



# INF3410 — Fall 2015

Book Chapter 1: Devices and Modelling



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PN Junction Properties

Field Effect Transistor Large Signal Models

Field Effect Transistor Small Signal Models

Field Effect Transistor 'Second Order' Properties

Short Remark on Passive Devices

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## Built in Potential/Junction Capacitance

$$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{\Phi_0}}} \quad (1.17)$$

$$C_{j0} = \sqrt{\frac{qK_S\epsilon_0}{2\Phi_0} \frac{N_A N_D}{N_A + N_D}} \quad (1.18)$$

$$\Phi_0 = U_T \ln \left( \frac{N_A N_D}{n_i^2} \right) \quad (1.6)$$

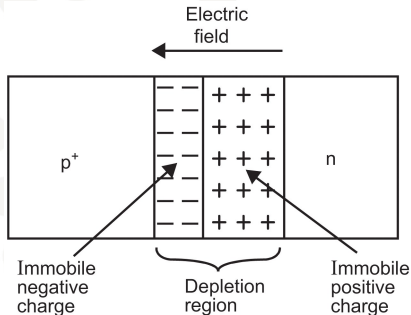


Figure 1.2  
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# nFET cross section and symbols

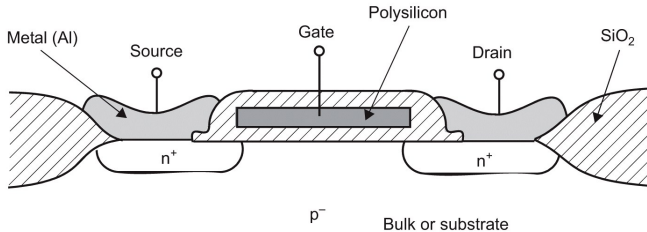


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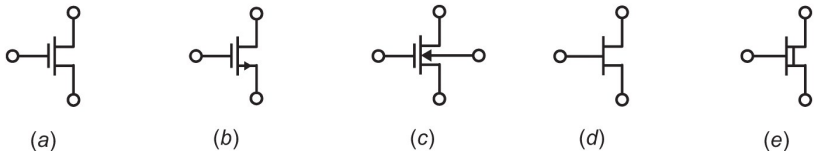


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## EKV Model

$$I_{DS} = I_F - I_R$$

for an NFET:

$$I_{F(R)} = I_S \ln^2 \left[ 1 + e^{\frac{V_G - V_{T0} - nV_{S(D)}}{2nU_T}} \right]$$

$$\text{where } I_S = 2n\beta U_T^2 \quad \beta = \mu C_{ox} \frac{W}{L}$$

Active region/in saturation:  $I_F \gg I_R$

Triode region/linear region:  $I_F \approx I_R$

## EKV Simplified in Weak Inversion

Weak inversion/subthreshold:

$$(I_F \ll I_S) = (V_G - nV_S < V_{T0})$$

$\Rightarrow$

$$I_F = 2n\beta U_T^2 e^{\frac{V_G - V_{T0} - nV_S}{nU_T}}$$

Book model (in saturation):

$$I_D = (n-1)\beta U_T^2 e^{\left(\frac{V_G - V_{tn}}{nU_T}\right)} \quad (1.121)$$



## EKV Simplified in Strong Inversion

Strong inversion/above threshold:

$$(I_F \gg I_S) = (V_G - nV_S > V_{T0})$$

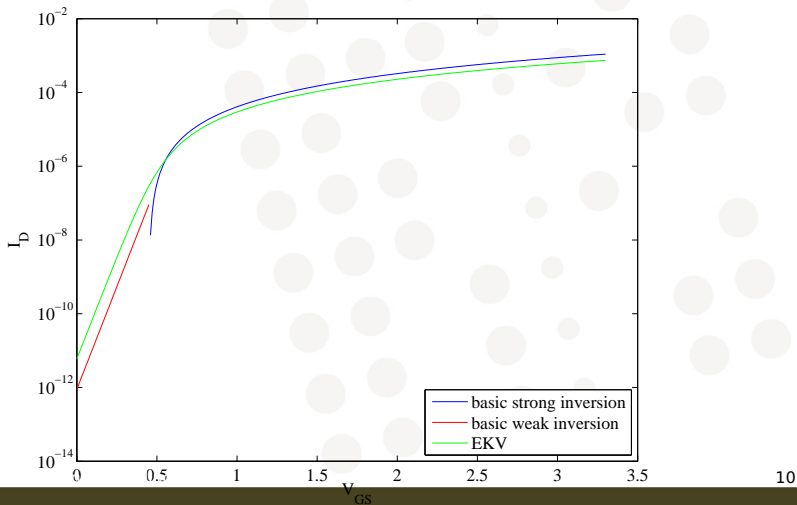
$\Rightarrow$

$$I_{F(R)} = \frac{\beta}{2n} (V_G - V_{T0} - nV_{S(D)})^2$$

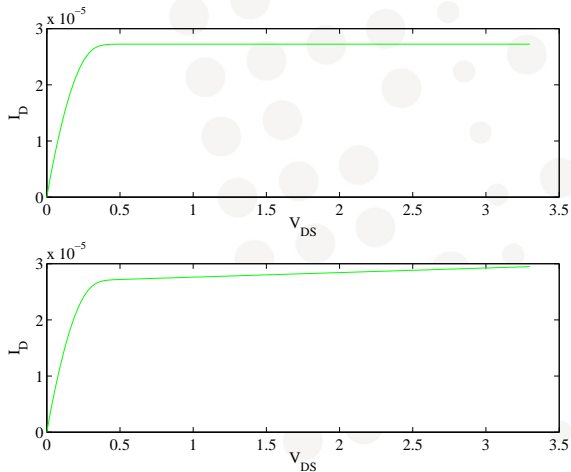
Book model (in saturation/active region):

$$I_D = \frac{\beta}{2} (V_G - V_S - V_{tn})^2 \quad (1.63)$$

# Basic Equation vs. EKV



# Channel Modulation/Early Effect Illustration



## Channel Modulation/Early Effect Formula

Strong Inversion

$$I_D = \frac{\beta}{2} (V_{eff})^2 [1 + \lambda(V_D - V_S - V_{eff})] \quad (1.67)$$

$$\lambda = \frac{k_{ds}}{2L\sqrt{V_{DS} - V_{eff} + \Phi_0}}$$

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Low Frequency Small Signal Model

High Frequency Small Signal Model

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## Small Signal Models

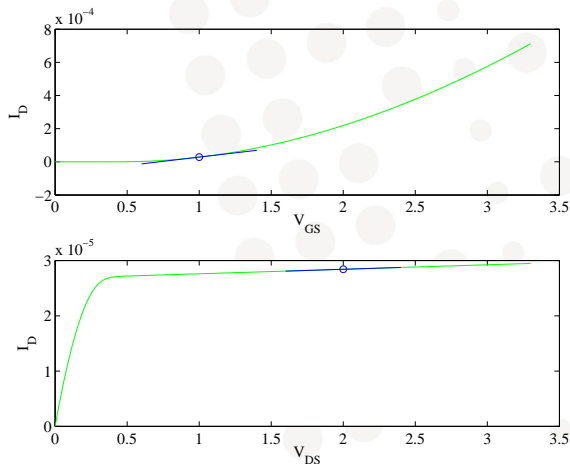
A linearized model that applies to a certain point of operation. All small signal variables (e.g.  $i_d$ ) are thus only the offset to the variables at this point of operation (e.g.  $I_D$ ). Sometimes the total of the two is referred to as  $i_D$  (though not in the Carusone book!).

$$i_D = I_D + i_d$$

Those so inclined may think of it as a first order Taylor expansion:

$$i_D(\vec{X} + \vec{x}) \approx I_D(\vec{X}) + \nabla I_D(\vec{X})\vec{x}^T = I_D(\vec{X}) + i_d(\vec{x})$$

# Linear Approximation/Small Signal Model



# Low Frequency Small Signal Model nFET

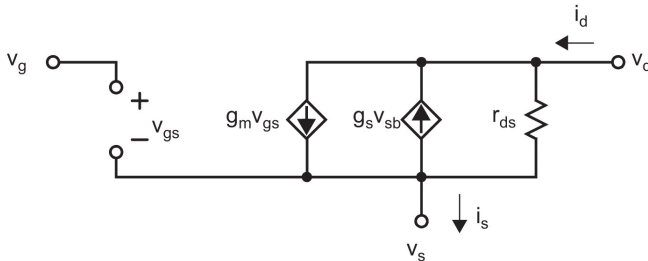


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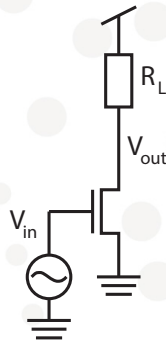


## Small Signal Model Parameters

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} \quad (1.77) \text{ (strong inversion)}$$
$$r_{ds} \approx \frac{1}{\lambda I_D} \quad (1.86)$$

## Example

In a simple common source amplifier with resistive load and a sine wave input with DC value 1.0V and an amplitude of 50mV, how does the output voltage look like? Use the parameters from the book in table 1.5 for a  $0.35\mu\text{m}$  process and a  $W=L=1\mu\text{m}$ .  $R_L=100\text{k}\Omega$ .

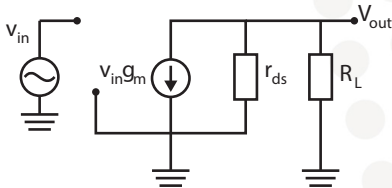


## Large Signal Model for Point of Operation

$$\begin{aligned} I_D &= \frac{1}{2} 190 \frac{\mu\text{A}}{\text{V}^2} \frac{1\mu\text{m}}{1\mu\text{m}} 0.2^2 \text{V}^2 \\ &= 3.8\mu\text{A} \end{aligned}$$

$$\begin{aligned} V_{out} &= V_{dd} - R_L I_D = 3.3\text{V} - 7.6 * 10^{-6} \text{A} * 100 * 10^3 \Omega \\ &= 3.3 - 0.76\text{V} = 2.54\text{V} \end{aligned}$$

## Small Signal Model (1/2)

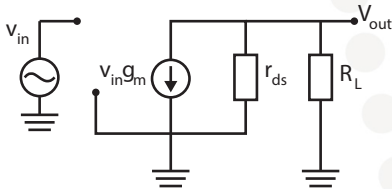


For a summary of the MOS FET equations see section 1.3.3, p42

$$\begin{aligned}
 g_m &= 190 \frac{\mu\text{A}}{\text{V}^2} \frac{1\mu\text{m}}{1\mu\text{m}} 0.2\text{V} \\
 &= 38 \frac{\mu\text{A}}{\text{V}}
 \end{aligned}$$

$$\begin{aligned}
 r_{ds} &= \frac{1}{\lambda I_D} \\
 &= \frac{1}{\lambda \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{eff}^2} \\
 &= \frac{1}{\frac{0.16 \frac{\mu\text{m}}{\text{V}}}{1\mu\text{m}} 190 \frac{\mu\text{A}}{\text{V}^2} \frac{1\mu\text{m}}{1\mu\text{m}} 0.2^2 \text{V}^2} \\
 &= 0.82 \frac{\text{V}}{\mu\text{A}} = 820\text{k}\Omega
 \end{aligned}$$

## Small Signal Model (2/2)



$$\begin{aligned}
 v_{out} &= -v_{in} g_m (R_L \parallel r_{ds}) \\
 &= -v_{in} 38 \times 10^{-6} \left( \frac{820 \times 100 \times 10^6}{820 \times 10^3 + 100 \times 10^3} \right) \\
 &= -v_{in} \times 38 \times 10^{-6} \times 89 \times 10^3 \\
 &= -v_{in} \times 3.38V
 \end{aligned}$$

$$A = \frac{V_{out}}{V_{in}} = 3.38$$

## Check Error: Large Signal Model for Max and Min

Large signal model:

$$V_{out}^{min,max} = (2.71V, 3.09V)$$

Small Signal Model:

$$V_{out}^{min,max} = 2.92 \pm 3.38 * 0.05 = (2.75V, 3.09V)$$

Why the discrepancy?

# High Frequency Small Signal Model nFET

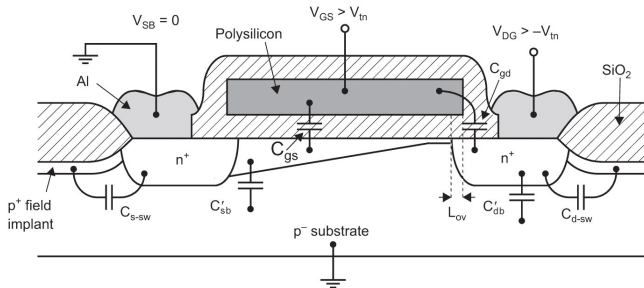


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# High Frequency Small Signal Model nFET

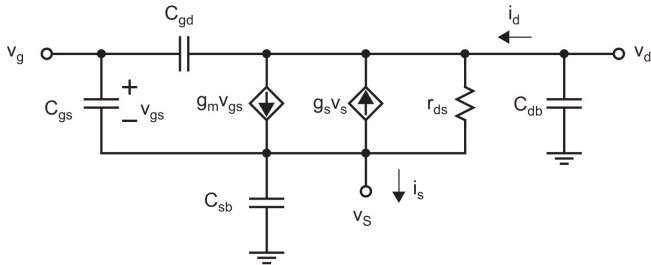


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## High Frequency Small Signal Model nFET

$$C_{gs} \approx \frac{2}{3} WLC_{ox} \quad (1.89)$$

$$C_{sb} \approx (A_S + A_{CH}) C_{js} \quad (1.92)$$

$$C_{gd} \approx C_{ox} WL_{ov} \quad (1.96)$$



## Unity-Gain Frequency (Intrinsic Speed)

Unity current gain ( $\frac{\partial I_{out}}{\partial I_{in}} = 1$ ), no external load, common source amplifier with ideal voltage source as 'load'

$$f_t \approx \frac{g_m}{2\pi(C_{gs} + C_{gd})} \approx \frac{3\mu_n V_{eff}}{4\pi L^2} \quad (1.116/117)$$

⇒ Higher speed for shorter transistors (small  $L$ ) and large  $V_{eff}$

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Mobility-Degradation

Short-Channel Effects

Leakage Currents

Short Remark on Passive Devices

# Mobility-Degradation/Velocity Saturation

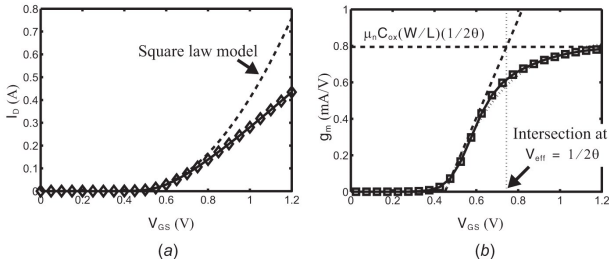


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$$I_D = \frac{1}{2} \beta V_{eff}^2 \frac{1}{[1 + (\theta V_{eff})^m]^{\frac{1}{m}}}, \quad g_m = \frac{1}{2} \beta \frac{1}{\theta} \text{ for } V_{eff} > \frac{1}{2\theta}$$

# Short-Channel Effects

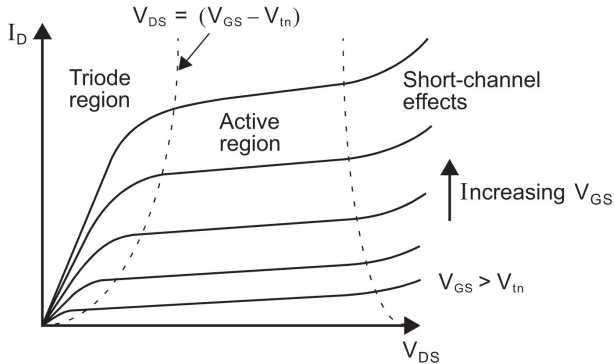


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## Leakage Currents

- ▶ subthreshold leakage
- ▶ junction leakage (strongly temperature dependent)
- ▶ gate leakage (depends on  $t_{ox} < 2\text{nm}$ : new technologies!)

# Leakage Currents

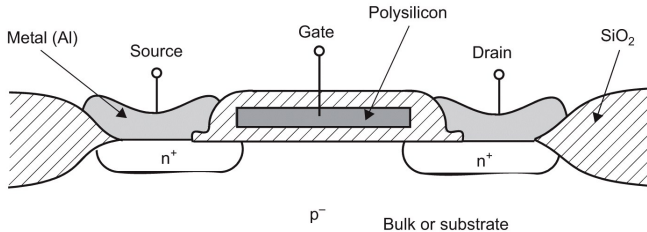


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# MOS capacitors, a word of caution

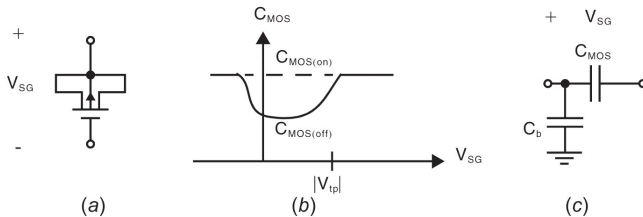


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All equations are summarized in the book section 1.3, starting from page 39.

Realistic parameters for a few technology nodes can be found on in table 1.5 on page 54.