K_t^{\alpha} (K_{G,t})^{\alpha_k}. \tag{E.27}$$

These conditions become using stationary (normalized) variables:

$$\tau_t^d \bar{r}_t^k = \mathrm{mc}_t \alpha \varepsilon_t H_t^{1-\alpha} \left(\frac{k_t}{\mu_{z+,t}\mu_{\psi,t}}\right)^{\alpha-1} \left(\frac{k_{G,t}}{\mu_{z+,t}\mu_{\psi,t}}\right)^{\alpha_k}, \tag{E.28}$$

$$\tau_t^d \vartheta_t R_t^f = \mathrm{mc}_t (1-\alpha) \varepsilon_t H_t^{-\alpha} \left(\frac{k_t}{\mu_{z+,t}\mu_{\psi,t}}\right)^{\alpha} \left(\frac{k_{G,t}}{\mu_{z+,t}\mu_{\psi,t}}\right)^{\alpha_k}. \tag{E.29}$$

Note that the definition of z_t^+ changes into the following one:

$$z_t^+ = \Psi_t^{\frac{\alpha+\alpha_k}{1-\alpha-\alpha_k}} z_t. \tag{E.30}$$

These normalized conditions can also be written as follows:

$$\mathrm{mc}_t = \frac{\tau_t^d \bar{r}_t^k}{\alpha \varepsilon_t \left(\frac{k_t}{\mu_{z+,t}\mu_{\psi,t}H_t}\right)^{\alpha-1} \left(\frac{k_{G,t}}{\mu_{z+,t}\mu_{\psi,t}}\right)^{\alpha_k}}, \tag{E.31}$$

$$\mathrm{mc}_t = \frac{\tau_t^d \vartheta_t R_t^f}{(1-\alpha) \varepsilon_t \left(\frac{k_t}{\mu_{z+,t}\mu_{\psi,t}H_t}\right)^{\alpha} \left(\frac{k_{G,t}}{\mu_{z+,t}\mu_{\psi,t}}\right)^{\alpha_k}}. \tag{E.32}$$

Equating Equations (E.31) and (E.32) and substituting the following resulting equation:

$$\frac{k_t}{\mu_{z+,t}\mu_{\psi,t}} = \frac{\vartheta_t R_t^f H_t}{\bar{r}_t^k} \frac{\alpha}{1-\alpha}, \tag{E.33}$$

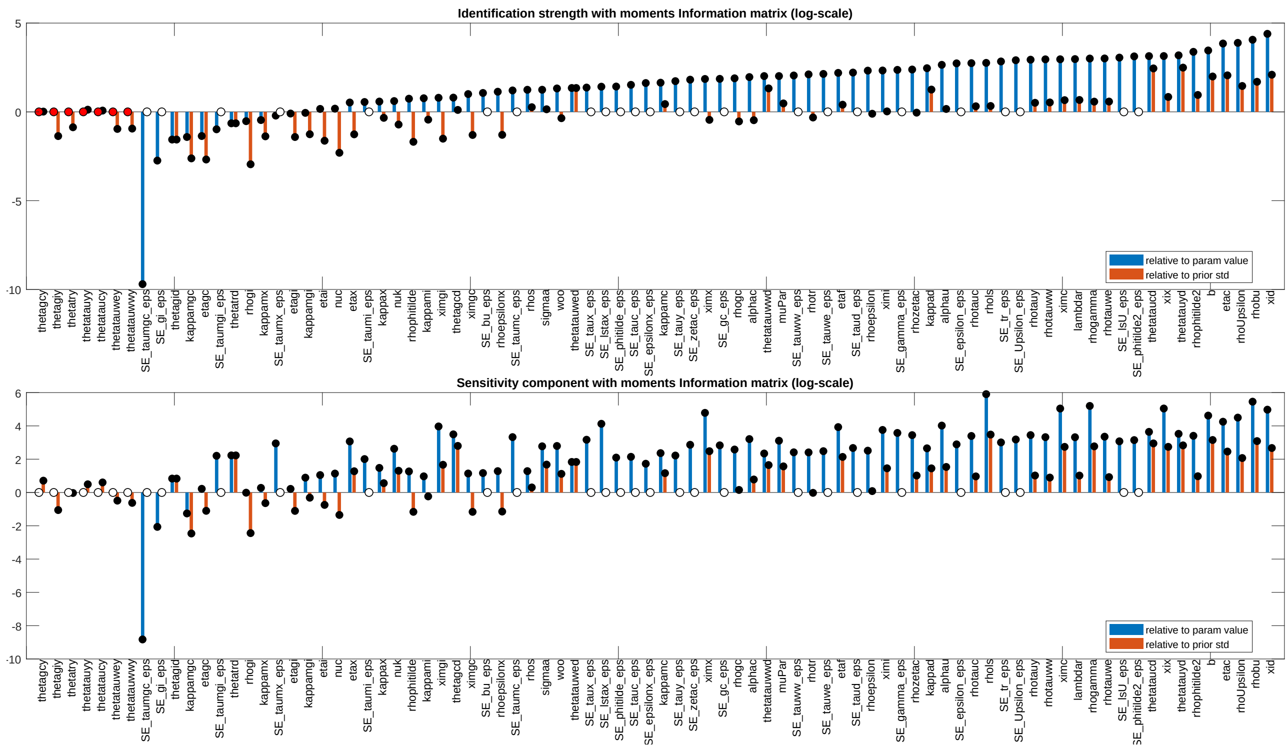
into Equation (E.31) leads to the following expression after some simple algebra:

$$\mathrm{mc}_t = \frac{\tau_t^d}{\varepsilon_t} \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\bar{r}_t^k\right)^{\alpha} \left(\vartheta_t R_t^f\right)^{1-\alpha} \left(\frac{k_{G,t}}{\mu_{z+,t}\mu_{\psi,t}}\right)^{-\alpha_k}. \tag{E.34}$$

F Additional Figures

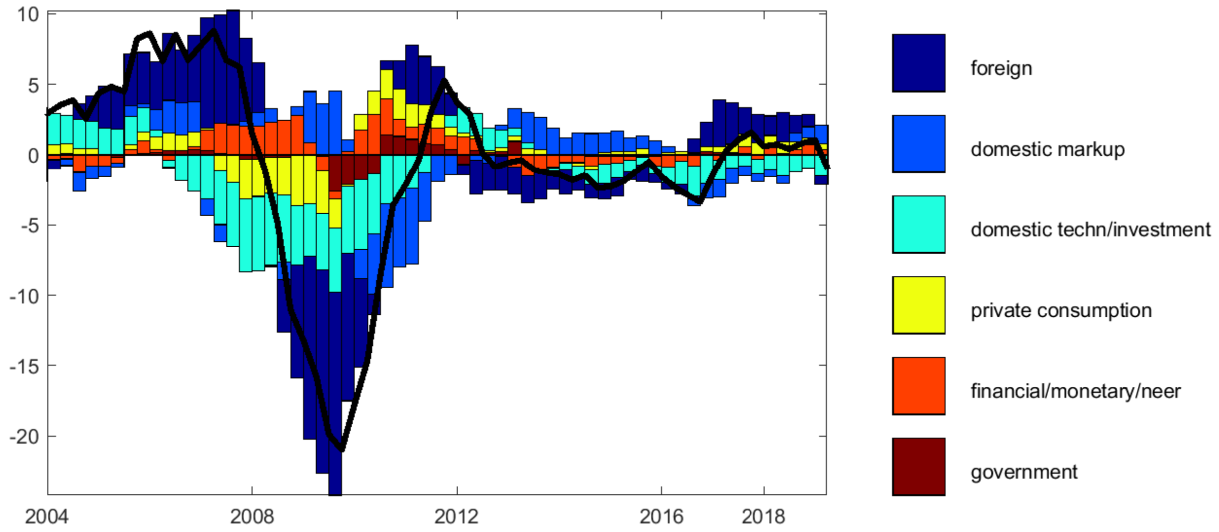
This appendix contains additional figures referenced in the main text.

Figure F.1: **Identification analysis of model parameters**



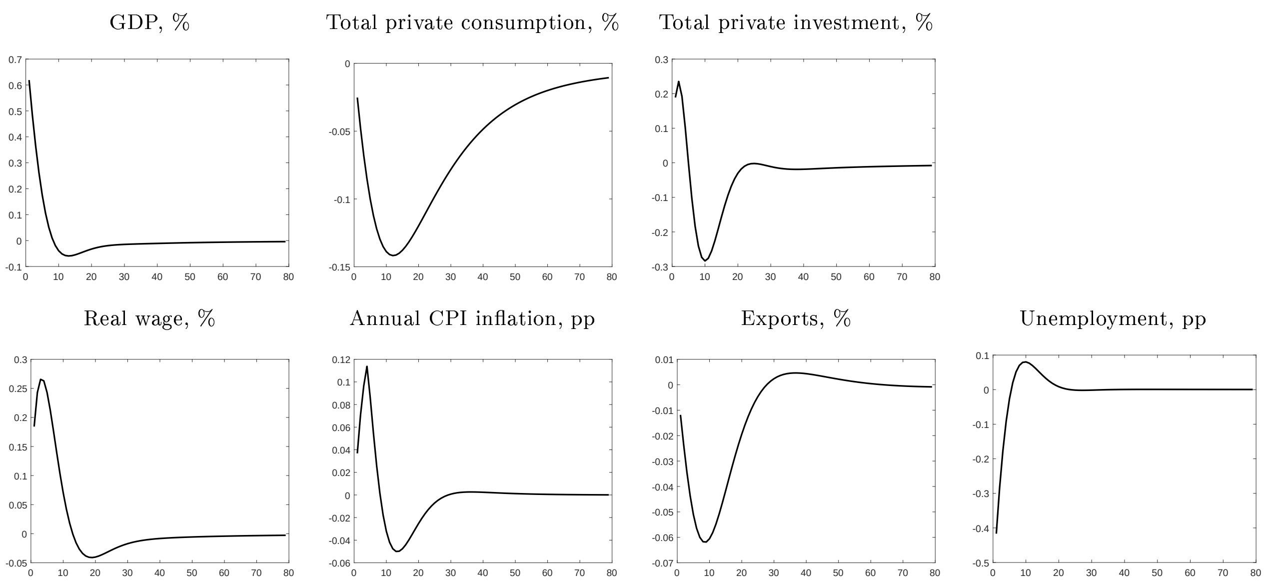
Notes: This figure depicts the fiscal model's parameter identification strength produced with the `dynare` identification command. Longer bars away from zero signal stronger identification. In particular, these results show that fiscal rule parameters in front of the output gap are weakly identified.

Figure F.2: **Historical shock decomposition of the annual GDP growth rate by the model of Bušs (2017)**



Notes: This figure shows the historical shock decomposition of the annual GDP growth rate, produced by the DSGE model without the fiscal block (Bušs, 2017). The model shocks are categorised in six groups – i) foreign shocks, ii) financial, monetary, and nominal effective exchange rate shocks, iii) domestic markup shocks, iv) domestic technology and investment-specific shocks, v) government-specific shocks, and vi) shocks to private consumption.

Figure F.3: **Effect of public consumption shocks in the model of Bušs (2017)**



Notes: This figure depicts GDP Y_t, total private consumption C_t, total private investment I_t, real wage $\bar{w}_t^p$, annual CPI inflation $\pi_{c,t}^{tax,yoy} = \sum_{i=0}^{3} 100\{\pi_{c,t-i}(1+\tau_{c,t-i})/(1+\tau_{c,t-i-1})\}$, exports X_t, and unemployment $1 - L_t$ in response to a shock in (wasteful) public consumption expenditure in the model of Bušs (2017). The shock size is calibrated to induce an increase in spending equal to 0.5% of final output in the steady state (Y) in period 1. The length of the impulse response functions is 80 quarters. The persistence of the public consumption expenditure shock is equal to 0.85.

References

Bernanke, B., Gertler, M. and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. In J. B. Taylor and M. Woodford (eds.), Handbook of Macroeconomics, vol. 1, 21, Elsevier Science, pp. 1341–1393.

Bušs, G. (2017). Wage formation, unemployment and business cycle in Latvia. Latvijas Banka Working Paper No. 1/2017.

Christiano, L. J., Eichenbaum, M. S. and Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. Journal of Political Economy, **113** (1), 1–45.

—, — and Trabandt, M. (2016). Unemployment and business cycles. Econometrica, **84** (4), 1523–1569.

Fisher, I. (1933). The debt-deflation theory of great depressions. Econometrica, **1** (4), 337–357.

Justiniano, A., Primiceri, G. E. and Tambalotti, A. (2011). Investment shocks and the relative price of investments. Review of Economic Dynamics, **14**, 102–121.