

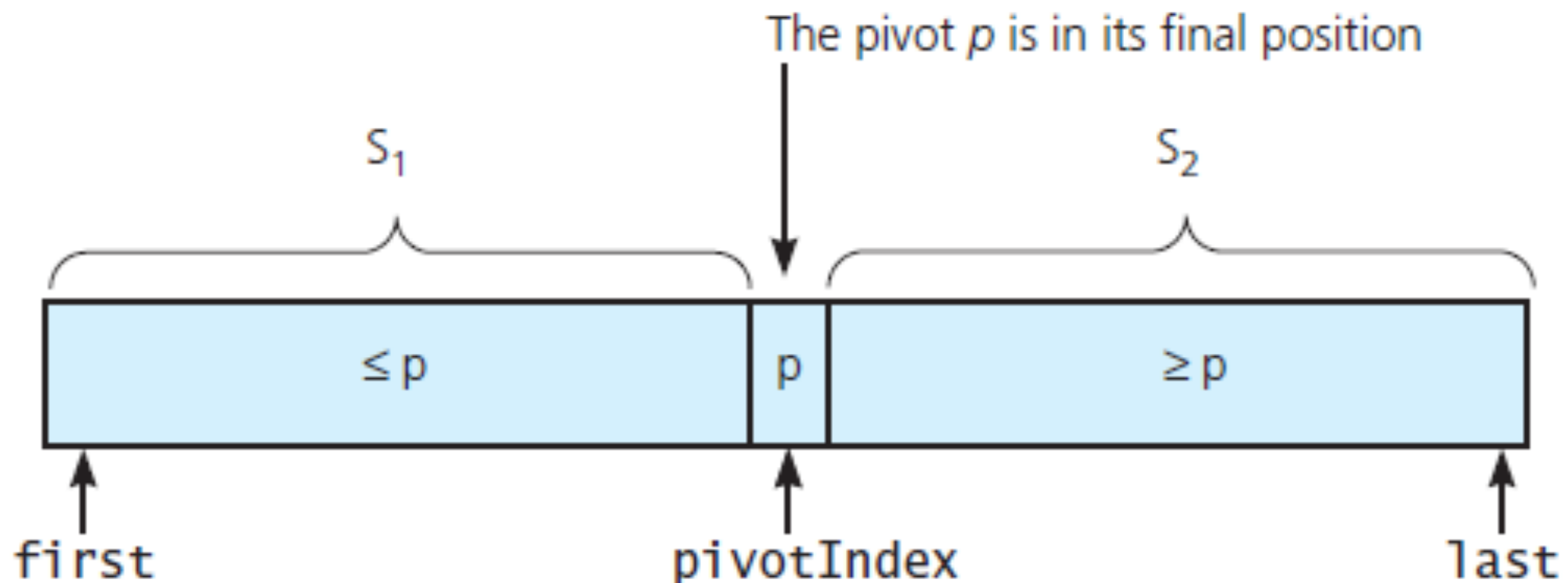
Quick Sort

CS110C

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quick sort

- Partition your array into two parts (S_1 and S_2), around a **pivot** p , such that:
 - all items in $S_1 \leq p$; all items in $S_2 \geq p$
- Note: this array isn't sorted, but we know that items in S_1 remain in S_1 after array is sorted, and likewise for S_2 . The pivot remains in its current position.



quick sort pseudocode

```
// perform quickSort on theArray between indices first and last
quickSort(theArray: ItemArray, first: integer, last: integer)
{
    if (first < last)
    {
        Choose a pivot p from theArray[first..last]
        Partition the items of theArray[first..last] about p, where
            p is at position pivotIndex

        // recursively sort first partition of array
        quickSort(theArray, first, pivotIndex-1)

        // recursively sort second partition of array
        quickSort(theArray, pivotIndex+1, last)
    }
}
```

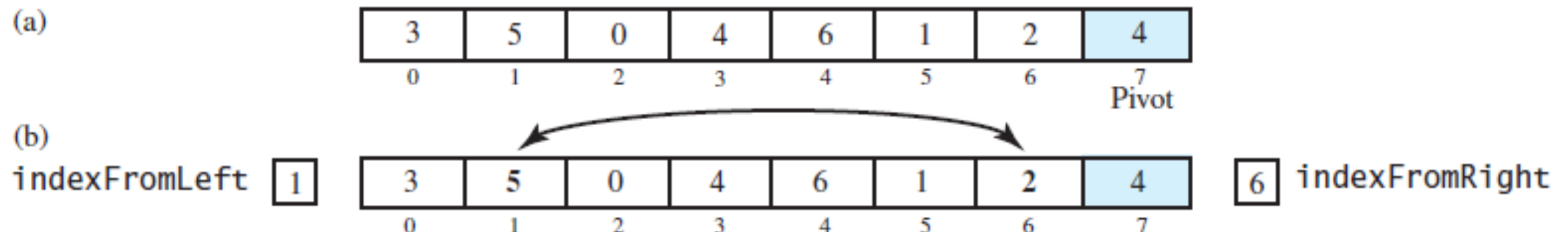
quick sort partitioning example

(a)

3	5	0	4	6	1	2	4
0	1	2	3	4	5	6	7

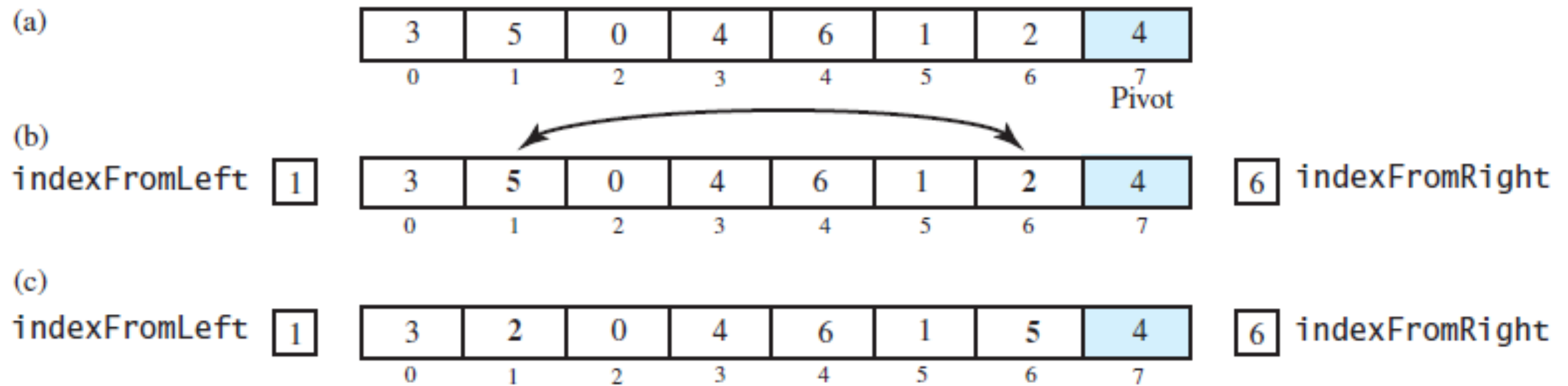
- Let's say we have chosen our pivot to be 4. Step (a) shows it swapped to the last position in array to get it out of the way while we do a partition.

quick sort partitioning example



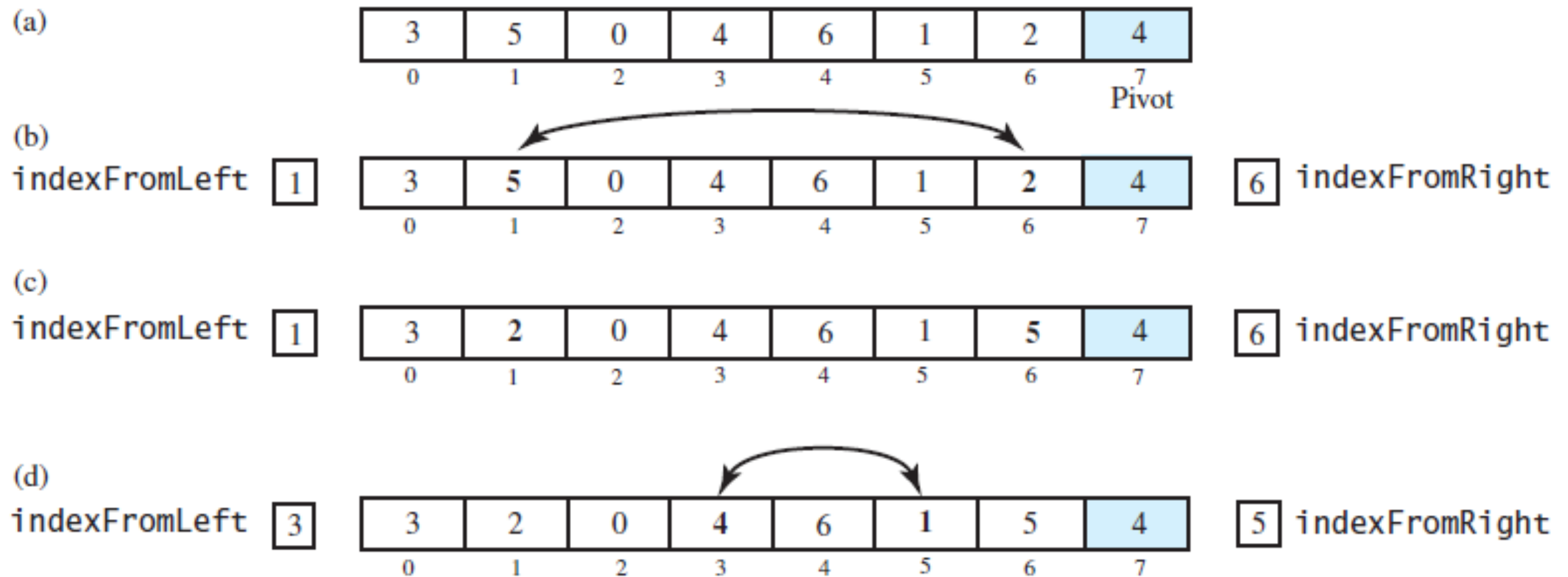
- Let's say we have chosen our pivot to be 4. Step (a) shows it swapped to the last position in array to get it out of the way while we do a partition.

quick sort partitioning example



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quick sort partitioning example



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quick sort partitioning

(e)

indexFromLeft 3

3	2	0	1	6	4	5	4
0	1	2	3	4	5	6	7

5 indexFromRight

quick sort partitioning

(e)

indexFromLeft 3

3	2	0	1	6	4	5	4
0	1	2	3	4	5	6	7

5 indexFromRight

(f)

indexFromLeft 4

3	2	0	1	6	4	5	4
0	1	2	3	4	5	6	7

3 indexFromRight

quick sort partitioning

(e)

indexFromLeft 3

3	2	0	1	6	4	5	4
0	1	2	3	4	5	6	7

5 indexFromRight

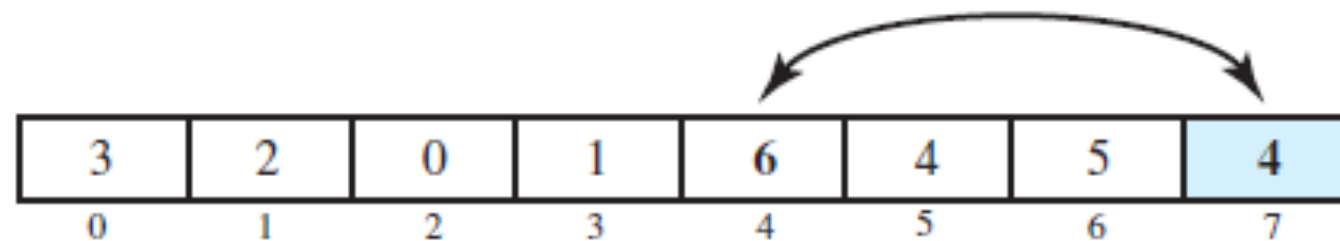
(f)

indexFromLeft 4

3	2	0	1	6	4	5	4
0	1	2	3	4	5	6	7

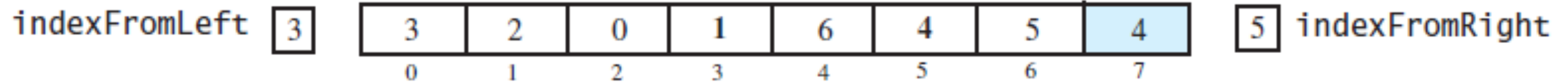
3 indexFromRight

(g)

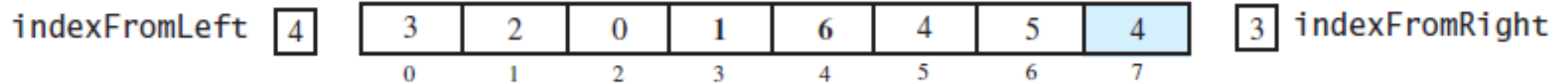


quick sort partitioning

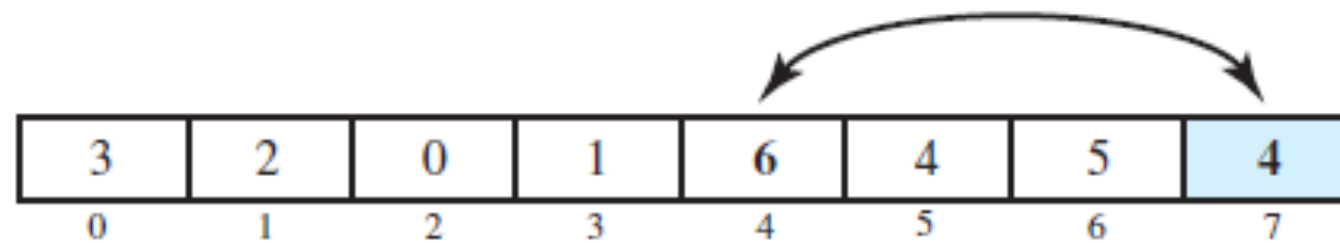
(e)



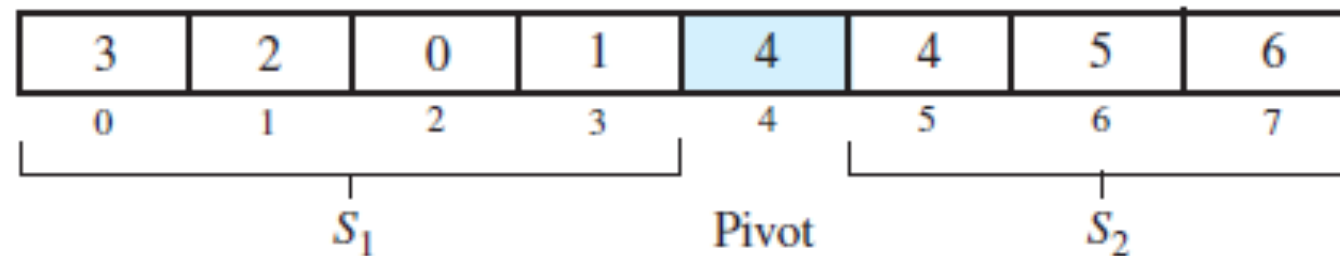
(f)



(g)



(h)



quick sort partitioning

```
template<class ItemType>
int partition(ItemType theArray[], int first, int last)
{
    // Choose pivot using median-of-three selection
    int pivotIndex = sortFirstMiddleLast(theArray, first, last);

    // Reposition pivot so it is last in the array
    std::swap(theArray[pivotIndex], theArray[last - 1]);
    pivotIndex = last - 1;
    ItemType pivot = theArray[pivotIndex];

    // Determine the regions S1 and S2
    int indexFromLeft = first + 1;
    int indexFromRight = last - 2;

    ... continued
```


quick sort partitioning

```
... continued
bool done = false;
while (!done)
{
    // Locate first entry on left that is >= pivot
    while (theArray[indexFromLeft] < pivot)
        indexFromLeft = indexFromLeft + 1;

    // Locate first entry on right that is <= pivot
    while (theArray[indexFromRight] > pivot)
        indexFromRight = indexFromRight - 1;

    if (indexFromLeft < indexFromRight)
    {
        std::swap(theArray[indexFromLeft], theArray[indexFromRight]);
        indexFromLeft = indexFromLeft + 1;
        indexFromRight = indexFromRight - 1;
    }
    else
        done = true;
} // end while

// Place pivot in proper position between S1 and S2, and mark its new location
std::swap(theArray[pivotIndex], theArray[indexFromLeft]);
pivotIndex = indexFromLeft;

return pivotIndex;
} // end partition
```

quick sort - pivot

- Selecting a pivot -- how?
 - Simplistic approach: pick the first element in the partition. But this can lead to undesirable performance.
 - Ideal approach: pick the median value. But that's impractical.
 - In practice, we at least try to avoid a bad pivot. We can sort the first, middle, and last entries, and use the middle one for the pivot, here 5. This is called **median-of-three** pivot selection

(a)

5	8	6	4	9	3	7	1	2
---	---	---	---	---	---	---	---	---

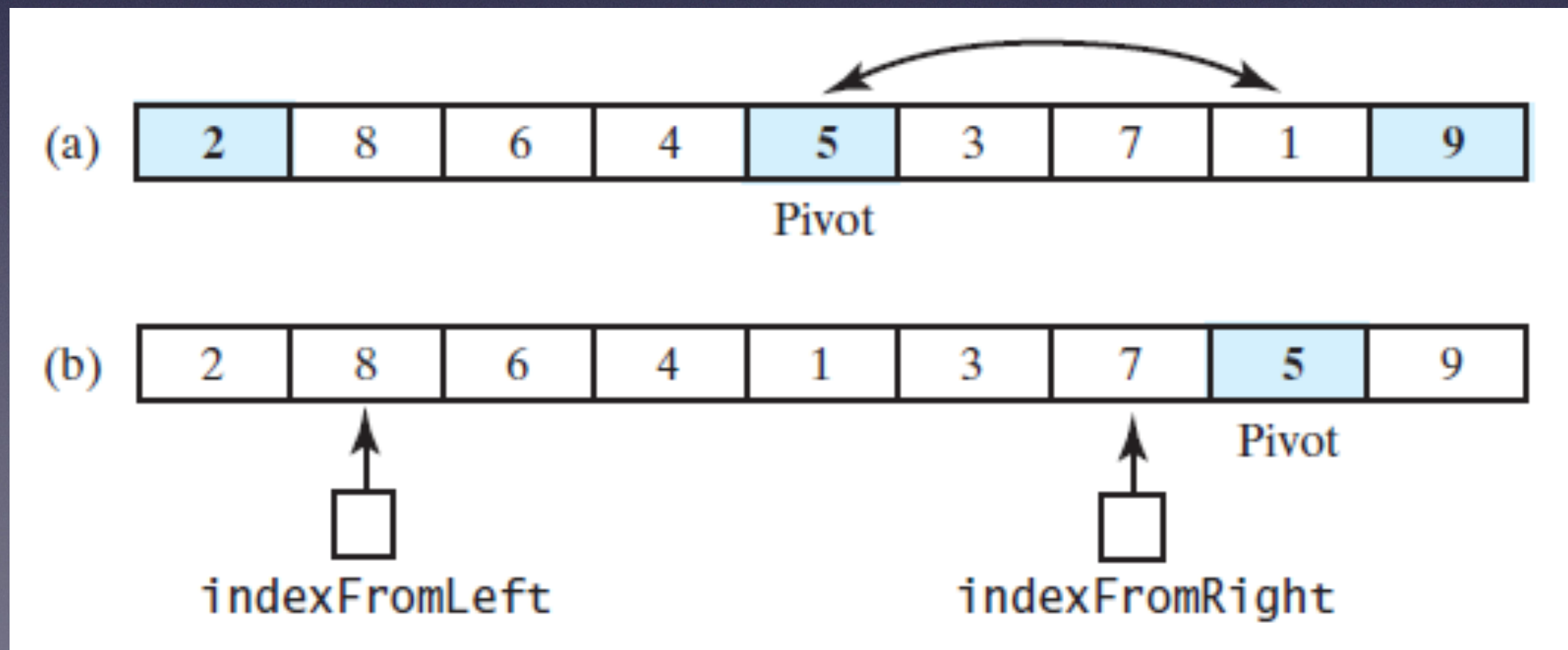
(b)

2	8	6	4	5	3	7	1	9
---	---	---	---	---	---	---	---	---

Pivot

quick sort - pivot

- Since we used median-of-three pivot selection, we know that the first item belongs in S1 and last item belongs in S2.
- Step b below shows the array right before partitioning (we've moved our pivot out of the way)



quick sort

```
quickSort(theArray: ItemArray, first: integer, last: integer)
{
  if (last-first + 1 < MIN_SIZE)
    insertionSort(theArray, first, last);
  else
  {
    // create partition: S1 | Pivot | S2
    int pivotIndex = partition(theArray, first, last);

    // Sort subarrays S1 and S2
    quickSort(theArray, first, pivotIndex - 1);
    quickSort(theArray, pivotIndex + 1, last);
  }
}
```


quick sort analysis

- The heavy lifting is the partitioning step. It requires at most n comparisons and is thus an $O(n)$ step.
- In the best case, every recursive call partitions the array into equally sized $S1$ and $S2$, and the quick sort is thus similar to merge sort in that it is halving the array each level. Thus there are $\log_2 n$ recursive levels and the total complexity is $O(n \log n)$.
- In a worst case, we could have an empty $S1$ or empty $S2$ after partitioning each time! The nonempty subarray decreases in size by only 1, so we could potentially have n levels. So the total complexity becomes $O(n^2)$.
- In an average case, with generally randomly placed numbers, a formal analysis can show $O(n \log n)$ performance.

quick sort vs merge sort

criteria	merge sort	quick sort	advantage
worst case efficiency	$O(n \log n)$	$O(n^2)$	merge sort
best / average case efficiency	$O(n \log n)$	$O(n \log n)$	none
needs extra storage	yes	no	quick sort

- Note: with a decent pivot selection algorithm like median-of-three, worst case performance for quick sort is rare