University of Oslo
Faculty of Mathematics and Natural Sciences

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Time: 9:00-13:00
This examination paper consists of * page(s)**
Permitted materials: All printed and written including approved (!) calculator

Make sure that your copy of this examination paper is complete before answering.

Chapter 3

Task 2 (4p): Figure 1 shows common gate gain stages with differrent loads and different serial resistances at the input. Derive small signal low frequency expressions for their gain $\frac{v_{out}}{v_{in}}$ in dependence of g_{m*} and r_{ds*} (where * stands for the indices 1-3) R_L and R_S. Neglect the body effect g_s and assume the transistor is in its active region/in saturation and that $g_m >> \frac{1}{r_{ds}}$. Also assume that pFETs and mFETs have the same g_m and $r_d s$ and that all circuits' point of operation is at the same large signal current through the common gate transistor

$$\frac{v_{out}}{v_{in}} \approx g_m(R_L||r_{ds}) * \frac{1}{1 + R_S g_m \frac{1}{1+\frac{R_L}{r_{ds}}}} \tag{1}$$

$$\frac{v_{outA}}{v_{in}} \approx g_m \frac{r_{ds}}{2} \tag{2}$$

$$\frac{v_{outB}}{v_{in}} \approx g_m(R_L||r_{ds}) \overset{R_L<<r_{ds}}{\approx} g_m R_L \tag{3}$$

$$\frac{v_{outC}}{v_{in}} \approx g_m \frac{r_{ds}}{2} * \frac{1}{1 + R_S g_m \frac{1}{2}} \tag{4}$$

$$\frac{v_{outD}}{v_{in}} \approx g_m(g_m r_{ds}^2||r_{ds}) \overset{g_m r_{ds}>>1}{\approx} g_m r_{ds} \tag{5}$$

Task 3 (2p): Order the circuits from the previous task according to their small signal low frequency voltage gain, i.e. make a list beginning with the circuit with the highest gain and ending with the lowest. In addition to the above assumptions also assume that $R_L = 0.5 * r_{ds}$ and $R_S = r_{in}$.

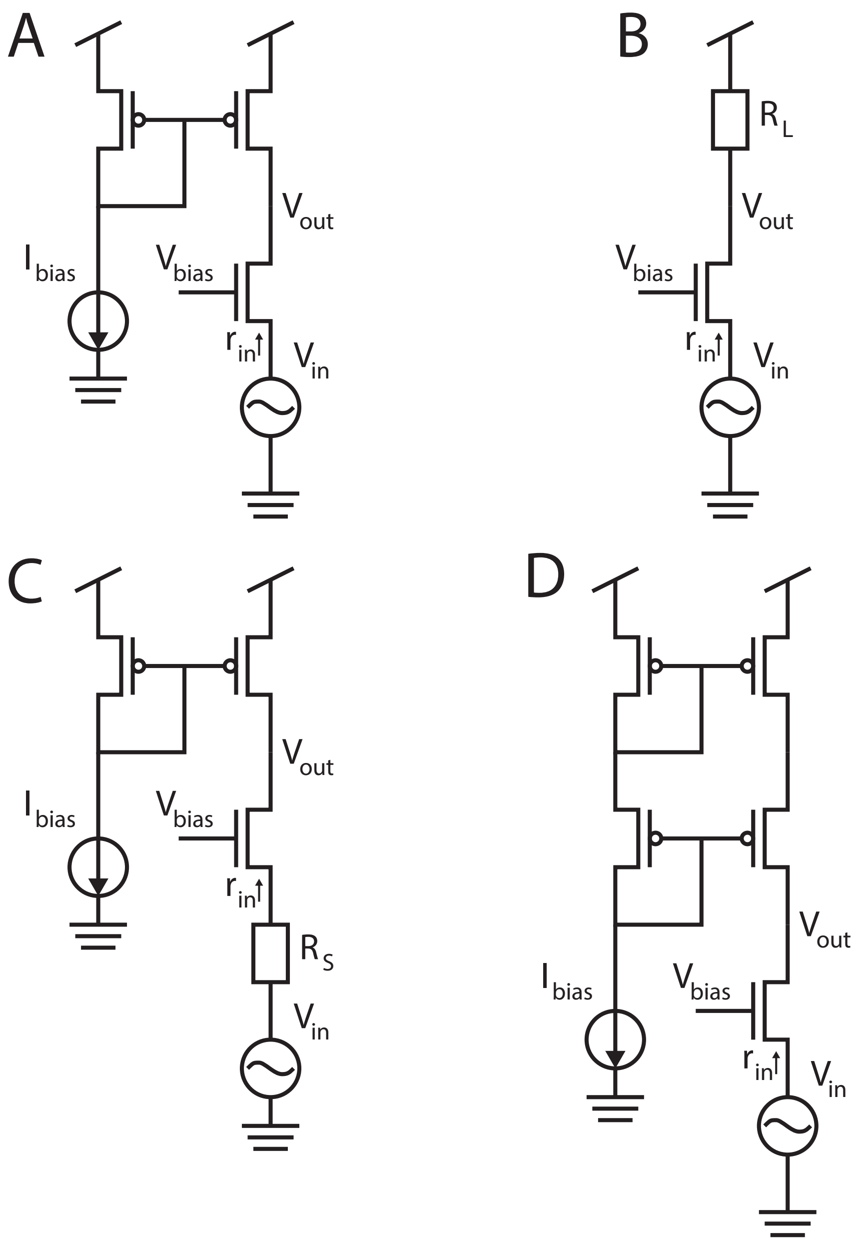

Figure 1: Common gate gain stages with different loads, and with input sources with different serial/output resistances

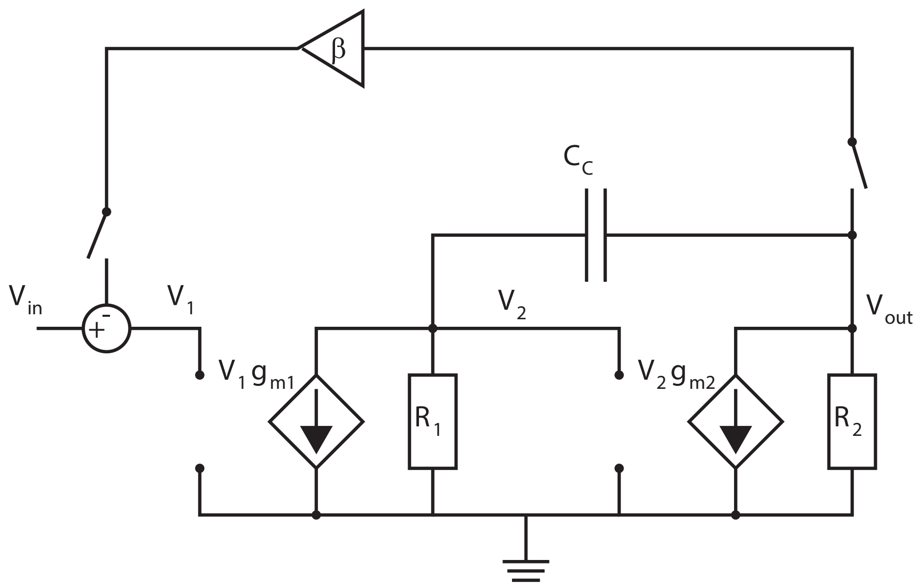

Figure 2: Two stage transconductance amplifier without neither load nor parasitic capacitances.

$$\frac{v_{outB}}{v_{in}} \approx g_m(R_L||r_{ds}) \overset{R_L=r_{ds}/2}{=} \frac{g_m r_{ds}}{3} \tag{6}$$

$$\frac{v_{outC}}{v_{in}} \overset{R_S=r_{in}}{\approx} g_m \frac{r_{ds}}{2} * \frac{1}{1+1} = g_m \frac{r_{ds}}{4} \tag{7}$$

Chapter 5

Task 6 (4p): Consider the two stage transconductance amplifier of figure 2. In this model, there are no capacitors other than the feedback capacitor between the output of the first and second stage. Derive the transfer function and explicitly write down the expressions for all poles and zeros.

$$V_x(G_1 + sC_C) + V_{in}g_{m1} = V_{out}sC \tag{8}$$

$$V_{out}(G_2 + sC_C) + V_x g_{m2} = V_x sC \tag{9}$$

$$V_x = \frac{V_{out}sC_C - V_{in}g_{m1}}{G_1 + sC_C} \tag{10}$$

$$V_{out}(G_2 + sC_C) + \frac{V_{out}sC_C - V_{in}g_{m1}}{G_1 + sC_C} g_{m2} = \frac{V_{out}sC_C - V_{in}g_{m1}}{G_1 + sC_C} sC_C \tag{11}$$

$$V_{out}(G_2 + sC_C) + V_{out}\frac{sCg_{m2}}{G_1 + sC_C} - V_{in}\frac{g_{m1}g_{m2}}{G_1 + sC_C} = V_{out}\frac{sCsC}{G_1 + sC_C} - V_{in}\frac{g_{m1}sC}{G_1 + sC_C} \tag{12}$$

$$V_{out}\left[(G_2+sC_C)+\frac{sC_Cg_{m2}}{G_1+sC_C}-\frac{sC_CsC_C}{G_1+sC_C}\right]=V_{in}\left[\frac{g_{m1}g_{m2}}{G_1+sC_C}-\frac{g_{m1}sC}{G_1+sC_C}\right] \tag{13}$$

$$V_{out}\left[(G_2+sC_C)(G_1+sC_C)+sC_Cg_{m2}-sC_CsC_C\right]=V_{in}\left[g_{m1}g_{m2}-g_{m1}sC\right] \tag{14}$$

$$\frac{V_{out}}{V_{in}}=\frac{g_{m1}g_{m2}-g_{m1}sC_C}{(G_2+sC_C)(G_1+sC_C)+sC_Cg_{m2}-sC_CsC_C} \tag{15}$$

$$\frac{V_{out}}{V_{in}}=\frac{g_{m1}g_{m2}(1-\frac{sC_C}{g_{m2}})}{G_2G_1+(G_1+G_2)sC_C+s^2C_C^2+sC_Cg_{m2}-s^2C_C^2} \tag{16}$$

$$\frac{V_{out}}{V_{in}}=\frac{g_{m1}g_{m2}}{G_1G_2}\frac{(1-\frac{sC_C}{g_{m2}})}{1+(R_2+R_1+R_1R_2g_{m2})sC_C} \tag{17}$$

$$z_1=-\frac{g_{m2}}{C_C} \tag{18}$$

Note: negative zero gives negativ phase shift!

$$\omega_1=\frac{1}{C_C(R_2+R_1+R_1R_2g_{m2})} \tag{19}$$

For z_1 to be at higher frequency than ω_1, $|\frac{z_1}{\omega_1}|$ needs to be bigger than 1:

$$|\frac{z_1}{\omega_1}|=g_{m2}*(R_2+R_1+R_1R_2g_{m2}) \tag{20}$$

So for the first pole to be at a lower frequency than the first zero it is sufficient that $g_{m2}R_2$ is bigger than 1, i.e. that there is gain in the second stage.

The gain at the zero is the DC gain divided by the zero frequency multiplied with the pole frequency if one considers the linearly approximated log-plot. However, if one does include the fact that there is a -3dB added right at a pole and +3dB right at a zero when one does a more accurate non-linearized plot Here then one has to add another factor $\sqrt{2}$ (corresponding to +3dB)!

$$|H(jz_1)|=g_{m1}g_{m2}R_1R_2*\frac{1}{C_C(R_2+R_1+R_1R_2g_{m2})}*\frac{C_C}{g_{m2}}*\sqrt{2}=\frac{g_{m1}R_1R_2}{R_2+R_1+R_1R_2g_{m2}}\sqrt{2} \tag{21}$$

The phase margin at z_1 is 45^o and $\frac{1}{\beta}$ needs to be $|H(jz_1)|$ to place the unity loop gain at exactly that frequency and bigger to have the unity loop gain at lower frequencies and a phase margin bigger than 45^o.

$$\beta\leq\frac{R_2+R_1+R_1R_2g_{m2}}{g_{m1}R_1R_2}\sqrt{2} \tag{22}$$

this can also be derived by simply solving the transfer function for $s=jz_1$

To make writing things down simpler:

$$A_0:=g_1g_2R_1R_2 \tag{23}$$

$$H(s)=A_0\frac{1-\frac{s}{z_1}}{1+\frac{s}{\omega_1}} \tag{24}$$

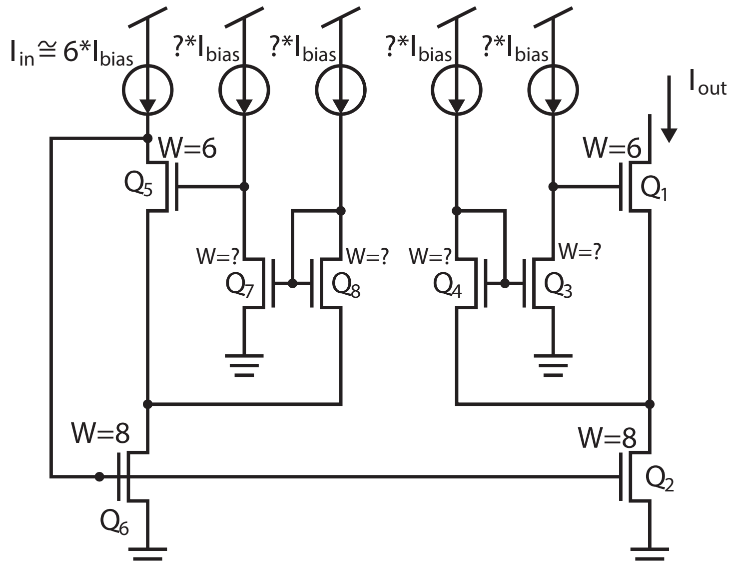

Figure 3: Advanced current mirror with wide-swing at output and enhanced output impedance

$$H(jz_1) = A_0 \frac{1-j}{1+j\frac{z_1}{\omega_1}} \overset{z_1>>\omega_1}{\approx} A_0 \frac{1-j}{j\frac{z_1}{\omega_1}} \tag{25}$$

$$|H(jz_1)| = A_0|| \frac{|1-j|}{|\frac{z_1}{\omega_1}|} = A_0 \frac{\sqrt{2}\omega_1}{z_1} \tag{26}$$

Chapter 6

Task 8 (4p): Figure 3 shows an adaptionof the wide-swing current mirror with enhanced output impedance from the book. All transistors have the same length and the widths are noted proportional to each other. Complete the parameters noted with '?' in the figure, i.e. what are the relative widths of Q_7, Q_8, Q_4, and Q_3, and what will the bias currents to those transistors have to be? The goal is that the output voltage swing at which the current mirror works (i.e. all transistors are in saturation/in their active region) shall be in the range from $2V_{eff}$ to Vdd.

Q_7, Q_8, Q_4, Q_3: W=2 $I_{bias8} = I_{bias4} = 2I_{bias}$
$I_{bias7} = I_{bias3} = 8I_{bias}$

Chapter 9

Task 9 (4p) Often transistors are used as pass-transistors to connect or disconnect an input, i.e. like a simple switch. Let us assume that an nFET of the 0.35μm process of table **??** with Vdd=3.3V is used as such a pass transistor and that it connects an input node at a point of operation of Vdd/2 to a black box circuit (with purely capacitive input impedance). It is set to be conducting, i.e. its gate voltage is at Vdd. Derive the noise spectral density function of the thermal noise *current* (you may ignore the flicker-noise!) through the pass transistor at room temperature. Hint 1: will this pass transistor operate in its triode- or active region? Note that there are different models for those two cases, one can be found in the table with noise models in the book and the other only in the text in the book! W=10μm, L=0.5μm. Sketch the noise spectral density function on a log-log plot, with the y-axis in dBm or V^2/Hz and the x-axis in $\log_{10}$(f). If you assume a noise bandwith of 1MHz, what will be the total rms noise power?

In the triode region:

$$r_{ds} = \frac{1}{\mu_n C_{ox} \frac{W}{L} V_{eff}} \tag{27}$$

$$r_{ds} = \frac{1}{190e - 6\mathrm{A/V^2} \frac{10e-6\mathrm{m}}{0.5e-6\mathrm{m}} (1.65 - 0.57)\mathrm{V}} = 244\Omega \tag{28}$$

$$r_{ds} = \frac{1}{190e - 6\mathrm{A/V^2} \frac{10e-6\mathrm{m}}{0.5e-6\mathrm{m}} (1.65 - 0.57)\mathrm{V}} = 244\Omega \tag{29}$$

$$I_d^2(f) = \frac{4kT}{r_{ds}} = 4 * 1.38e - 23 * 293/244 = 6.6e - 23 \tag{30}$$

$$\int_0^{1e6} I_d^2(f)\mathrm{d}f = 6.6e - 17 \tag{31}$$