Queue Analysis Project

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1. Introduction

The following section provides readers with an overview of the project detailed in this report, as well as useful information for those unacquainted with queueing theory that may aid in understanding the report's contents.

1.1 Purpose

The purpose of this project is to gain experience with queueing theory and random number generation by utilizing them to generate and analyze models, then draw conclusions about their performance. Concepts from random number generation that are applied in the project include testing for uniformity and goodness-of-fit. Concepts from queueing theory that are applied in the project include notation, metrics, tools, and evaluation techniques.

1.2 Problem Description

This project addresses the modelling, simulation, and analysis of a single server queue, a multi-server queue, and a queue network. The single server queue, denoted in this report as an M/M/1 or M/G/1 queue, can be equated to an airport with a single runway to service incoming airplanes where the queue is the airspace above the airport itself. The multi-server queue, denoted as an M/M/c/N queue, is analogous to an airport with c runways to service incoming airplanes and a maximum system capacity (airspace capacity above the airport plus the runway capacity) of N airplanes. The queue network, which involves a single queue that branches into two other queues after the initial service, can be thought of as an airport with one runway that has two terminals available for the offloading of passengers.

For each of the schemes detailed above, some or all of the generated queue metrics ρ, L, w, and w_Q are compared with their respective theoretical values for theoretical $\rho = 0.1, 0.2, \ldots, 0.8$. Due to the simplicity of the M/M/1 queue, some additional analysis is performed with it, which includes the comparison of its metrics against the respective metrics from the M/G/1 queue to determine any differences in performance when service times are changed from a Markovian distribution to an arbitrary distribution. The exponential distribution used by the Java Modelling Tools (JMT) software, used to model interarrival times for this project, is also being analyzed in the case of the M/M/1 queue and confidence intervals are discussed for the case of theoretical $\rho = 0.5$ [1].

As a preliminary check, the performance of JMT's random number generator is analyzed to verify its uniformity; without this attribute, other conclusions relying on randomness become less trustworthy (see section 2.1).

If this project's problem domain were a real-world airport scenario, the results gathered would be helpful in predicting the performance of the airport's runway(s), airspace, and – in the case of the queue network – passenger terminals given different configurations, arrival rates, and service time distributions.

2. Methodology

Each model created and evaluated for this project was implemented using JMT [1]. The following sections detail the configurations used in each case and should be sufficient to repeat the simulations discussed.

2.1 General Configurations

Certain configurations in JMT are constant throughout each of the models evaluated in this project. To prevent redundancy, the following subsections detail steps that should be taken while constructing the individual models in sections 2.3, 2.4, and 2.5. Section 2.2 is an exception because it tests the JMT RNG, rather a queueing model.

2.1.1 Airplane Class

To have the source in each model generate airplanes as a Poisson process, create a customer class through the Customer Classes window accessed via Define -> Customer Classes in the upper menu. Ensure that its type is "open" with its distribution being exponential with a λ of 0.1 such that the mean is 10 to have a theoretical queue utilization of 10% (see Figure 1).

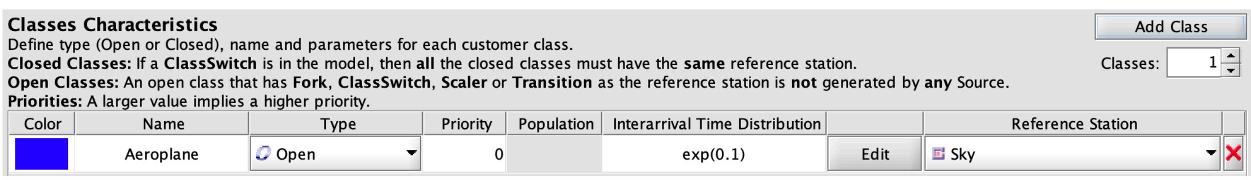


Figure 1: Customer class, "Aeroplane," configuration.

The "Aeroplane" class is the only class used for models in this project.

Note that, in the case of the triple runway with limited system capacity model, the exponential distribution should be set such that the mean is 3.33 to maintain consistent server utilizations with other models.

2.1.2 Model Performance Metrics

Performance statistics gathered directly from models in this project include the average number of customers in the system, L; the average time spent waiting in the airspace per customer, w_Q; the average total time in the system, w; and the average server utilization, ρ. To enable the collection of these statistics for each model, open JMT's Performance Indices window by selecting Define -> Performance Indices. From the drop-down menu, select response time, number of customers, queue time, and utilization. Finish configuring each selection as shown below, ensuring that the "Stat. Res." option is selected so that results are logged to a comma-separate value (CSV) file (see Figure 2).

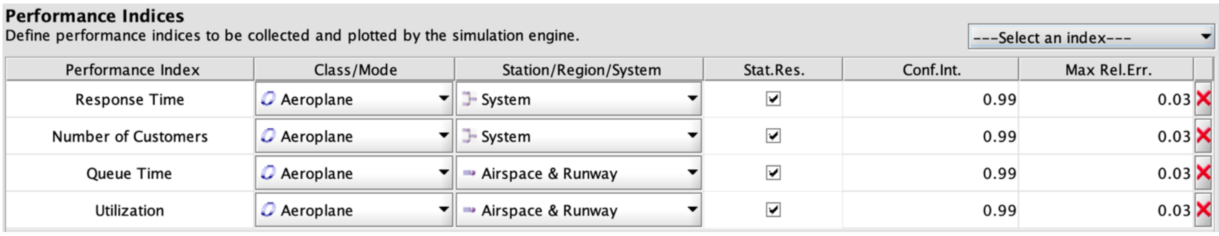


Figure 2: Configuration of simulation queue metrics.

Save for the single runway, dual terminal model, these metrics suffice for model analysis. In this case, a slightly different configuration is used; it is detailed in section 2.5.

Note that the triple runway, limited system capacity model adds the drop rate metric. This is detailed in section 2.4.

2.1.3 Simulation Parameters

In each case, simulation parameters, like maximum population size and maximum simulation time, can be configured as one sees fit – this project, however, set the parameters to be as shown below (see Figure 3).

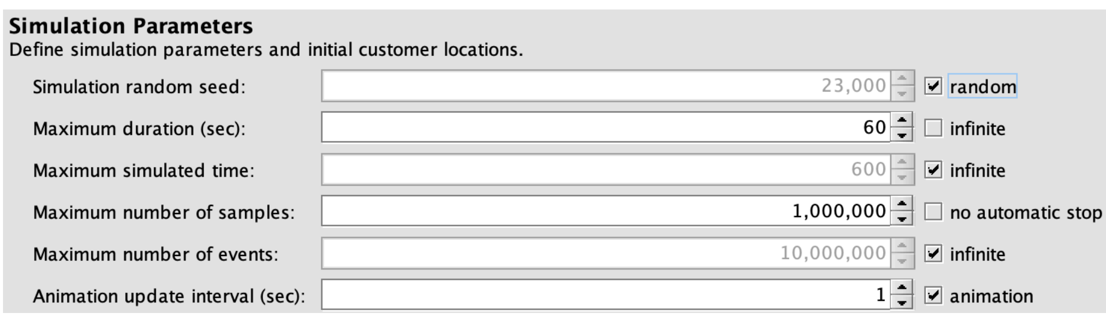


Figure 3: Simulation parameters used for each simulation in this project.

The above parameters are appropriate for recreating results seen in this report.

2.1.4 Automating Interarrival Time Increments

As discussed in section 1.2, for each model detailed in sections 2.2, 2.3, and 2.4, performance statistics are gathered for theoretical server utilizations $\rho = 0.1, 0.2, \dots, 0.8$. JMT's built-in "What-if" analysis tool is useful for automating this process and provides data formatted in a convenient manner. To enable the tool, select Define -> What-if from the upper menu. In the window, select "Enable What-If analysis," then select arrival rates from the drop-down menu. As seen in the following sections, only the one class of "Airplane" exists in each model, therefore it should be ensured that "Change the arrival rate of one open class" is enabled. To increment the mean arrival rate from the Poisson process as desired, configure the remaining entries as follows (see Figure 4).

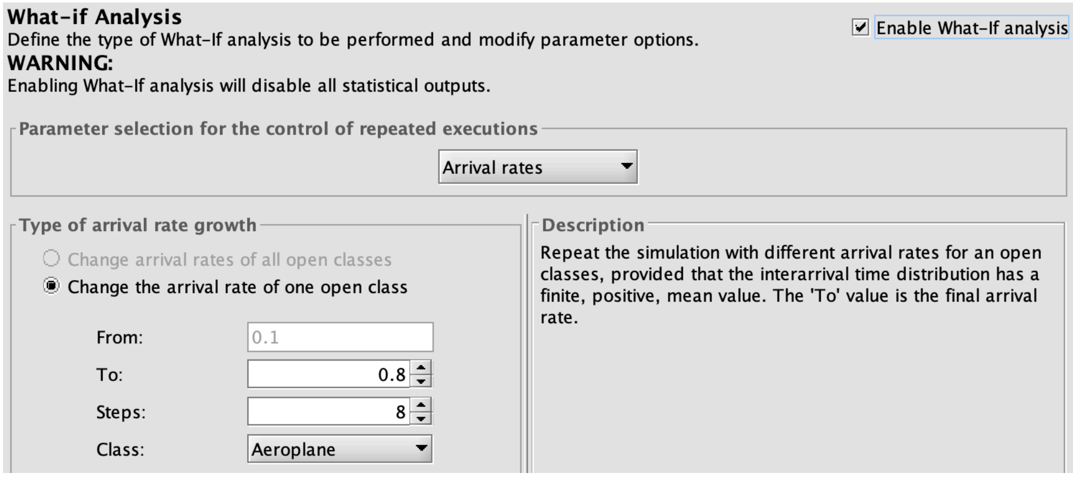


Figure 4: JMT's "What-if" analysis tool configuration to automate incrementation of server utilization.

By using this tool to automate incrementation of server utilization, analysis becomes far less tedious.

Note that, in order to keep the theoretical server utilizations of the triple runway with limited system capacity model at $\rho = 0.1, 0.2, \ldots, 0.8$, the "From" and "To" fields must be changed such that "From" specifies 0.3 and "To" specifies 3.0.

2.2 Random Number Generator

Testing JMT's random number generator for uniformity is a necessary step in verifying the correctness of the simulation software before moving on to the modelling of queueing systems. It is not feasible to review the JMT source code for correctness of its RNG, but it can be tested through the use of a simulation in itself.

To test the RNG, three JMT simulation components are required: a source, a logger, and a sink. Each component must be connected together in the order listed. The source is intended to generate mock customers with interarrival times governed by a uniform distribution, U(0, 1). The logger writes each interarrival time of a customer to a comma-separated value (CSV) file for later analysis (see section 3.1). The sink removes the customers from the system. The format for the system in JMT looks as follows (see Figure 5).

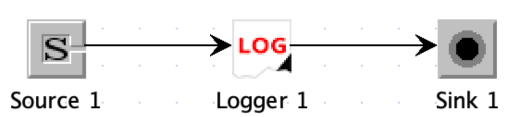


Figure 5: Simple JMT system for logging interarrival times.

A customer class must be created in JMT that will be generated by the source. This is done by opening the Customer Classes window through Define -> Customer Classes in the upper menu. Add a class via "Add Class," ensuring that the class has the "open" type, and selecting the U(0,

1) as its distribution (see Figure 6).

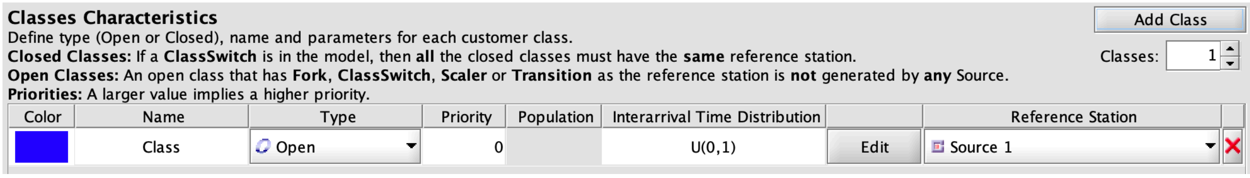


Figure 6: Customer class, "Class," configuration.

Selecting the logger component, tick the box for "Interarrival time (same class)" so that it plots the interarrival times of customers to a CSV file. Note the path that the CSV file is going to be saved to, which is located at the bottom of the logger window (see Figure 7).

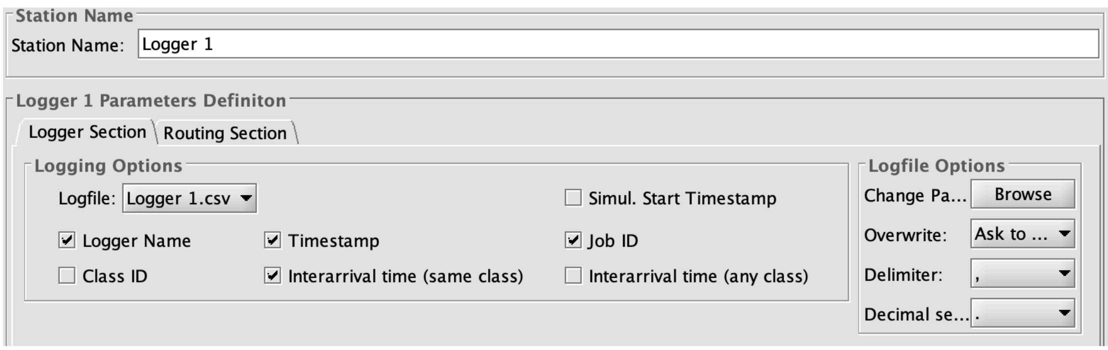


Figure 7: Logger configuration with "Interarrival time (same class)" selected.

The simulation can then be run by selecting the green arrow in the upper menu. Section 3.1 details the analysis of the resulting interarrival time data.

2.3 Single Runway

As stated in section 1.2, the case in which an airport has one runway to service airplanes is modelled as both an M/M/1 and M/G/1 queue; the difference between the two is the distribution that the service times follow. Both models' arrival times follow a Poisson distribution, but the former's service time distribution is exponential and the latter's service time distribution is uniform with a mean of 1, U(0, 2).

To model the M/M/1 and M/G/1 queues with JMT, three components are required: a source, a queue, and a sink. These components are linked together in the order listed. The source generates the airplanes, or customers, at a rate governed by the chosen distribution. From there, the airplanes are directed to wait in the airspace (queue) until the runway (service station) is available. When the runway is available for an airplane, that airplane will be removed from the airspace and land. The service time for landing is governed by the chosen distribution, much like the arrival times in the source. When the service time has elapsed for the current airplane, that airplane enters the sink and is removed from the system (see Figure 8).

Figure 8: Single runway queueing system.

For the M/M/1 queue, the queue component must be configured to service landing airplanes on an exponential distribution. To do this, open the queue component's configuration window by selecting the queue. Open the Service Section and ensure that the distribution is exponential (see Figure 9).

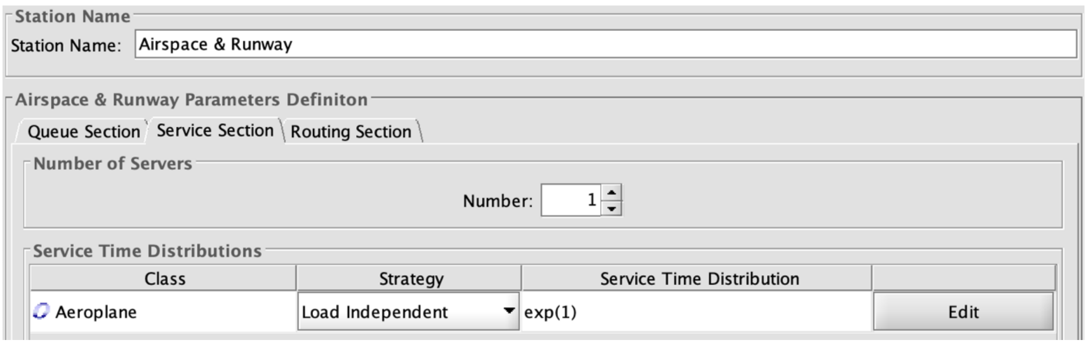


Figure 9: M/M/1 queue component service time configuration.

For the M/G/1 queue, the queue component must be configured to service airplanes on a uniform distribution with a mean of 1. To do this, open the queue component's configuration window by selecting the queue. Open the Service Section and ensure that the distribution is uniform with a mean of 1 (see Figure 10).

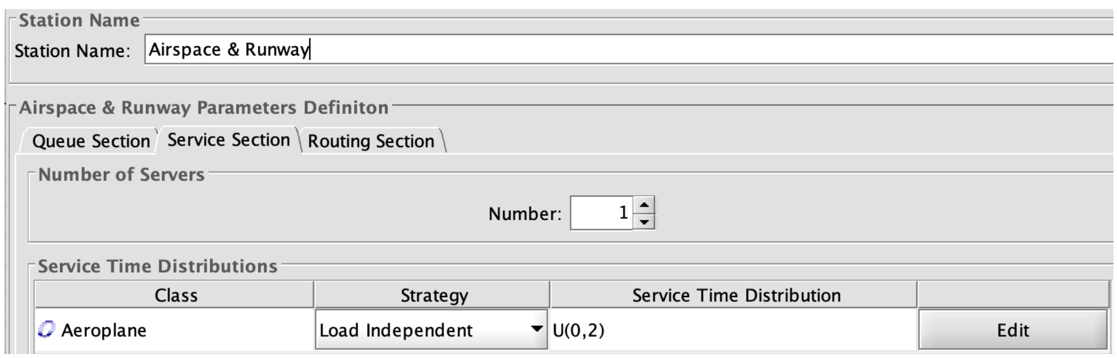


Figure 10: M/G/1 queue component service time configuration.

In the case of exponentially distributed service times on the runway (M/M/1), both the interarrival times of the source and service times of the queue's service station must be logged to a comma-separated value (CSV) file for later analysis in section 3.2.1. Two of JMT's loggers are used to achieve this goal – one placed in between the source and the queue and the other placed in between the queue and the sink. As in section 2.1, ensure that each of the two loggers is writing interarrival times to its own CSV file and note the path of each CSV file for later reference. The overall appearance of the M/M/1 system with the loggers looks as follows (see Figure 11).

Figure 11: Single runway queueing system adjusted for logging of interarrival and service times.

With the creation of the M/M/1 or M/G/1 queue model and completion of general configurations in section 2.1, the simulation can be initialized by selecting the green arrow in JMT's upper menu. With the loggers in the M/M/1 model, the CSV files will be saved upon simulation completion.

2.4 Triple Runway with Limited System Capacity

Detailed in this section is the creation of a model for an airport with three runways and a total airport and runway capacity of twenty airplanes. The setup is equivalent to that of the M/M/1 and M/G/1 system without the logger components, except for the configuration of the "Airspace & Runway" queue component. To prevent redundancy, please reference section 2.3 figures 8-9 and related writings for setup instructions.

To modify the queue such that it contains three runways instead of the default one, as well as a limited system capacity of twenty airplanes, open the queue configuration window by selecting the queue component. In the queue section, set capacity to finite and enter twenty into the space for "max no. customers," while ensuring that the drop rule is set to "Drop" (see Figure 12).

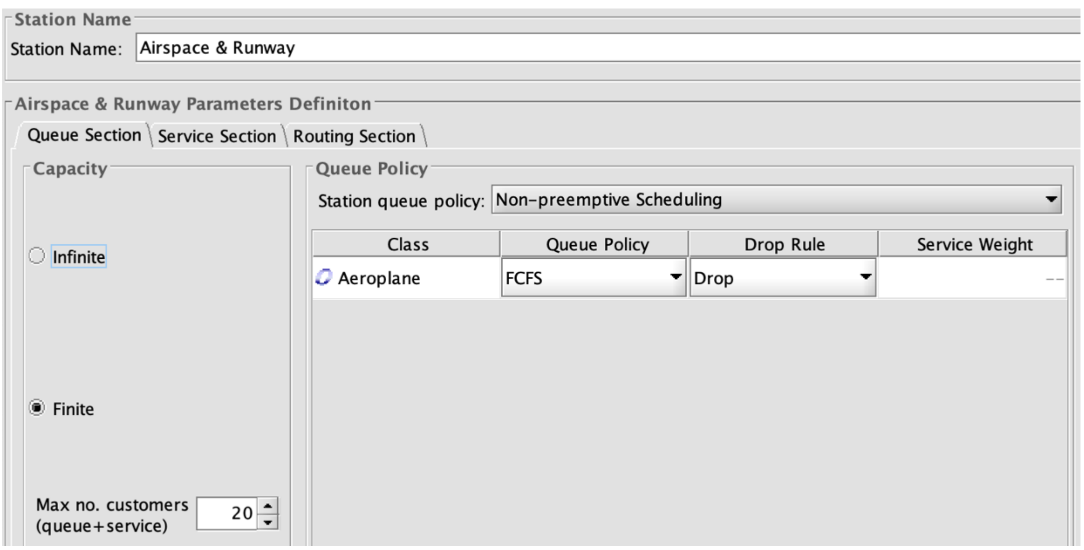


Figure 12: Airspace and runway queue component queue configuration for an M/M/3/20 system.

In the service section of the queue configuration window, increase the number of servers to three (see Figure 13).

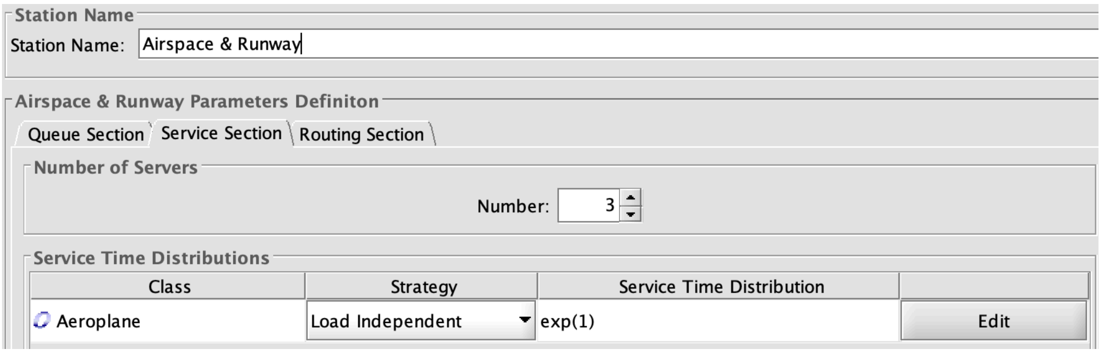


Figure 13: Airspace and runway queue component service configuration for an M/M/3/20 system.

When completing the general configurations in section 2.1, ensure that the drop rate performance index is also selected when configuring metrics for data collection. With the outlined steps and general configurations in section 2.1 complete, the simulation is now ready to be executed. Selecting the green arrow in the upper menu initiates this action.

2.5 Single Runway, Dual Terminal Network

This section explains how the model for a network of three M/M/1 queues was generated, as well as how the performance indices for the collection of data from simulations was configured due to it being different than other models.

One source, three queues, and two sinks are required for this model. In reality, only one sink is needed, but the collection of data becomes significantly easier with two sinks as detailed shortly (see Figure 14).

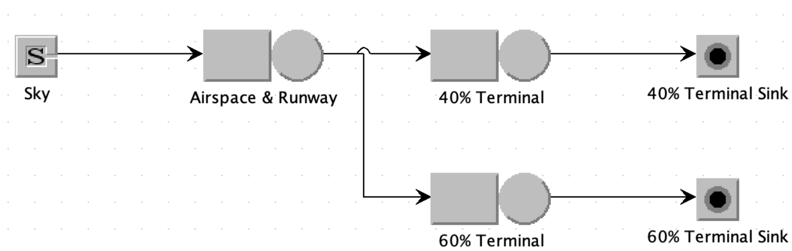


Figure 14: Network of queues with two streams.

The "Airspace & Runway" queue must be configured in the same fashion as in the M/M/1 system, except that the routing must be changed from random to 60% in one stream and 40% in the other stream. To do this, select the queue and navigate to the "Routing Section." Select "Probabilities" from the drop-down menu, then enter 0.4 into one text box and 0.6 into the other text box on the right (see Figure 15).

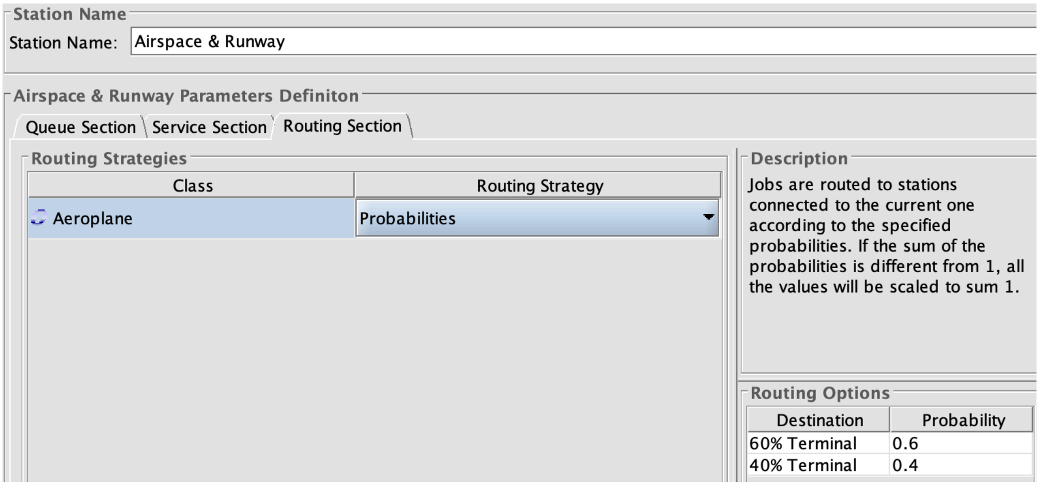


Figure 15: Configuration of the "Airspace & Runway" queue's routing for the network.

Both of the passenger terminal queues can be configured just as the "Airspace & Runway" queue in the M/M/1 system.

Instead of configuring the performance indices as stated for other models in section 2.1.2, select response time per sink two times, specifying the 40% stream's sink in one index and the 60% stream's sink in the other index. Ensure "Stat. Res." is selected to log to a CSV file (see Figure 16).

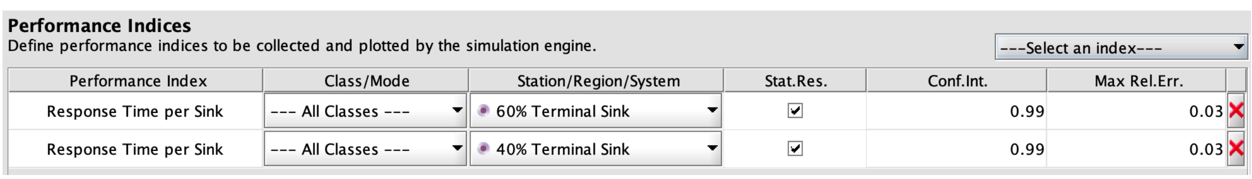


Figure 16: Performance metrics for later analysis of the response times of individual streams in the queue network.

After finishing all other general configurations, the simulation is ready to be run by selecting the green arrow in the upper menu.

3. Analysis

In section 2, the methodology by which statistics for each of the models discussed in this report were gathered is discussed; this section presents the collected statistics and makes brief observations on them, noting elements that appear particularly interesting.

3.1 Random Number Generator

To test whether the JMT RNG is a satisfactory approximation of a uniform distribution, U(0, 1), the data collected via CSV was used in a Chi-square test and Kolmogorov-Smirnov test. The parameters and results of the Chi-square test are shown below (see Figure 17).

$$n = 100$$

$$k = 15$$

$$v = k - s - 1 = 14$$

$$\chi_0^2 = 17.0693 \qquad \chi_{0.05,14}^2 = 23.7$$

$$p\ value = 0.2525$$

Figure 17: Parameters and results of the Chi-square test for JMT RNG uniformity.

The Python script used in this test can be found in Appendix A. By the above score, the null hypothesis for the random number generator is not rejected due to the $\chi_0^2 < \chi_{0.05,14}^2$, therefore it cannot be deemed insufficient for this project by the Chi-square test. The parameters and results of the Kolmogorov-Smirnov test are displayed below (see Figure 18).

$$n = 50$$

$$D = 0.1511 \qquad D_{crit} = 0.1885$$

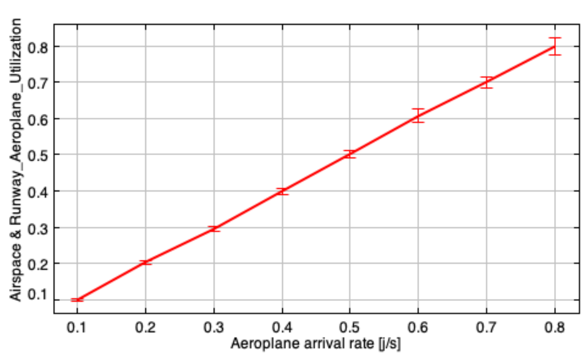
$$p\ value = 0.1839 \qquad \alpha = 0.05$$

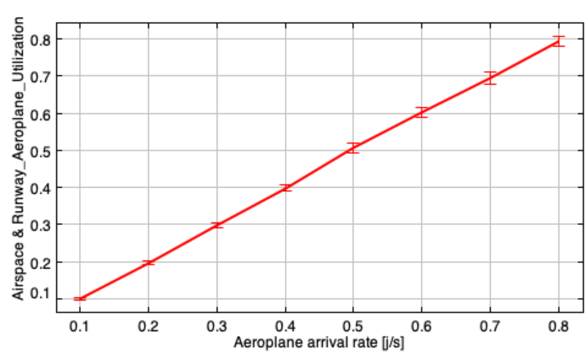
Figure 18: Parameters and results of the Kolmogorov-Smirnov test for JMT RNG uniformity.

Code used to perform this test can be found in Appendix A. The null hypothesis is not rejected by the Kolmogorov-Smirnov test, judging by the results above, because $D < D_{crit}$. The random number generator cannot be deemed insufficient for this project by the Kolmogorov-Smirnov test.

3.2 Single Runway

A graph of the average server utilization, ρ, for the M/M/1 airspace and runway is show below on the left, while the same for the M/G/1 airspace and runway is shown to the right (see Figures 19 and 20).





Figures 19 & 20: Average server utilization, ρ, over eight trials for the M/M/1 system (left) and M/G/1 system (right).

Both the M/M/1 queueing system and the M/G/1 queueing system maintained the expected linear trend in the relationship between arrival rate and server utilization. The corresponding values to the M/M/1 system's graph on the left are shown below (see Figure 21).

Simulation Results

*	0.100 j/s	0.200 j/s	0.300 j/s	0.400 j/s	0.500 j/s	0.600 j/s	0.700 j/s	0.800 j/s
Mean Value	0.0992	0.2028	0.2962	0.4001	0.5015	0.6075	0.6992	0.7992
Max (Conf.Int.)	0.1015	0.2078	0.3033	0.4085	0.5132	0.6257	0.7159	0.8217
Min (Conf.Int.)	0.0969	0.1978	0.2891	0.3917	0.4899	0.5893	0.6825	0.7767

Figure 21: Values corresponding to the above graph for average server utilization in the M/M/1 system over eight trials.

The theoretical server utilizations for the M/M/1 system of 0.1, 0.2, …, 0.8, fit within the corresponding confidence intervals of the actual values, therefore the simulation appears consistent with the theoretical model. The exact values corresponding to the M/G/1 system's graph for server utilization are displayed below (see Figure 22).

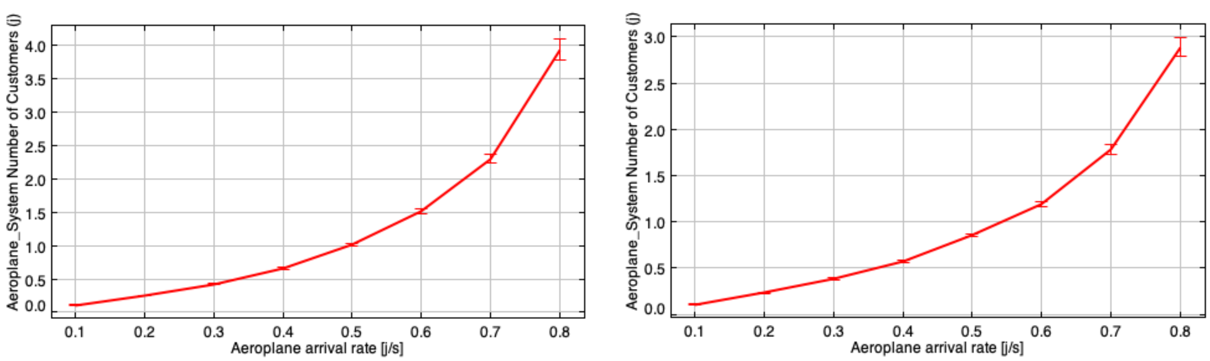
Simulation Results

*	0.100 j/s	0.200 j/s	0.300 j/s	0.400 j/s	0.500 j/s	0.600 j/s	0.700 j/s	0.800 j/s
Mean Value	0.1006	0.1972	0.2986	0.3988	0.5057	0.6011	0.6950	0.7933
Max (Conf.Int.)	0.1035	0.2013	0.3058	0.4067	0.5193	0.6139	0.7109	0.8059
Min (Conf.Int.)	0.0976	0.1932	0.2913	0.3909	0.4920	0.5882	0.6791	0.7807

Figure 22: Values corresponding to the above graph for average server utilization in the M/G/1 system over eight trials.

As in the case of the M/M/1 system, the theoretical server utilizations for the M/G/1 system of 0.1, 0.2, …, 0.8 fit within the corresponding confidence intervals of the actual values. The simulation is consistent with the theoretical calculations in terms of ρ.

The graph corresponding to the total number of airplanes in the M/M/1 system, L, is shown below on the left, while the same for the M/G/1 system is shown below on the right (see Figures 23 and 24).



Figures 23 & 24: Avg. number of airplanes in the system, L, over eight trials for the M/M/1 system and M/G/1 system.

As depicted, the average number of airplanes in the M/M/1 system and the M/G/1 system can be seen to grow exponentially over the eight trials. Exact values corresponding to the M/M/1 system's graph on the left are detailed below (see Figure 25).

Simulation Results								
*	0.100 j/s	0.200 j/s	0.300 j/s	0.400 j/s	0.500 j/s	0.600 j/s	0.700 j/s	0.800 j/s
Mean Value (j)	0.1095	0.2521	0.4204	0.6628	1.0142	1.5093	2.3014	3.9357
Max (j) (Conf.Int.)	0.1125	0.2565	0.4312	0.6755	1.0391	1.5485	2.3658	4.0956
Min (j) (Conf.Int.)	0.1064	0.2477	0.4096	0.6502	0.9893	1.4701	2.2369	3.7758

Figure 25: Values corresponding to the above graph for average airplanes in the M/M/1 system over eight trials.

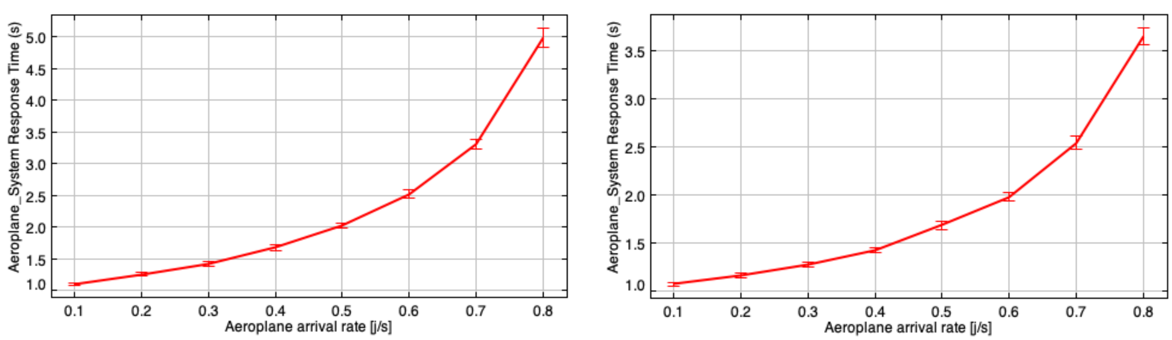
Theoretical *L* values for the arrival rates 0.1, 0.5, and 0.8 were calculated to be 0.1111, 1.0000, and 4.0000, respectively. Each of these fits within the confidence interval of its corresponding actual value, therefore the simulation appears to agree with the theory. The exact values corresponding to the M/G/1 system's graph are detailed below (see Figure 26).

Simulation Results								
*	0.100 j/s	0.200 j/s	0.300 j/s	0.400 j/s	0.500 j/s	0.600 j/s	0.700 j/s	0.800 j/s
Mean Value (j)	0.1070	0.2309	0.3816	0.5727	0.8525	1.1884	1.7812	2.8873
Max (j) (Conf.Int.)	0.1098	0.2372	0.3922	0.5882	0.8702	1.2189	1.8309	2.9884
Min (j) (Conf.Int.)	0.1042	0.2245	0.3710	0.5572	0.8348	1.1579	1.7315	2.7862

Figure 26: Values corresponding to the above graph for average airplanes in the M/G/1 system over eight trials.

Theoretical values for *L* at average arrival rates of 0.1, 0.5, and 0.8 are higher with respect to the measured values at those arrival rates at 0.1111, 1.0000, and 4.0000, respectively. The gap increases as the arrival rate increases and theoretical values fall outside of the confidence intervals shown above.

Graphs for the M/M/1 system and M/G/1 system's average time spent in the system, *w*, are found below, respectively (see Figures 27 and 28).



Figures 27 & 28: Average time spent in the system for the M/M/1 system (left) and M/G/1 system (right).

The results seen for *w* over the eight trials strongly coincide with those seen for *L* in each case. For both the M/M/1 and M/G/1 systems, exponential growth is evident. The exact values corresponding to the M/M/1 system's graph on the left can be seen below (see Figure 29).

Simulation Results								
*	0.100 j/s	0.200 j/s	0.300 j/s	0.400 j/s	0.500 j/s	0.600 j/s	0.700 j/s	0.800 j/s
Mean Value (s)	1.1049	1.2582	1.4201	1.6790	2.0243	2.5161	3.3064	4.9834
Max (s) (Conf.Int.)	1.1251	1.2857	1.4516	1.7215	2.0700	2.5815	3.3784	5.1313
Min (s) (Conf.Int.)	1.0847	1.2307	1.3885	1.6364	1.9786	2.4506	3.2344	4.8356

Figure 29: Values corresponding to the above graph for average time spent in the M/M/1 system per airplane.

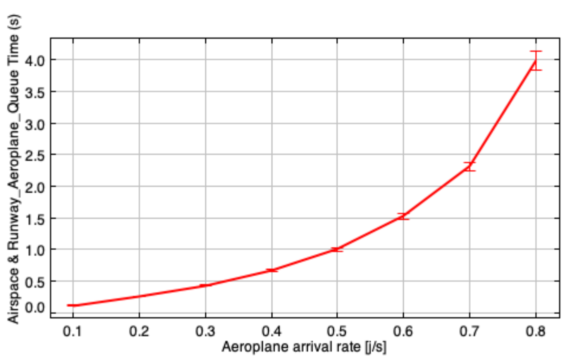
For the arrival rates of 0.1, 0.5, and 0.8, the respective theoretical w values are 1.1111, 2.0000, and 5.0000, each of which fall within the corresponding confidence intervals. The simulation continues to produce results seemingly consistent with the theory of M/M/1 queues. Values for the M/G/1 system's graph on the right can be found below (see Figure 30).

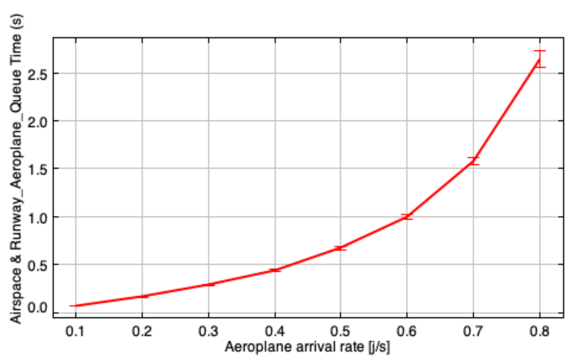
Simulation Results								
*	0.100 j/s	0.200 j/s	0.300 j/s	0.400 j/s	0.500 j/s	0.600 j/s	0.700 j/s	0.800 j/s
Mean Value (s)	1.0753	1.1634	1.2759	1.4264	1.6879	1.9813	2.5416	3.6399
Max (s) (Conf.Int.)	1.0927	1.1861	1.3007	1.4537	1.7327	2.0251	2.6142	3.7276
Min (s) (Conf.Int.)	1.0579	1.1407	1.2511	1.3992	1.6430	1.9375	2.4691	3.5521

Figure 30: Values corresponding to the above graph for average time spent in the M/G/1 system per airplane.

The theoretical w values at average arrival rates of 0.1, 0.5, and 0.8 are 1.1111, 2.0, and 5.0, respectively. As in the case of L, theoretical w values are higher than the measured values and the gap increases as the arrival rates increase.

The graphs corresponding to the M/M/1 and M/G/1 system's w_Q metric are shown below in the order mentioned (see Figures 31 and 32).





Figures 31 & 32: Average time spent waiting in the airspace for the M/M/1 system (left) and M/G/1 system (right).

Following the trend seen in the previous two metrics, both the M/M/1 system and the M/G/1 system experience exponential growth over the eight trials. Exact values for the M/M/1 system's graph on the left are shown below (see Figure 33).

Simulation Results								
*	0.100 j/s	0.200 j/s	0.300 j/s	0.400 j/s	0.500 j/s	0.600 j/s	0.700 j/s	0.800 j/s
Mean Value (s)	0.1109	0.2524	0.4286	0.6643	0.9981	1.5200	2.3071	3.9845
Max (s) (Conf.Int.)	0.1136	0.2577	0.4372	0.6826	1.0261	1.5626	2.3767	4.1304
Min (s) (Conf.Int.)	0.1083	0.2472	0.4199	0.6461	0.9701	1.4775	2.2374	3.8387

Figure 33: Values corresponding to the above graph for average time spent waiting in the airspace in the M/M/1 system.

In agreement with each of the other metrics, the theoretical values of 0.1111, 1.0000, and 4.0000 for arrival rates of 0.1, 0.5, and 0.8, respectively, fit within the corresponding confidence intervals above. Values for the M/G/1 system's graph on the right are displayed in the following figure (see Figure 34).

Simulation Results								
*	0.100 j/s	0.200 j/s	0.300 j/s	0.400 j/s	0.500 j/s	0.600 j/s	0.700 j/s	0.800 j/s
Mean Value (s)	0.0739	0.1660	0.2862	0.4415	0.6714	0.9939	1.5766	2.6406
Max (s) (Conf.Int.)	0.0755	0.1698	0.2920	0.4516	0.6888	1.0185	1.6170	2.7269
Min (s) (Conf.Int.)	0.0724	0.1621	0.2804	0.4315	0.6539	0.9693	1.5362	2.5543

Figure 34: Values corresponding to the above graph for average time spent in waiting in the airspace in the M/G/1 system.

Following the trends for L and w, w_Q theoretical values are higher than the measured counterparts at 0.1111, 1.0000, and 4.0000 for average arrival rates of 0.1, 0.5, and 0.8, respectively.

For the case of the M/M/1 system, a histogram depicting the system state for the case in which the theoretical server utilization, ρ, is 0.5 was gathered separately from the automated incremental analysis. This histogram is displayed below for the interest of the reader (see Figure 35).

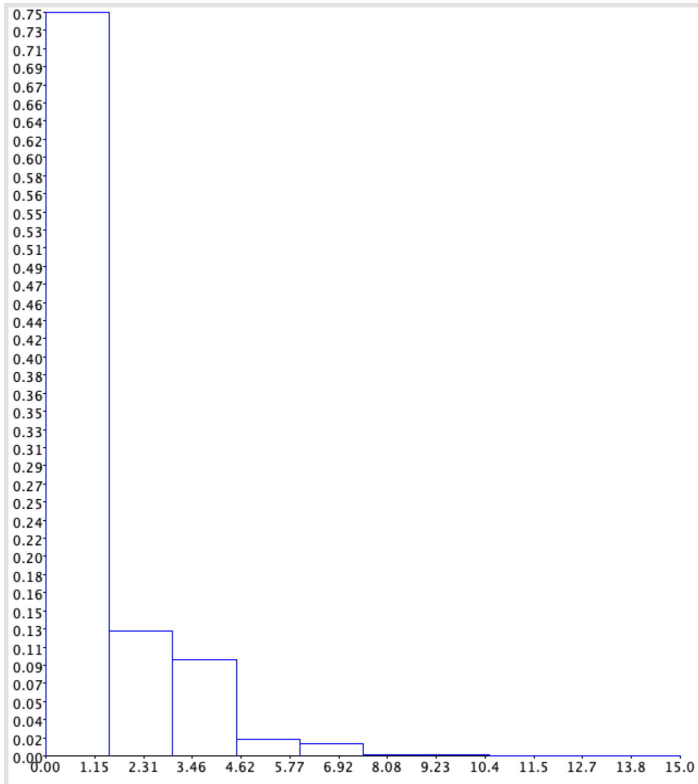


Figure 35: A histogram depicting the system state for the M/M/1 system when the theoretical server utilization is 0.5.

The histogram contains ten bins in the range zero to fifteen. The results shown, in which there are between zero and one airplane in the system 75% of the time, agree with those of the M/M/1 system's graph for average number of airplanes in the system, where there is approximately one airplane in the system for a server utilization of 0.5. The overall shape of the histogram appears exponential, which is expected.

3.2.1 Interarrival and Service Time Distribution Testing

The interarrival and service times in the CSV files generated by the M/M/1 system's loggers at a theoretical utilization of 0.5 were analyzed using a Chi-square test and a Kolmogorov-Smirnov test. Both the interarrival and service times were expected to fit the exponential distribution, as set in JMT. The results and parameters of the Chi-square test on the interarrival times are shown below (see Figure 36).

$$n = 100$$

$$k = 11$$

$$v = k - s - 1 = 9$$

$$\chi_0^2 = 11.7258 \qquad \chi_{0.05,9}^2 = 16.9$$

$$p\ value = 0.2292 \qquad \alpha = 0.05$$

Figure 36: Parameters and results of the Chi-square test for JMT interarrival exponential distribution goodness-of-fit.

The Python script that performed this test can be found in Appendix B. With the $\chi_0^2 < \chi_{0.05,9}^2$, the null hypothesis is not rejected and the fit of the interarrival times to the expected exponential distribution is satisfactory when $\alpha = 0.05$ using the Chi-square test. The results and parameters of the Kolmogorov-Smirnov test on the interarrival times are displayed below (see Figure 37).

$$n = 50$$

$$D = 0.08113 \qquad D_{crit} = 0.1885$$

$$p\ value = 0.8590 \qquad \alpha = 0.05$$

Figure 37: Parameters and results of the Kolmogorov-Smirnov test for JMT interarrival exponential distribution goodness-of-fit.

Python code used in this test can also be found in Appendix B. Seen above, $D < D_{crit}$, therefore the null hypothesis is not rejected and the fit of the service times to the expected exponential distribution is satisfactory by the Kolmogorov-Smirnov test.

The results and parameters of the Chi-square test on the service time distribution are shown below (see Figure 38).

$$n = 100$$

$$k = 11$$

$$v = k - s - 1 = 9$$

$$\chi_0^2 = 13.2193 \qquad \chi_{0.05,9}^2 = 16.9$$

$$p\ value = 0.1529 \qquad \alpha = 0.05$$

Figure 38: Parameters and results of the Chi-square test for JMT service time exponential distribution goodness-of-fit.

With $\chi_0^2 < \chi_{0.05,9}^2$, the null hypothesis is not rejected and the fit of the interarrival times to the expected exponential process is satisfactory when $\alpha = 0.05$, as in the case of the interarrival times for the Chi-square test. The null hypothesis is not rejected. The results and parameters of the Kolmogorov-Smirnov test on the service times are shown below (see Figure 39).

$$n = 50$$

$$D = 0.1182 \qquad D_{crit} = 0.1885$$

$$p\ value = 0.4584 \qquad \alpha = 0.05$$

Figure 39: Parameters and results of the Kolmogorov-Smirnov test for JMT service time exponential distribution goodness-of-fit.

As in the case of the interarrival times, $D < D_{crit}$, therefore the null hypothesis is not rejected and the fit of the service times to the expected exponential distribution is satisfactory by the Kolmogorov-Smirnov test.

3.2.2 Confidence Interval Testing

This section details confidence interval testing on each of the four collected metrics for the M/M/1 system at a theoretical utilization of 0.5 to practice accepting or rejecting a model based its outputs. To obtain the data for these tests, the model of the M/M/1 system was run five times repeatedly in JMT using the "What-if" analysis tool with an unchanging interarrival time. Parameters and results for each metric look as follows (see Figure 40).

	n	$\boldsymbol{\alpha}$	$\boldsymbol{t_{\frac{\alpha}{2},n-1}}$	$\boldsymbol{\varepsilon}$	$\boldsymbol{\overline{Y}}$	**S**	**Conf. Int.**	**Best-Case Error**	**Worst-Case Error**
ρ	5	0.05	2.78	0.01	0.50034	0.00118870	[0.498688, 0.501992]	0.0013123	0.001992
L	5	0.05	2.78	0.1	1.00610	0.00431509	[1.000102, 1.012098]	0.000102	0.012098
w	5	0.05	2.78	0.02	2.00358	0.00420083	[1.997741, 2.009419]	0.002259	0.009419
w_Q	5	0.05	2.78	0.02	1.00544	0.00560562	[0.997648, 1.013232]	0.002352	0.013232

Figure 40: A table containing parameters and results for confidence interval testing in an M/M/1 system.

Due to there being no real-world data to use for the actual μ_0 values, the theoretical expected values calculated for the metrics in an M/M/1 system are used. For the arrival rate of 0.5, the theoretical values are 0.5, 1.0, 2.0, and 1.0 for metrics ρ, L, w, and w_Q, respectively. In the cases of ρ, w, and w_Q, the theoretical values are inside the above confidence intervals and the worst-case error is less than the corresponding chosen "close enough" value, ε. Therefore, the null hypothesis (the model's mean in each case equals the true mean) and the model are accepted for an arrival rate of 0.5. In the case of L, the theoretical value is not inside the above confidence interval, but the worst-case error is less than ε so the null hypothesis and model are accepted for an arrival rate of 0.5.

3.3 Triple Runway with Limited Airspace Capacity

As in the case of the single runway M/M/1 and M/G/1 system, the triple runway with limited system capacity was simulated over eight theoretical server utilizations. This analyzed system is analogous to an M/M/3/20 queueing system.

The graph corresponding to this M/M/3/20 system's actual server utilizations can be seen below (see Figure 41).

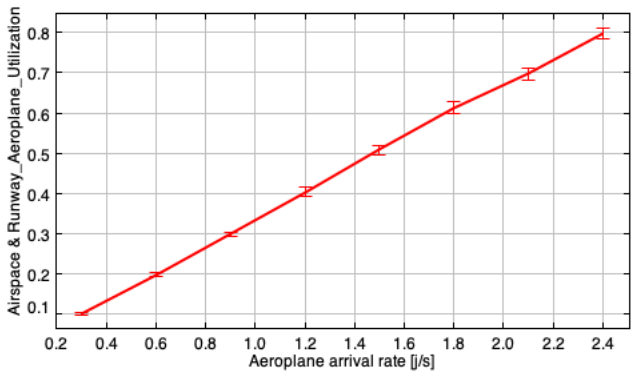


Figure 41: Graph displaying the average server utilization in the M/M/3/20 queueing system over eight trials.

The graph shows the expected linear relationship between arrival rate and server utilization. The actual values for each of the simulations corresponding the above graph are shown below (see Figure 42).

Simulation Results								
*	0.300 j/s	0.600 j/s	0.900 j/s	1.200 j/s	1.500 j/s	1.800 j/s	2.100 j/s	2.400 j/s
Mean Value	0.0998	0.1972	0.2981	0.4035	0.5079	0.6121	0.6970	0.7972
Max (Conf.Int.)	0.1025	0.2015	0.3040	0.4151	0.5194	0.6271	0.7125	0.8106
Min (Conf.Int.)	0.0972	0.1930	0.2922	0.3920	0.4965	0.5972	0.6816	0.7837

Figure 42: Values corresponding to the above graph for server utilization.

Each of the theoretical values for ρ – 0.1, 0.2, …, 0.8 – fit within the corresponding confidence intervals for the actual values, therefore the simulation appears to produce predictable results in this metric.

The graph corresponding with the average number of airplanes in the system, L, in each of the eight simulations is presented in the following figure (see Figure 43).

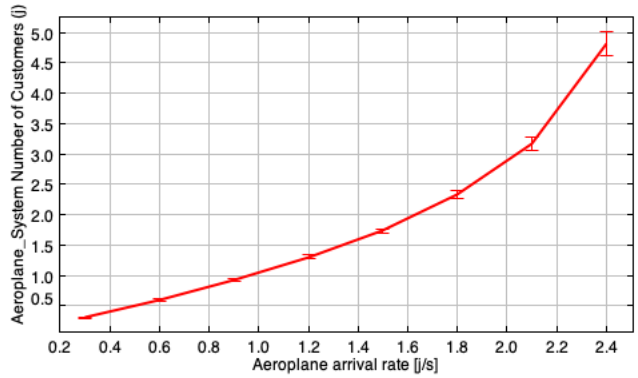


Figure 43: Graph displaying the average airplanes in the M/M/3/20 system throughout the eight trials.

The exact values corresponding to the above graph are displayed below (see Figure 44).

Simulation Results

*	0.300 j/s	0.600 j/s	0.900 j/s	1.200 j/s	1.500 j/s	1.800 j/s	2.100 j/s	2.400 j/s
Mean Value (j)	0.3000	0.5969	0.9206	1.2990	1.7306	2.3365	3.1673	4.8101
Max (j) (Conf.Int.)	0.3087	0.6145	0.9476	1.3325	1.7602	2.4039	3.2775	5.0104
Min (j) (Conf.Int.)	0.2913	0.5792	0.8935	1.2655	1.7010	2.2690	3.0572	4.6098

Figure 44: Values corresponding to the above graph for average airplanes in the system.

The theoretical L values at arrival rates of 0.3, 1.5, and 2.4 are 0.3004, 1.7369, and 4.7509, respectively, which all fall into the corresponding confidence intervals. The model appears to perform predictably in this metric, as well.

Results for the w_Q metric, the average time spent waiting in the airspace per airplane, are shown in the following figure (see Figure 45).

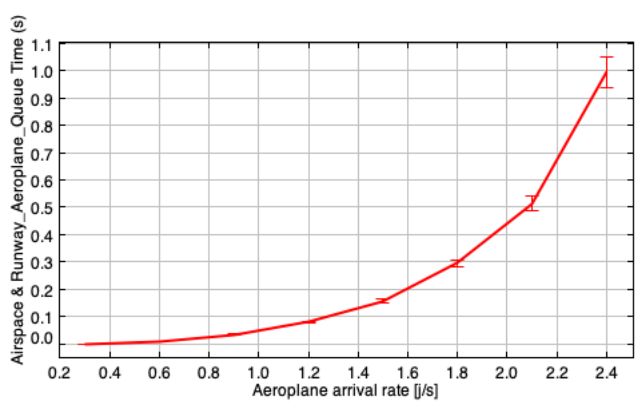


Figure 45: Graph displaying the average time spent in the airspace (queue) of the M/M/3/20 system over eight trials.

Exact values corresponding to the above graph are displayed below (see Figure 46).

Simulation Results								
*	0.300 j/s	0.600 j/s	0.900 j/s	1.200 j/s	1.500 j/s	1.800 j/s	2.100 j/s	2.400 j/s
Mean Value (s)	6.33E-4	9.90E-3	0.0345	0.0799	0.1565	0.2947	0.5127	0.9939
Max (s) (Conf.Int.)	6.53E-4	0.0106	0.0364	0.0830	0.1632	0.3081	0.5399	1.0488
Min (s) (Conf.Int.)	0.0	9.17E-3	0.0327	0.0768	0.1498	0.2814	0.4856	0.9389

Figure 46: Values corresponding to the above graph for average time spent in the airspace per airplane.

Theoretical w_Q values for arrival rates of 0.3, 1.5, and 2.4 were calculated to be 0.0014, 0.1579, and 0.9854. In the same fashion as the previous metrics, each of the theoretical values fit within the confidence intervals of the respective measured values and the model behaves predictably.

The graph of the w metric in each of the eight trials, which indicates the average total time spent in the system for an airplane, is displayed below (see Figure 47).

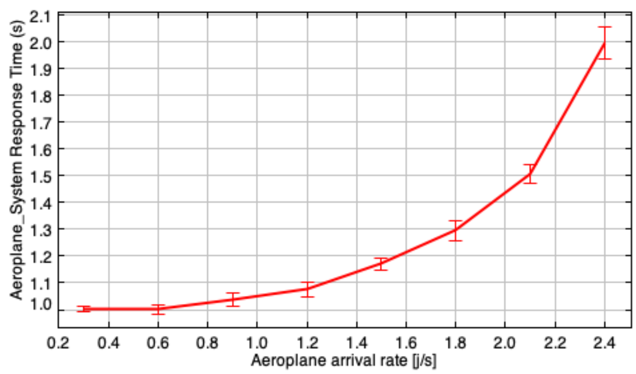


Figure 47: Graph displaying the average total time spent in the M/M/3/20 system per airplane over eight trials.

Values corresponding to the above graph are shown below (see Figure 48).

Simulation Results								
*	0.300 j/s	0.600 j/s	0.900 j/s	1.200 j/s	1.500 j/s	1.800 j/s	2.100 j/s	2.400 j/s
Mean Value (s)	1.0030	1.0008	1.0381	1.0757	1.1698	1.2967	1.5087	1.9947
Max (s) (Conf.Int.)	1.0150	1.0179	1.0626	1.1019	1.1940	1.3341	1.5444	2.0551
Min (s) (Conf.Int.)	0.9911	0.9837	1.0137	1.0495	1.1457	1.2593	1.4731	1.9343

Figure 48: Values corresponding to the above graph for average total time spent in the system per airplane.

With theoretical values for w at arrival rates of 0.3, 1.5, and 2.4 being 1.001, 1.1579, and 1.9854, respectively, the model appears to continue behaving as expected in terms of the total time spent in the system per airplane.

Lastly, the drop rate for each of the eight trials can be seen in the following figure (see Figure 49).

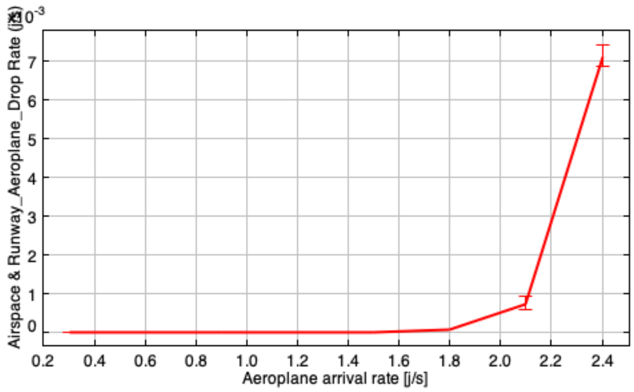


Figure 49: Graph for average drop rate in the M/M/3/20 system over eight trials.

The exact values corresponding to the above graph are shown below (see Figure 50).

Simulation Results

*	0.300 j/s	0.600 j/s	0.900 j/s	1.200 j/s	1.500 j/s	1.800 j/s	2.100 j/s	2.400 j/s
Mean Value (j/s)	0.0	0.0	0.0	0.0	0.0	6.90E-5	7.19E-4	7.11E-3
Max (j/s) (Conf.Int.)	0.0	0.0	0.0	0.0	0.0	-	9.40E-4	7.40E-3
Min (j/s) (Conf.Int.)	0.0	0.0	0.0	0.0	0.0	-	5.83E-4	6.85E-3

Figure 50: Values corresponding to the above graph for average drop rate of the M/M/3/20 system.

At average arrival rates of 0.3, 1.5, and 2.4, the theoretical drop rates are 0.0000, 0.0000, and 0.0071, respectively. Each of these align with the measured counterparts in the figure above and fall within the corresponding confidence interval, therefore the model appears to reflect the theoretical equations with respect to drop rate.

3.4 Single Runway, Dual Terminal Network

In this section, the model of three M/M/1 queues in a network is analyzed. This model is analogous to an airport with a single runway to serve incoming airplanes with two terminals for the offloading of passengers after landing. One of the terminals is favoured with 60% of incoming airplanes being directed to it, while the other terminal has 40% of airplanes directed to it. The total response time for each of the two streams is reviewed and compared against theoretical values.

A graph of the average total response time over eight trials for the stream using the terminal with 60% favourability is shown below (see Figure 51).

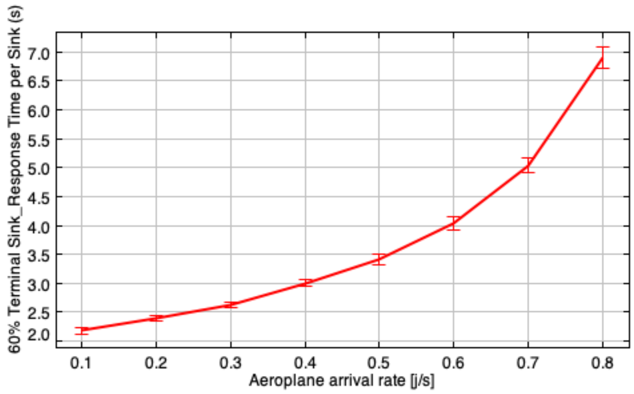


Figure 51: Graph of the average response time, w, for the stream with a 60% favourability queue in the network over eight trials.

The exact values corresponding to the above graph are displayed below (see Figure 52).

Simulation Results								
*	0.100 j/s	0.200 j/s	0.300 j/s	0.400 j/s	0.500 j/s	0.600 j/s	0.700 j/s	0.800 j/s
Mean Value (s)	2.1824	2.3904	2.6288	2.9996	3.4097	4.0390	5.0382	6.9005
Max (s) (Conf.Int.)	2.2416	2.4313	2.6727	3.0585	3.5086	4.1558	5.1599	7.0788
Min (s) (Conf.Int.)	2.1232	2.3495	2.5850	2.9407	3.3109	3.9222	4.9166	6.7223

Figure 52: Values corresponding to the above graph for average response time over eight trials.

Theoretical values for this stream's *w* metric corresponding to the arrival rates of 0.1, 0.5, and 0.8 were calculated as 2.1749, 3.4286, and 6.9231, respectively. Each theoretical value falls within the confidence interval for the related actual values, therefore this stream within the network appears to behave as predicted.

A graph of the average total response time over eight trials for the stream using the terminal with 60% favourability is shown below (see Figure 53).

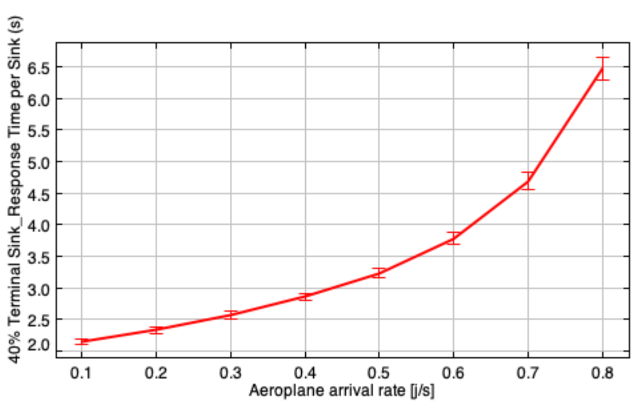


Figure 53: Graph of the average response time, w, for the stream with a 40% favourability queue in the network over eight trials.

The exact values corresponding to the above graph are displayed below (see Figure 54).

Simulation Results								
*	0.100 j/s	0.200 j/s	0.300 j/s	0.400 j/s	0.500 j/s	0.600 j/s	0.700 j/s	0.800 j/s
Mean Value (s)	2.1481	2.3316	2.5686	2.8545	3.2289	3.7769	4.6880	6.4677
Max (s) (Conf.Int.)	2.1879	2.3807	2.6220	2.9137	3.3080	3.8693	4.8256	6.6423
Min (s) (Conf.Int.)	2.1083	2.2824	2.5152	2.7953	3.1498	3.6846	4.5504	6.2931

Figure 54: Values corresponding to the above graph for average response time over eight trials.

The calculated theoretical values for this stream at arrival rates of 0.1, 0.5, and 0.8 were 2.1528, 3.2500, and 6.4706, respectively. With each of these values falling within the corresponding confidence interval above, this stream within the network also appears to behave as predicted by the theoretical model.

4. Conclusion

The results provided in the queueing system metrics for each of the M/M/1, M/M/3/20, and queue network models aligned with theoretical predicted values, thus supporting their validity. The one area where this did not occur was with the M/G/1 system, where theoretical values consistently fell outside the corresponding confidence intervals of the actual results. This may be because the M/G/1 theoretical equations provide estimates for arbitrary service time distributions and are thus less reliable than equations describing systems with exponentially distributed service times. The comparisons of actual values with theoretical values can be found in sections 3.2 to 3.4.

The null hypothesis for the uniformity of the random number generator in Java Modelling Tools could not be rejected by both of the tests run against it, providing confidence in its performance. The same result held true for JMT's exponential distribution in terms of interarrival times and service times, where their respective Chi-square tests and Kolmogorov-Smirnov tests passed. These tests can be found in sections 3.1 and 3.2.1.

Overall, this project was successful at providing experience in designing models that apply to real-world scenarios, running simulations on those models, and performing analysis with the results of simulations. In addition to this, it allowed for the testing of a random number generator for uniformity, as well as the testing of an exponential distribution for goodness-of-fit, both of which utilized the Chi-square and Kolmogorov-Smirnov tests. The knowledge and skills acquired through the project will be useful in future academia and employment.

5. References

[1] B. Marco, “Java Modelling Tools - JMT,” *Java Modelling Tools - Introduction*. [Online]. Available: http://jmt.sourceforge.net/. [Accessed: 06-Apr-2020].

[2] “NumPy,” *NumPy*. [Online]. Available: https://numpy.org/. [Accessed: 06-Apr-2020].

[3] “Pandas,” *Pandas*. [Online]. Available: https://pandas.pydata.org/. [Accessed: 06-Apr-2020].

[4] “SciPy,” *SciPy.org*. [Online]. Available: https://www.scipy.org/. [Accessed: 06-Apr-2020].

Appendix A – Random No. Generator Test Script

Open-source Python libraries used in this script include NumPy, Pandas, and SciPy [2, 3, 4].

```
import numpy as np
import pandas as pd
from random import sample
from scipy import stats
import sys

if len(sys.argv) < 2:
    print("Path to CSV file must be provided")
    sys.exit()

df = pd.read_csv(sys.argv[1])
pop = list(df["INTERARRIVAL_SAMECLASS"])

ks_test_sample_count = 50
cs_test_sample_count = 101

D = stats.kstest(sample(pop, ks_test_sample_count), 'uniform', alternative='two-
sided')

num_bins = 15
intervalSize = 1.0 / num_bins
bins = np.zeros(num_bins)
for row in sample(pop, cs_test_sample_count):
    idx = int(row / intervalSize)
    bins[idx] += 1
cs = stats.chisquare(bins, f_exp=None, ddof=0)

print("Kolmogorov-Smirnov sample statistic for " + str(ks_test_sample_count) + "
samples: " + str(D))
print("Chi-square score for " + str(cs_test_sample_count) + " samples: " + str(cs))
```

Appendix B – Expon. Distribution Test Script

Open-source Python libraries used in this script include NumPy, Pandas, and SciPy [2, 3, 4].

```
import numpy as np
import pandas as pd
from random import sample
from scipy import stats
import sys


def expon_cdf(x, lambda_exp):
    return 1 - np.exp(-1 * lambda_exp * x)


def indices_to_combine(f_exp):
    print("This is the array of expected frequencies with their indices:\n" +
str(list(zip(range(0, len(f_exp)), f_exp))))
    raw = input("What indices should be kept the same or be combined? With commas
separating "
                "values and colons separating lists (i.e. 0,1:2:3,4,5,6:7,8:9).\n")
    indices_array = raw.split(":")
    return np.array([np.array(indices.split(",")).astype(np.int) for indices in
indices_array])


def combine(arr, combine_indices):
    new_arr = []
    for indices in combine_indices:
        sum = 0.
        for index in indices:
            sum += arr[index]
        new_arr.append(sum)
    return np.array(new_arr)


if len(sys.argv) < 2:
    print("Path to CSV file must be provided")
    sys.exit()

df = pd.read_csv(sys.argv[1])
pop = list(df["INTERARRIVAL_SAMECLASS"])

ks_test_sample_count = 50
cs_test_sample_count = 100

D = stats.kstest(sample(pop, ks_test_sample_count), stats.expon.cdf, args=(0, 2),
alternative='two-sided')

num_bins = 15
max_bin_end = 5.0
lambda_exp = 0.5
interval_size = max_bin_end / num_bins
interval_boundaries = np.arange(0., max_bin_end, interval_size)
interval_boundaries = np.append(interval_boundaries, np.inf)
interval_probabilities = np.array([expon_cdf(x2, lambda_exp) - expon_cdf(x1,
lambda_exp)
                                   for x1, x2 in zip(interval_boundaries[:-1],
interval_boundaries[1:])])
f_exp = interval_probabilities * cs_test_sample_count
```

```
combine_indices = indices_to_combine(f_exp)
f_exp_adjusted = combine(f_exp, combine_indices)
print("Original expected frequencies: ", f_exp)
print("Adjusted expected frequencies: ", f_exp_adjusted)
bins = np.zeros(num_bins)
for row in sample(pop, cs_test_sample_count):
    idx = int(row / interval_size)
    if idx >= num_bins:  # anything past the maximum bin gets binned in the maximum
bin
        idx = num_bins - 1
    bins[idx] += 1
bins_adjusted = combine(bins, combine_indices)
print("Original bins: ", bins)
print("Adjusted bins: ", bins_adjusted)
cs = stats.chisquare(bins_adjusted, f_exp=f_exp_adjusted, ddof=1)

print("Kolmogorov-Smirnov sample statistic for " + str(ks_test_sample_count) + "
samples: " + str(D))
print("Chi-square score for " + str(cs_test_sample_count) + " samples: " + str(cs))
```