INF3410 — Fall 2015

Book Chapter 3: Basic Current Mirrors and Single-Stage Amplifiers

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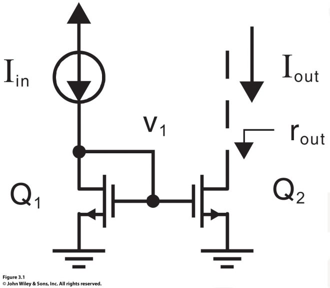
Simple Current Mirror



When will $I_{out} = I_{in}$, $I_{out} \neq I_{in}$, $I_{out} \approx I_{in}$, $I_{out} \approx xI_{in}$?

Diode Connected nMOS

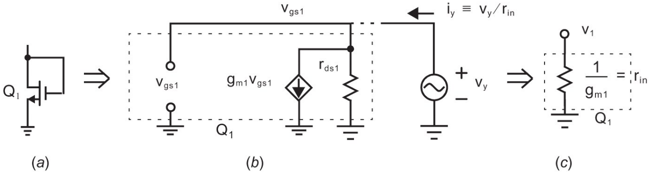


Figure 3.2

Is the simplified small signal model very useful here? For example when you want to know the voltage when you apply a given current?

Current Mirror Small Signal Model

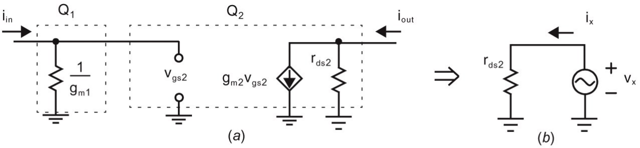


Figure 3.3

How do you describe the effect of different $\frac{W}{L}$ ratios of the two transistors in terms of the small signal model?

Current Mirror Small Signal Model, Example 3.1 (1/2)

A detour solution (shorter in the book):

$$
\begin{aligned}
V_{eff} = V_{in} - V_{t0} &= \sqrt{\frac{2}{\beta} I_{in}} \\
&= \sqrt{\frac{2}{10\mu\text{m}/0.4\mu\text{m} * 190\mu\text{A/V}^2} * 100\mu\text{A}} \\
&= 205\text{mV}
\end{aligned}
$$

$$V_{in} = 205\text{mV} + 570\text{mV} = 775\text{mV}$$

Current Mirror Small Signal Model, Example 3.1 (2/4)

Continued detour solution (shorter in the book):

$$
\begin{aligned}
I_{out} &= \frac{1}{2}\beta V_{eff}^2 \\
&= \frac{10\mu\text{m}/0.4\mu\text{m} * 190\mu\text{A/V}^2 * 205\text{mV}^2}{2} \\
&= 100\mu\text{A}
\end{aligned}
$$

Current Mirror Small Signal Model, Example 3.1 (3/4)

Wrong (!!!) approach:

$$\begin{aligned} g_m &= \beta * V_{eff} = 10\mu\text{m}/0.4\mu\text{m} * 190\mu\text{A/V}^2 * 0.205\text{V} \\ &= 974\mu\text{A/V} \end{aligned}$$

That was still correct but ...

$$I_{out} = V_{in} * g_m = 755\mu\text{A}$$

... is a wrong conclusion!!!

Current Mirror Small Signal Model, Example 3.1 (4/4)

Continued correct detour solution:

$$\begin{aligned} r_{out} &= \frac{1}{\lambda I_{out}} \\ &= \frac{L}{\lambda L I_{out}} \\ &= \frac{0.4\mu\text{m}}{0.16\mu m/V * 100\mu\text{A}} \\ &= 0.025\text{V}/\mu\text{A} = 25\text{k}\Omega \end{aligned}$$

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Common-Source Amplifier with active load

Active: current mirror or FET with fixed gate voltage as load
Passive: a resistor to Vdd (normally lower output impedance or excessive voltage across resistor)

What are the conditions for this circuit to work as an amplifier? When will it not work?

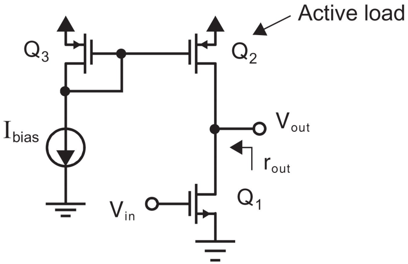


Figure 3.4

Small Signal Model

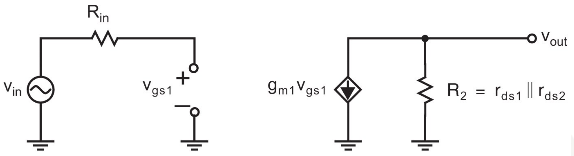


Figure 3.5

$$A_V = \frac{v_{out}}{v_{in}} = -g_m R_2 = -g_m (r_{ds1} \| r_{ds2})$$

(R_{in} not well motivated in the book: makes only sense with including parasitic capacitances. It is actually attributed to a non-ideal voltage source, i.e. not the transistor.)

Example 3.2 (1/3)

$$
\begin{aligned}
g_{m1}^2 &= 2\beta I_{bias} = 2 * 190\frac{\mu\text{A}}{\text{V}^2} * \frac{12\mu\text{m}}{0.5\mu\text{m}} * I_{D/bias} \\
&= 9120\frac{\mu\text{A}}{\text{V}^2} * I_{D/bias} \\
r_{ds1/2} &= \frac{L}{\lambda L I_{D/bias}} = \frac{0.5\mu\text{m}}{0.16\frac{\mu\text{m}}{\text{V}}} * \frac{1}{I_{D/bias}} \\
&= 3.125\text{V} * \frac{1}{I_{D/bias}}
\end{aligned}
$$

Example 3.2 (2/3)

$$
\begin{aligned}
A_V^2 = 15^2 &= (g_{m1} * (r_{ds1} \parallel r_{ds2}))^2 = \left(g_{m1} * \frac{r_{ds1}}{2}\right)^2 \\
&= 9120\frac{\mu\text{A}}{\text{V}^2} * I_{D/bias} * [3.125\text{V}]^2 * \left(\frac{1}{2I_{D/bias}}\right)^2 \\
&= 22266\mu\text{A} * \frac{1}{I_{D/bias}} \\
I_{D/bias} &= \frac{22266\mu\text{A}}{15^2} = 99\mu\text{A}
\end{aligned}
$$

Example 3.2 (3/3)

$$
\begin{aligned}
V_{eff} = V_{in/DC} &= \sqrt{\frac{2I_D}{\beta}} \\
&= \sqrt{\frac{2 * 99\mu\mathrm{A}}{190\frac{\mu\mathrm{A}}{\mathrm{V}^2} * \frac{12\mu\mathrm{m}}{0.5\mu\mathrm{m}}}} \\
&= 208.4\mathrm{mV}
\end{aligned}
$$

This V_{eff} is the DC level (large signal value) of the input! For the V_{eff} of the pFETs one would need to use a different μC_{ox} resp. β!

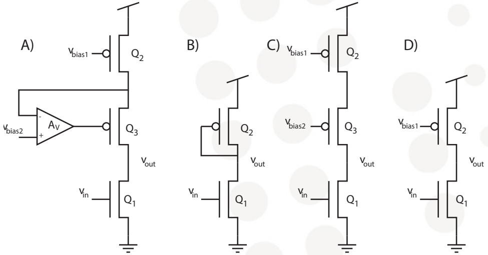
Example 3.2 alternative

For Common-Source with active load where $r_{ds1} = r_{ds2}$:

$$
\begin{aligned}
A_V &= \frac{A_i}{2} = \frac{g_{m1} r_{ds1}}{2} \approx \frac{1}{\lambda V_{eff}} \\
V_{eff} &= \frac{1}{\lambda 15} = \frac{0.5\mu\text{m}}{0.16\frac{\mu\text{m}}{\text{V}}15} \\
&= 208.4mV \\
I_D &= \frac{1}{2}\beta V_{eff}^2 = 99\mu\text{A}
\end{aligned}
$$

Exam 2013, task 2 and 3

We'll have a look at the circuits in B) and D) in the tasks 2 and 3. Circuits A) and C) you do not yet understand, but maybe we can try to get an first intuitive feeling for them as well ...



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Simple Current Mirror
Common-Source Amplifier
Common-Drain Amplifier with active load / Source Follower
Common-Gate Amplifier with active load
Source-Degenerated Current Mirrors
Cascode Current Mirrors
Cascode Gain Stage
Diff Pair and Gain Stage

Source Follower

What are the conditions for this circuit to work? Would you call it an amplifier?

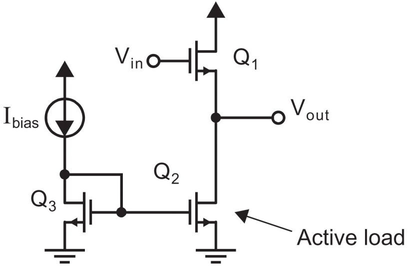


Figure 3.6

Small Signal Model

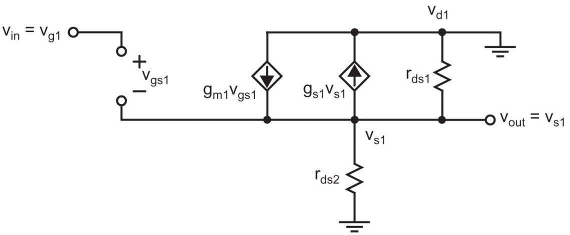
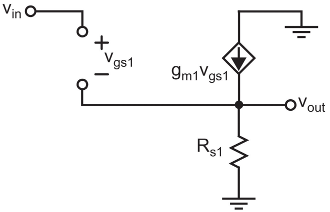


Figure 3.7

Simplified Small Signal Model



This circuit is used to follow a voltage but with an offset and small output resistance.

$$
\begin{aligned}
A_V &= g_{m1}\left(\frac{1}{g_{m1}} \parallel \frac{1}{g_{s1}} \parallel r_{ds1} \parallel r_{ds2}\right) \\
&= \frac{g_{m1}}{g_{m1} + g_{s1} + \frac{1}{r_{ds1}} + \frac{1}{r_{ds2}}} \\
&\approx 1
\end{aligned}
$$

(but actually somewhat smaller)

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Common Gate Amplifier

Similar to common-source amplifier but with a less than infinite input resistance. Good for e.g terminating resistance for input from a (e.g. 50Ω) cable or current input (transresistance).

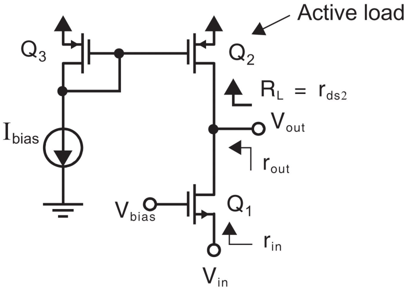

Figure 3.9

Small Signal Model

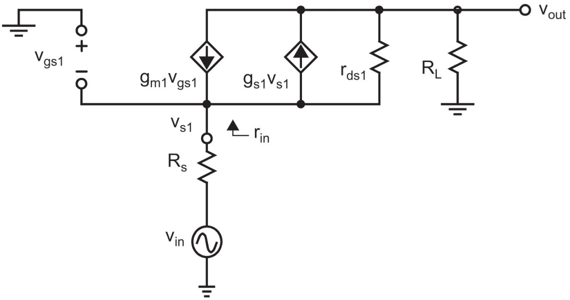

Figure 3.10

Simplified Small Signal Model and A_V

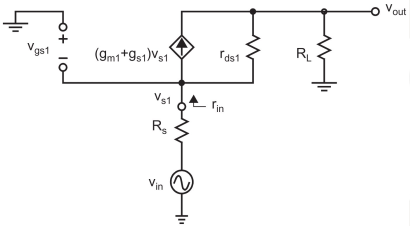

Figure 3.11

A_V is approximately the same as for the comon source amplifier.

$$(v_{out} - v_{s1})g_{ds1} + v_{out}G_L - v_{s1}(g_{m1} + g_{s1}) = 0$$

$$A_V = \frac{v_{out}}{v_{s1}} = \frac{g_{m1} + g_{s1} + g_{ds1}}{G_L + g_{ds1}} \approx \frac{g_{m1}}{G_L + g_{ds1}} = g_{m1}(R_L \parallel r_{ds1})$$

Small Signal Model and r_{in} (1/2)

$$
\begin{aligned}
i_{in} &= v_{in}(g_{m1}+g_{s1}) - (v_{out}-v_{in})g_{ds1} \\
&= v_{in}(g_{m1}+g_{s1}+g_{ds1}) - \frac{v_{in}(g_{m1}+g_{s1}+g_{ds1})}{G_L+g_{ds1}} g_{ds1} \\
&= v_{in}(g_{m1}+g_{s1}+g_{ds1}) \left(1 - \frac{g_{ds1}}{G_L+g_{ds1}}\right) \\
&= v_{in}(g_{m1}+g_{s1}+g_{ds1}) \frac{G_L}{G_L+g_{ds1}} \\
&= v_{in}(g_{m1}+g_{s1}+g_{ds1}) \frac{1}{1+\frac{g_{ds1}}{G_L}}
\end{aligned}
$$

Small Signal Model and r_{in} (2/2)

$$\begin{aligned} g_{in} &= \frac{i_{in}}{v_{in}} = (g_{m1} + g_{s1} + g_{ds1})\frac{1}{1 + \frac{g_{ds1}}{G_L}} \\ &\approx g_{m1}\frac{1}{1 + \frac{g_{ds1}}{G_L}} \end{aligned}$$

$$r_{in} \approx \frac{1}{g_{m1}}\left(1 + \frac{g_{ds1}}{G_L}\right)$$

A realistic voltage source at input

Since the input resistance is not infinit, one needs to be careful here: if considering a resistance R_S between ideal source and common-gate amplifier, e.g. a 50Ω cable or simply a realistic source with $r_{out} > 0$, A_V will appear smaller! That is to say when you first meaure v_{in} without connecting your source to the amplifier, then connect the amp, and then measure v_{out}, you will not see the A_V computed before, but A'_V (still called A_V in the book)!

$$\frac{v_{s1}}{v_{in}} = \frac{r_{in}}{r_{in} + R_S} \quad \Rightarrow A'_V = A_V \frac{r_{in}}{r_{in} + R_S}$$

$$A'_V \approx \frac{g_{m1}(R_L \parallel r_{ds1})}{1 + R_S \frac{g_{m1}+g_{s1}+g_{ds1}}{1+R_L/r_{ds1}}}$$

Content

Source-Degenerated Current Mirror

A first attempt to provide higher output resistance (i.e. a better current source). Note: requires higher voltage at output to work at all!

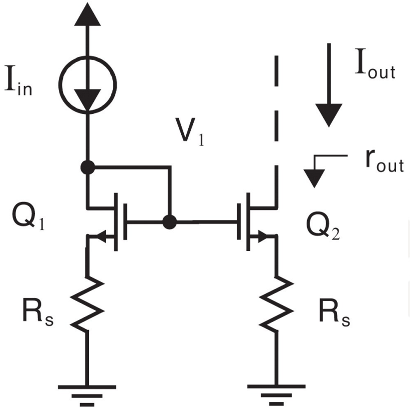

Figure 3.12

Small Signal Model

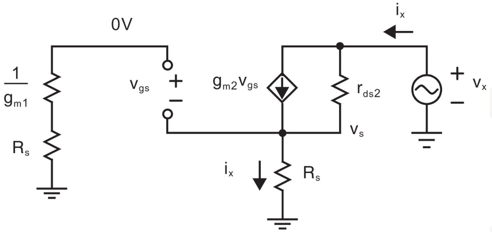

Figure 3.13

Important point: to compute small signal r_{out} the input current can be considered constant and thus the small signal input current source is an open circuit, i.e. $v_{gs} = -v_s$.

$$r_{out} = r_{ds2}\left[1 + R_S(g_{m2} + g_{s2} + g_{ds2})\right] \approx r_{ds2}\left[1 + R_S(g_{m2} + g_{s2})\right]$$

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Cascode Current Mirror

A second attempt to provide higher output resistance (i.e. a better current source). Note: also requires higher voltage at output to work at all but is less dependent on (large signal) current!

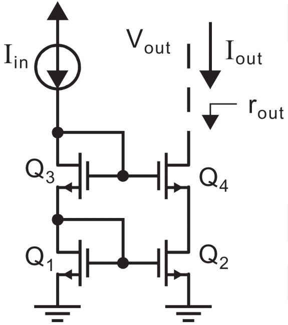

Figure 3.14

Small Signal r_{out}

The analysis is based on the source degenerated current mirror where former R_S is replaced by r_{ds2}

$$r_{out} = r_{ds4}\left[1 + r_{ds2}(g_{m4} + g_{s4} + g_{ds4})\right] \approx r_{ds4}r_{ds2}g_{m4} \quad (3.37)$$

$$V_{OUT} \overset{!}{>} 2V_{eff} + V_{tn} \quad (3.42)$$

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Cascode Gain Stage

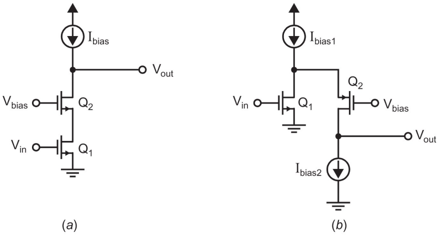

Figure 3.15

Offers large gain for a single stage (depends on quality of current source!) and limits voltage accross the input drive transistor (e.g. avoiding short channel effects).

Small Signal Model

The small signal analysis makes use of the previous common source and common gate analysis, simply multiplying the prvious two voltage gains:

$$\frac{v_{out}}{v_{in}} = \frac{v_{s2}}{v_{in}}\frac{v_{out}}{v_{s2}} \quad (3.54)$$

$$\approx -g_{m1}g_{m2} * (r_{ds1} \parallel r_{in2})(r_{ds2} \parallel R_L)$$

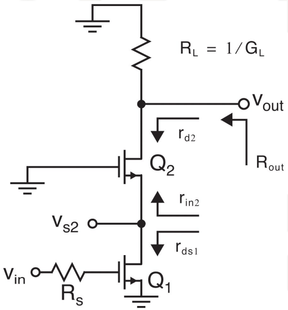

Figure 3.16

Small Signal Model Folded Cascode

Similar to the telescopic cascode gain stage but non inverting and with an extra load resistor (current source) diminishing the gain:

$$\frac{v_{out}}{v_{in}} = \frac{v_{s2}}{v_{in}} \frac{v_{out}}{v_{s2}} \quad (3.54) \approx g_{m1} g_{m2} (r_{ds1} \parallel r_{in2} \parallel R_L)(r_{ds2} \parallel R_L)$$

Assuming High Quality Current Mirrors

$$\frac{v_{out}}{v_{in}} \approx -g_{m1}g_{m2}(r_{ds1} \parallel r_{in2})(r_{ds2} \parallel R_L) \approx -\frac{1}{2}g_m^2 r_{ds}^2$$

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Diff Pair

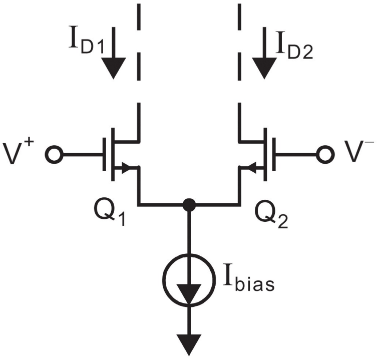
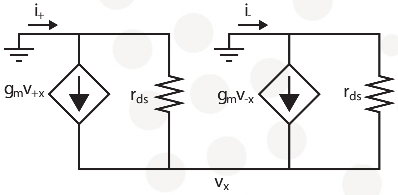


Figure 3.17

Realizes a differential input for most integrated amplifiers: immunity to noise/offsets that affect both inputs (e.g. pick-up noise on twisted-pair cables etc.)

Small signal model

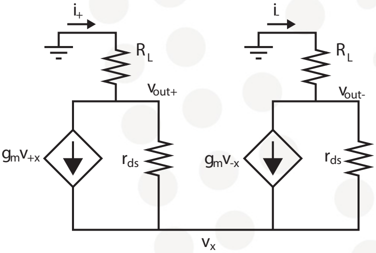
The book uses the T-model. Here comes an alternative deduction based on the 'normal' model.



$$i_{+/-} = (v_{+/-} - v_x)g_m - v_x\frac{1}{r_{ds}}$$

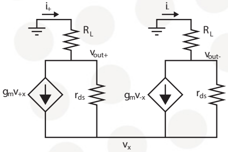
$$i_+ - i_- = (v_+ - v_-)g_m \quad (3.69)$$

Small signal model with resistive loads on each branch



$$i_{+/-} = -v_{out+/-}\frac{1}{R_L} = (v_{+/-} - v_x)g_m + (v_{out+/-} - v_x)\frac{1}{r_{ds}}$$

Small signal model with resistive loads on each branch



$$-v_{out+/-}\left(\frac{1}{R_L}+\frac{1}{r_{ds}}\right)=v_{+/-}g_m-v_x\left(g_m+\frac{1}{r_{ds}}\right)$$

$$\begin{aligned}-(v_{out+}-v_{out-}) &= \frac{(v_+-v_-)g_m}{\frac{1}{R_L}+\frac{1}{r_{ds}}}\\ &\approx (v_+-v_-)g_mR_L\end{aligned}$$

Differential Gain Stage, Transcunductance Amp

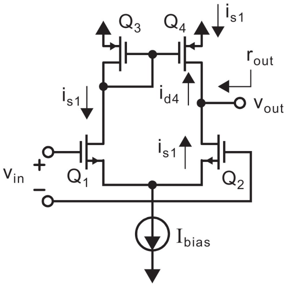


Figure 3.19

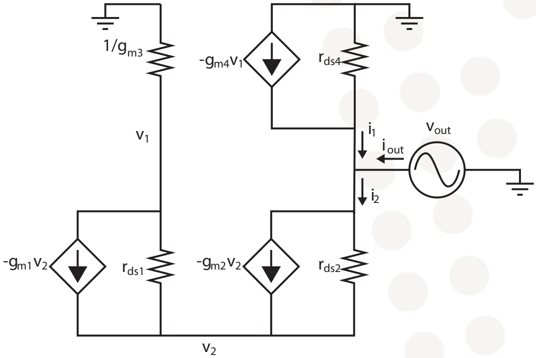
The simple analysis: consider that the current mirror faithfully copies the current through the left branch and consider a voltage source at the output, then the current into that voltage source is exactly the previous $i_{out} = i_+ - i_-$. With no voltage source, the difference in current has to flow trough the output resistance $\Rightarrow v_{out} = i_{out} r_{out}$. With (3.69) and (3.78)

$v_{out} = g_m v_{in} r_{out} \Rightarrow$

$A_V \approx g_m (r_{ds2} \parallel r_{ds4})$ which is (3.79).

More Careful Analysis for r_{out} (1/4)

(different from the book)

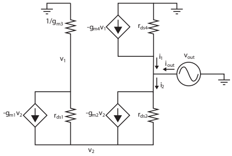


$$i_{out} = i_2 - i_1$$

More Careful Analysis for r_{out} (2/4)

$$i_1 = -\frac{1}{r_{ds4}} v_{out} - g_{m4} v_1$$

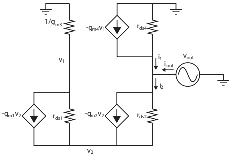
$$\begin{aligned} i_2 &= \frac{1}{r_{ds2}}(v_{out} - v_2) - g_{m2} v_2 \\ &\approx \frac{1}{r_{ds2}} v_{out} - g_{m2} v_2 \end{aligned}$$



$$\begin{aligned} i_2 &= (v_2 - v_1)\frac{1}{r_{ds1}} + g_{m1} v_2 \\ &\approx v_1 \frac{1}{r_{ds1}} + g_{m1} v_2 \end{aligned}$$

$$i_2 = v_1 g_{m3}$$

More Careful Analysis for r_{out} (3/4)



Using 2. and 3. term on previous page for i_2:

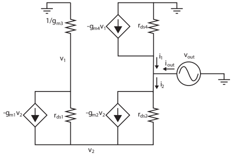
$$v_2 \approx \frac{g_{m3}}{g_{m1}} v_1$$

using 1. and 2. term on previous page for i_2 and $g_{m1} = g_{m2}$

$$\begin{aligned} v_{out} \frac{1}{r_{ds2}} &\approx v_1 \frac{1}{r_{ds1}} + g_{m3} v_1 + \frac{g_{m3} g_{m2}}{g_{m1}} v_1 \\ &\approx 2 g_{m3} v_1 \end{aligned}$$

$$v_1 \approx v_{out} \frac{1}{2 g_{m3} r_{ds2}}$$

More Careful Analysis for r_{out} (4/4)



Finally substituting all into $i_{out} = i_2 - i_1$ using the simplest expression for i_2 (3rd on page (2/4))

$$i_{out} = v_{out}\frac{1}{2r_{ds2}} + v_{out}\frac{1}{2r_{ds2}} + \frac{1}{r_{ds4}}v_{out}$$

$$i_{out} = v_{out}\left(\frac{1}{r_{ds2}} + \frac{1}{r_{ds4}}\right)$$