INF3410 — Fall 2015

Book Chapter 5: Feedback Amplifiers

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Ideal Model of Negative Feedback
Dynamic Response of Feedback Amplifiers
Common Feedback Amplifiers

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Ideal Model of Negative Feedback

Dynamic Response of Feedback Amplifiers

Common Feedback Amplifiers

General Concept

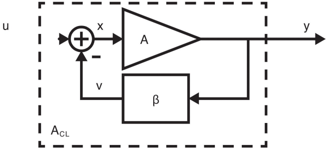

Figure 5.1
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$$A_{CL} = \frac{y}{u} = \frac{A}{1+A\beta} = \frac{A}{1+L}$$

where $L := A\beta$

Why would anyone want to reduce gain in such a way? Because:

$$A_{CL} \approx \frac{1}{\beta} \quad \text{for large } L$$

... and β can be designed with good reliable matching!

Gain Sensitivity / Gain Error (variant 1)

$$
\begin{aligned}
\Delta A_{CL} &= \frac{A+\Delta A}{1+(A+\Delta A)\beta} - \frac{A}{1+A\beta} \\
&= \frac{(A+\Delta A)(1-A\beta) - A[1+(A+\Delta A)\beta]}{[1+(A+\Delta A)\beta](1+A\beta)} \\
&= \frac{\cancel{A} + \Delta A + \cancel{A^2\beta} + \cancel{A\Delta A\beta} - \cancel{A} - \cancel{A^2\beta} - \cancel{A\Delta A\beta}}{1+A\beta+A\beta+A^2\beta^2+\Delta A\beta+A\Delta A\beta^2} \\
&= \frac{\Delta A}{(1+A\beta)^2+\Delta A\beta+A\Delta A\beta^2} \quad \text{wrong in book?} \\
&\ll \Delta A \;\; (\text{for} \;\; L \gg 1)
\end{aligned}
$$

Gain Sensitivity / Gain Error (variant 2)

$$A' = A(1+\epsilon)$$

$$\begin{aligned} A'_{CL} &= \frac{A(1+\epsilon)}{1+A\beta(1+\epsilon)} \\ &= \frac{A}{\frac{1}{1+\epsilon}+A\beta} \\ &\approx A_{CL} \quad (\text{for } A\beta \gg 1) \end{aligned}$$

Bandwidth

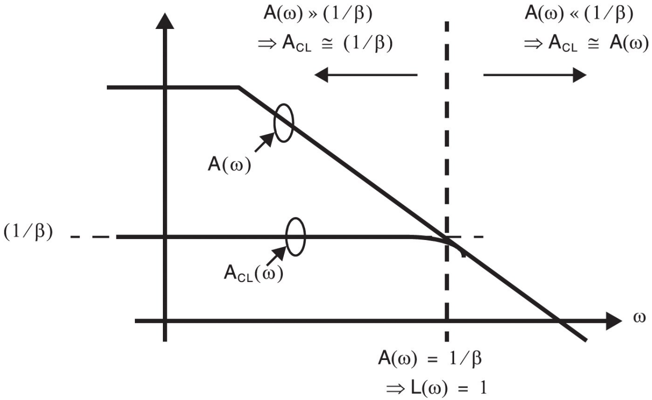

Figure 5.2

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Linearity

As a side effect, the simplification to a linear system is even more valid than before, as the amplifier input x is strongly attenuated with respect to u. Hence, the linear region with respect to the input is larger in the closed loop amplifier. Maybe not so surprising as with the reduced gain a larger input is necessary to fill the entire output range of the amplifier, i.e. the linear range with respect to x is still the same.

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The Crux with Positive Feedback Caused by Phase Shift

Positive feedback is obviously problematic. Note that negative feedback with -180^o phase shift actually becomes positive feedback! If your loop gain L is bigger than 1 at that point you'll certainly be in trouble!

Stability of Feedback Amplifiers with Low-Pass Characteristics

General: If the loop gain is smaller than 1, the closed loop gain will be finite, even if the phase shift is -180^o

$$\forall\omega : \left\{(\angle L(j\omega) = -180^o \wedge |L(j\omega)| < 1) \Rightarrow |A_{CL}(j\omega)| < \infty\right\}$$

Low pass: Unity loop gain frequency ω_t needs to have phase shift not yet at -180^o (then the loop gain will be smaller than 1 when the phase shift becomes -180^o)

$$\exists\omega_t : \{|L(j\omega_t)| = 1 \wedge \angle L(j\omega_t) > -180^o\} \Rightarrow \forall\omega : \{A_{CL}(j\omega) < \infty\}$$

But note that even so $|A_{CL}(j\omega_t)|$ with $\angle A_{CL}(j\omega_t)$ close to -180^o can still be much larger than the DC gain!

Bode Plots of Loop Gain $L(j\omega)$

The *phase margin* tells you how far away from -180^o the phase shift is at the unity gain frequency of the loop gain: It's a measure to how close to instability the circuit is and how *resonant* it becomes around its cut-off frequency.

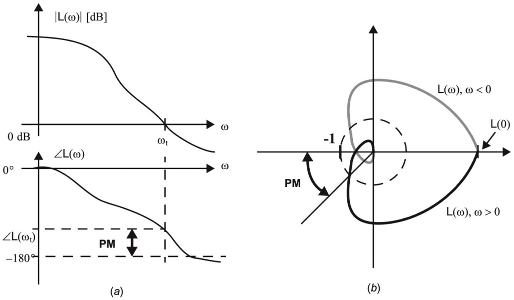

Figure 5.4

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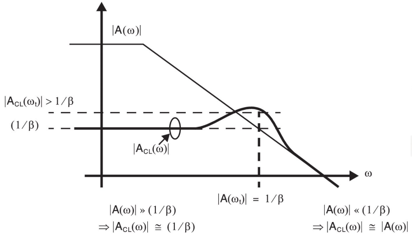
Closed Loop Gain $A_{cl}(j\omega)$ with PM less than 60^o



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$$|A_{cl}(j\omega_t)| = \frac{1}{\beta\sqrt{2(1-\cos\theta)}}$$

For PM $\theta < 60^o$ it follows that $|A_{cl}(j\omega_t)| > \frac{1}{\beta}$.

Ignoring Poles and Zeros $\gg \omega_t$

For analysis of 'in-band' performance and stability, feedback systems can be simplified to neglect poles and zeros beyond the loop unity gain frequency. Actually, systems where the loop unity gain frequency is beyond the second pole, will have extremely little phase margin at best and very likely be unstable and of little practical interest.

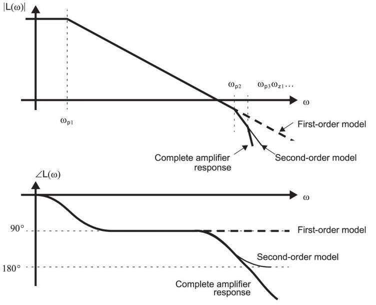

Figure 5.7

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For First Order Feedback Systems

First order filters do not have a problem as their phase shift will not decend below -90^o

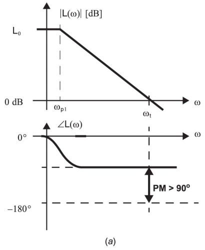

Figure 5.8
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$$\omega_t \approx L_0\omega_1 \quad \text{for } \omega_t \gg \omega_1 \quad (5.25)$$

$$\tau = \frac{1}{\omega_{-3dB}} = \frac{1}{\omega_t} \quad (5.29,\ \text{settling time})$$

Second Order Feedback Systems

For 2nd order filters we still do not have a problem as their phase shift will not decend below -180^o. However, the phase margin might be very small and can approximate 0^o!

$$L(s) = \frac{L_0}{(1+\frac{s}{\omega_1})(1+\frac{s}{\omega_{eq}})} \quad (5.34)$$

For $\omega_1 \ll \omega_t$, the phase margin θ is:

$$\theta = 90^o - \tan^{-1}(\frac{\omega_t}{\omega_{eq}}) \quad (5.40)$$

PM, $\frac{\omega_t}{\omega_{eq}}$, Q-factor, and percentage step response overshoot

Table 5.1 The relationship between PM, ω_t/ω_{eq}, Q factor, and percentage overshoot

PM (Phase margin)	ω_t/ω_{eq}	Q factor	Percentage overshoot for a step input
55°	0.700	0.925	13.3%
60°	0.580	0.817	8.7%
65°	0.470	0.717	4.7%
70°	0.360	0.622	1.4%
75°	0.270	0.527	0.008%
80°	0.175	0.421	-
85°	0.087	0.296	-

Table 5.1
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Higher-Order Feedback Systems

If more than two poles and/or zeros exist in the loop gain $L(s)$ that are close to ω_t one might still get away by approximating the entire system as a second order low-pass filter for the purpose of a stability analysis with (5.34) by either defining ω_{eq} analytically as:

$$\frac{1}{\omega_{eq}} = \sum_{i=2}^{n} \frac{1}{\omega_i} - \sum_{i=1}^{m} \frac{1}{z_i} \quad (5.50)$$

or by simulating and determine the frequency ω_{eq} as the frequency at which the phase shift becomes -135^o

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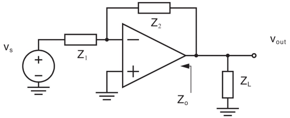
General Opamp Feedback Configurations

The 'desired' closed loop gain $\frac{1}{\beta}$ (so named in the book) is A_{cl} obtained for $A = \infty$.



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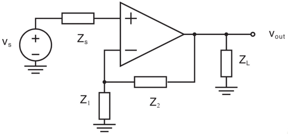

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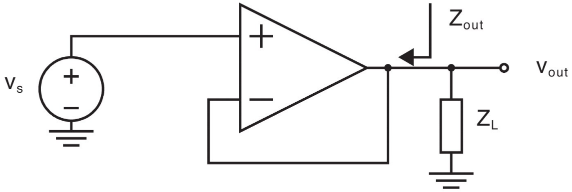
Inverting opamp configuration. $\frac{1}{\beta} = -\frac{Z_2}{Z_1}$

Non-inverting opamp configuration. $\frac{1}{\beta} = \frac{Z_2}{Z_1} + 1$ (5.57)

Voltage follower

A noteworthy special case of non-inverting opamp configuration



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Ambiguity in Separating Feed-Forward A and Feed-Back β Components

Determining the part of the circuit attributing for A_{ol} and β is not always simple and unambiguous in the more general case. L on the other hand is unambiguous and for stability analysis (phase margin and bandwith) one only needs to know L.

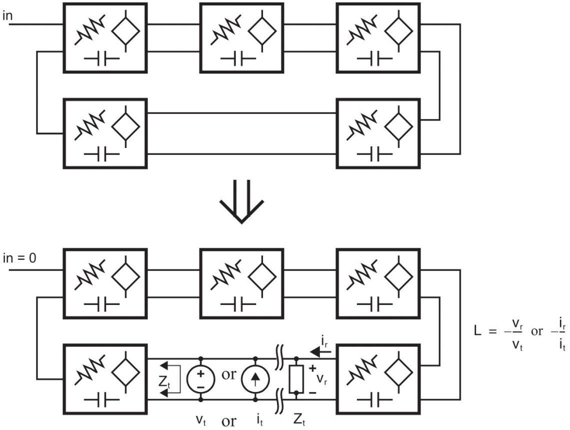
Finding the Loop-Gain L (text)

1. In a linear model (e.g. small signal), set any independent sources to zero (like for the small signal analysis, but now including the input!)
2. Break the loop. On the 'back' side, replace the input impedance that you just cut away on the 'forward' side. Select a break point that make your life simple!
3. Insert a test signal v_t or i_t on the 'forward' side and get an expression for the v_r or i_r on the 'back' side in dependency of v_t/i_t.

The loop gain L is then

$$L = -\frac{v_r}{v_t} = -\frac{i_r}{i_t} \quad (5.53)$$

Finding the Loop-Gain *L* (illustration)



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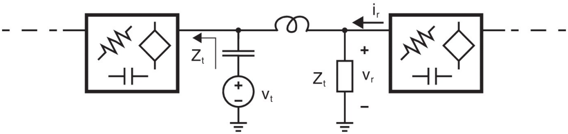
Finding the Loop-Gain L (simulation)

The problem applying this in transistor level simulation is to maintain the DC biassing on the input where you break the circuit. This can be solved by using a big inductance instead of a break, which is transparent at DC but becomes a effective open circuit at higher frequencies. Different from the book: you may just pt the voltage source v_t in series after the inductance.



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Example 5.8

5.8 is a very generally applicable model of an inverting opamp feedback configuration.

1. Demonstrated on the white board...
2. Breaking the circuit somewhere else on the white board....

Gain Error

Since A is less than infinity, the closed loop gain will be less than $\frac{1}{\beta}$, rephrasing (5.12):

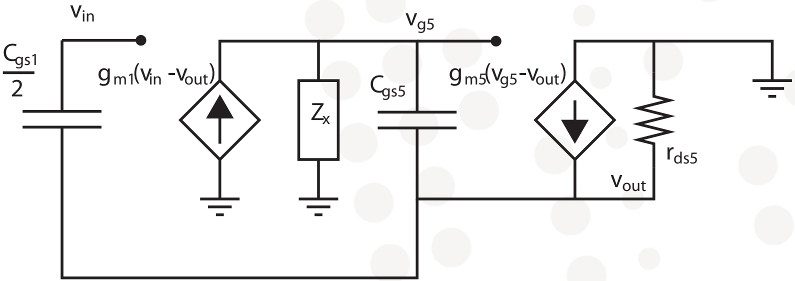
$$A_{CL} = \frac{1}{\beta}\frac{L(s)}{1+L(s)} \quad (5.51)$$

and the relative error is

$$1 - \frac{L(s)}{1+L(s)} \quad (5.52)$$

Example 5.9

Here with the hybrid π model rather than the T-model (in the book).



Example 5.9

For $L(0)$ we ignore capacitors:

$$v_{g5}\left(\frac{1}{Z_x}\right) - g_{m1}(v_{in/t}) = 0 \qquad |1)$$

$$v_{out}\left(\frac{1}{r_{ds5}}\right) - g_{m5}(v_{g5} - v_{out/r}) = 0 \qquad |2)$$

$$v_{out}\left(\frac{1}{r_{ds5}}\right) - g_{m5}(Z_x g_{m1} v_{in} - v_{out}) = 0 \quad |1) \text{ in } 2)$$

$$v_{out}\left(\frac{1}{r_{ds5}} + g_{m5}\right) = g_{m1} g_{m5} Z_x v_{in}$$

$$\frac{v_{out/r}}{v_{in/t}} = \frac{g_{m1} g_{m5} Z_x}{\frac{1}{r_{ds5}} + g_{m5}} \approx g_{m1} Z_x$$

Example 5.9

For $A_{cl}(0)$ we ignore capacitors:

$$v_{g5}\left(\frac{1}{Z_x}\right) - g_{m1}(v_{in} - v_{out}) = 0 \qquad |1)$$

$$v_{out}\left(\frac{1}{r_{ds5}}\right) - g_{m5}(v_{g5} - v_{out}) = 0 \qquad |2)$$

$$v_{out}\left(\frac{1}{r_{ds5}}\right) - g_{m5}(Z_x g_{m1}(v_{in} - v_{out}) - v_{out}) = 0 \quad |1) \text{ in } 2)$$

$$v_{out}\left(\frac{1}{r_{ds5}} + g_{m1}g_{m5}Z_x + g_{m5}\right) = g_{m1}g_{m5}Z_x v_{in}$$

$$\frac{v_{out}}{v_{in}} = \frac{g_{m1}g_{m5}Z_x}{\frac{1}{r_{ds5}} + g_{m1}g_{m5}Z_x + g_{m5}} \approx 1$$

Inverting Opamp as Transimpedance Amp

Transimpedance: $\frac{v_{out}}{i_{in}}$

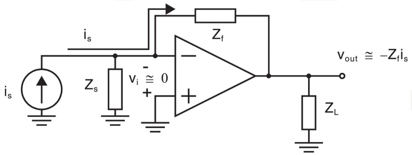


Figure 5.20
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$$\frac{v_{out}}{i_{in}} = \frac{1}{\beta} = -Z_f$$