Trigonometric Formulas

Euler's Formulas

For all $x \in \mathbb{R}$:

$$e^{ix} = \cos x + i \sin x, \quad e^{-ix} = \cos x - i \sin x.$$

Conversely:

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}, \quad \sin x = \frac{e^{ix} - e^{-ix}}{2i}.$$

Fundamental Relation

$$\cos^2 x + \sin^2 x = 1, \quad |e^{ix}| = 1.$$

Symmetries

$$\cos(-x) = \cos x, \quad \sin(-x) = -\sin x.$$

Shifts

$$\cos(x + 2\pi) = \cos x, \quad \cos(x + \pi) = -\cos x,$$

$$\sin(x + \pi) = -\sin x, \quad \sin(x + 2\pi) = \sin x,$$

$$\cos\left(x + \tfrac{\pi}{2}\right) = -\sin x, \quad \sin\left(x + \tfrac{\pi}{2}\right) = \cos x.$$

Special Values

For $n \in \mathbb{Z}$:

$$\cos(n\pi) = (-1)^n, \quad \sin(n\pi) = 0.$$

Addition Theorems

$$\cos(x + y) = \cos x \cos y - \sin x \sin y,$$

$$\sin(x + y) = \sin x \cos y + \cos x \sin y.$$

Product Formulas

$$\cos x \cos y = \tfrac{1}{2}\big(\cos(x-y) + \cos(x+y)\big),$$

$$\sin x \sin y = \tfrac{1}{2}\big(\cos(x-y) - \cos(x+y)\big),$$

$$\sin x \cos y = \tfrac{1}{2}\big(\sin(x+y) + \sin(x-y)\big).$$

Double-Angle Formulas

$$\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x,$$

$$\sin 2x = 2\sin x \cos x.$$

Square Formulas

$$\cos^2 x = \tfrac{1}{2}(1 + \cos 2x), \quad \sin^2 x = \tfrac{1}{2}(1 - \cos 2x).$$

Derivatives

$$\frac{d}{dx}(\cos x) = -\sin x, \quad \frac{d}{dx}(\sin x) = \cos x.$$

Integrals

$$\int \cos x \, dx = \sin x + C, \quad \int \sin x \, dx = -\cos x + C.$$

Special Values

$$\cos(0) = 1, \quad \sin(0) = 0,$$

$$\cos\big(\tfrac{\pi}{4}\big) = \tfrac{\sqrt{2}}{2}, \quad \sin\big(\tfrac{\pi}{4}\big) = \tfrac{\sqrt{2}}{2},$$

$$\cos\big(\tfrac{\pi}{2}\big) = 0, \quad \sin\big(\tfrac{\pi}{2}\big) = 1,$$

$$\cos(\pi) = -1, \quad \sin(\pi) = 0.$$