Trigonometriske formler

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Eulers formler

For alle $x \in \mathbb{R}$ gælder

$$e^{ix} = \cos x + i \sin x \qquad \text{og} \qquad e^{-ix} = \cos x - i \sin x.$$

Omvendt kan både cosinus og sinus udtrykkes ved hjælp af den komplekse eksponentialfunktion:

$$\cos x = \frac{1}{2}(e^{ix} + e^{-ix}) \qquad \text{og} \qquad \sin x = \frac{1}{2i}(e^{ix} - e^{-ix}).$$

Grundrelation

$$\cos^2 x + \sin^2 x = 1$$

Det medfører at, for alle $x \in \mathbb{R}$:

$$|e^{ix}| = 1.$$

Symmetrier

$$\cos(-x) = \cos x \qquad \text{og} \qquad \sin(-x) = -\sin x.$$

Forskydning

$$\begin{aligned} \cos(x + 2\pi) &= \cos x, & \sin(x + 2\pi) &= \sin x, \\ \cos(x + \pi) &= -\cos x, & \sin(x + \pi) &= -\sin x, \\ \cos\left(x + \frac{\pi}{2}\right) &= -\sin x, & \sin\left(x + \frac{\pi}{2}\right) &= \cos x. \end{aligned}$$

Specielle værdier

For $n \in \mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ gælder

$$\cos(n\pi) = (-1)^n, \qquad \sin(n\pi) = 0.$$

Additionstheoremer

$$\begin{aligned} \cos(x + y) &= \cos x \cos y - \sin x \sin y, \\ \sin(x + y) &= \sin x \cos y + \cos x \sin y. \end{aligned}$$

Produktformler

$$
\begin{aligned}
\cos x \cos y &= \frac{1}{2}(\cos(x-y) + \cos(x+y)),\\
\sin x \sin y &= \frac{1}{2}(\cos(x-y) - \cos(x+y)),\\
\sin x \cos y &= \frac{1}{2}(\sin(x+y) + \sin(x-y)).
\end{aligned}
$$

Fordobblingsformler

$$
\begin{aligned}
&\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x\\
&\sin 2x = 2\sin x \cos x.
\end{aligned}
$$

Kvadratformler

$$
\begin{aligned}
\cos^2 x &= \frac{1}{2}(1 + \cos 2x)\\
\sin^2 x &= \frac{1}{2}(1 - \cos 2x)
\end{aligned}
$$

Afledte

$$(\cos x)' = -\sin x \qquad \text{og} \qquad (\sin x)' = \cos x$$

Integraler

$$\int \cos x \, dx = \sin x + C \qquad \text{og} \qquad \int \sin x \, dx = -\cos x + C$$

Særlige værdier

$$
\begin{aligned}
&\cos(0) = 1, \qquad \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}, \qquad \cos\left(\frac{\pi}{2}\right) = 0, \qquad \cos(\pi) = -1,\\
&\sin(0) = 0, \qquad \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}, \qquad \sin\left(\frac{\pi}{2}\right) = 1, \qquad \sin(\pi) = 0.
\end{aligned}
$$