Preliminary Assignment

CCMVI2085U Machine Learning for Predictive Analytics in Business

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I will review the business mathematics in lecture 02 and please take some time to complete this preliminary assignment before that. **It should be noted that this preliminary assignment is NOT included in your course assessment. It is mainly for you to recall the mathematical foundations as well as self-evaluate so that I can have a better understanding of your mathematical background for the course.** If you have any questions or need further advice, please get in touch with me at bc.acc@cbs.dk.

1. Roll a die and then flip that number of fair coins. What is the probability of "We get exactly 3 heads"?
2. If a random variable X follows a Poisson distribution $X \sim \text{Poi}(\lambda)$, please prove the following equations:
   (a) $\sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!} = 1$;
   (b) $\mathbb{E}(X) = \lambda$;
   (c) $\text{Var}(X) = \lambda$.
3. If a random variable X follows a Binomial distribution $X \sim \text{Bin}(n, p)$. Let $\lambda = np$. Suppose that n is very large and p is very small, please prove that $X \sim \text{Poi}(\lambda)$.
4. If a random variable X follows an exponential distribution, $X \sim \text{Exp}(\lambda)$, please prove the following equations::
   (a) $\int_0^{\infty} \lambda e^{-\lambda x} dx = 1$;
   (b) $\mathbb{E}(X) = \frac{1}{\lambda}$;
   (c) $\text{Var}(X) = \frac{1}{\lambda^2}$.
5. Let $x_1, \cdots, x_n$ be a set of independently and identically distributed (i.i.d). random variables and each variable has a population mean μ and a finite variance σ^2. Please prove the following limiting behaviour:

$$\lim_{n\to\infty} \sqrt{n}\left(\frac{\sum_{i=1}^{n} x_i}{n} - \mu\right) \sim N(0, \sigma^2).$$

6. Prove that the sample variance $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{X})^2$ is an unbiased estimator, i.e., $\mathbb{E}(s^2) = \sigma^2$.
7. From past experience, a professor knows that the test score of a student taking his final examination is a continuous random variable with mean 75.
   (a) Give an upper bound for the probability that a student's score will be at least as large as 85.
   (b) Suppose, in addition, that the professor knows that the variance of a student's test score is equal to 25. Give a lower bound to the probability that a student will score between 65 and 85?
8. Coin number one comes up heads with probability 0.6 and coin number two with probability 0.5. A coin is continually flipped until it comes up tails, at which time that coin is put aside and we start flipping the other one.
   (a) What proportion of flips use coin 1?
   (b) If we start the process with coin 1, i.e., flip 1 uses coin 1, what is the probability that coin 2 is used on the fifth flip?