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# Documentation for TULIP 2

K. Henriksson

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## Abstract

This booklet gives an introduction to the TULIP, which is a program for fitting interatomic potentials to physical data of different phases and lattices.

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## 1 Introduction

The program TULIP tries to fit a interatomic potential to *e.g.* given physical properties of compounds *etc.*. Physical properties are for instance lattice parameters, cohesive energy, bulk modulus and elastic constants.

Input is essentially an initial guess for potential parameters, a list of compounds and their desired properties (lattice parameters, cohesive energy, mixing energy, bulk modulus, elastic constants, ...), and technical specifications to guide the fitting. Using the parametrization, compounds are relaxed in a molecular dynamics (MD) simulation (MDS), and then the specified physical properties are calculated. The fitting routine tries to minimize the merit function. The fitting method can be selected.

## 2 Source code

0. Prerequisites: Install the `spglib` package from <http://spglib.sourceforge.net/> in a standard location and remember the location of installed files.

1. Local users only: Download the `libutils` source code into a folder:

```
git clone /home/phys-data/people/koehenri/repos/libutils.git
```

The `libutils` directory will be created. Descend into it and follow the instructions in the Readme file to make and install the library.

2. Local users only: Download the `tulip2` source code into a folder:

```
git clone /home/phys-data/people/koehenri/repos/tulip2.git
```

The `tulip2` directory will be created. Descend into it and follow the instructions in the Readme file to make and install the fitting code.

3. The static version `tulip_static` can be copied to other computer systems having the same architecture and be run there without any additional effort. The shared library version requires adding the `libutils` library to the library path.

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To download updated versions of these codes descend into the root directories (e.g. `tulip2`) and execute

```
git pull
```

This will fetch and merge updated code with your local copy.

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The `tulip2/examples` directory contains some ready-made examples, which can be run without additional editing to get a better feel for how the code works.

### 3 Program arguments

```
TULIP version 2 (c) Krister Henriksson 2013-
Purpose: Fit data to an interatomic potential.
Usage:
    tulip arguments [options]
Arguments:
    -pf file           Path to file containing potential information.
    -gf file           Path to file containing geometry information.

Options:
    -sf file           Path to file containing technical specifications about the calculations.
    -ro               Only calculate properties of reference compounds, then exit. Default: not used.
    -nof             Only calculate properties of reference and read-in compounds, then exit. Default: not used.
    -xyz             Use traditional XYZ format when writing XYZ files. Default: not used.
                    The extended XYZ format (http://jrkermode.co.uk/quippy/io.html#extendedxyz)
                    is used by default.

    -dfitpropn        Show information about fitting of properties. Here 'n' must be
                    an integer. Supported: 0-4. 0: debug fitting method. 1-4: debug deeper.
                    lying methods used by the fitting method. Default: not used
                    NOTE: 0 also shows some info about the initial Chi^2 object.
    -dfitpotn         Show information about fitting of potentials. Here 'n' have a similar
                    role as for fitting of the properties.
                    NOTE 1: 0 also shows some info about the initial Chi^2 object.
                    NOTE 2: 'fitpot0' is always set to true, others are false by default.

    -dforces          Debug the forces. Default: not used
    -dpressure        Debug the pressure. Default: not used
    -dmdsprop         Debug MDS runs of the structures. Default: not used
    -dall             Activate all debugging options (top level only). Default: not used

    -mif             Suggest an initial fit and exit. Default: not used

    -omp              Request maximal number of threads (8) for any OpenMP parts.
    -omp_nt num       Request 'num' number of threads for any OpenMP parts. Default: 1
```

#### Main arguments:

<code>-pf potinfofile</code>	This file contains info about the elements and interactions.
<code>-gf geominfile</code>	This file contains the info about the compounds which are to be fitted.

#### Recommended options to always use:

<code>-sf specsinfofile</code>	This file contains settings steering the calculation of properties of read-in compounds and the fitting process.
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#### Useful options:

<code>-ro</code>	Calculate properties of reference (single-species) compounds and quit.
<code>-nof</code>	Calculate properties of reference and read-in compounds and quit. This is useful when parametrization is finalized and high-accuracy values of properties are desired (use long MD relaxation times!).
<code>-mif</code>	Obtain an initial partial fit to start from.
<code>-dmdsprop</code>	Show progress of all MD relaxations of all compounds.

Note: The OpenMP options can be used, but the OpenMP parallelization has not been extensively debugged. Some cache trashing is likely to occur, degrading performance.

## 4 Potentials

In TULIP the following potentials are subject to fitting: (i) the ABOP (Analytical Bond Order Potential). The EAM (Embedded Atom Method) potential is understood by the program, but this type of interactions cannot be fitted.

### 4.1 ABOP potential

The Brenner-Tersoff or ABOP potential gives the total energy of a system of atoms as

$$V = \frac{1}{2} \sum_i \sum_j f_{c,ij} (V_{R,ij} - B_{ij} V_{A,ij}) = \sum_i \sum_{j>i} f_{c,ij} (V_{R,ij} - \bar{B}_{ij} V_{A,ij}), \quad (1)$$

Here

$$\bar{B}_{ij} = \frac{B_{ij} + B_{ji}}{2} \quad (2)$$

Note:

$$V_{ij} \equiv f_{c,ij} (V_{R,ij} - B_{ij} V_{A,ij}) \quad (3)$$

may not be equal to  $V_{ji}$ .

The repulsive ( $R$ ) and attractive ( $A$ ) parts are

$$V_{R,ij} = \frac{D_0}{S-1} \exp \left[ -\beta \sqrt{2S} (r_{ij} - r_{0,ij}) \right] \quad (4)$$

$$V_{A,ij} = \frac{SD_0}{S-1} \exp \left[ -\beta \sqrt{2/S} (r_{ij} - r_{0,ij}) \right] \quad (5)$$

with

$$r_{ij} = |\mathbf{r}_{ij}| = |\mathbf{r}_i - \mathbf{r}_j| \quad (6)$$

The cutoff-function is

$$f_c(r) = \begin{cases} 1, & r \leq R - D, \\ \frac{1}{2} \left( 1 - \sin \left( \frac{\pi}{2} \frac{r-R}{D} \right) \right), & |R - r| < D \\ 0, & r \geq R + D \end{cases} \quad (7)$$

Hence, full interaction is felt when  $r < R - D$ , and no interaction when  $r > R + D$ , making the cutoff distance  $r_c = R + D$ . The derivative is

$$f'_c(r) = \begin{cases} 0, & r \leq R - D, \\ \frac{1}{2} \left(1 - \frac{\pi}{2D} \cos\left(\frac{\pi}{2} \frac{r-R}{D}\right)\right), & |R - r| < D \\ 0, & r \geq R + D \end{cases} \quad (8)$$

The bond-order parameter is defined as

$$B_{ij} = (1 + \chi_{ij})^{-\frac{1}{2}} \quad (9)$$

where

$$\chi_{ij} = \sum_{k, k \neq i, k \neq j} f_{c,ik} g_{ijk} \omega_{ijk} \exp[\alpha_{ijk}(r_{ij} - r_{ik})] \quad (10)$$

If the Brenner form is used then

$$\omega_{ijk} = \exp[-\alpha_{ijk}(r_{0,ij} - r_{0,ik})] \quad (11)$$

and  $\omega_{ijk}$  is **not** a separate parameter. Also,

$$g_{ijk} = \gamma \left(1 + \frac{c^2}{d^2} - \frac{c^2}{d^2 + (h + \cos \theta_{ijk})^2}\right) \quad (12)$$

where  $\theta_{ijk}$  is the angle between the bonds  $ij$  and  $ik$ .

## 4.2 EAM potentials

EAM potentials can as of now not be fitted, but only be used together with other potentials. The total energy of a solid is in the EAM formalism

$$V = \frac{1}{2} \sum_i \sum_{j, j \neq i} V_2(r_{ij}) + \sum_i a_s F_s(\rho_{s,i}^a) + \sum_i a_p F_p(\rho_{p,i}^a) + \sum_i a_d F_d(\rho_{d,i}^a). \quad (13)$$

Here  $a_i$  is 1 if the band  $i$  is included, else it is 0. Also,

$$\rho_i^a = \sum_{j=1, j \neq i} \rho(r_{ij}), \quad (14)$$

where  $\rho(r)$  is the atomic electron density at distance  $r$  from an atom. The program recognized the EAM versions displayed in table 1.

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The EAM potentials are read in from files. The format of these is shown in tables 2-4.

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**Note:** The  $N_r$  points  $(r, V_2)$  in the EAM files are read in as  $r = 0, \dots, (N_r - 1)dr$ . To avoid any problems, we must have  $N_r \times dr > r_{cut}$ . For instance, use  $N_r \times dr = r_{cut} + 10 \times dr$ .

Table 1: Recognized EAM flavors.

EAM-s	EAM with only $s$ embedding energy
EAM-p	EAM with only $p$ embedding energy
EAM-d	EAM with only $d$ embedding energy
EAM-sp	EAM with $s$ and $p$ embedding energies
EAM-sd	EAM with $s$ and $d$ embedding energies
EAM-spd	EAM with $s$ , $p$ , and $d$ embedding energies

Table 2: EAM file format when a single embedding energy is used. Note: First and second lines are ignored.

Comment line
Z1 Z2 mass1 mass2 latpar1 latpar2 latname1 latname2
Nrho drho Nr dr rcut
(Nrho points of $F_s$ or $F_p$ or $F_d$ )
(Nr points of $V_2$ )
(Nr points of $\rho_s$ or $\rho_p$ or $\rho_d$ )

Table 3: EAM file format when two embedding energies are used. Example is for  $s$  and  $d$  embedding energies. Note: First and second lines are ignored.

Comment line
Z1 Z2 mass1 mass2 latpar1 latpar2 latname1 latname2
Nrho drho Nr dr rcut Nrhos drhos
(Nrho points of $F_d$ )
(Nr points of $V_2$ )
(Nr points of $\rho_d$ )
(Nrhos points of $F_s$ )
(Nr points of $\rho_s$ )

## 5 Read-in files

All specifications about the fitting procedure, physical properties, and calculations in general are given in read-in files: (i) file containing settings for the potentials, (ii) file containing

Table 4: EAM file format when three embedding energies are used. Note: First and second lines are ignored.

```

Comment line
Z1 Z2 mass1 mass2 latpar1 latpar2 latname1 latname2
Nrhd drhd Nr dr rcut Nrhd drhd Nrhd drhd
(Nrhd points of  $F_d$ )
(Nr points of  $V_2$ )
(Nr points of  $\rho_d$ )
(Nrhd points of  $F_s$ )
(Nr points of  $\rho_s$ )
(Nrhd points of  $F_p$ )
(Nr points of  $\rho_p$ )

```

physical properties, and (iii) file containing technical specifications about the calculations. The last file is optional.

The delimiters separating tokens in the read-in files are: tabulator, space, and the following characters: `:`, `(`, `)`, `[`, `]`, `=`. The point is not a delimiter. Character strings which are not part of the input value for an option are simply ignored.

## 5.1 File: Elements and interactions information

An overview of which potentials that can be read in from tabulated data in  $(x, y)$  format and those which can be specified via parameters only is shown in Table 5.

Table 5: R = Can be read in from data file. A = Can be specified by parameters.

Potential	R	A
EAM-*	Yes	No
ABOP	No	Yes

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Complete example:

```

# MUST BE FIRST:
# Number of elements:
nelem = 3

# MUST BE SECOND:
# Element names:
elem(1) = Fe
elem(2) = Cr
elem(3) = C

```

```
# Optional:
atomtype(Fe) = 1
atomtype(Cr) = 2
atomtype(C) = 3

# Masses
mass(Fe) = 55.8470
mass(Cr) = 51.9961
mass(C) = 12.0110

# Reference lattices:
lat(Cr, Cr) = BCC
lat(Fe, Fe) = BCC
lat(C, C) = GRA

# Supported ref. lattices:
# DIM1 (homomer), DIM2 (heteromer), SC, BCC, BCC-P, FCC, FCC-P,
# DIA, HCP, GRA, GRP (graphene)
# The ...-P versions refer to alternate structures with
# non-Cartesian primitive vectors.

a(Cr, Cr) = 2.87
a(Fe, Fe) = 2.87
#####
# Use accurate values for C, otherwise the graphite might explode due
# to massive pressure!!! Also, use small enough time step!!!
#####
a(C, C) = 1.46 rNN for graphite (GRA) DIA: 3.55647765821
c(C, C) = 6.689
# Also possible:
# bpa(C, C) = ...
# cpa(C, C) = ...

# Interaction types:
iac(Fe, Fe) = ABOP
iac(Cr, Cr) = ABOP
iac(C, C) = ABOP
iac(Fe, Cr) = ABOP symmetric
iac(Fe, C) = ABOP symmetric
iac(Cr, C) = ABOP symmetric

# Fit this interaction? The interaction must be analytical.
fit(Fe, Cr) = yes symmetric

# Repulsive potentials (ZBL):

use_rep_core(Fe, Fe) = yes
bfermi(Fe, Fe) = 10
rfermi(Fe, Fe) = 1.0
```



```
use_rep_core( Cr, Cr ) = yes
bfermi( Cr, Cr ) = 12
rfermi( Cr, Cr ) = 1.7

use_rep_core( C, C ) = yes
bfermi( C, C ) = 8
rfermi( C, C ) = 0.6

use_rep_core( Fe, Cr ) = no
bfermi( Fe, Cr ) = 10
rfermi( Fe, Cr ) = 1

use_rep_core( Fe, C ) = yes
bfermi( Fe, C ) = 10
rfermi( Fe, C ) = 1

use_rep_core( Cr, C ) = yes
bfermi( Cr, C ) = 8
rfermi( Cr, C ) = 1.2

# Parameters for fixed interactions

potpar( Fe, Fe ):D0 = 1.5
potpar( Fe, Fe ):r0 = 2.29
potpar( Fe, Fe ):beta = 1.4
potpar( Fe, Fe ):S = 2.0693109
potpar( Fe, Fe ):gamma = 0.0115751
potpar( Fe, Fe ):c = 1.2898716
potpar( Fe, Fe ):d = 0.3413219
potpar( Fe, Fe ):h = -0.26
potpar( Fe, Fe ):R = 3.15
potpar( Fe, Fe ):D = 0.2

potpar(Cr, Cr ):D0 = 4.04222081
potpar(Cr, Cr ):r0 = 2.13018547
potpar(Cr, Cr ):beta = 1.62158721
potpar(Cr, Cr ):S = 3.36793914
potpar(Cr, Cr ):gamma = 0.02388562
potpar(Cr, Cr ):c = 1.03288255
potpar(Cr, Cr ):d = 0.13813230
potpar(Cr, Cr ):h = -0.28569237
potpar(Cr, Cr ):R = 3.2
potpar(Cr, Cr ):D = 0.20

potpar(C, C ):D0 = 6.0
potpar(C, C ):r0 = 1.39
potpar(C, C ):beta = 2.1
potpar(C, C ):S = 1.22
potpar(C, C ):gamma = 2.0813e-4
potpar(C, C ):c = 330.0
potpar(C, C ):d = 3.5
potpar(C, C ):h = 1.0
potpar(C, C ):R = 1.85
```

```
potpar(C, C ):D = 0.15

potpar(Fe, C ):D0 = 3.95000634
potpar(Fe, C ):r0 = 1.53426579
potpar(Fe, C ):beta = 1.82109816
potpar(Fe, C ):S = 1.43035110
potpar(Fe, C ):gamma = 0.07485571
potpar(Fe, C ):c = 1.11674155
potpar(Fe, C ):d = 0.94663188
potpar(Fe, C ):h = -0.18665305
potpar(Fe, C ):R = 2.6
potpar(Fe, C ):D = 0.20

potpar( Cr, C ):D0          = 2.77620074
potpar( Cr, C ):r0          = 1.81289285
potpar( Cr, C ):beta        = 2.00816371
potpar( Cr, C ):S           = 2.04637644
potpar( Cr, C ):gamma       = 0.00068830
potpar( Cr, C ):c           = 3.93353757
potpar( Cr, C ):d           = 0.17497204
potpar( Cr, C ):h           = -0.17850001
potpar( Cr, C ):R           = 2.95
potpar( Cr, C ):D           = 0.1

# Parameters for fittable interactions

      potpar( Fe, Cr ):D0 = 3.48049488
min: potpar( Fe, Cr ):D0 = 0.1
max: potpar( Fe, Cr ):D0 = 10.0

      potpar( Fe, Cr ):r0 = 2.16998952
min: potpar( Fe, Cr ):r0 = 1.0
max: potpar( Fe, Cr ):r0 = 5.0

      potpar( Fe, Cr ):beta = 1.75467567
min: potpar( Fe, Cr ):beta = 1.0
max: potpar( Fe, Cr ):beta = 5.0

      potpar( Fe, Cr ):S = 2.28661503
min: potpar( Fe, Cr ):S = 1.1
max: potpar( Fe, Cr ):S = 5.0

      potpar( Fe, Cr ):gamma = 0.15766130
min: potpar( Fe, Cr ):gamma = 1e-5
max: potpar( Fe, Cr ):gamma = 1e5

      potpar( Fe, Cr ):c = 0.48531613
min: potpar( Fe, Cr ):c = -1e5
max: potpar( Fe, Cr ):c = 1e5

      potpar( Fe, Cr ):d = 0.31427413
min: potpar( Fe, Cr ):d = -1e5
max: potpar( Fe, Cr ):d = 1e5

      potpar( Fe, Cr ):h = -0.69
```

```

min: potpar( Fe, Cr ):h = 1.0
max: potpar( Fe, Cr ):h = 1.0

      potpar( Fe, Cr ):R = 3.10
min: potpar( Fe, Cr ):R = 2.0
max: potpar( Fe, Cr ):R = 2.0

      potpar( Fe, Cr ):D = 0.15
min: potpar( Fe, Cr ):D = 1.0
max: potpar( Fe, Cr ):D = 1.0

# ABOP alpha and omega parameters

#####
##
## NOTE: There are NO ABOP alpha, omega, 2mu parameters used by default.
##       Only the specified ones are used.
##       Defaults: alpha=2mu=0.0 and omega=1.0 as constants.
##
##       If an omega parameter is specified it is taken as an
##       independent parameter (non-Brenner form), otherwise
##       it is constructed from alpha parameters --- if they are
##       specified --- as
##
##       "omega_ijk" = exp( alpha_ijk*(r0_ij - r0_ik) )
##
#####

# Fixed:

abop_alpha( Fe, Fe, Fe ) = 0.0
abop_omega( Fe, Fe, Fe ) = 1.0

abop_alpha( Cr, Cr, Cr ) = 1.39662066
abop_omega( Cr, Cr, Cr ) = 1.0

abop_alpha( C, C, C ) = 0.0
abop_omega( C, C, C ) = 1.0

abop_alpha( Cr, Cr, C ) = 0.8640643600
abop_alpha( Cr, C, Cr ) = -1.7520448300
abop_alpha( C, Cr, Cr ) = 0.6122158900
abop_alpha( C, C, Cr ) = 0
abop_alpha( C, Cr, C ) = 0
abop_alpha( Cr, C, C ) = 0

abop_omega( Cr, Cr, C ) = 1.6402877600
abop_omega( Cr, C, Cr ) = 0.2939996300
abop_omega( C, Cr, Cr ) = 0.4190507900
abop_omega( C, C, Cr ) = 1
abop_omega( C, Cr, C ) = 1
abop_omega( Cr, C, C ) = 1

```

```
# Fittable:
```

```
    abop_alpha( Fe, Fe, Cr ) = 1.0
min: abop_alpha( Fe, Fe, Cr ) = 100.0
max: abop_alpha( Fe, Fe, Cr ) = 100.0
```

```
    abop_alpha( Fe, Cr, Fe ) = 1.0
min: abop_alpha( Fe, Cr, Fe ) = 100.0
max: abop_alpha( Fe, Cr, Fe ) = 100.0
```

```
    abop_alpha( Cr, Fe, Fe ) = 1.0
min: abop_alpha( Cr, Fe, Fe ) = 100.0
max: abop_alpha( Cr, Fe, Fe ) = 100.0
```

```
    abop_alpha( Cr, Fe, Cr ) = 1.0
min: abop_alpha( Cr, Fe, Cr ) = 100.0
max: abop_alpha( Cr, Fe, Cr ) = 100.0
```

```
    abop_alpha( Fe, Cr, Cr ) = 1.0
min: abop_alpha( Fe, Cr, Cr ) = 100.0
max: abop_alpha( Fe, Cr, Cr ) = 100.0
```

```
    abop_alpha( Cr, Cr, Fe ) = 1.0
min: abop_alpha( Cr, Cr, Fe ) = 100.0
max: abop_alpha( Cr, Cr, Fe ) = 100.0
```

```
    abop_omega( Fe, Fe, Cr ) = 1.0
min: abop_omega( Fe, Fe, Cr ) = 100.0
max: abop_omega( Fe, Fe, Cr ) = 100.0
```

```
    abop_omega( Fe, Cr, Fe ) = 1.0
min: abop_omega( Fe, Cr, Fe ) = 100.0
max: abop_omega( Fe, Cr, Fe ) = 100.0
```

```
    abop_omega( Cr, Fe, Fe ) = 1.0
min: abop_omega( Cr, Fe, Fe ) = 100.0
max: abop_omega( Cr, Fe, Fe ) = 100.0
```

```
    abop_omega( Cr, Fe, Cr ) = 1.0
min: abop_omega( Cr, Fe, Cr ) = 100.0
max: abop_omega( Cr, Fe, Cr ) = 100.0
```

```
    abop_omega( Fe, Cr, Cr ) = 1.0
min: abop_omega( Fe, Cr, Cr ) = 100.0
max: abop_omega( Fe, Cr, Cr ) = 100.0
```

```
    abop_omega( Cr, Cr, Fe ) = 1.0
min: abop_omega( Cr, Cr, Fe ) = 100.0
max: abop_omega( Cr, Cr, Fe ) = 100.0
```

## 5.2 File: Compounds information

In order to specify a physical property to be fitted, the keywords in example below must be used.

Keywords are grouped into sets than begin with the string LAT on a separate line. Properties in each set refer to the same structure/lattice/geometry.

**All properties that are specified (=set) are activated as fitting targets.**

Complete listing of options for a lattice: Note that . . . means that numerical values (single or several ones) are expected, and \*\*\* means that a string is needed.

```
LAT                                <= starts readin of new lattice info

name          = ...    string
file          = ...    LAT file, string
elements      = ...    element names (e.g. W H), strings
Ndesired      = ...    number of cells in direction a, three integers
Neven_desired = ...    0 for false, 1 for true, three integers
Nodd_desired  = ...    0 for false, 1 for true, three integers

# Lattice parameters
a              = ...
w_a           = ...    weight
b              = ...
u_b           = ...    uncertainty
c              = ...    default: weight: 1.0

# weight for any property: w_***
# uncertainty for any property: u_***

# Lattice parameter relationships:
bpa           = ...
cpa           = ...

# Dimer bond distance (only for dimers):
# r0           = ...

# Angles (radians):
angle_ab      = ...
angle_ac      = ...
angle_bc      = ...

# Atomic volume (cubic Angstroms):
Vatom         = ...

# This compound should be considered ground/reference state when calculating
# cohesive energies:
Ecoh_delta_refcomp = true
```

```
# If no cohesive energies used, then skip this setting.

# Change in cohesive energy relative to reference:
Ecoh_delta    = ...  (>0.0, i.e. more unstable than reference)

# Formation energy Ef in 'mixing energy' Emix = Ef/natoms form:
Emix          = ...

# Bulk modulus B (GPa):
B             = ...

# Pressure derivative of B:
Bp           = ...

# Elastic constants (all):
C11          = ...
C12          = ...
C13          = ...
C14          = ...
C15          = ...
C16          = ...
C22          = ...
C23          = ...
C24          = ...
C25          = ...
C26          = ...
C33          = ...
C34          = ...
C35          = ...
C36          = ...
C44          = ...
C45          = ...
C46          = ...
C55          = ...
C56          = ...
C66          = ...
# A SPGLIB call will determine space group and which Cij are correct.
# Wrong ones will be turned off.

# Options:
option: no_heating           Do not use temperatures > 0 K.
option: fixed_geometry       Do not allow atom positions in compounds to change.
option: quench_always        Always scale velocities to 0.

# Force handling:
frc_file       = ***  string
# => use atomic forces, get them from specified file
frc_use        = ***  boolean
# => use atomic forces
frc_use_w      = ***  boolean, use weights for force components (default)
# frc_use_u    = ***  boolean, use weights for force components
#
# Format of forces file: Lines of
#   fx  fy  fz  dfx  dfy  dfz
# in eV/fs, where dfx, dfy, dfz is interpreted as weights/uncertainties,
```

```

# depending on which of frc_use_w/frc_use_u is set.
#

# Options that still exist but are deprecated:
# Fmax          = ...    try to achieve this largest atomic force
#                  in relaxed compound
# Pmax          = ...    try to achieve this largest Cartesian pressure
#                  in relaxed compound
# displmax      = ...    try to achieve this largest atomic displacement
#                  in relaxed compound
# Rationale: Prefer parametrizations that minimize forces, pressures, and
# displacements.
# Note: Usually the MD relaxation achieves these goals already, and specifying
# these options may interfere with the fitting (see the merit function discussion).

# Compound-specific MD options, overrides any others specified elsewhere:
# Complete list (example values):

mds_skint       = 1.0      # Angstrom
mds_seed        = 12345
mds_ndump       = 10      # dump info every ndump steps (if -dmdsprop option)

mds_tstart      = 0.0
mds_tend        = 2000.0
mds_dt          = 3.0
mds_max_dt      = 3.0

mds_Tstart      = 0.5     starting temperature T (K)
mds_btc_tau     = 10.0    Berendsen time constant for T control (fs)
mds_btc_T0      = 0.0     desired T (K)

mds_bpc_tau     = 80.0    Berendsen time constant for P control (fs)
mds_bpc_P0      = 0.0     desired P (GPa)
mds_bpc_scale   = 50.0    scaling constant, usually on the order of bulk modulus (GPa)

# mds_quench_tstart = 2000
# mds_quench_rate   = 1.0  # quenching rate (K/fs), negative value => heating

mds_error_T_gt   = 1e6     Fatal error if T gets over this limit (K).
mds_error_dt_lt  = 0.01    Fatal error if dt gets under this limit (fs).
mds_error_boxlen_gt = 1e4  Fatal error if any boxlen gets over this limit (Angstrom).

```

**Note:** If a boolean value of logical true is to be input, use yes, Yes, true, True or the first letter in these strings. Any other string evaluates to logical false.

**Note:** If neither weight nor uncertainty is specified for any property a default weight of 1 is used. The weights  $w_i$  are normalized:  $\sum_i w_i = 1$ , where the sum goes over properties with weights only, properties with uncertainties are not taken into account.

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**Example:**

```
LAT                                     <= triggers readin of new compound data

name      = dimer
file      = in/dimer.lat
elements  = W H

r0 = 1.5                                Desired bond distance in dimer
w_r0 = 0.0001                           weight

Ecoh      = -0.123
u_Ecoh    = 0.0068                      uncertainty (w and u can be freely mixed
                                         for different properties)

option: no_heating                      Do not use temperatures > 0 K.


LAT                                     <= triggers readin of new compound data

name      = bcc
file      = in/bcc.lat
elements  = W
a         = 2.9
w_a       = 1.0
Ecoh_delta_refcomp = true
C11       = 80.1605326176
w_C11     = 20
C12       = 20.4233027208
w_C12     = 20
C44       = 25.072362326
w_C44     = 20


LAT                                     <= triggers readin of new compound data

name      = octa
file      = in/octa.lat
elements  = W H
Ndesired  = 2 2 2
Neven_desired = 1 1 1
Nodd_desired = 0 0 0
a         = 2.9
Emix      = 0.001

mds_seed  = 123
mds_tend  = 5000.0
mds_dt    = 3.0
mds_max_dt = 3.0

mds_Tstart = 0.5  starting temperature T (K)
mds_btc_tau = 10.0 Berendsen time constant for T control (fs)
mds_btc_T0 = 0.0  desired T (K)
```



```

mds_bpc_tau    = 80.0  Berendsen time constant for P control (fs)
mds_bpc_P0     =  0.0  desired P (GPa)
mds_bpc_scale  = 50.0  scaling constant, usually on the order of bulk modulus (GPa)

```

## 5.3 LAT files

### 5.3.1 Basic structure

Format of LAT file:

Comment	Arbitrary comment.
L	Overall scaling constant.
a1 a2 a3 optional-string	Components of the first primitive vector U1.
b1 b2 b3 optional-string	Components of the second primitive vector U2.
c1 c2 c3 optional-string	Components of the third primitive vector U3.
format	internal or direct
Nbasis	number of basis vectors
S1 B11 B12 B13 constr1	Species and vector components of first basis atom.
S2 B21 B22 B23 constr2	Species and vector components of second basis atom.
...	

The real primitive vectors are  $\mathbf{v}_i = L\mathbf{u}_i$ .

The strings  $S_i$  — e.g. Cr, H, W — specify the species of the basis atom.

If optional-string is present and is pbc, then the corresponding direction is considered periodic, i.e. it is a true primitive vector in an infinite lattice. Other strings are ignored.

If the format keyword is scaled or Scaled or letter s or S then e.g. the  $j$ :th basis vector is

$$\mathbf{b}_j = (L * B_{j1}) \mathbf{e}_x + (L * B_{j2}) \mathbf{e}_y + (L * B_{j3}) \mathbf{e}_z$$

This is equivalent to **Cartesian** in VASP.

If instead the format keyword is internal or Internal or letter i or I then the basis vector is

$$\mathbf{b}_j = (L * B_{j1}) \mathbf{u}_1 + (L * B_{j2}) \mathbf{u}_2 + (L * B_{j3}) \mathbf{u}_3.$$

This is equivalent to **Direct** in VASP.

### 5.3.2 Constraints

The constr1, constr2, etc. are atomic constraint strings (each atom can be given constraints). There are three possible constraint types:

1. Fix atom: fix
2. Constrain atom to move in one direction only: freedir u1 u2 u3, where the  $u_i$  are coordinates. Example: freedir 0 0 1 allows motion in  $z$  direction only.

3. Constrain atom to move in a plane only: `freeplane u1 u2 u3`, or `freeplanevecs v1 v2 v3 w1 w2 w3`, where the  $u_i$  are coordinates of the plane's normal vector, and the  $v_i, w_i$  are coordinates of two vectors lying in the plane.

All direction vectors are normalized automatically. Only one option is valid for any atom.

### 5.3.3 Specifying the origin

An explicit origin can be specified on the line after the last basis atom. The first string has to start with lower- or upper-case O, then three coordinates must be given. This origin will be used when the simulation cell is created.

### 5.3.4 Examples

Example 1: Fe-Y dimer in vacuum, non-periodic boundaries:

```
Fe-Y dimer
  10.0  <= scaling constant
1.0  0.0  0.0    primitive vector U1
0.0  1.0  0.0    primitive vector U2
0.0  0.0  1.0    primitive vector U3
  Scaled
  2
Y   0.0000000000  0.0000000000  0.0000000000
Fe  0.2000000000  0.0000000000  0.0000000000 freedir 1 0 0
origin: 0.0 0.0 0.0
```

Example 2: CrC in the NaCl crystal form:

```
#
  4.0741512214
0.0  0.5  0.5    pbc    primitive vector U1
0.5  0.0  0.5    pbc    primitive vector U2
0.5  0.5  0.0    pbc    primitive vector U3
  Internal
  2
C   0.0  0.0  0.0
Cr  0.5  0.5  0.5
```

Scripts are provided to convert between POSCAR, XYZ and LAT formats.

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A simulation box with box lengths  $L_1, L_2, L_3$  is constructed from a compound with primitive vectors  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ , so that all box lengths are more than twice the largest cutoff radius for the elements occurring in the compound:

$$L_i = N_i v_i \geq 2r_c \quad (15)$$

Here  $N_i$  is initially the corresponding value in the `Ndesired` setting, if specified.

\*   \*   \*

Predicted lattice parameter  $\tilde{a}$  is calculated as

$$\tilde{a} = (\tilde{L}_1/L_1)a, \quad (16)$$

where  $a$  is the read-in lattice parameter in the file listing the compounds,  $L_1$  is the initial length of the simulation box in the  $\mathbf{v}_1$  direction, and  $\tilde{L}_1$  is the relaxed length of the simulation box in the same direction.

\*   \*   \*

**Note: There must always be at least one basis vector.** The following example for bcc Fe should be helpful:

```
Comment: BCC Fe with primitive vectors.
-0.25  0.25  0.25  pbc
 0.25 -0.25  0.25  pbc
 0.25  0.25 -0.25  pbc
  Internal
  1
Fe  0.0  0.0  0.0
```

Using a conventional cubic cell, the lattice is

```
Comment: BCC Fe.
1.0  0.0  0.0  pbc
0.0  1.0  0.0  pbc
0.0  0.0  1.0  pbc
  Internal
  2
Fe  0.0  0.0  0.0
Fe  0.25 0.25 0.25
```

## 5.4 Optional file: Technical specifications

```
# -----
# Calculating properties of compounds
# -----

# Bulk modulus
prop:BM_rel_sys = yes      relax a strained frame
prop:BM_fmin    = -0.01
prop:BM_fmax    =  0.01
prop:BM_Nf      =  10      number of points to use for (V,E) curve
```

```

prop:BM_ef      = 1e-10    error ratio for each energy valye

# Elastic constants
prop:C_rel_sys  = yes
prop:C_fmin     = -0.01
prop:C_fmax     = 0.01
prop:C_Nf       = 10
prop:C_ef       = 1e-10

# -----
# Fitting properties of compounds
# -----
prop:fitmet      = LM      options: CG, PM, GN, LM, DL, SM, DE, PS, BC, GS, or SA
prop:nitermin    = 5       do at least this many iterations
prop:nitermax    = 100    do a maximum of this many iterations
prop:niterrestart = 20     restart every 20th iteration

#### Negative values means that it will not be used when testing for convergence:
prop:functolabs  = 1e-5    convergence when absolute merit function value is less
                           than this
prop:functolrel  = -1e-5   ... change in merit function value ...
prop:gradtolabs  = 1e-5    ... absolute gradient value ...
prop:steptolabs  = 1e-5    ... absolute step size ...
prop:steptolrel  = -1e-5   ... change in step size ...

prop:dogleg_radius    = 0.2    initial trust region radius
prop:dogleg_minradius = 1e-5   exit when radius gets this low
prop:barrier_scale     = 0.0    scaling constant for barrier penalty function,
                               to keep parameter values inside min/max interval
prop:simann_delta_rel  = 0.2    initial displacements in coordinate directions
prop:use_data_scales   = true   use/do not use scaled values in the merit function

# -----
# General MDS settings
# -----
prop:mds_skint = 1.0          Angstrom
prop:mds_seed  = 12345
prop:mds_ndump = 10           dump info every ndump steps (if -dmdsprop option)

prop:mds_tstart = 0.0
prop:mds_tend   = 2000.0
prop:mds_dt     = 0.5
prop:mds_max_dt = 1.0

prop:mds_Tstart = 300.0      starting temperature T (K)

prop:mds_btc_tau = 20        Berendsen time constant for T control (fs)
prop:mds_btc_T0  = 0         desired T (K)

prop:mds_bpc_tau = 100       Berendsen time constant for P control (fs)
prop:mds_bpc_P0  = 0         desired P (GPa)
prop:mds_bpc_scale = 100     scaling constant, usually on the order of bulk modulus
                               (GPa)

```

```

prop:mds_quench_tstart = 100
prop:mds_quench_rate   = 1.0   quenching rate (K/fs), negative value => heating

# -----
# MDS settings for reference compounds
# -----

# Use this to copy MDS settings for reference compounds from the general settings
# specified above:
# prop:ref:mds = prop:mds

prop:ref:mds_skint = 1.0      Angstrom
prop:ref:mds_seed  = 12345
prop:ref:mds_ndump = 10      dump info every ndump steps (if -dmdsprop option)

prop:ref:mds_tstart = 0.0
prop:ref:mds_tend   = 3000.0
prop:ref:mds_dt      = 5.0
prop:ref:mds_max_dt  = 5.0

prop:ref:mds_Tstart = 1.0    starting temperature T (K)

prop:ref:mds_btc_tau = 20    Berendsen time constant for T control (fs)
prop:ref:mds_btc_T0  = 0     desired T (K)

prop:ref:mds_bpc_tau = 80    Berendsen time constant for P control (fs)
prop:ref:mds_bpc_P0  = 0     desired P (GPa)
prop:ref:mds_bpc_scale = 80  scaling constant, usually on the order of bulk modulus
                             (GPa)

prop:ref:mds_quench_tstart = 100
prop:ref:mds_quench_rate   = 1.0   # quenching rate (K/fs), negative value => heating

# #####
# Fitting potential(s)
# #####

pot:fitmet      = DL      options: CG, PM, GN, LM, DL, SM, DE, PS, BC, GS, or SA
pot:nitermin    = 5
pot:nitermax    = 100
pot:niterrestart = 20     restart every 10th iteration

#### Negative values means that it will not be used when testing for convergence:
pot:functolabs  = 1e-5
pot:functolrel  = -1e-5
pot:gradtolabs  = 1e-5
pot:steptolabs  = 1e-5
pot:steptolrel  = -1e-5

pot:dogleg_radius = 0.2
pot:dogleg_minradius = 1e-5
pot:barrier_scale  = 0.0    barrier value
pot:simann_delta_rel = 0.2

```

```

pot:use_data_scales = true      scaling option

INFO:

# Gradient-based fitting methods:
# -----
# CG = conjugate gradients (ls)
# PM = Powell's method (ls)
# GN = Gauss-Newton (mi)
# LM = Levenberg-Marquardt (mi)
# DL = Powell's dog-leg method (mi) (usually most robust)
# SA = simulated annealing

#
# Population-based fitting methods:
# -----
# SM = simplex method
# DE = differential evolution
# PS = particle swarm method
# BC = bee colony method
# GS = gravitational search method

# mi: uses matrix inversion
# ls: uses line-search, usually implies slow fitting

```

The keyword `prop` is relevant to the calculations in which lattice parameter, cohesive energy, bulk modulus and other properties are calculated and/or fitted for the read-in structures using a given potential parameter set.

On the other hand, the keyword `pot` is relevant to the potential fitting itself, *i.e.* the evolution of the potential parameters.

## 6 Calculation of physical properties

Predicted lattice parameter  $\tilde{a}$  is calculated as

$$\tilde{a} = (\tilde{L}_1/L_1)a, \quad (17)$$

where  $a$  is the read-in lattice parameter in the compounds file (the file listing all the compounds to be used),  $L_1$  is the initial length of the simulation box in the  $\mathbf{v}_1$  direction, and  $\tilde{L}_1$  is the relaxed length of the simulation box in the same direction.

$L_i$  are determined from  $\mathbf{v}_i$  as  $L_i = N_{\text{desired}}(i) \times v_i$ , if the `Ndesired` option is used. Otherwise  $N_i$  is determined from the requirement  $L_i = N_i v_i \geq 2 \max(r_c)$ , where  $\max(r_c)$  is the largest cutoff radius between species present in the compound. In this calculations the options `Neven_desired` and `Nodd_desired` are considered, if given in the compounds file.

Predicted lattice parameters  $\tilde{b}, \tilde{c}$  are calculated in a similar way.

$$\tilde{b}/a = (\tilde{L}_2/L_2)/(\tilde{L}_1/L_1) \times b/a \quad (18)$$

$$\tilde{c}/a = (\tilde{L}_3/L_3)/(\tilde{L}_1/L_1) \times c/a \quad (19)$$

Cohesive energy:

$$\tilde{E}_{\text{coh}} = \tilde{E}_P/N \quad (20)$$

$$\widetilde{\Delta E}_{\text{coh}} = \tilde{E}_{\text{coh}} - \tilde{E}_{\text{coh}}^{\text{ref}} \quad (21)$$

Here  $N$  is the number of atoms in the compound, and  $\tilde{E}_{\text{coh}}^{\text{ref}}$  is the cohesive energy of the reference compound, if used.

Mixing energy:

$$\tilde{E}_{\text{mix}} = \tilde{E}_f/N \quad (22)$$

$$\tilde{E}_f = \tilde{E}_p - \sum_s N(s) E_{\text{coh}}(s) \quad (23)$$

Here  $N(s)$  is the number of atoms of species  $s$  in the compound, and  $E_{\text{coh}}(s)$  is the cohesive energy of the ground state of species  $s$ . Parametrization for species  $s$  must be given. For instance, if  $s$  is C, then ground state might be GRA (graphite).

Bulk modulus  $B$  and its pressure derivative  $B'(P)$  is calculated from  $(V(\varepsilon), E_p(\varepsilon))$  data using the Birch-Murnaghan equation of state

$$\begin{aligned} E = E_0 + \frac{9}{16} V_0 B_0 \left( \left( \frac{V_0}{V} \right)^{2/3} - 1 \right)^2 \\ \times \left[ \left( \left( \frac{V_0}{V} \right)^{2/3} - 1 \right) B'_0 + 6 - 4 \left( \frac{V_0}{V} \right)^{2/3} \right] \end{aligned} \quad (24)$$

The volume is  $V(\varepsilon) = (1 + \varepsilon)V_0 = (1 + f)^3 L_1(0) L_2(0) L_3(0)$ , with  $f \in [-f_{\text{max}}, f_{\text{max}}]$ , usually  $f_{\text{max}} \sim 0.01$ .

Each box — for a given value of  $f$  — may be allowed to relax (`prop:BM_rel_sys = yes`), or not (`prop:BM_rel_sys = no`).

Elastic constants  $C_{ij}$  are calculated using volume-non-conserving strains  $\varepsilon_{ij}$ . Given a strain matrix

$$[\varepsilon_{ij}] = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \quad (25)$$

the transformation matrix is

$$F = 1_3 + [\varepsilon_{ij}] = \begin{bmatrix} 1 + \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & 1 + \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & 1 + \varepsilon_{33} \end{bmatrix} \quad (26)$$

Any  $\varepsilon_{ij}$  can take only three values:  $-\varepsilon, 0, \varepsilon$ . The exact structure of  $F$  depends on which elastic constants are being calculated.

The atomic positions are in essence transformed as

$$r' = Fr, \quad (27)$$

where  $r$  is a column vector containing the Cartesian coordinates  $x, y, z$ . From the calculations the data points  $(\varepsilon, E_p(\varepsilon))$  are obtained. A second degree polynomial in strains is fitted to this data to obtain the elastic constants.

Each box — for a given value of  $\varepsilon$  — may be allowed to relax (`prop:C_rel_sys = yes`), or not (`prop:C_rel_sys = no`).

## 7 Data fitting

Given compounds and their desired properties, as well as technical specifications on how to compute them, the program tries to obtain a best fit of potential parameters. The best fit is achieved by minimizing the merit/cost function

$$\chi^2 = \frac{1}{2} \sum_i f_i^2 \quad (28)$$

$$f_i = t_i \frac{\tilde{Y}_i - Y_i}{s_i} \quad (29)$$

Here  $\tilde{Y}_i$  is the predicted value of property  $Y_i$ , e.g. the bulk modulus of the second compound (if two or more compounds are used and this compound has a bulk modulus specified as a fitting target). The  $t_i$  is either a weight,  $t_i = w_i$ , if a weight is used for property  $Y_i$ , and  $t_i = 1/u_i$  if an uncertainty is used.

Note: The weights  $w_i$  are normalized:  $\sum_i w_i = 1$ , so that properties using uncertainties have zero weight in this normalization sum.



The  $s_i = 1$  if scales are not used (`use_data_scales = false` in the specifications file), and  $s_i = Y_i$  if scales are used. By using scaled properties all are on an equal footing.

Assume the predicted value is  $\tilde{Y}_i = Y_i + \delta Y_i$ , then  $(\tilde{Y}_i - Y_i)/s_i = \delta Y_i/Y_i$  if scales are used, and  $(\tilde{Y}_i - Y_i)/s_i = \delta Y_i$  otherwise. Suppose the relative change  $\delta Y_i/Y_i = 50\%$  for two properties, but the properties themselves have values of 1 and  $10^{-7}$ . The former choice (using scales) gives balanced contributions to the merit function, but the latter does not. In worst case scenario properties with small values will have a small impact on the fitting process.

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Consider now the use of any of the deprecated options, e.g. `Fmax`. This option may be used to try to guide the fitting process to accept parametrizations giving small forces of relaxed compounds. Consider now e.g. `Fmax = 10-10`. Let the predicted value be e.g.  $10^{-7}$ . The term  $f_i$  (with  $t_i = 1$  and using scaling,  $s_i = Y_i$ ) is

$$\frac{10^{-7} - 10^{-10}}{10^{-10}} \sim 10^3,$$

i.e. probably much larger than for any of the conventional properties. In a gradient-based search the moves in parameter space will tend to focus on minimizing the forces, paying less attention to other properties, e.g. differences in cohesive energies.

Lesson: Do not use `Fmax`, `Pmax` or `displmax`. Rely on the MD relaxation instead, to provide zero forces and zero pressure. In fact, the MD run can be made to scale the pressure to any desired value (`mds_bpc_P0`), so it's better to use that one.

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The **gradient** of the merit function is

$$g_j = \frac{\partial \chi^2}{\partial x_j} = \sum_i f_i \frac{\partial f_i}{\partial x_j} \equiv \sum_i f_i J_{ij} = \sum_i J_{ji}^T f_i = -h_j \quad (30)$$

where  $h_j$  are components of the **antigradient**, and

$$J_{ij} = \frac{\partial f_i}{\partial x_j} = \frac{t_i}{s_i} \frac{\partial \tilde{Y}_i}{\partial x_j} \quad (31)$$

## 8 Convergence conditions

Convergence criteria in the technical specifications file are of the form (prop, for fitting compound properties):

```
prop:functolabs = 1e-5    convergence when absolute merit function value is less
                        than this
prop:functolrel = -1e-5   ... change in merit function value ...
prop:gradtolabs = 1e-5    ... absolute gradient value ...
prop:steptolabs = 1e-5    ... absolute step size ...
prop:steptolrel = -1e-5   ... change in step size ...
```

Note: A **negative** value means that criterion is **not used**.

- `functolabs` refers to  $\min(|\chi^2|)$
- `functolrel` refers to  $\min(|\Delta\chi^2|/|\chi^2|)$ , where  $\Delta$  signifies the change (in the merit function in this case) during the last step.
- `gradtolabs` refers to  $\min(|\nabla_x\chi^2|)$ , where  $\nabla_x\chi^2$  is the gradient of the merit function with respect to the parameters.
- `steptolabs` refers to  $\min(|\mathbf{h}|)$ , where  $\mathbf{h}$  is the last step taken in parameter space.
- `steptolrel` refers to  $\min(|\Delta\mathbf{h}|/|\mathbf{h}|)$

## 9 Parameters

The parameters are  $x_1, x_2, \dots, x_M$ .

Parameter limits  $x_i^{\min} = x_i^{\max} = 0$  means that the parameter is **unconstrained** and can vary freely in the interval  $(-\infty, \infty)$ .

The limits  $x_i^{\min} = x_i^{\max} \neq 0$  means that the parameter is **fixed**. The default for any parameter is  $x_i^{\min} = x_i^{\max} = 1$ .

The limits  $x_i^{\min} < x_i^{\max}$  means that the parameter is **constrained** in the given interval.

## 10 Constraint method

In order to forcibly constrain parameter values a barrier penalty function can be used. The option `barrier_scale =  $\mu$`  with a value  $> 0$  switches on the penalty function  $U$ :

$$U = - \sum_i \mu \ln \left\{ \frac{1}{s_i} \left( \left[ \frac{1}{2}(x_i^{\min} - x_i^{\max}) \right]^2 - \left[ x_i - \frac{1}{2}(x_i^{\min} + x_i^{\max}) \right]^2 \right) \right\} < 0 \quad (32)$$

The total cost function is then

$$\chi_{\text{tot}}^2 = \chi^2 + U \quad (33)$$

A complication with this scheme is that with small weights  $w_i$  one may get  $\chi_{\text{tot}}^2 = \chi^2 + U < 0$ . This may interfere with the convergence criteria  $\min(|\chi^2|)$ , since  $\chi_{\text{tot}}^2 < 0 < \min(|\chi^2|)$  as soon as  $|U|$  starts to dominate over  $|\chi^2|$ .

With  $\mu = 0$  the program still tries to stop parameter values from going outside their limits, but a penalty function is not used.