Collateralized Loan Obligations

A Structural Model for Pricing Tranche Spreads in a Corporate Loan Securitization

A thesis presented for the degree of
MSc in Advanced Economics and Finance

Authors

Kristian Rumph Frederiksen (133708)

Rasmus Falk Hoffmann (133683)

Supervisor

David Lando

Total Page Count: *117*
Character Count: *188,847*

Department of Finance
Copenhagen Business School

17 May 2021

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Kristian Rumph Frederiksen

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Abstract

In this paper, we analyze the source and magnitude of mispricing, if Collateralized Loan Obligation (CLO) tranches are rated as if the collateral pool has no prepayment risk, while the CLO investors price the tranches correctly, anticipating portfolio changes from collateral prepayments. The paper provides an example of an European CLO structure and its lifecycle. By adapting existing structural credit risk models, we create a framework for pricing CLO tranche spreads. Specifically, we model two cases: a base case without prepayments and a case with prepayment and make the assumption that rating agencies do not incorporate prepayment risk in their rating methodology. Using this assumption, we find a ”mispricing” in line with the excess spreads between CLO tranches and equivalently rated corporate bonds observed in empirical data. When making parameter assumptions lowering the prepayment rate, e.g., loan rating changes or increasing refinancing costs, we find that the mispricing decreases and vice versa.

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Part I

Introduction

1 Introduction

A Collateralized Loan Obligation (CLO) is a securitization of loans, bundled together, and held by a separate legal entity referred to as a Special Purpose Vehicle (SPV). This entity invests and manages a diversified pool of mainly senior secured corporate loans, financed by a number of note classes, i.e. tranches, with different seniority and claims on the assets. These tranches are typically rated by independent rating agencies, using the conventional credit ratings from AAA (Aaa) to B, which are given on basis of the underlying pool of collateral loans. CLOs were introduced in the late 1980s, and experienced much interest from investors in the years leading up to the Financial Crisis around 2007-08, alongside other asset-backed securities (ABS) such as Mortgage-Backed Securities (MBS) and other types of Collateralized Debt Obligation (CDO). During the Financial Crisis all types of asset-backed securities, including CLOs, experienced difficulties. As a result, the amount of new CLOs issues almost vanished and the amount of new issues went from USD 140bn in 2007 to close to zero around 2009-2010. The CLO market has since regained its popularity and the global amount of outstanding CLO obligations have doubled to USD 800bn in 2020 (IMF, 2020).

The high demand from institutional investors can be explained by the fact that institutional investors have ratings-based capital requirements and hereby *reach for yield*. This means that the investors seek the most high-yielding assets given their risk budgets. Arbitrage CLOs are structured to absorb the excess supply of risky debt that cannot be bought by banks or pension funds with capital requirements on their balance sheets. The structuring of the risky debt into new separate legal entities is financed by equity and multiple debt classes or tranches with different seniorities. These tranches come with better credit ratings compared to the loans in the collateral, and thereby mitigate the regulatory constraints imposed on the financial investors. Additionally, the CLO tranches come with higher yields compared to corporate bonds with the same credit rating and duration. The combination of lower capital requirement and higher yields entail high demand from institutional investors, like banks, insurers, and pension funds (Cordell et al., 2021).

Cordell et al. (2021) investigate the performance of CLOs and find that the difference between the Internal Rate of Return (IRR) on CLO tranches and similar floating-rate corporate bonds is around 65 bps per year for AAA and AA tranche, 90 to 105 bps per year for mezzanine (A and BBB tranches) and over 200 bps for the junior tranches (BB and B). Numerous academic papers have investigated the large risk premia on CLO tranches, relative to corporate bonds. Coval et al. (2009) examine how risk factors affect the CLO tranche spreads and find that since the default risk of senior tranches is concentrated in systematically adverse economic states, investors should demand larger risk premia for holding such structured claims than for holding comparable corporate bonds. Similarly, Elkamhi et al. (2020) investigate how systematic risk relates to the large credit spreads on CLO tranches and finds that the high spreads are a fair reflection of the systematic risk from the correlation of loan defaults. Other explanations of the higher spreads are illiquidity (Hendershott et al., 2020) and information asymmetry between investors and the collateral manager (Cordell et al., 2021). The findings from these papers can explain some of the increased risk in CLOs, but, in general, fail to explain the relatively high spreads on the most senior tranches.

A review of the rating agencies' approaches shows that most agencies do not make concrete assumptions about the risk of prepayment on the collateral assets (Moody's Investors Service, 2020). Leveraged loans are typically callable at any time and often without prepayment fees (S&P Global Market Intelligence, 2021b). Hence, there is a risk that the "good" companies repay their debt prematurely, while the "bad" companies remain in the collateral pool. This impacts the performance of the CLO as the newly purchased loans may provide lower yields for the same amount of risk relative to the redeemed loans. In this thesis, we will focus on explaining the higher spreads on the CLO tranches by examining the prepayment risk on the collateral assets.

1.1 Research Question

The motivation for our thesis is to investigate how collateral prepayment risk affects the spreads on the CLO tranches and whether this risk can explain the relatively high spreads on the most senior tranches in a corporate loan securitization. Specifically, our goal is to develop a structural model for pricing CLO tranche spreads by taking into account the possibility that the the collateral assets are repaid prematurely. The question guiding this research is:

> *How can CLO tranche spreads be priced and how are they affected by collateral prepayments?*

To answer this research question, it is necessary to understand the structure and characteristics of a Corporate Loan Obligation and then analyze the most important aspects of pricing its liabilities. To fully answer the research question, the thesis will therefore set the following objectives:

- Describe the structure and lifecycle of a CLO
- Investigate the developments in the CLO market
- Develop a structural model to analyze and price spreads on the CLO tranches, including prepayment risk
- Analyze and discuss how the tranche spreads are affected by collateral prepayments, using the structural model developed.

1.2 Structure of the Thesis

The thesis is divided into four parts and is structured as follows.

Part I (Introduction) introduces the motivation and research question of this thesis in chapter 1. In chapter 2, we examine the existing literature on structured finance and CLO performance.

Part II (On Collateralized Loan Obligations) gives an introduction to the concept of Collateralized Loan Obligation (CLO) and an analysis of the European CLO market. Chapter 3 provides an introduction to the structure and characteristics of a CLO and presents the various participants included in the process of creating and managing a CLO. In chapter 4, we take a closer look at the collateral assets of a CLO and present the most common asset in a CLO portfolio: Leveraged Loans. Chapter 5 continues the introduction of a CLO by presenting the lifecycle of a CLO from the warehousing period to the amortization period. Finally, in chapter 6, we analyze the European CLO market and the characteristics of the new issuances in the period from 2013 to marts 2021.

Part III (A Structural Model) presents the structural model, we use to analyze the research question of the thesis and the results from the analysis. Chapter 7 introduces the theoretical foundation needed to develop a structural model. Using the knowledge from chapter 7, we develop a structural model for pricing spreads on tranches of a CLO in chapter 8. The model is then used in chapter 9, where we investigate, how early prepayment of leveraged loans affects the spreads on the CLO tranches. Furthermore, chapter 9 includes a sensitivity analysis of the results and an analyze of the

Part IV (Concluding Remark) includes with a discussion of the found results and the model in chapter 10, before a conclusion is presented in chapter 11.

2 Literature Review

In this chapter, we provide a review of literature on CLOs. We focus on a subset of papers related to structured finance and CLO performance, among a diverse field of study. We have chosen to do so, because the combination of these two areas enables us to investigate our research question.

2.1 Structured Finance

In this section, we examine and present a selection of the existing literature on structured finance. We begin with the definition from Jobst (2005) on structured finance:

> *”Structured finance encompasses all advanced private and public financial arrangements that serve to efficiently refinance and hedge any profitable economic activity beyond the scope of conventional forms of on-balance sheet securities (debt, bonds, equity) in the effort to lower cost of capital and to mitigate agency costs of market impediments on liquidity. In particular, most structured investments (i) combine traditional asset classes with contingent claims, such as risk transfer derivatives and/or derivative claims on commodities, currencies or receivables from other reference assets, or (ii) replicate traditional asset classes through synthetication.”*

At the time of Jobst defining structured finance, Asset-Backed Securities were on a rise. The paper’s purpose was to inform a more specific debate about the regulatory challenges posed by the process of securitization and of structured finance in general, given the asset exposure and the dynamics of credit risk transfer.

Around the same time, Cuchra and Jenkinson (2005) published a paper on the importance of structured finance, more specifically structured debt. In this paper, Cuchra and Jenkinson provide the first systematic empirical analysis of tranching. This comes after various papers have proposed explanations on why pooling and tranching is used. In the paper, the authors conduct a systematic empirical analysis of European securitizations from the period between 1987 and 2003. Using this data, they test multiple hypotheses regarding the determinants of tranching and investigate the impact of tranching on pricing. In the paper, they find that the greater degree of asymmetric information is associated with a higher optimal number of tranches. Furthermore, they find that different explanations of tranching are responsible for the creation of different groups of tranches in the same deal. Finally, they investigate the effect of tranching on the pricing of issues at launch. From their analysis, the results show that tranching might be successful in remedying problems of market segmentation and suggest that structuring allows issuers to exploit market factors using the asymmetric information as an advantage.

In a later study Brennan et al. (2009), investigate the inefficiencies and gains that can be obtained from securitization by pooling and tranching loans. These gains are referred to as marketing gains, and they arise from selling the structured debt at yields that only reflect their credit ratings and therefore do not distinguish between systematic and non-systematic risk. They use a model based on Merton (1974). They find, that by issuing subordinated debt with bond collateral, equilibrium yields on a securitization should be significantly higher than that of an equivalently rated corporate issuer.

Contrarian Views

Not all views on structured finance is positive. After the onset of the Financial Crisis, Coval et al. (2009) published a paper examining structured assets, focusing on how they were incorrectly seen as ”safe” assets. The paper examines how securitization has been used to allow trillions of dollars of risky assets to be transformed into securities that were considered safe, and argue, that two key features have fueled the growth of structured finance: First, the authors show that most securities have received high credit ratings, only if the rating agencies were confident about their own ability to estimate the default probabilities and correlations of the underlying securities. Here they find that small imprecision’s in the parameter estimates can lead to major variation in the default risk of the tranches, which can be sufficient to cause a AAA-rated security to default with reasonable likelihood. At the same time, they find that another defective feature of securitization is that it substitutes risks that are largely diversifiable for highly systematic risks. This results in the securities produced through structuring having far less chance of surviving a severe economic downturn than traditional corporate securities with equal rating. Coval et al. argue that the result from these two features are extremely fragile assets, at their rating. They end the paper by predicting that the ability to create large quantities of AAA-rated securities using securitization is likely to be forever diminished, as the rating process evolves to better account for parameter and model uncertainty.

However, since the Coval et. al., made their prediction and after the collapse of the CLO market during the Financial Crisis, the market for CLOs have rebounded since 2011 for the US and 2013 for the European market. In chapter 6, we present a detailed analysis of the development in the European CLO market. Following this rebound, a new field of study focused on the performance of CLOs have emerged.

2.2 CLO Performance

The performance of collateralized loan obligations can be measured in various ways. In this section, we describe how Cordell et al. (2021) analyse CLO performance, by examining returns on the CLO's equity tranche and the relatively high promised spreads on the debt tranches compared to equivalently rated corporate bonds. The paper investigates which market imperfections the CLO's are designed to address and examine various imperfections to evaluate how they affect the performance of the CLO's collateral, and hence, the CLO claims. The first part of the paper explores the abnormal returns to the equity holders. This is a result of the adjusted price differential between leveraged loans in the collateral pool and the secured tranches issued to finance the SPV. The result shows that the return, net of fees, are similar to the return generated from a diversified portfolio of loan mutual funds. Given this, they find that in aggregate, the CLO manager has neither an informational advantage nor superior skill for selecting leveraged loans compared to other market participants.

In the second part of the paper, the authors investigate the performance of CLO tranches. Here they find that debt tranches on CLOs are promising significantly higher spreads compared to similarly rated and duration-matched corporate bonds. They propose that a reason for this could be the unmeasured risk exposure rather than abnormal return on the portfolio.

As mentioned in the introduction, several papers have analysed different reasons for the higher spread between CLO tranches and equivalently rated corporate bonds. Elkamhi et al. (2020) find that the high returns on CLO tranches could be explained by compensation for systematic risk. By applying a structural model to Business Development Companies (BDC), they extract market-implied correlations for the BDC loan portfolios and generate fair spreads between CLO tranches and an implied benchmark. Doing this, they find that the systematic risks, from correlated loan defaults is correctly reflected in the large credit spreads on CLOs, in the period after the global financial crisis.

Similarly, Wojtowicz (2014) examine the risk and return characteristics of CDOs by using the market standard model. Using this approach, they find that fair spreads on CDO tranches are much higher than fair spreads on similarly-rated corporate bonds. The results imply that credit ratings are not sufficient for pricing tranches and comment on the limitations of rating methodologies that are solely based on real-world default probabilities, that do not correctly price systematic risk premia. They show that CDO debt tranches have large exposures to systematic risk, and hereby, their ratings and prices are likely to decline when credit conditions deteriorate.

In the following parts of this thesis, we build a structural model to investigate how spreads can be priced using an adapted approach from Brennan et al. (2009). Additionally, we investigate the CLO tranche spreads and try to explain the spread differential between CLO tranches and equivalently rated corporate bonds, using assumptions about collateral prepayments.

Part II

On Collateralized Loan Obligations

3 Structure of Collateralized Loan Obligations*

Collateralized Loan Obligation (CLO) has a complicated structure. As our goal in this thesis is to investigate the spreads on CLO tranches, we will begin our analysis by presenting the structure and characteristics of a CLO. In this chapter, we start by presenting concepts of securitization, Asset-Backed Securities (ABS), and Special Purpose Vehicle (SPV). After a short introduction to these concepts, the chapter continues with a presentation of the structure of a CLO including the various participants in the process of creating and managing a CLO.

3.1 Securitization

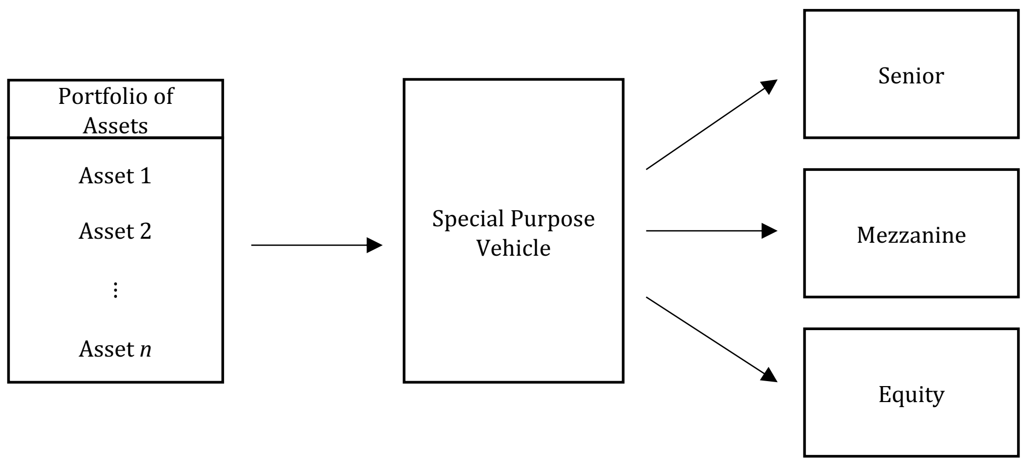
Securitization is a risk management tool used to reduce the idiosyncratic risk linked to the default of individual assets. The use of securitization began in the 1960s in the U.S., where banks had trouble keeping up with the pace of the demand for residential mortgages. To deal with this problem, the banks created mortgage-backed securities, which are portfolios of mortgages, where the interest and principal payments generated from the portfolio are packed as securities and sold to investors. By doing this, the banks originated the mortgages, but sold the risk and hereby removed them from their balance sheets. Hereby, the banks could increase their lending faster than their deposits were growing. During the 1980s the use of securitization spread from the mortgage market to other asset class and investors from all over the world started investing in the securities (Hull, 2017).

*Several sections in this chapter is based on informal talks with market participants and material from investment banks acting as underwriters and arrangers for CLO managers: Barclays (2016), BofAML (2016), Deutsche Bank (2016), Wells Fargo (2017) and Guggenheim Investments (2019).

3.1.1 Asset-Backed Securities

From the beginning of 2000, and until the eruption of the Financial Crisis in 2007-2008, the most used type of securitization was Asset-Backed Securities (ABS) (Hull, 2017). ABS consist of a portfolio income producing assets such as loans. These assets are originated by a bank and sold to a special vehicle, where the cash flows are allocated to tranches. The tranches are typically divided into three categories: Senior, Mezzanine, and Equity tranches, where senior tranches have the highest seniority (See figure 3.1).

Figure 3.1: Asset-Backed Security - Structure



Source: Hull (2017)

The payments (interest and principal) to the tranches are not guaranteed. Rather, the payments will be realized if there are no defaults in the pool of assets. The cash flows are allocated using a waterfall model, where the principal and interest payments are allocated following the seniority of the tranches. In the situation of any defaults in the pool of assets, the subordinated tranche is the first to take a loss. If the loss in the underlying pool of assets exceeds the principal of the subordinated tranche, the losses are then borne by the mezzanine tranche and so on. By using the waterfall model in the SPV, the risky assets are transformed into tranches with different risks, where the subordinated tranche is the riskiest (no rating), and the senior tranche is the less risky (AAA-rated). The details of the CLO tranches will be elaborated in section 3.3.4

3.1.2 Collateralized Debt Obligations

ABS can be of different types depending on the characteristics of the underlying pool of assets in the structure. The most common type is Collateralized Debt Obligation (CDO), which is divided into categories depending on the type of debt instrument in the pool of assets. In table 3.1, the most popular types of CDOs are presented.

Table 3.1: Types of CDOs

Type	Description
CLO	Collateralized Loan Obligations are CDOs in which the pool of assets are broadly syndicated loans, typically leveraged loans
CBO	Collateralized Bond Obligations consists of bonds and tranche notes from other CDOs
CMO	Collateralized Mortgage Obligations also referred to as Mortgage-Backed Securities (MBS) are composed of mortgage bonds

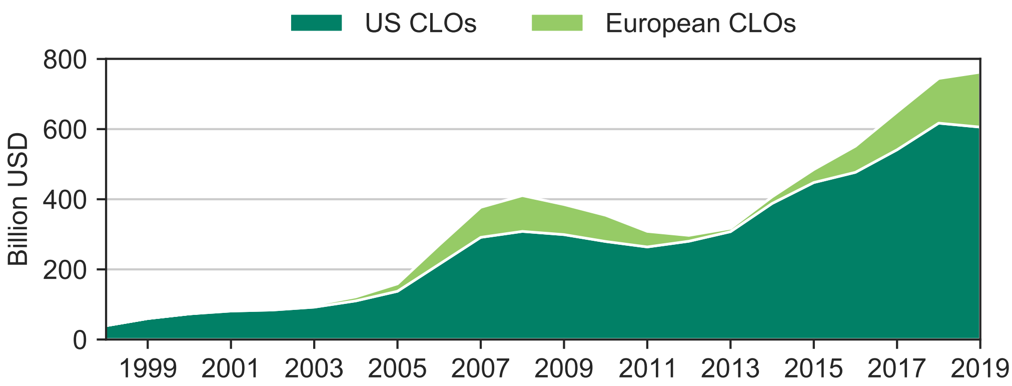
Source: Hull (2017)

3.2 Collateralized Loan Obligation

A CLO is a Special Purpose Vehicle (SPV) that holds, invests, and manages a diversified pool of syndicated leveraged loans and bonds through investments in rated debt and equity tranches. The product originated in the late 1980s as a way for banks to bundle leveraged loans together and hereby creating a product that provides varying degrees of risk and return for the investors. Only a few big banks issued CLOs in the 1990s, and the market remained relatively small until the early 2000s (PineBridge Investments, 2019). In this period, institutional investors were looking for higher yielding investments due to low interest rates, which resulted in a growth in the global outstanding of CLOs from around 80 billion dollars in 2000 to 400 billion dollars in 2008 (figure 3.2).

The number of new issuances came to a halt during the Financial Crisis with the collapsing subprime Mortgage-Backed Security (MBS) sector and the number of new CLO issuances was close to 0 in 2009 (S&P Global Market Intelligence, 2021a). In the following years, the US market and European

Figure 3.2: US and EU CLOs outstanding



Source: Own creation on data from IMF (2020)

CLO market developed differently, where US new issuances restarted in 2011, meanwhile the European new issuances first began to increase in 2013 (figure 3.2).

In response to the arising problems during the Financial Crisis, a new structure was introduced. This structure was named CLO 2.0 and had more conservative features to prevent the massive downgrade and huge losses in market value, that had occurred during the Financial Crisis. The new features were:

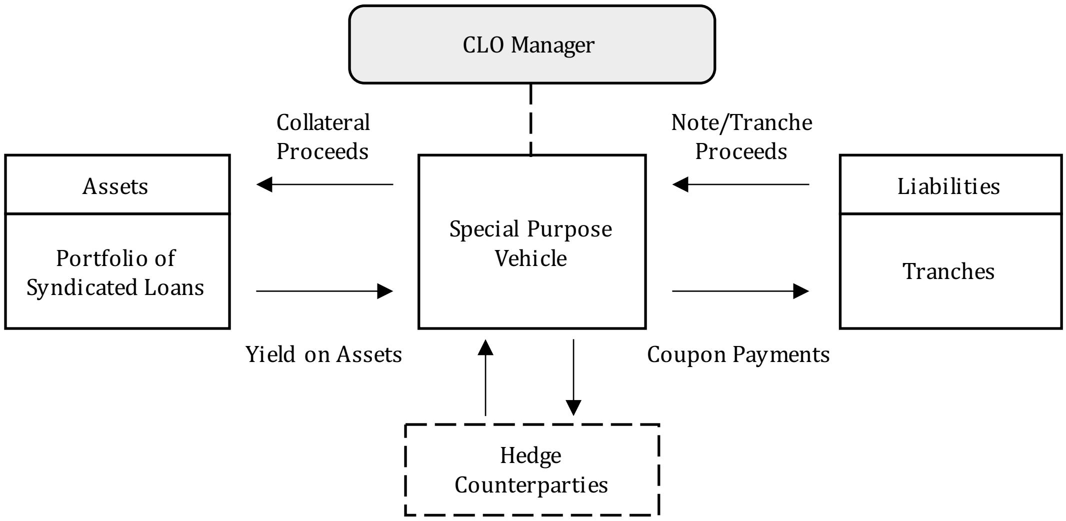
- Revised rating methodologies for the rating agencies
- Change in the tranches with smaller senior tranches and larger equity tranches.
- Shorter reinvestment periods
- Retention rules for the CLO manager to obtain convergence of interest between the investors and the manager.

Source: Ostrum Asset Management (2018).

3.3 Structure

The structure of a CLO is divided into four sections and is presented in figure 3.3. A CLO begins with a CLO manager, who manages a SPV. The manager acquires and manages an underlying pool of assets. These assets typically consist of leveraged loans and bonds and the SPV receives interest proceeds and principal from these loans. Using securitization, the CLO sell the payments from the pooled assets to investors in differentiated tranches with different risk and returns. This process is conducted with a number of counterparties.

Figure 3.3: Typical CLO Structure



Source: Own creation

3.3.1 CLO Manager

The CLO Manager (also referred to as Asset or Collateral Manager) is responsible for acquiring and managing the pool of underlying assets throughout the life of the CLO. The manager is incentivized to maximize the return of the underlying pool of assets, given some restrictions in relation to the composition of the assets, and in return, the manager receives fees (senior, junior and incentive fees) linked to the performance of the CLO (Barclays, 2016). The process of acquiring the assets and the structure of the fees will be explained in chapter 5.

Risk Retention

As a part of the new CLO 2.0 structure, risk retention was presented to align the incentives of the investors and the CLO manager. In Europe and US, the retention rules were presented as a part of the Capital Requirements Directive in 2011 and the Wall Street Reform and Consumer Protection Act of 2010 (also known as Dodd-Frank Act). The new rules require the CLO manager to retain a credit of at least 5 % in the securitization. By doing this, the managers are required to provide their own capital in order to issue new CLOs (Ostrum Asset Management, 2018). The idea behind the risk retention is to align the interest of the CLO manager with the investors in the CLO tranches. The rules were introduced as a way of reducing agency problems that occurred in many of the CLO 1.0 structures prior to the crisis in 2008-09 (Barclays, 2016). The 5 % retention may be held as either 1) a vertical slice, where the manager owns a piece of all the trances (total minimum 5 % of the total issuance), 2) a horizontal slice, where the manager owns 5 % of the equity tranche or 3) a combination of the horizontal and vertical creating a L shaped slice (Deloitte, 2017).

3.3.2 Hedge Counterparties

When establishing and managing a CLO, several counterparties are involved (Deloitte, 2018). Below, the most important of these are listed and described:

The Trustee is typically an investment bank acting as a fiduciary agent for the CLO investors. This includes maintaining custody of the collateral assets and their cash flows, with respect to the cash flow structure and coverage tests.

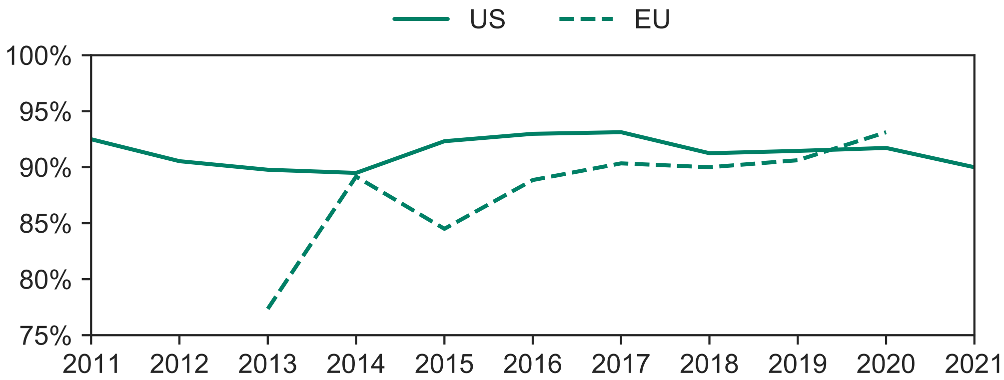
The Placement Agent is typically a commercial or investment bank hired by the CLO to structure and sell the CLO liabilities by privately placements. Also, the Placement Agent often provides the warehouse facility and is responsible for marketing of the CLO.

The Credit Rating Agencies assign ratings to the CLO tranches, around the closing date and monitor the portfolio on the basis on information from the CLO manager and trustee. The various rating agencies and theirs methods are described in more detail in section 6.4.

3.3.3 Assets

The CLO Manager sets some limits for the various types of assets to be included in the pool of assets. Typically, the CLO manager is restrained to hold a minimum of 90 % of secured senior obligation in aggregate, where a minimum of these obligations should be secured senior loans. Further, the CLO manager has limits for the maximum amount of a type of asset the portfolio can contain. This includes for example 30 % Cov-Lite loans and 10 % of Collateral Manager Portfolio Companies. Figure 3.4 shows the development in the set requirements by the CLO manager for the minimum percentage amount of senior secured in the pool of assets for the US and EU CLO market.

Figure 3.4: Senior Secured Loans (Minimum %)



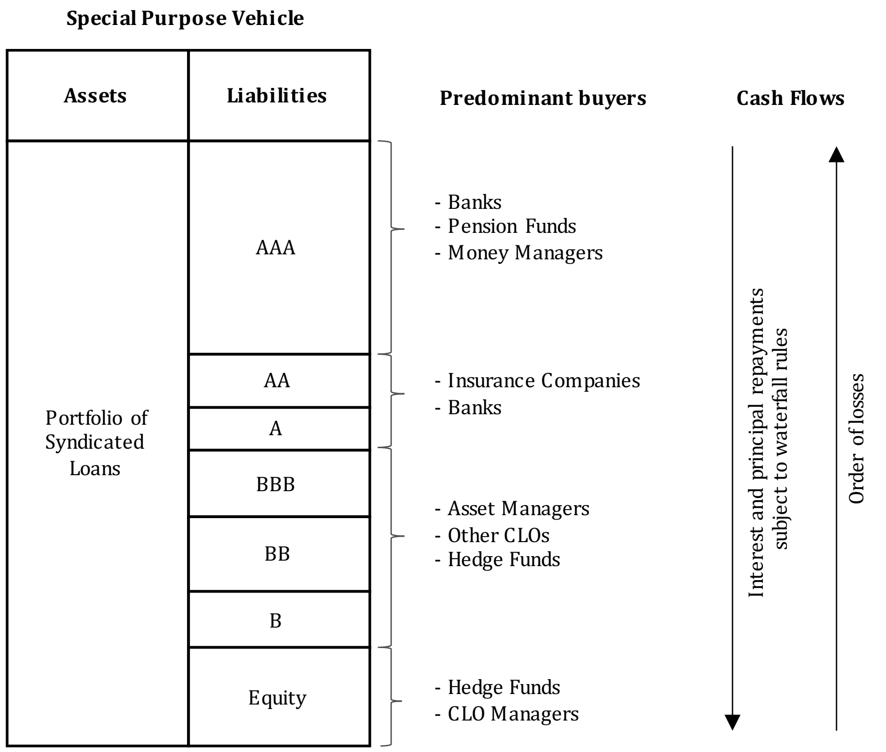
Source: Own creation on data from S&P Global Market Intelligence (2021a)

Figure 3.4 illustrates the development in the minimum requirement for senior secured loans in the underlying pool of assets. The development in the US market is stable in the period around 90 %. For the European market, it can be seen that around the introduction of CLO 2.0, the minimum requirement is around 80 %. In the following years, the requirements increase and in 2020 it is similar to the US at around 90 %. In chapter 4, we give a more detailed introduction to the collateral of the CLO and the characteristics and features of leveraged loans.

3.3.4 Liabilities

Given the portfolio of loans, the CLO manager creates a number of tranches. The various CLO tranches offer varying levels of exposure to default risk of the underlying portfolio and offers therefore also different spreads. The tranches can be divided into three categories: Senior (AAA-A), mezzanine (BBB-B), and equity (non-rated). The categories can be divided in several tranches and several tranches can have the same rating. Typically, there are between 7-9 tranches, which consist of ratings from AAA-B and a non-rated equity (S&P Global Market Intelligence, 2021a). Figure 3.5 depicts the structure of a typical CLO and the predominant buyers of the tranches.

Figure 3.5: CLO Liabilites and Investors



Source: Own creation based on Barclays (2016) and Guggenheim Investments (2019).

Alongside the tranches and the typical investor, the figure also shows the order of the cash flow payments and the order of losses. The principal and interest cash flows follow a waterfall structure in which the most senior tranche will get paid first, followed by the second most senior tranche. Opposite the cash flows structure, the order of losses is from the bottom to the top meaning that if the first tranche to default is the equity tranche. Therefore, the spreads will also be highest for the least senior tranches compared to more senior tranches.

Investors

From figure 3.5, it can be seen that there are various investors for the CLO tranches. These includes financial institutions such as banks, insurers, and pension funds, but also other structured credit (eg. other CLOs) and hedge funds. Finally, the equity tranche has investments from the CLO manager, which is a result of the risk retention rule. The majority of the funding for the CLO comes from the AAA and AA rated tranches, where most of the investors are financial institutions. The high demand from financial institutions is due to the fact that the institutions are constrained by rating-based capital requirements and hereby seek high-yield debt within each rating category. Financial institutions are constrained due to capital requirements on their balance sheets, and they can therefore not hold the risky loans themselves. Additionally, the CLO tranches come with higher yields compared to corporate bonds that have the same credit rating and duration. The combination of lower capital requirement and attractive yields results in banks and insurers being the primary investors of the most senior tranches of the CLO (Cordell et al., 2021).

3.4 Call Provisions

Most CLOs have incorporated flexibility for the equity investor to call the deal and hereby be less vulnerable to asset-liability mismatch. As fiduciary for the equity investors, the CLO manager, besides actively managing the risk and spread of the collateral, has some choices to increase the return on equity. These choices are manifested as the right to redeem the CLO-liabilities. Furthermore, if the collateral falls short of some coverage tests, a mandatory redemption shall occur. In general, the manager can exercise three types of call options:

Table 3.2: Types of Call Options

Type	Description
Call	The CLO manager liquidates all collateral at the existing market value and repay lenders and shareholders with the proceeds (if any)
Reset	The CLO manager sells the collateral to a new SPV and then redeem the legacy CLO in full. The new SPV is then issued with the asset from the former, but with new maturity and new WACD
Refinance	The CLO manager reduces the margin applicable to an entire class of existing loans, but keeps the original maturity of the CLO

Source: NEPC LLC (2015)

The refinance feature was introduced in post-financial crisis structures, i.e. CLO 2.0s and provides the manager a time and cost-efficient alternative to reset, as full par value redemption of the entire class is unnecessary. Only lenders disagreeing will need to be repaid at the applicable rate. Tranches are redeemed at par plus any accrued and unpaid deferred interest. For the subordinated tranches, the redemption price differs from the other tranches as the price is either 100 percent of the outstanding principal amount or, if greater, the pro-rata share of the aggregate proceeds of liquidation of the collateral or realization of the security thereover in such circumstances, remaining following application thereof in accordance with the priorities of payment.

4 Collateral Assets

In chapter 3, we presented the structure of a CLO including the pool of underlying assets. The assets in a CLO are also referred to as collateral assets. In the following chapter, we present the development of the collateral assets for the European CLO market. This includesan analysis of the requirements for the pool of assets set by the CLO manager as well as a presentation of leveraged loans.

4.1 Eligible Asset Criteria

As described in section 3.3.3, the CLO managers set requirements for the collateral portfolio. To ensure that the collateral portfolio stays within these requirements, the CLO transactions contains a hierarchy of rules and test. The rules can be divided into two categories: Eligibility criteria and Collateral quality and portfolio test. The collateral quality and portfolio test are also referred to as coverage test and will be described in section 5.4.2 (Barclays, 2016).

The eligibility criteria are defined at the asset level, and the assets must satisfy these criteria at the time they are added to the portfolio. In table 4.1, the requirement for the asset portfolio is presented for the EU market in the period from 2013 to 2020.

Table 4.1: Requirements for Assets in European CLO New Issues

European New Issues	2013	2014	2015	2016	2017	2018	2019	2020
Sr Secured Loans (min)	77.4%	89.2%	84.5%	88.9%	90.4%	90.0%	90.6%	93.1%
Floating Rate (min)	87.0%	91.0%	91.0%	91.2%	90.0%	90.0%	90.0%	87.5%
Cov-Lite Loans (max)	21.7%	27.0%	26.1%	32.4%	37.5%	30.0%	35.7%	38.8%
High Yield Bonds (max)	13.9%	5.8%	10.0%	8.8%	30.0%	10.0%	23.8%	30.0%
Sr Unsecured Loans (max)	10.3%	8.6%	10.0%	7.4%	9.4%	10.0%	9.4%	5.5%
Fixed Rate (max)	13.0%	9.0%	9.0%	8.8%	9.7%	10.0%	9.1%	12.5%
CCC+ Rated or Below (max)	7.7%	7.5%	7.5%	7.5%	7.5%	7.5%	7.5%	7.5%

Notes: All numbers are arithmetic averages for European CLOs issued between 2013-2020.
Source: Data from S&P Global Market Intelligence (2021a)

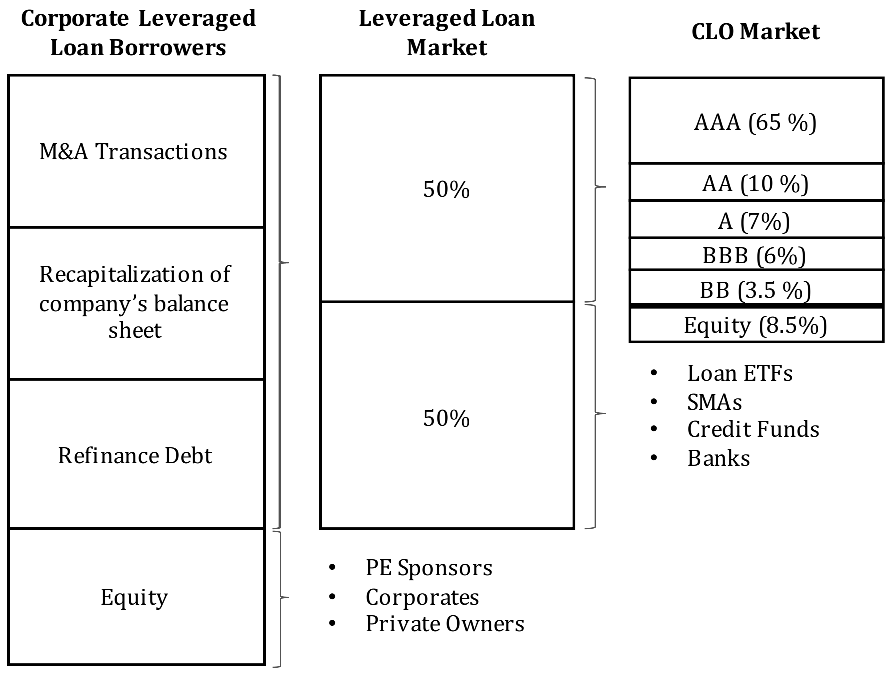
From the table, we find that the criteria for senior secured loans have grown since 2013, and in 2020 the average European CLO had requirements on a minimum of 93.13 %. Furthermore, around 90 % of the assets have to be floating rate. We refer to these loans as leveraged loans. The CLO can contain other types of assets such as bonds, fixed-rate loans, structured finance, etc. in addition. However, we focus on leverages loans in the paper.

4.2 Leveraged Loans

The biggest part of the underlying pool of assets consists of leveraged loans. A leveraged loan is a commercial loan provided by one or several commercial or investment banks focussing on lower credit borrowers. There are various definitions of when a loan becomes a leveraged loan. Some practitioners use a yield-spread cut-off as a definition, where loans with a spread over LIBOR/EURIBOR + 150 BP would be defined as leveraged loans while others refer to credit rating. Using credit rating's leveraged loans are often defined as loans with at Ba1 or lower using Moody's or at BB + or below using S&P or Fitch (Altman et al., 2019, p. 22)

The leveraged loan market experienced big difficulties during the Financial Crisis in 2008-09 and the gross issuances of leveraged loans fell from 1,800 trillion dollars in 2007 to 900 trillion dollars in 2008. After some years with reduced gross issuances, the market started growing again and in the period from 2009 to 2019, the global leveraged loans outstanding have grown with 78 % to around 5,500 trillion dollars (IMF, 2020). The loans are typically used for purposes such as M&A transactions, recapitalization of a company's balance sheet, or refinancing of debt. The leveraged loans investor market consists of three primary investors: Banks, Finance companies, and Institutional investors (figure 4.1).

Figure 4.1: Leveraged Loans and CLOs



Source: Guggenheim Investments (2019)

From the figure, we find that 50 % of the leveraged loans are bought by CLOs. Therefore, the drop in the issuances of new leveraged loans had a large effect on the CLOs during the Financial Crisis (see section 6.2).

4.2.1 Types of Leveraged Loans

Leveraged loans cover a variety of individual loan types and structures. Traditionally, the four main types of syndicated loan facilities are Revolving debt, Term debt, Letter of credit, and Acquisition or equipment line.

Revolving Credit Facilities allow borrowers to draw down, repay, and reborrow. The facility acts much like a corporate credit card, except that borrowers are charged an annual fee on unused amounts. The loans typically have a maturity of less than a year and are in general limited to the investment-grade market. Given this, banks can offer the issuers a lower unused fee than with longer maturities (S&P Global Market Intelligence, 2021b).

Term Loans are installment loans, where the borrower can draw a fixed amount during a very short commitment period, and receive an amount upfront. The borrowers can then repay it based on either a scheduled series of repayments or a one-time lump-sum payment at maturity (S&P Global Market Intelligence, 2021b). Term loans can be divided into amortizing term loans (term loan A) and institutional term loans (term loan B, C, D). Term loans A are amortizing loans, meaning that they have a progressive repayment schedule. The maturity is typically six years or less. The loans are most often syndicated to a bank together with revolving debt (S&P Global Market Intelligence, 2021b). Term loans B, C, and D are designed for institutional investors and have longer maturity at 7-9 years. As opposed to term loan A, the term loan for institutional investors has bullet repayment BofAML (2016). Term loan B, C, D includes second-lien and covenant-lite loans.

Letter of Credit are guarantees provided by the bank group to pay off debt or obligations if the borrower is unable to do this (S&P Global Market Intelligence, 2021b).

Acquisition or Equipment Lines are credits where there may be drawn down for a given period to purchase a specific asset or to make an acquisition. The issuer pays a fee during the commitment period and the lines get repaid over a specified period (S&P Global Market Intelligence, 2021b).

4.2.2 Covenant-lite Loans

Another important asset in the collateral assets is covenant-lite loans, also referred to as cov-lite loans. Cov-lite loan is a variant of leveraged loans which build on less protective covenants compared to 'normal' leveraged loans. This means that the cov-loans typically are more flexible for the borrower, and therefore also riskier for the lender. Therefore, the borrower can often lend more compared to loans with financial covenants (S&P Global Market Intelligence, 2021b). As can be seen in table 4.1, cov-lite loans are limited to around 40 % in 2021.

4.2.3 Coupons and Maturity

Leveraged loans are structured with a variable interest payment, and uses a base rate, typically the benchmark rates LIBOR or EURIBOR, and an additional pre-determined spread. As mentioned previously, a loan is categorized as a leveraged loan, when the spread above the base rate is more than 150bps. Leveraged loans typically have a maturity of 5-7 years but can have a maturity up to 10 years (S&P Global Market Intelligence, 2021b).

Margin Ratchet

Leveraged loans are often constructed with margin ratchets. A margin ratchet is a mechanism whereby the initial margin is reduced when the issuer achieves a better financial position. Some key financial ratios are determined to valuate the position. If these are held, the margin will be lower. Contrary, if the financial position is worsened, the margin will return to its original level.

4.2.4 Non-Call Features

The leveraged loans are FRNs, and therefore they have the option of being prepaid at any time without penalty. This means that the borrower can call the loan when it is advantageous to refinance at a better rate, recapitalize the business and pay themselves a dividend, or float the business publicly through an IPO. However, in the recent market standards in both Europe and the US a soft call provisions or non-call period has been integrated. The non-call feature is a fee that requires the issuer to pay a penalty premium if the loan is called in this period. Often this is a six-month with a prepayment fee of 1% (S&P Global Market Intelligence, 2021b).

An alternative to the soft call is hard call protection. Hard call protection is a provision, in which the issuer of the loan cannot exercise the call before the specified date. This is also referred to as a non-call period and is similar to the non-call protection in the CLO structure described in section 3.4.

4.2.5 Credit Risk

When pricing leveraged loans, the arrangers have to evaluate the risk inherent in a loan. The principal credit risk factors are default risk and loss-given default risk when buying loans are. These risks can be examined in different ways and rating, collateral coverage, seniority, etc. are among the more primary.

Default risk is the likelihood of a borrower being unable to pay interest or principal on time. The likelihood is based on parameters such as industry segment, the condition of the industry, and other economic variables. The default risk is most often expressed used rating from various rating agencies(S&P Global Market Intelligence, 2021b). The ratings range from AAA-CCC (S&P) or Aaa to C (Moody's), where AAA and Aaa are the most creditworthy and CCC and C are the least. The rating can also be divided into two segments: Investment grade, which covers loans rated from BBB or higher, and speculative grade, which covers ratings from BB or lower.

Loss Given Default (LGD) risk measures the loss that the lender is likely to incur in the event of default. The investors in leveraged loans assess the LGD based on the collateral backing the loan and the seniority of the loan compared to other debt and equity subordinated to the loan. This can also be thought of as the recovery risk. In the process of evaluating LGD, the lenders will look at financial covenants. Financial covenants enforce minimum financial performance measures against the borrower. These covenants can be divided into two categories: Maintenance and Incurrence. Maintenance ensures that the issuer must pass several financial performance tests, while incurrence is tested only if an issuer takes an action such as issuing new debt or making acquisitions. If the issuer fails these tests, the lenders can renegotiate the terms of a loan (S&P Global Market Intelligence, 2021b).

4.2.6 Default and Restructuring

Leveraged loans have two primary ways to default. First, a loan can technical default. Technical defaults occur when the issuer violates a loan agreement, e.g. a financial covenant test. In these situations, the lender can accelerate the loan and force the issuer into bankruptcy. Secondly, a loan can payment default, which occurs when a company either misses a interest or principal payment (S&P Global Market Intelligence, 2021b).

Default Rate and Recovery Rates for European Leveraged Loans

To investigate the historical development in leveraged loan defaults, we use the European Leveraged Loan Index (ELLI). The index presents the trailing 12-month default rate in Europa as a percentage of the principal amount. Looking at the development, we find that in the years after the Financial Crisis erupted, the default rates for the index increased from under 2 % in 2008 and up over 10 % in 2009. In the following years, the default rate fluctuated between 2 % and 6 % until 2015, where the default rate was at 2 %. After some years with a stable default rate on approximately 2 %, the default rate dropped to 0 % in 2018 and 2019. Since the beginning of 2020, the default has gone up and is in November 2020 back at around 2 % (Standard and Poor, 2020b).

If we instead take a look at the Recovery Rate (RR), we find that historically, the leveraged loans have had better recovery compared to bonds as they are typically secured and have stronger covenants (Deutsche Bank, 2016). This can be seen from table 4.2, where both using issuer weighted and volume weighted results in first lien bank loan having a higher recovery rate than first lien bonds.

Table 4.2: Average Global Debt Recovery Rates (1983-2015)

Type	Issuer Weighted	Volume-Weighted
1st Lien Bank Loan	66.6%	62.3%
2nd Lien Bank Loan	31.8%	27.6%
Sr. Unsecured Bank Loan	47.1%	40.2%
1st Lien Bond	53.4%	53.4%
2nd Lien Bond	49.7%	47.4%
Sr. Unsecured Bond	37.6%	33.7%

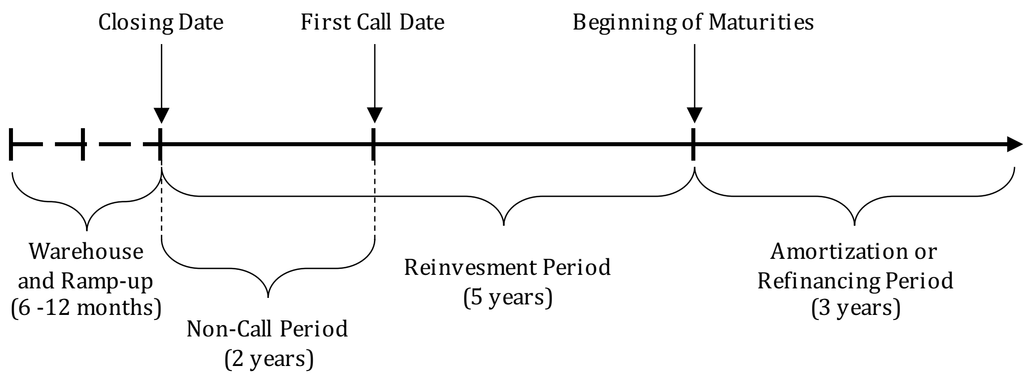
Note: Measured by post-default trading prices
Source: BofAML (2016)

5 Lifecycle of a Loan Securitization

The following chapter presents the lifecycle of a CLO. The section builds on the information introduced in the previous chapters about the structure of a CLO and the collateral assets. The purpose of the chapter is to give a presentation of the various stages of the lifecycle, which we use in the structural model in chapter 8.

The total lifecycle of a Collateralized Loan Obligation (CLO) is usually around eight to 10 years and can be divided into three distinct phases from the time where the SPV is initially established. Figure 5.1 illustrates a typical lifecycle of a CLO.

Figure 5.1: Typical CLO Lifecycle



The first phase is the warehousing and ramp-up period, which usually is around six to 12 months. In this phase, a warehousing provider finances a temporary credit facility to finance the CLO manager's acquisition of leveraged loans. The credit facility is expected to be repaid on the *closing date*, i.e., the date when the committed proceeds accrue to the SPV.

The second phase is the reinvestment period. The reinvestment period begins at the effective date and continues until the first maturities of the most senior tranches. This period is usually five years in length and can be subdivided into two periods: a non-call period, where the CLO manager is prohibited from using the earlier described call provision, and a callable period, typically starting two years after the effective date. In this period the CLO manager can refinance some or all of the outstanding tranches in the structure, subject to the same waterfall regime as described in section 5.4.

The final phase of the CLO lifecycle is the amortization period, which begins after the reinvestment period ends. During this final period, proceeds from the loan portfolio may no longer be reinvested in new assets, and installments are made to the CLO-lenders as cash accrues to the SPV from repayments of the loans. Installments on the tranches are paid subject to the waterfall regime, where no repayments are made to the junior tranche before the more senior tranches are paid in whole.

5.1 Warehouse and Ramp-up Period

As described in section 3, all CLOs are initiated by establishing a separate legal entity or SPV. Immediately after the creation, an arranger/underwriter often in the form of an investment bank provides temporary financing to this entity. This is known as a warehousing credit facility, such that it can begin purchasing collateral in the form of leveraged loans. This process begins several months before the final issuance of the CLO notes and even before commitments are made for these notes.

At some point in the intermediate period between the establishment of the warehouse and the time of the final issuance of the CLO notes, the CLO manager and its arrangers initiate a process of marketing and gauging the sentiment from potential investors (see subsection 3.3.4). This process ends in a *pricing*, where primary market investors commit to purchasing an amount of the notes offered.

Around six to seven weeks after the pricing date, the final commitment materializes and the CLO *closes*, i.e., proceeds from the CLO notes accrue to the SPV. At the closing date, the CLO is yet to be fully invested and the collateral manager will accelerate ramping up, i.e., investing in assets until the collateral reaches the target amount.

One of the important reasons for the warehouse period with incremental investments in collateral is the opportunity for the CLO manager to capture assets in the primary market, i.e., when syndicated loans are initially offered to the market. As described in the leveraged loan section 4.2, loan purchases are more likely to offer discount and rebates when bought in the primary markets. Without a warehouse period, the very short ramping period entails a risk of not being able to allocate the note proceeds in an orderly manner.

5.2 Reinvestment Period

The reinvestment period is the predetermined period, detailed in the legal prospectus of the CLO, where the manager is expected to reinvest the interest and principal received from the collateral. The reinvestment period is usually four to six years in length, with the majority of European CLOs having a reinvestment period of five years and begins at the closing date. At the same time, as the warehouse facility is repaid with the proceeds from the issuance and interest begin accruing on the liabilities. As described above, the CLO is yet to be fully invested as of closing and thus the manager will accelerate the collateral purchases immediately after closing, as a high degree of collateralization is an important objective and in the interest of the manager.

An important characteristic of the CLO tranches are the embedded call option, where the CLO manager can repay and refinance the notes after the non-call period has ended (described in section 3.4). This is similar to the callable feature of corporate bonds, where lenders are protected from the issuer repaying the bond prematurely. The non-call period is typically two years and after this period, the CLO manager may repurchase and refinance its liabilities subject to the call structure in the legal documentation.

5.3 Amortization Period

Following the end of the Reinvestment Period, the portfolio principal proceeds and sales proceeds from the assets are used to repay the tranches sequentially. The repayment of the tranches starts with the most senior tranches and then follows the order of seniority. The remaining life is determined by the frequency and size of the collateral principal payments. This is affected by the maturities of the underlying assets and whether the underlying obligors redeem or default on their obligations prior to the scheduled maturity, thereby accelerating the principal repayments. As more senior tranches get repaid, the

Weighted Average Cost of Debt (WACD) will start increasing as the relatively cheaper funding gets paid down. As the WACD increase, the equity investors will be incentivised to call the CLO as described in section 3.4 (Deutsche Bank, 2016).

5.4 Cash Flows

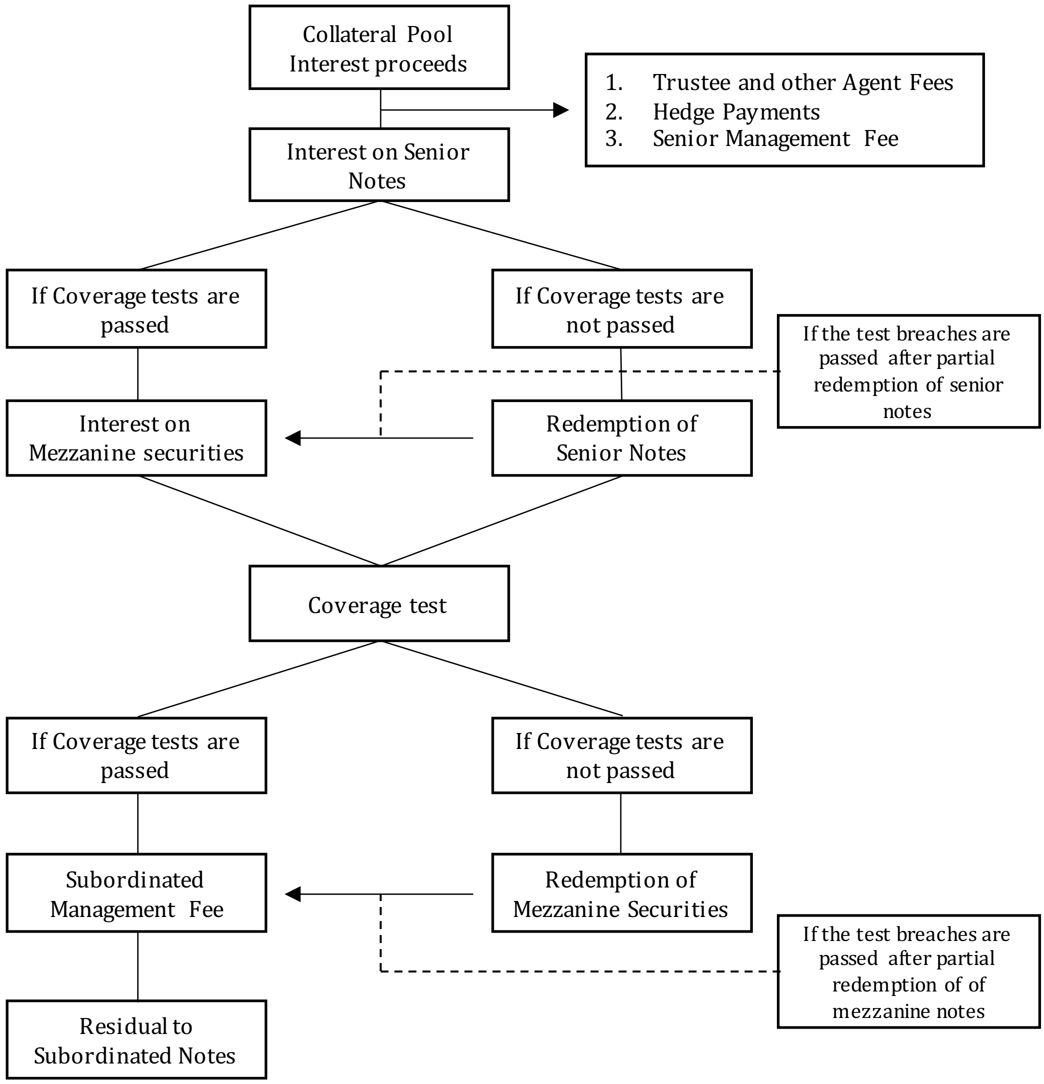
The cash flows of a CLOs are structured as transactions, whereby the income is generated by the underlying collateral and is used to pay the tranche holders and the equity investors in a predefined sequence. The payments to the tranche holders and equity investors are paid sequentially starting with the most senior tranche and ending with the equity tranche to meet the priority of payments. This is also known as a payment waterfall. The payment made to the tranche holders are divided into interest and principal payments. This means that the structure has two sub-waterfalls in the structure. The waterfall of interest includes the priority of payments of interest to tranches and the payment of fees, while the waterfall of principal relates to the repayment of the tranches. The repayments are paid using prepayment and recovery cash flows from the underlying pool of loans. The interest cash flows are depicted in figure 5.2. The payment waterfall has a similar structure, but instead of the interest coverage (IC) test, there are performed overcollateralization (OC) test.

5.4.1 Fees and Incentives

The waterfall structure includes several fees for the CLO manager, the trustee and other agents. Before, the principal and interest are paid to the tranche holders. The trustee and other agents and the senior management fee to the CLO are paid. At a later point in the structure, the CLO manager will obtain a subordinated management fee, if the payments to all tranche holders are successfully paid in regard to the IC and OC tests.

Collateral Management Fees are divided into three types: Senior Management Fee, Subordinated Management Fee, and Incentive Collateral Management Fee. The CLO manager receives a senior fee, which is paid regardless of the performance of the CLO. This fee is paid as a percentage of the underlying portfolio of loans' par value (0.15% per annum). Additionally, the CLO

Figure 5.2: CLO - Waterfall Structure



Source: Own creation based on PineBridge Investments (2019).

manager receives a subordinated management fee, when senior and mezzanine tranches have been paid. Finally, the manager is entitled to an incentive fee on each payment date, if the Incentive Collateral Management Fee IRR Threshold has been met or surpassed. The fee is equal to 20% of any interest proceeds and principal proceeds that would otherwise be available to distribute to the subordinated noteholders/Equity tranche.

5.4.2 Coverage Tests

The CLO manager is required to test the portfolio by using a series of coverage tests. These tests are conducted to help ensure that the cash flows generated by the underlying loans meet the obligations in the CLO tranches. The coverage tests have the purpose to determine whether senior tranches are sufficiently protected. If the CLO fails a test, cash flows are directed to senior tranches until a deal is back in compliance with the test.

Over-Collateralization (OC)

The OC test measures the adequacy of the collateral in the CLO supporting each class of notes. The purpose of the test is to protect the tranche holders against a decline in the value of the portfolio collateral. The test is performed using the ratio of the principal collateral value over the outstanding liabilities/notes (Deutsche Bank, 2016).

$$\text{OC(Tranche)} = \frac{\text{Total principal of collateral loans}}{\text{Aggregate debt amount outstanding at the tranche-level}}$$

The threshold varies for the tranches depending on the seniority, where the subordinated tranches have a lower OC threshold compared to more senior tranches. If the OC test fails, the interest and principal payments are diverted from more junior tranches to pay down the liabilities of the liabilities in order of seniority until the OC test is over the threshold again (Deutsche Bank, 2016).

Interest Coverage (IC)

The IC test compares the interest income received against the liabilities to ensure sufficient coverage. This is done to protect the tranche holders against a decline in interest income from the underlying portfolio (Deutsche Bank, 2016). Like the OC test, each class has its own test and threshold.

$$\text{IC(Tranche)} = \frac{\text{Interest proceeds from Collateral}}{\text{Interest due on the tranche and tranches senior to it}}$$

If the test fails, the procedure is similar to the OC test, as the CLO manager is required to divert interest and principal cash flows from more junior classes to pay down liabilities in order of seniority. This continues until the ratio is above the stated values in the CLO prospectus (Deutsche Bank, 2016).

6 The European CLO Market

Up until now, we have focused on describing the structure, assets, and lifecycle of a CLO. Based on the obtained knowledge, the following section provides an analysis of the European CLO market in the period from 2013 and until now. This period is also referred to as CLO 2.0. The analysis will be split up into three sections. First, we investigate the development in the European CLO market since the introduction in 1999 and through up and down periods. Secondly, we examine the new issuances of CLO 2.0 and look at the development in the characteristics such as tranche size, non-call periods, spreads, prepayment, etc. Finally, we analyze the rating agencies and ratings in relation to the CLO market. The findings of the chapter will be used to develop the structural model of a CLO in chapter 8 and to compare the results from the model.

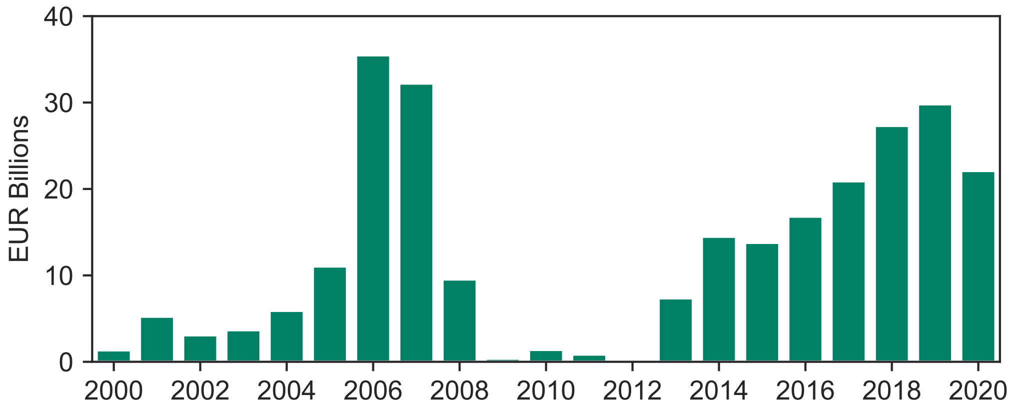
6.1 Data

The data used for the following analysis are from the CLO Global Databank provided by Standard and Poor (S&P). The databank contains information about new issues of American and European CLOs. The databank is divided into two datasets, where the first one is about the number of new CLOs and volume and cover the period from 1999 to 2021. The second dataset is more detailed and contains data about the CLO's tranches, number of tranches, tranche size, rating, coupon payment, etc. This dataset only covers the period from 2013 and until marts 2021. We will therefore focus on analyzing this period in the next section.

6.2 Development in the European CLO Market

Since the beginning of the 21st century, the European CLO market has experienced ups and downs. The development in new issuances of CLOs in Europe is depicted in figure 6.1 between 1999 and March 2021.

Figure 6.1: New CLO Issuances - Europe



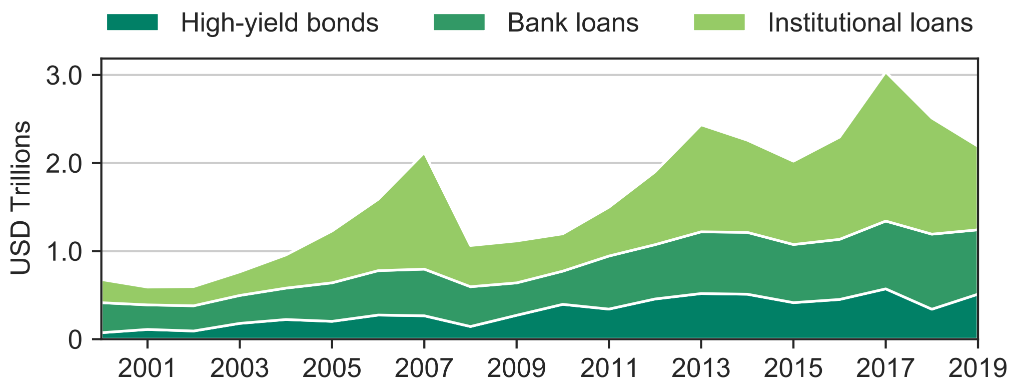
Source: Own creation based on (IMF, 2020)

After a slow start following the introduction of CLOs at the US market in late 1980, the European CLO market started growing at the beginning of the 2000s. In the following period, the market got affected by the Dot-com bubble, which resulted in a decrease from 2001 to 2002. Shortly after, the market recovered and started increasing again with the annual issuances by six from 2002 and until the onset of the Financial Crisis in 2006, where the market peaked with new issuances of EUR 35 billion. In the years around the Financial Crisis, the ABS market became the center of attention, where especially the MBS market experienced big difficulties. Similarly, the European CLO market was affected, and the new issuances fell from 30 billion EUR in 2007 to close to 0 in 2009. This continued in the following years. Coval et al. (2009) pointed out that:

> *"The ability to create large quantities of AAA-rated securities from a given pool of underlying assets is likely to be forever diminished, as the rating process evolves to better account for parameter and model uncertainty."*

In 2013, the European issuances restarted as a result of various factors. First, the availability of collateral changed. In the years following the Financial crisis, the available collateral in the form of leveraged loans was a scarce source (figure 6.2).

Figure 6.2: Global Gross issuance of High-Yield Bonds and Leveraged Loans



Source: Own creation based on IMF (2020)

From 2010 and onwards, the global gross issuances of leveraged loans started to grow again. This was primarily a result of two factors: 1) more legacy CLOs (CLO 1.0) were called by the equity tranche and 2) a rising supply of institutional leveraged loans, which were driven by M&A, LBO, and refinancing activity (Unicredit, 2015). Combined, this resulted in the institutional loans rising from the beginning of 2010 and around 2012, it was back at the pre-crisis level in 2007.

6.2.1 From CLO 1.0 to CLO 2.0

Another factor that affected the restart of the CLO market was the introduction of a new structure for CLOs called CLO 2.0. The structure was introduced in 2013 and included new regulations regarding risk retention. The regulation concerning risk retention was changed, and the rules required the CLO manager to purchase and retain a minimum of 5% of the issued CLO (see section 3.3.1).

In general, the structure of CLO 2.0 became more robust compared to pre-crisis equivalents. This included greater subordination levels, stricter collateral rules, shorter reinvestment periods, and new retention rules for the subordinated tranche (Unicredit, 2015). The most important changes are depicted in table 6.1.

Table 6.1: CLO 1.0 vs. CLO 2.0

	CLO 1.0 (Pre-crisis)	CLO 2.0 (Post-crisis)
Senior Attachment Point	Around 30%	At least 40%
Reinvestment period	5 years +	4 years
Non-call period	5-6 years	2 years
Repricing (Refi)	Not permitted	After Non-Call period
Risk retention	Risk not retained	Risk retention

Source: Own creation based on Deutsche Bank (2016) and Unicredit (2015).

From the table above, we find that the post-crisis structures have become more conservative in regards to the subordination levels. In CLO 1.0 issuances, the original credit enhancement levels for the AAA tranches were typically around 30-40 %, and especially in 2007 and 2008 the AAA attachment points were less than 30 % (Unicredit, 2015). Since, the senior attachment point have increased to average around 60 % in 2021 (section 6.3). Similarly, the equity tranche has increased from CLO 1.0 to 2.0. The structural shift in the structure is reflected in the CLO 2.0 leverage, which is the deal size divided by its equity size. In 2007-08, the leverage of European CLOs where around 10x to 15 x. In 2013, the leverage was around 5x, but through 2014 and 2015, the deals hit 10x leverage. This rapid rise in the leverage trending back to pre-crisis level can be a result of the investors becoming familiar with the new structure and having observed a solid performance of the CLO 2.0 (Unicredit, 2015). In 2020, the average leverage were 10x (S&P Global Market Intelligence, 2021a). Other changes to the structure include shorter Weighted Average Life (WAL) for all tranches except for single B tranches, introduction of fixed-rate tranches, and shorter reinvestment period and non-call periods (Unicredit, 2015).

In the years following the presentation of the new structure, the number of new issuances rose from 20 in 2013 to 72 in 2019 with a total volume of EUR 35 billion (figure 6.1). These number are higher than in the years up to the Financial crisis.

6.3 CLO 2.0

In this section, we analyze the new issuances in the period from January 2013 and until March 2021 based on the CLO 2.0. First, this section presents a general overview of the average issuances for each year in the period including size, maturity, reinvestment period, etc. Secondly, we take a closer look at the construction of the liabilities. This includes an analysis of the tranches, tranche spreads, prepayment, and defaults. Table 6.2 presents statistics for the CLO issuances between 2013 and 2021.

From the table, we find similar trends as presented in section 6.2. The numbers of issuances have been increasing since 2013 with the exception of 2020, where Covid-19 affected the CLO markets in march 2020 (Cordell et al., 2021). Furthermore, we find that the average size of the European CLOs has been stable in the interval between 370-420 million EUR for the period. Again, the CLO size in 2020 is an outlier with an average of 335, which we assume to be a result of the Covid-19 pandemic. Similar to the average size, the average maturity of the CLOs has been stable over the period. Next, the table presents the development in the reinvestment period and non-call period. We find a similar trend as presented in table 6.1. Since the introduction of CLO 2.0, the reinvestment period has been around four years compared to five or more years in CLO 1.0. Similarly, the CLO 2.0 non-call period falls drastically compared to CLO 1.0 and is in the period around 2 years and in some cases closer to 1 year. The final part of the table presents the development in the Weighted Average Cost of Debt (WACD). The WACD or the funding cost is the weighted average of the spreads paid to the different tranche investors. Looking at the trend, we find that the WACD has been decreasing through the period from around 200 bps to 165 bps in 2021. We will examine the spreads on the tranches closer in section 6.3.3.

6.3.1 Tranche size

Next, we investigate the development in the tranche size. In Europe, the CLOs create tranches with ratings from AAA to single B and a non-rated equity tranche. Often the liabilities in the CLO can have several tranches with the same rating and the typical CLO have between 7-9 (S&P Global Market Intelligence, 2021a). In this section, we will focus on the aggregate tranches in relation to the rating, meaning that if there are two tranches with AAA

Table 6.2: EUR CLO Issuance Statistics

Year	**Count**	**Size**	**Maturity**	**Reinvestment**	**Non-Call**	**WACD**
	No.	*EURm*	*Yrs*	*Yrs*	*Yrs*	*bps*
2013	*20*	*369*	*12.31*	*3.75*	*2.05*	*201*
		(85.4)	*(1.07)*	*(0.44)*	*(0.23)*	*(18.2)*
2014	*35*	*414*	*13.14*	*4.00*	*2.00*	*208*
		(77.6)	*(0.74)*	-	-	*(7.0)*
2015	*34*	*406*	*13.01*	*3.93*	*1.97*	*208*
		(64.0)	*(2.46)*	*(0.25)*	*(0.18)*	*(18.4)*
2016	*41*	*410*	*13.21*	*3.92*	*1.97*	*216*
		(49.2)	*(0.75)*	*(0.48)*	*(0.16)*	*(21.3)*
2017	*51*	*410*	*13.18*	*3.98*	*1.98*	*159*
		(50.1)	*(0.73)*	*(0.4)*	*(0.15)*	*(11.8)*
2018	*66*	*413*	*13.11*	*4.3*	*1.98*	*162*
		(36.6)	*(0.41)*	*(0.33)*	*(0.28)*	*(20.9)*
2019	*72*	*414*	*13.04*	*4.43*	*1.95*	*194*
		(34.7)	*(0.40)*	*(0.56)*	*(0.27)*	*(10.3)*
2020	*66*	*335*	*12.87*	*3.21*	*1.24*	*209*
		(70.4)	*(0.67)*	*(0.99)*	*(0.40)*	*(29.0)*
2021*	*17*	*388*	*13.57*	*4.25*	*1.49*	*165*
		(42.8)	*(0.51)*	*(0.32)*	*(0.24)*	*(7.4)*
Total	**402**	**396**	**13.06**	**3.98**	**1.83**	**191**
		(62.4)	*(0.98)*	*(0.68)*	*(0.39)*	*(28.4)*

Source: Data from S&P Global Market Intelligence (2021a), includes both EUR and GBP denominated CLO's. Size is in EUR equivalents.
Notes: (1) All columns except 'Year' and 'Count' are simple equal-weighted arithmetic averages. (2) Standard deviations in parentheses. (*) Data is only available for the period from January 1st, 2013 until March 19th, 2021.

rating, we are looking at the characteristic of the total AAA tranche. In table 6.3, the average tranche size given rating is presented for new issuances in Europe.

Table 6.3: Average Tranche Sizes

Rating		2013	2014	2015	2016	2017	2018	2019	2020	2021
AAA	Size	209.3	236.4	231.4	238.1	235.8	245.7	250.1	199.0	234.4
	%	55.7%	57.7%	57.0%	57.6%	57.5%	58.8%	60.2%	58.5%	60.4%
AA	Size	40.8	47.6	45.9	49.6	51.2	43.3	41.1	36.1	37.6
	%	10.9%	11.6%	11.3%	12.0%	12.5%	10.4%	9.9%	10.6%	9.7%
A	Size	24.2	24.6	24.0	23.8	25.7	27.6	26.8	24.0	26.9
	%	6.5%	6.0%	5.9%	5.8%	6.3%	6.6%	6.4%	7.1%	6.9%
BBB	Size	18.3	20.9	20.8	19.7	20.5	23.2	25.5	20.7	25.4
	%	4.9%	5.1%	5.1%	4.8%	5.0%	5.6%	6.2%	6.1%	6.5%
BB	Size	23.6	28.9	26.0	24.9	23.4	25.3	22.4	17.5	19.9
	%	6.3%	7.1%	6.4%	6.0%	5.7%	6.1%	5.4%	5.1%	5.1%
B	Size	12.8	13.2	11.9	11.1	11.6	12.0	10.3	8.0	10.8
	%	3.4%	3.2%	2.9%	2.7%	2.8%	2.9%	2.5%	2.4%	2.8%
Equity	Size	46.6	37.9	46.3	46.1	42.0	40.7	38.9	34.7	33.4
	%	12.4%	9.3%	11.4%	11.2%	10.2%	9.7%	9.4%	10.2%	8.6%

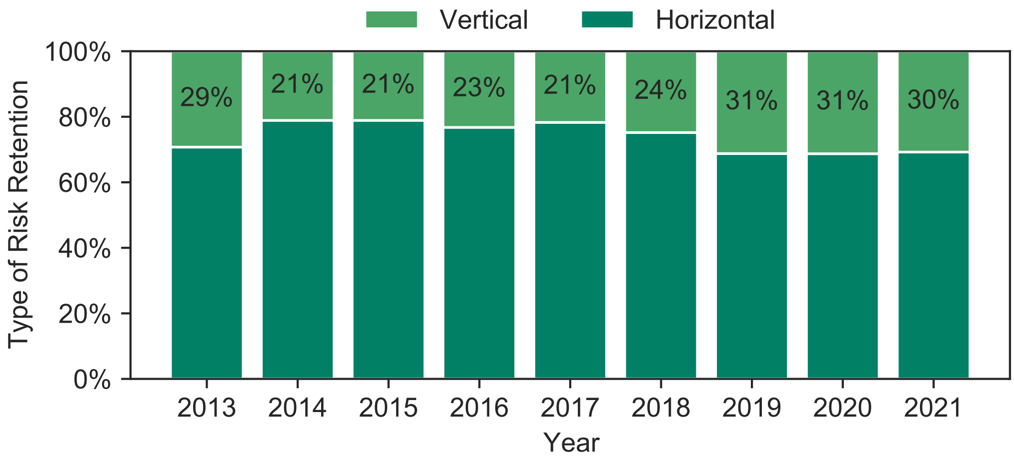
Source: Own creation on data from S&P Global Market Intelligence (2021a)

Looking at the table, we find that the percentage size of tranches throughout the period is stable. The AAA tranche is the largest and accounts for up 60 % of the total liabilities, while the other tranches except for the equity accounts for 10 % or less. The biggest fluctuations are in the AAA and equity tranche, which is a result of the new CLO 2.0 structure.

6.3.2 Risk retention

As described in section 3.3.1, the CLO manager is required to retain a credit of at least 5%. The CLO manager can do this by holding either a vertical slice or a horizontal slice. In figure 6.3, the risk retention for the new issuances of EU CLOs is presented.

Figure 6.3: Risk Retention, European CLOs



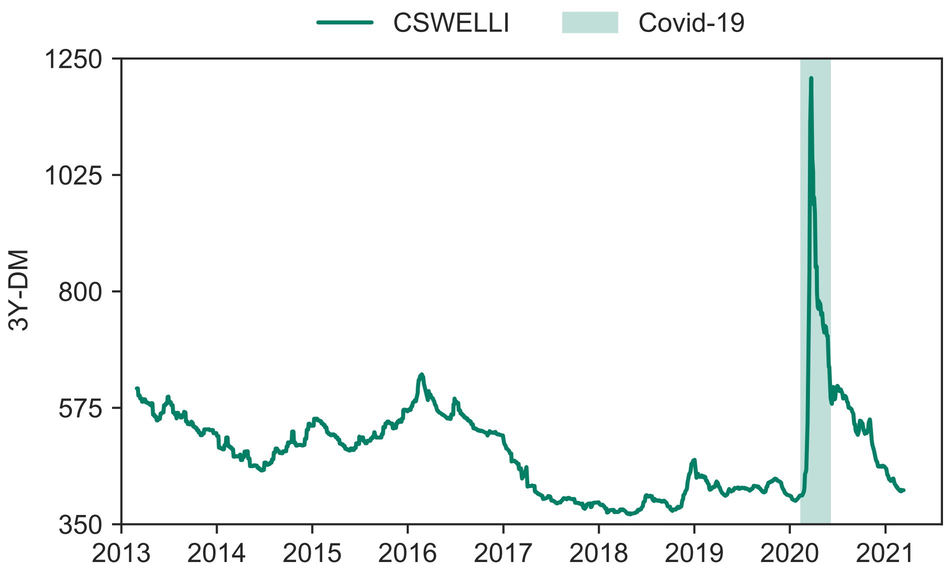
Source: S&P Global Market Intelligence (2021a)

The figure displays that since the introduction of risk retention with the new issuances in 2013, the use of vertical and horizontal risk retention has been stable with most using horizontal at around 70 % and around 30 % using vertical. The third possibility of L-shaped slice is not used in Europe, but can instead be observed in the US CLO market (S&P Global Market Intelligence, 2021a).

6.3.3 Margins

CLOs are structured to generate arbitrage by taking advantage of the difference between the payments associated with the collateral pool and the payments to the tranche holders. In figure 6.4, the 3 years Discount Margin (DM) is depicted. The DM is the average expected return above the reference rate that is earned for a floating-rate security, and the size of the DM depends on the price of the floating-rate security.

Figure 6.4: European Leveraged Loans Spreads



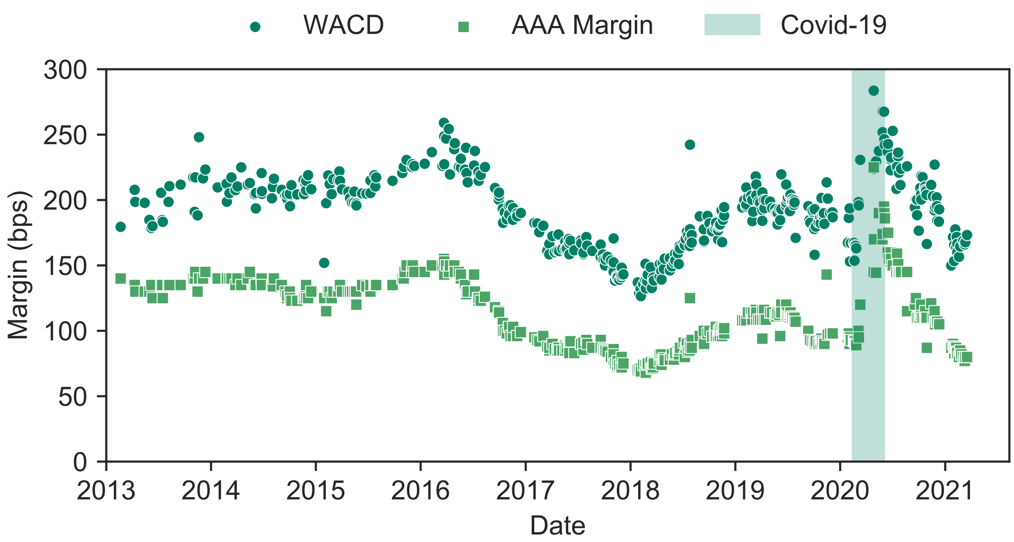
Source: Credit Suisse Western European Leveraged Loan Index, 3-year discount margin, from Credit Suisse (2021)

The figure builds on data from Credit Suisse Western European Leveraged Loan Index, which is designed to mirror the Western European leveraged loan market. The data in the index covers the period from January 1998 and until March 2021, but as we focus on the period between 2013 and 2021, the figure depicts this period. The index sets some rules for inclusion, which are as follows: 1) The issuer has assets located in, or revenues derived from Western Europe, 2) Loan facilities must be rated BB or lower. If unrated, the initial spread level must be LIBOR/EURIBOR plus 125 bps or higher, 3) The tenor must be at least one year, and 4) Minimum outstanding balance is $100 million dollars (Credit Suisse, 2021)

By examining figure 6.4, we find that the DM on the collateral has been falling throughout the period starting at around 600 bps in 2013. In the following years, the margin fluctuated between 450 and 650 bps between 2014 and 2016. From 2016 and until February/March 2020, the margin has been decreasing and was at the lowest in May 2018 at 370 bps. As a result of the Covid-19 pandemic, the leveraged loan market was hit hard by the uncertainty of the future, and the margin skyrocket to 1,200 bps on the 23rd of March 2020. Afterwards, the market has normalized, and in March 2021 the margin is close to being at the same level as in primo 2020.

Next, we look at the funding margins, which are the payments associated with tranches. The tranches have assigned different coupon payments dependent on the credit rating of the tranche. The AAA is the most senior, and therefore has the lowest coupon payment, while the single-B tranche is the least senior and most risky, and therefore has the highest coupon payment. To illustrate the average cost to the tranches, we use Weighted Average Cost of Debt (WACD). In figure 6.5, the WACD and the AAA margin is presented in the period from 2013 and until March 2021.

Figure 6.5: Weighted Average Cost of Debt and AAA margin



Source: Own construction on data from S&P Global Market Intelligence (2021a)

From the figure, we find a trend similar to the leveraged loans. In the period from 2013 and until 2016, the WACD is stable around 200 bps until mid 2015. Similar to the margin on collateral, the WACD rises in 2016, before it starts to fall between 2016 and 2017. The WACD continuous to decrease until 2018, where the WACD is around 150 bps. Again, there is an increase in the cost in the period up to 2020 before the new issuances stop at the beginning of the Covid-19 pandemic. Similar to the margin on collateral, the WACD has returned to pre-Covid-19 levels.

6.3.4 Redemptions in Practice

As introduced in section 3.4, the investors have the optionality to redeem the CLO liabilities. In the data from S&P Global Market Intelligence (2021a), we find three situations, where the CLO liabilities are redeemed before maturity. The two first cases are the Refinance and Reset as presented in table 3.2. The final case is a combination of Reset and Refinance, which we denote as Refi/Reset. In this case, the CLO structure is either first Refinanced, where one or more tranches have their margin reduced, and then at a later point the CLO is reset and hereby all existing notes are replaced or the situation can also be reversed, where the CLO is reset first, and then refinanced. In table 6.4, the number of refinance, reset, and refi/reset are presented for the European market.

Table 6.4: Redemptions in the European Market

	Refinance	Resets	Refi/Reset	Total
Number	45	61	38	144

Source: S&P Global Market Intelligence (2021a)

In the period from 2013 and until March 2021, there have been a total of 144 different CLOs, in which the equity have used their optionality. Of these CLOs some have reset up to three times.

6.3.5 Defaults

Since the introduction of CLOs in the late 1980s, the global CLO market has had a strong credit performance. In the period from 1996 and until 2019 only 21 (0.7 %) of the European CLOs tranches rated by S&P have defaulted, and globally the number of defaulted CLO tranches is 61 (0.4%). In table 6.5, the number of defaulted tranches per year is shown for the period from 1996 and until 2019.

Table 6.5: Global CLO Annual Default (Count)

1996-2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	Total
2	3	4	11	2	5	10	2	8	7	4	3	61

Source: Standard and Poor (2020a)

From the figure, we find that the defaults are concentrated in the period after the Financial Crisis, and were at the highest in 2011 with 11 tranches defaulting. All of the defaulted CLO tranches are from the CLO 1.0 vintage (table 6.6), and a total of 1.5 % of the CLO 1.0 tranches have defaulted. In the second section of table 6.6, the default given the rating category is presented.

Table 6.6: Default by Generation and Rating category

Generation	Ratings (no.)	Defaults (no.)	Defaults (%)
CLO 1.0	1,443	21	1.5
CLO 2.0	1,370	0	0
Rating	Ratings (no.)	Defaults (no.)	Defaults (%)
AAA	777	0	0
AA	532	0	0
A	456	0	0
BBB	489	3	0.6
BB	389	17	4.4
B	170	1	0.6
Total	2,813	21	0.7

Source: Standard and Poor (2020a)

Looking at the table, we find that zero tranches with rating AAA to A have defaulted in the period. The 21 defaults are divided between the BBB, BB, and B tranches, where the most come from the BB-rated tranche.

If we compare the credit performance of CLOs with the performance of corporates and speculative-graded (spec-grade) corporates, we find that the default rate for CLO tranches remains below corporates and spec-grade corporate for each year for the period from 2000 and until 2019. Even when the spec-grade tranches of the CLO are compared to the spec-graded corporates, we find that the default rate for spec-graded corporates is much higher. The highest annual global spec-grade default for CLO occurred in 2002 reaching 2.56 %, while the corporate default rate was around 10 % the same year.

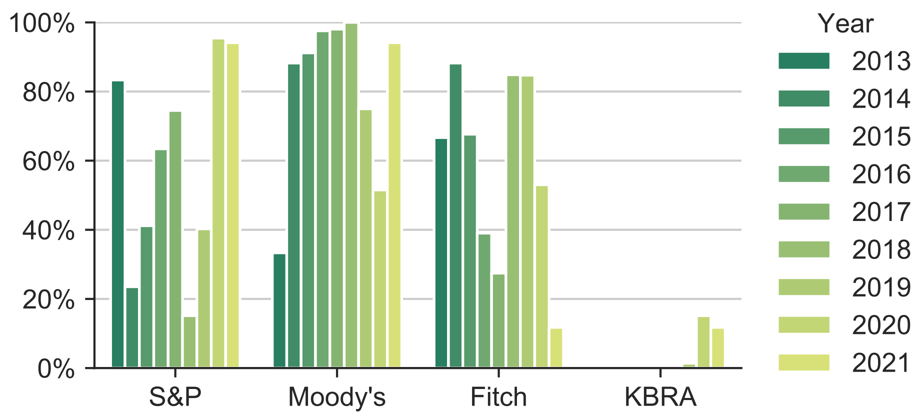
6.4 Rating and Rating agencies

In chapter 4, we described ratings as a measure for investors to evaluate credit risk. In this section, we briefly present the approached of the most commonly used rating agencies and introduce some important concepts.

6.4.1 Rating Agencies

Credit risk arises from the probability that borrowers and counterparties may default (Hull, 2017). There are several companies or agencies, who evaluate the credit risk. The three biggest credit rating agencies are Standard & Poors (S&P), Moody's and Fitch. These companies accounts for 90 % of the ratings conducted, and in the CLO market, they cover nearly 100 % of new issuances in Europe. In figure 6.6, the rating agencies used at issuance of new CLOs are presented for the European market.

Figure 6.6: European CLO Ratings at Issuance



Source: S&P Global Market Intelligence (2021a)
Note: Usually there are more than one rating agencies included in a new issues. Therefore, the sum of the agencies per year can be higher than 100%.

Looking at the figure, we find the development in the use of the rating agencies. Looking at Moody's and Fitch, we find that during the period, they have rated close to 90 % and 80% of all new issuances, while S&P was small at the beginning, but has increased to close to 100% in 2020 and 2021.

6.4.2 Rating Methodology

The rating agencies have various methodologies to rate the likelihood of default. The ratings are assigned on four general criteria:

1. Default/Loss Probability of the Portfolio
2. Cash Flow modeling
3. Collateral manager
4. Legal documentation

The biggest difference between the agencies is their approach to modeling the cash flow (Deutsche Bank, 2016):

Standard & Poors (S&P) use a three step approach to rate the cash flows from the CLOs. First, they conduct a supplemental stress test, which includes a test of the largest obligor and industries in relation to default. Secondly, they apply a CLO evaluator in which they use a Monte Carlo simulation adjusted for certain correlation assumptions to find the cumulative default rate for a given portfolio. Finally, at step three, they model the cash flows and the associated cash flows are stresstested using different default timing scenarious based on the Weighted Average Life (WAL) of the portfolio (Deutsche Bank, 2016).

Moody's bases their ratings on Expected Loss (EL), which is found using a cash flow model consisting of two elements: 1) Determining the default distribution of a portfolio of assets and 2) calculating the EL under each default scenario using the cash flow model. To determine the portfolio's default distribution, Moody's relies on the binomial distribution and three key metrics of the CLO portfolio's assets: Weighted average default probability (WARF), Weighted average life (WAL), and a diversity score. The WARF is calculated as the par-weighted average rating factor of each of the assets in the portfolio. Similarly, the WAL of the portfolio is found using the remaining par-weighted lives of the individual assets. Given the WARF and the WAL, the default probability is found using the idealized expected default rate table. Using the default probability, the diversity score, and the recovery rate, the expected loss for each tranche is found. This is done by incorporating the above parameters into a model that calculates expected loss for each rated CLO liability (Moody's Investors Service, 2020).

Fitch uses a portfolio credit model approach, which is similar to a Monte Carlo simulation. Various inputs such as asset type, county, industry, etc. are combined with the default probability, recovery rate, asset correlation and asset par value assumptions in the model. Fitch then uses a Gaussian copula function to generate the distribution of possible portfolio rates (Deutsche Bank, 2016).

6.4.3 Cumulative Default Rates and Expected Loss

In this section we provide a brief introduction S&P and Moody's use of cumulative default rates and expected loss for credit ratings.

Both rating agencies report either historical or idealized cumulative default probability tables, presenting the cumulative default rate for a given period, based on a specific initial rating. In table 6.7 below, we have data from 1981 to 2020, and as an example a corporate with initial credit rating AA has a chance of 0.02% of defaulting within the first year, 0.06% within the first two, etc. The conditional probability or hazard rate, of a corporate defaulting between two years is computed as the difference between the cumulative default rates for any two years. In table 6.7 S&P presents their historical observed default rates and in panel A of table 6.8, the idealized cumulative default rate by Moody's are presented.

From the tables, we see that firms with an initial investment-grade rating has increasing hazard rates over time, while the more speculative rated firms, have decreasing hazard rates over time. This is because of rating migration, where the creditworthiness of the investment-grade companies deteriorates, whereas for the initially risky firms it improves, hence the longer a firm survives, the higher the likelihood of changes in its financial condition.

Another important measure, which is used by Moody's for assigning ratings, is Expected Loss (EL), which is computed as:

$$\text{EL} = \text{Probability of Default} \times \text{Loss Given Default} \times \text{Exposure at Default}$$

This means that EL equals the chance of default multiplied with what is lost in the case of default and the exposure at default. The EL can be interpreted as a percentage of loss on which the exposure is applied to create an absolute number (Moody's Investors Service, 2020). In panel B. of table 6.8, we have presented Moody's idealized cumulative expected loss rates.

Table 6.7: S&P Global Corporate Average Cumulative Default Rates (%)

Rating	1	2	3	4	5	6	7	8	9	10
AAA	0.00	0.03	0.13	0.24	0.34	0.45	0.51	0.59	0.64	0.70
AA	0.02	0.06	0.11	0.21	0.30	0.41	0.49	0.56	0.63	0.70
A	0.05	0.13	0.22	0.33	0.46	0.60	0.76	0.90	1.05	1.20
BBB	0.16	0.43	0.75	1.14	1.54	1.94	2.27	2.61	2.94	3.24
BB	0.63	1.93	3.46	4.99	6.43	7.75	8.89	9.90	10.82	11.64
B	3.34	7.80	11.75	14.89	17.35	19.36	20.99	22.31	23.50	24.62

Note: From S&P's historical cumulative default probabilities, we see an irregularity in the relation between ratings and default rates: The default probability of an AAA-rated firm after six years is 0.45%, while the default probability for an AA-rated company is 0.30%. This is a result of either a low historical number of defaults, as described in (S&P Global Ratings, 2021).
Source: S&P Global Market Intelligence (2021a), for the period 1981-2020

Table 6.8: Moody's Idealized Cumulative Default and Expected Loss Rates

Panel A: Moody's Idealized Cumulative Expected Default Rates

Rating	1	2	3	4	5	6	7	8	9	10
AAA	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01
Aa	0.00	0.01	0.03	0.05	0.07	0.09	0.11	0.14	0.16	0.20
A	0.01	0.07	0.22	0.35	0.47	0.58	0.71	0.83	0.98	1.20
Baa	0.17	0.47	0.83	1.20	1.58	1.97	2.41	2.85	3.24	3.60
Ba	1.56	3.47	5.18	6.80	8.41	9.77	10.70	11.66	12.65	13.50
B	7.16	11.67	15.55	18.13	20.71	22.65	24.01	25.15	26.22	27.20

Panel B: Moody's Idealized Cumulative Expected Loss Rates

Rating	1	2	3	4	5	6	7	8	9	10
AAA	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
Aa	0.00	0.00	0.01	0.03	0.04	0.05	0.06	0.07	0.09	0.11
A	0.01	0.04	0.12	0.19	0.26	0.32	0.39	0.46	0.54	0.66
Baa	0.09	0.26	0.46	0.66	0.87	1.08	1.33	1.57	1.78	1.98
Ba	0.86	1.91	2.85	3.74	4.63	5.37	5.89	6.41	6.96	7.43
B	3.94	6.42	8.55	9.97	11.39	12.46	13.21	13.83	14.42	14.96

Source: Moody's Investor Service (2016), for the period 1981-2016

Part III

A Structural Model

7 Theoretical Foundation

In this chapter, we present the models that form the theoretical foundations, necessary to develop an appropriate model of a CLO securitization. We begin the chapter by broadly describing how credit risk can be modeled using different approaches, namely, structural and reduced-form models. Next, we elaborate on the notion of contingent claim pricing and how this topic relates to structural models in general. Hereafter, we introduce Merton's Model (1974) and an important extension with both systematic and idiosyncratic risk in more detail. Finally, the concept of Monte Carlo simulations are briefly introduced.

7.1 Credit Risk

Credit risk concerns the possibility that a security issuer does not meet its obligations by either being incapable of repaying its loans or meeting its contractual agreements. This risk can be modeled using various types of models, where the two broader categories being structural models and reduced-form or intensity models.

7.1.1 Structural Models

Structural models are derived from theory, and for credit risk specifically, they are characterized by the assumptions about how the firm asset value evolves. The central model is from the seminal work by Robert C. Merton (1974) and is based on the principles of option pricing (Black and Scholes, 1973). Merton's model is described in more detail in section 7.3. Altman et al. (2019) subdivide structural models into two distinct generations of models.

First-generation structural models include the original Merton's model, with the basic intuition that default occurs if the market value of the firm's assets is lower than its liabilities at maturity. Thus, the payoffs to debt holders are the smaller of the face value of the debt or the market value of firm assets and payoffs to shareholders following a similar option-like nature. Other first-generation models include those of Black and Cox (1976), Geske (1977), and Vasicek (1984). These models do all have in common that they involve assumptions about the structural characteristics of the firm: asset volatility and capital structure. All first-generation models involve refinements on the original Merton's framework, by removing one or more of the unrealistic assumptions underlying it. A key characteristic of first-generation models is an endogenous Recovery Rate (RR). In the basic Merton's model, the RR is typically inversely related to the Probability of Default (PD). This inverse relation is consistent with empirical findings, but also very simplified, as any recovery in case of default is only realized at maturity a characteristic clearly at odds with reality (Altman et al., 2019). Also, the lognormal characteristics in the basic Merton model tend to overstate recovery rates, relative to a more fat-tailed distribution of asset returns (Altman et al., 2004).

Second-generation structural models have been developed in response to the difficulties of Merton's framework, namely the unrealistic assumption that default can only occur at maturity. In second-generation models is triggered when the value of the firm's assets reaches a threshold level, which dependent on the model is either exogenously or endogenously given. Second-generation models include Hull and White (1995), Longstaff and Schwartz (1995), Leland and Toft (1996), and Collin-Dufresne and Goldstein (2001). An empirical analysis of several of these models is found in Eom et al. (2004). Unlike most of the first-generation models, the recovery rate in these models is exogeneous and often independent of firm asset value. Longstaff and Schwartz (1995) argue for using historical default and recovery rates to form reliable estimates of RR, whereas Collin-Dufresne and Goldstein (2001) assumes a more general model, with a stochastic default threshold, allowing for a mean-reverting process for the leverage ratio, consistent with empirical findings (Altman et al., 2019). Leland (1994) and Leland and Toft (1996) are considered endogenous bankruptcy-barrier models, where default happens when firm asset value hits an endogenous default boundary, which is found by maximizing equity value.

Although second-generation models present improvements to the first-generation ones, they still suffer from three main drawbacks: (i) They require assumptions about the evolution of firm asset value, (ii) they do not incorporate credit-rating changes, at odds with reality, as firms are most often downgraded before a default occurs and (iii) structural models assume that firm value evolves in continuous time and as a result default events can be predicted with high accuracy just prior to default. Duffie and Lando (2000) introduce a model with incomplete information, which introduces jumps into the dynamics of the model and thereby allows for sudden "surprise" defaults.

7.1.2 Intensity Models

The second type of credit risk models is Intensity models. We will in this thesis focus on structural models, specifically an extension to Merton's model, and intensity models are therefore only briefly introduced. Intensity models have been developed to overcome the shortcoming of structural models and to be more applicable to real-world modeling. Intensity models are also known as reduced-form models and are characterized by a different approach to modeling, where endogenous variables are evaluated in terms of observable exogenous variables.

Examples of intensity models are: Litterman and Iben (1991), Jarrow and Turnbull (1995), Jarrow et al. (1997), Lando (1998), and Duffie and Singleton (1999). The common characteristic of the intensity models is that these models do not condition default on the value of the firm, and thus, no assumptions about asset dynamics have to be made. Instead, explicit assumptions on the dynamics of PD and RR have to be made, either separately or collectively and this is where the term 'intensity model' comes from. The models assume a stochastic process for the default intensity (or hazard rate), i.e. the arrival rate of default. For each instant in these models, there is some probability that a firm defaults (conditional on no earlier default), and these intensities are then used for pricing the credit risk. An advantage of the intensity models is that they allow for 'surprise' defaults.

Intensity models differ from structural models in how recovery rates are treated and there is large heterogeneity in how they treat RR. Some efforts have been made to combine the advantages of structural and intensity models: One attempt can be found in Zhou (2001), where jumps are introduced to the evolution of asset value. In this model variation in RR is endogenously generated, consistent with the empirical relation between recovery rates and credit ratings (Gupton et al., 2000; Altman et al., 2005).

7.2 Contingent Claims

An important characteristic of structural models is that equity and debt are viewed as contingent claims on the asset of the firm. "A *contingent claim* is an asset whose payoff depends upon the value of another 'underlying' asset, the value of which is exogenously determined $(\ldots)$" (Brennan, 1979).

Thus, if the firm asset value is exogenously given and not affected by the funding mix, the firm can issue two types of claims: equity and debt. In the simple model, the firm issues a zero-coupon loan at time $t = 0$, with maturity T and face value B. Under these assumptions, the payoffs on the claims at time $t = T$ are:

$$D(T) \equiv \min(B, V(T)) = B - \max(B - V(T), 0) \tag{7.1}$$

$$E(T) \equiv \max(V(T) - B, 0) \tag{7.2}$$

These payoffs are identical to those of European options at expiration, where the payoff on equity is identical to that of a call option with strike price B, and the payoff on debt is identical to the payoff from holding two securities: a risk-free bond with face value B and a put option with the same strike price. This is also the finding that led to the development of the Black-Scholes-Merton (BSM) methodology and the Merton's model, described in more detail in sections 7.3 and 7.4.

Table 7.1: Claim Payoffs for Different Realizations of $V(T)$

Tranche	i	$[0, \bar{\mathcal{B}}_1)$	$[\bar{\mathcal{B}}_1, \bar{\mathcal{B}}_2)$	$[\bar{\mathcal{B}}_2, \infty)$
Senior	1	$V(T)$	$\mathcal{B}'_1$	$\mathcal{B}'_1$
Subordinated	2	–	$V(T) - \bar{\mathcal{B}}_1$	$\mathcal{B}'_2$
Equity		–	–	$V(T) - \bar{\mathcal{B}}_2$

Note: Payoffs to each tranche in a stylized three tranche firm: $V(T)$ denotes the asset value at time T. $\mathcal{B}'_i$ denotes the incremental thickness of each tranche, whereas $\bar{\mathcal{B}}$ denotes the cumulative principal at the subscript-level (from most to least senior), e.g: $\bar{\mathcal{B}}_2 \equiv \mathcal{B}'_1 + \mathcal{B}'_2$.

7.2.1 Tranching and Subordinated Debt

The payoffs outlined in equations 7.1 and 7.2 can be generalized to a case where the firm issues debt of different seniorities. This is summarized in table 7.1, where the payoffs to 'senior' debt with a higher claim than 'subordinated' (or 'junior') debt, as well as the payoff to equity, is illustrated for different realizations of asset value $V(T)$. The option-like payoff structure is the same as in the single-class debt case outlined in table 7.1.

Table 7.1 is inspired by table 2.1 in Lando (2004), but the notations have been changed slightly. In this thesis, when the calligraphic $\mathcal{B}$ is used, we refer to the face value of tranched debt.

Acknowledging that the payoffs for a corporate issuer with debt tranches of different seniority have similar option-like characteristics as the payoff on claims from a single-tranche issuer, the payoffs can be generalized to represent these characteristics. Also, the option representation can be extended to the I-tranche case. This representation is provided in table 7.2 below.

Table 7.2: Option Representation of Claim Payoffs

Tranche	Payoffs
$i=1$	$\min\left(V(T)\,,\,\bar{\mathcal{B}}_1\right)$
$i=2$	$\max\left(V(T)-\bar{\mathcal{B}}_1\,,0\right)-\max\left(V(T)-\bar{\mathcal{B}}_2\,,0\right)$
$\vdots$	$\vdots$
$i=I$	$\max\left(V(T)-\bar{\mathcal{B}}_{I-1}\,,0\right)-\max\left(V(T)-\bar{\mathcal{B}}_I\,,0\right)$
Equity	$\max\left(V(T)-\bar{\mathcal{B}}_I\,,0\right)$

Note: The payoffs from immediate redemption to each tranche and equity. Notation is identical to table 7.1, i.e. $\bar{\mathcal{B}}_I \equiv \left(\sum_{i=1}^{I}\mathcal{B}'_i\right)$, where $\mathcal{B}'_i$ is the size of tranche i.

7.2.2 CLO's as Contingent Claims on Contingent Claims

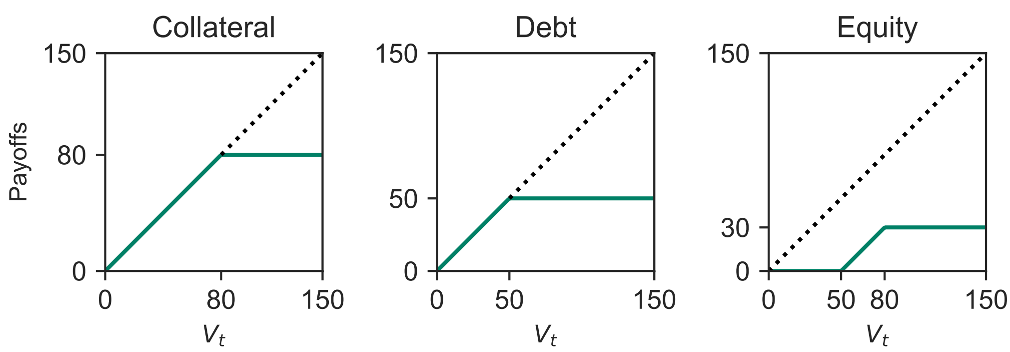
Following Merton (1974), the standard approach has been to use the Black-Scholes-Merton (BSM) methodology to price the claims on an asset whose underlying asset is assumed to follow a Geometric Brownian Motion, with log-normally distributed asset returns. Assuming, for now, that this provides a useful approximation for the assets of a corporate issuer, i.e., a non-financial firm with outstanding debt, then it is self-evident that this cannot hold for a securitization of corporate debt.

If a CLO holds collateral in the form of loans from corporate issuers with the aforementioned attribute of log-normally distributed asset returns and those loans have non-linear payoffs, like those represented in table 7.1 and 7.2. The claims on a CLO must be “contingent claims on contingent claims”, whose upside is limited at the principal for the zero-coupon debt case and as such is not consistent with the unlimited upside implied by a log-normal distribution.

To illustrate the payoffs on CLO claims, figure 7.1 shows the payoff on collateral, debt, and equity as a function of the asset value of the single corporate issuer, whose loan is held as collateral in the securitization. This figure is adapted from figure 1 in Nagel and Purnanandam (2019), who describes the “contingent claims on contingent claims” characteristic for banks.

In conclusion, we note that CLO assets, being contingent claims themselves, follow very different processes than assumed in the classical Merton’s model and instead evolve in a non-linear and asymmetric fashion with an altered risk profile. However, if the assumption underlying Merton’s model provides for an useful approximation on corporate issuers, then all hope is not lost, and we can still generate valuable results on securitizations.

Figure 7.1: CLO Payoffs from Different Realizations of V_t



Note: Payoffs at maturity for a CLO with total debt outstanding of $\mathcal{B} = 50$. The only collateral is a single zero-coupon loan from a corporate issuer with asset value V_t and outstanding debt of $B = 80$. In this case the payoff on the loan is ‘capped’ at B and the collateral clearly have an asymmetric payoff structure, affecting the payoff on the CLO claims.

7.3 Merton's Model

Next, we outline Merton's model (1974) and the BSM framework and describe how contingent claims can be priced using the model. The main assumptions are an arbitrage free and complete market. The latter is an important assumption for the model and entails that all contingent claims can be replicated. Both assumptions are necessary to price securities using the risk-neutral probability measure, Q. Specifically, if the assumptions hold, the fundamental theorems of asset pricing implies that contingent claims can be priced as the discounted expected payoff under this measure.

If we want to price contingent claim on a corporate issuer whose assets follow a Geometric Brownian Motion (GBM), represented by the following Stochastic Differential Equation (SDE):

$$dV(t) = \mu V(t)dt + \sigma V(t)dZ(t), \quad V_0 = V(0) \tag{7.3}$$

where $V(t)$ is the asset value at price t, μ is the drift, $\sigma > 0$ is the volatility and $Z(t) \sim \mathcal{N}(0, t)$ represents a standard Brownian motion under the physical measure, P, with zero mean and variance t.

Letting V_0 denote the starting value of assets, then the solution to equation 7.3, in the Itô sense, is:

$$V(t) = V_0 \exp\left[\left(\mu - 0.5\sigma^2\right)t + \sigma Z(t)\right], \quad V_0 = V(0) > 0 \tag{7.4}$$

This implies that the increments in the logarithm of asset values are independent normally distributed random variables, i.e.:

$$\ln V(t) - \ln V(s) \overset{iid}{\sim} \mathcal{N}\left(\mu - 0.5\sigma^2 t,\, \sigma^2(t-s)\right)$$

Another assumption in the model is the existence of a money-market account with a constant risk-free rate, r_f, whose price evolves deterministically as: $\beta(t) = \exp[r_f\, t]$, meaning that one unit invested in the money-market account at time 0 will grow to $\exp[r_f\, t]$ at time t if no money is withdrawn.

If, for now, we assume that an economy consist just of these two assets, then the initial price of a contingent claim with payoff $X(T) = X(V(T))$, at time T, is equal to:

$$X_0 = \mathbb{E}^Q\left[X(T)\right]\, e^{-r_f T}, \quad X_0 = X(0) \tag{7.5}$$

where Q is the equivalent martingale measure or risk-neutral measure. Then, if μ is replaced by r_f in equation 7.4, we find the dynamics of V represented under the risk-neutral measure as:

$$V^Q(t) = V_0 \exp\left[\left(r_f - 0.5\sigma^2\right) t + \sigma Z(t)\right] \tag{7.6}$$

A critical assumption about these asset dynamics is independence from any financing decisions made by the asset manager or firm owner. If the firm issue two types of claims, at time $t = 0$: debt and equity, where for tractability we assume that debt is a zero-coupon loan with face value B and maturity T. Then, the payoffs to these claims, at time T, are represented by equation 7.1 and 7.2 from the previous section:

$$D(T) \equiv \min\left(B, V(T)\right) = B - \max\left(B - V(T), 0\right)$$

$$E(T) \equiv \max\left(V(T) - B, 0\right)$$

Assuming familiarity with the Black and Scholes (1973) approach to pricing European options on an asset following a Geometric Brownian Motion, and recognizing the option-like payoffs on the equity and debt claims, we can then price the claims at all periods, $t = 0, \ldots, T-1$, using the framework, where equity is priced as a call option on the firm's assets and debt is priced as a risk-free bond and a put option on the firm's assets.

For now, we have assumed a single-class of debt, but as described in section 7.2, it is straightforward to extend the framework to multiple classes of debt with different seniorities. The important assumptions are no bankruptcy costs in case of default, no taxes and no dependence between funding mix and asset value.

Black-Scholes Option Pricing

Under the aforementioned assumptions, and given the asset price $V_0 = V(0)$, constant volatility σ and risk-free rate r_f, we can price a European call option using the Black-Scholes 1973 equation:

$$C^{BS}(V(t), B, T, t, \sigma, r_f) = V(t)\Phi(d_1) - Be^{-r_f T}\Phi(d_2) \tag{7.7}$$

where Φ denotes the cumulative distribution function of the standard normal distribution and:

$$d_1 = \frac{\ln\left(V(t)/B\right) + \left(r_f + 0.5\sigma^2\right)(T - t)}{\sigma\sqrt{T - t}}$$

$$d_2 = d_1 - \sigma\sqrt{T - t}$$

For simplicity, we denote the Black-Scholes prices of a call and put option, with identical expiry and strike price, C^{BS} and P^{BS} respectively. Hence, the put-call parity can be expressed as:

$$C^{BS}(V(t)) - P^{BS}(V(t)) = V(t) - Be^{-r_f(T-t)} \tag{7.8}$$

From this, we can represent the equity and debt prices in Merton's model:

$$\begin{aligned} D(t) =& Be^{-r(T-t)} - P^{BS}(V(t)) \\ E(t) =& C^{BS}(V(t)) \end{aligned}$$

Yields and Spreads in Merton's Model

Using the values from Merton's model, the continuously compounded yield from period t to maturity T can be computed as:

$$y\left(t,T\right) = \frac{1}{T-t} \ln \frac{B}{D\left(t\right)} \tag{7.9}$$

and the yield spread is computed by subtracting the risk-free rate:

$$s\left(t,T\right) = y\left(t,T\right) - r_f \tag{7.10}$$

7.4 An Extension to Merton's Model

The original Black-Scholes-Merton (BSM) framework involves pricing contingent claims for a given volatility and drift of the underlying asset, with no specification about whether the volatility comes from systematic or idiosyncratic risk. However, when pricing financial securities in general, we know conceptually that only systematic risk should be rewarded, as idiosyncratic risk can be diversified away.

Capital Asset Pricing Model (CAPM)

Before presenting the extension to Merton's Model, we will briefly present the intuition of the Capital Asset Pricing Model, which relates the expected return on assets with the volatility of the return. The model separates risk into two components: systematic risk, which relates to the market returns, and idiosyncratic risk, which is unique to specific assets and can be diversified away by holding a sufficiently large portfolio of different assets.

The CAPM formula for expected returns on asset i is:

$$\mathbb{E}\left[r_i\right] = r_f + \beta_i \mathbb{E}\left[r_m - r_f\right] \tag{7.11}$$

where β_i is the CAPM beta coefficient linking asset returns and volatility to the market, $\mathbb{E}[r_m]$ is the expected return on the market portfolio and r_f is the risk-free rate. An asset's beta is a way to measure the sensitivity of its return to returns from the market. (Hull, 2017).

Merton's Model with Beta*

Building on a continuous-time CAPM framework and assuming that the market portfolio follows a Geometric Brownian Motion, then the asset value of the market portfolio, V_m, evolves under the physical probability measure as:

$$dV_m(t) = r_m V_m(t)dt + \sigma_m V_m(t)dZ_m(t) \tag{7.12}$$

The market portfolio represents purely systematic risk, meaning all uncertainty is driven by a single Brownian motion, $Z_m(t) \sim \mathcal{N}(0,t)$. The market portfolio's instantaneous Sharpe ratio is:

$$\lambda_m = \frac{(r_m - r_f)}{\sigma_m} \tag{7.13}$$

Assuming the CAPM holds, and noting that only idiosyncratic risk is rewarded, we should expect a higher Sharpe ratio on the market portfolio, than on any single asset.

Returning to the Merton's with both systematic and idiosyncratic risk, the asset value of a firm j is assumed to evolve as:

$$dV_j(t) = \mu_j V_j(t)dt + V_j(t)\left(\beta_j \sigma_m dZ_m(t) + v_j dZ_j(t)\right) \tag{7.14}$$

Then, analogously we find the solution to this SDE, in the Itô sense, is:

$$V_j(t) = V_{j,0} \exp\left[\left(\mu_j - 0.5\sigma_j^2\right)t + \beta_j \sigma_m Z_m(t) + v_j Z_j(t)\right] \tag{7.15}$$

where firm j's initial asset value is $V_{j,0} > 0$ with a drift of $\mu_j = r_f + \beta_j(r_m - r_f)$ and total volatility:

$$\sigma_j = \sqrt{\beta_j^2 \sigma_m^2 + v_j^2}$$

which is the volatility one should use when pricing claims on firm j's assets.

Using this extension to Merton's model, we can construct a scenario in which two firms have different idiosyncratic risks, and hence different default probabilities, but have the same total volatility, and hence the same yield spreads. Or the opposite scenario, where the two firms have identical default probabilities, but different yield spreads. Finally, we can analyze loans from both low- and high beta firms in a portfolio, which is an important feature for our model of a securitization.

*This subsection follows the description of Merton's Model with Beta in Lando (2021) closely.

7.5 Monte Carlo Simulation

Monte Carlo methods are a broad class of numerical algorithms that are based on simulations of random variables to generate outcomes on specific events. Specifically for our project, we use the numerical package NumPy in Python to generate random outcomes of stochastic variables, which are used as inputs for our models described throughout this paper.

By generating a sufficiently high number of outcomes, we can compute reliable approximations of the expected payoff on the securities under investigation, under both the physical P and risk-neutral Q measures.

Law of Large Numbers (LLN)

To compute reliable approximations on the distribution and moments of a random variable, we can take advantage of the law of large numbers: If X is a random variable, e.g. a payoff on a contingent claim at a specific time, then the expected value $\mathbb{E}(X)$, can be estimated using Monte Carlo Simulation. By generating identically independently distributed samples of X: $X_1, \ldots, X_n$, we can approximate $\mathbb{E}(X)$ by the sample average, $\bar{X}$:

$$\mathbb{E}[X] \approx \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \tag{7.16}$$

then, by the law of large numbers, we find that $\bar{X} \to \mathbb{E}(X)$ as $n \to \infty$.

Applying Monte Carlo Simulation for Pricing Contingent Claims

Monte Carlo methods are widely used in finance for pricing contingent claims or derivatives. In this thesis, we will use Monte Carlo methods to simulate the sample paths of a corporate issuer, whose asset value follows a Geometric Brownian Motion (GBM). As an intuitive example of this approach, we begin by assuming that the assets of firm j follow a GBM, as represented by equation 7.4.

Noting that this expression have a single stochastic component: $Z(t) \sim \mathcal{N}(0, t)$, the distribution of which, is also the distribution of $Z_x\sqrt{t}$, where Z_x is a standard normal random variable with zero mean and unit variance. Hence, we can represent the asset value, at the terminal period $t = T$, under the risk-neutral measure, Q, as:

$$V^Q(T) = V_0 \exp\left[\left(r_f - 0.5\sigma^2\right)t + \sigma Z\sqrt{T}\right] \tag{7.17}$$

Hence, the logarithm of the asset value is normally distributed and the discounted expectation of the payoff on a contingent claim on the asset is an integral with respect to the lognormal density of $V(T)$. As an example: if a firm's assets evolve as presented in equation 7.17, then the value of its equity is:

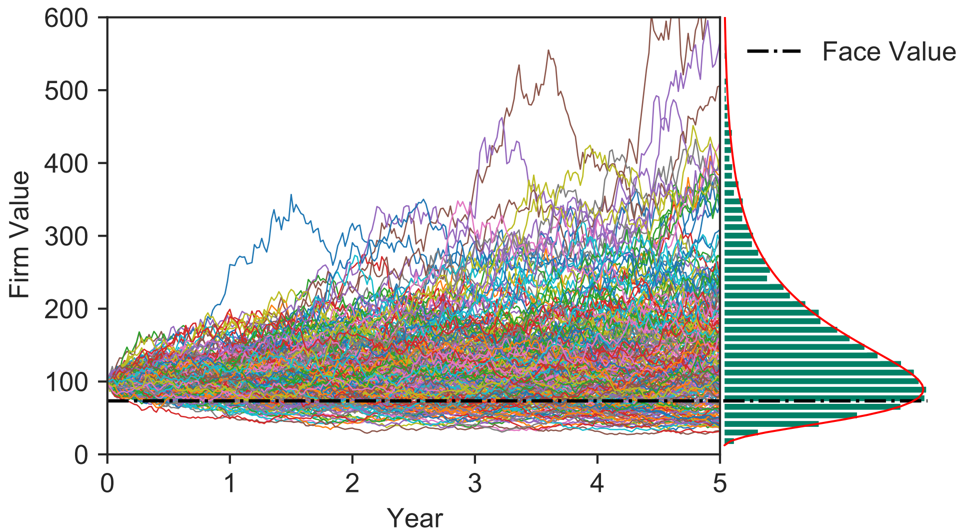
$$E_0 = \mathbb{E}[\max(V(T) - B, 0)]e^{-r_f T} \tag{7.18}$$

which can be approximated by computing the estimator $\hat{E}_0$ using a very simple algorithm inspired by Glasserman (2003):

Simple Monte Carlo Algorithm
for $i = 1, \ldots, n$:
    Generate Z_i
    set $V_i^Q(T) = V_0 \exp\left[\left(r_f - 0.5\sigma^2\right)t + \sigma Z_i \sqrt{T}\right]$
    set $E_{0,i} = \max\left[V_i^Q(T) - B, 0\right]e^{-r_f T}$
Compute $\hat{E}_0 = (E_{0,1} + \ldots + E_{0,n})/n$

The estimator $\hat{E}_0$ is unbiased for $n \geq 0$, and its also strongly consistent, as $\hat{E}_0 \to E_0$, as $n \to \infty$.

Figure 7.2: Monte Carlo Simulations of a Geometric Brownian Motion



Note: $N = 250{,}000$ sample paths of the asset values from our simulations in chapter 8. The parameters for the GBM, under the physical measure, P, are: $V_0 = 100$, $\mu = 0.091$, total volatility $\sigma = 0.274$ and face value of $B = 73.44$, implied by the default probability of $\pi(k, T) = 17.35\%$ for a rating k and maturity $T = 5$. The log-normal distribution is evident from the histogram on the right-hand side.

8 A Structural Model of the CLO

In the following chapter, we develop a model that incorporates features from the CLO structure to price and determine spreads on CLO tranches. Specifically, the aim of the model is to investigate how the introduction of prepayment to the CLO affects the spreads on the CLO tranches.

The chapter is divided into six sections, where we will expand the model step by step to incorporate the prepayment risk. The first section introduces the dynamics of a corporate issuer. Herein, we build a model using Merton's model 1974 to determine how the spreads on the debt changes when we introduce callable loans. Using the dynamics of a corporate loan issue, the next section expands our framework to include several corporate issuers. This part of the model builds on the approach presented by Brennan et al. (2009). Next, we create the base case, in which we simulate and price the tranches of a SPV without prepayment. Finally, we introduce collateral prepayments into the model. To do this, , we add assumptions to the model and change the simulation process. Using these changes, we find the values of the SPV, when we introduce collateral prepayments.

Adjusted Default Rates

We will use the corporate average cumulative default rates as an input in our model. As presented previously in section 6.4, the default rates of AAA and AA are contradictive since the cumulative default rate from year 3 and until year 6 is higher for AAA than for AA. S&P argues this to be a result of the underlying population being very small, resulting in counter-intuitive historical default rates. However, this does not imply that AAA rated companies are riskier than AA rated companies. To deal with this problem, we have modified the default rates for the AAA rated companies. This is done by using the relationship between AAA and AA rated companies in the US market and use this factor to adjust the default rates for the European market. The result is presented in table 8.1, where the adjusted default rates are marked as bold.

Table 8.1: Adjusted Historical Default Rates

Rating	1	2	3	4	5	6	7	8	9	10
AAA	0.00	0.03	**0.10**	**0.19**	**0.27**	**0.37**	**0.41**	**0.46**	**0.52**	**0.58**
AA	0.02	0.06	0.11	0.21	0.30	0.41	0.49	0.56	0.63	0.70
A	0.05	0.13	0.22	0.33	0.46	0.60	0.76	0.90	1.05	1.20
BBB	0.16	0.43	0.75	1.14	1.54	1.94	2.27	2.61	2.94	3.24
BB	0.63	1.93	3.46	4.99	6.43	7.75	8.89	9.90	10.82	11.64
B	3.34	7.80	11.75	14.89	17.35	19.36	20.99	22.31	23.50	24.62

Note: Adjusted default rates are marked as **bold**. They are adjusted to fix the problem with counter-intuitive default rates, as described .
Source: Own creation on data from S&P Global Market Intelligence (2021a)
Period: 1981-2020

8.1 Dynamics of a Corporate Issuer

We start the model by developing a model of the collateral assets. To do this, we model begin by modeling the dynamics of a corporate issuer and how the value of its outstanding debt evolves.

8.1.1 Single Debt Issue

We start by examining a single firm with asset values following a Geometric Brownian Motion (GBM), with the analytical representation:

$$V(t) = V_0 \exp\left[\left(\mu - 0.5\sigma^2\right)t + \sigma Z(t)\right] \tag{8.1}$$

where Z_t is a Brownian motion. In this section, μ and σ are decomposed into idiosyncratic and market components, by the intuition of the Capital Asset Pricing Model (CAPM), where β is the CAPM coefficient and $(r_m - r_f)$ is the market excess return. The two parameters are found as:

$$\mu = r_f + \beta(r_m - r_f)$$
$$\sigma = \sqrt{\beta^2\sigma_m^2 + v^2}$$

We start by assuming that the firm issues a single class of zero-coupon debt. The face value of the debt, B, consistent with a default probability of π, is found using the properties of the log-normal distribution of the GBM from equation 8.1:

$$B(\pi) \equiv \frac{V_0}{\exp\left(-\Phi^{-1}[\pi]\,\sigma\sqrt{T} - (\mu - 0.5\sigma^2)\,T\right)} \tag{8.2}$$

where Φ denotes the Cumulative Distribution Function (CDF) for the standard normal distribution and π is the default probability threshold.

Similarly, the fair market value of debt at time 0, D_0, is determined using Merton's model (Merton, 1974):

$$D(t) = B\,e^{-r_f\,(T-t)}\,\Phi\left(d_2^Q\right) + V(t)\,\Phi\left(-d_1^Q\right) \tag{8.3}$$

where:

$$d_1^Q(t) = \frac{\ln(V(t)/B) + (r_f + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \tag{8.4}$$

$$d_2^Q(t) = d_1^Q - \sigma\sqrt{T-t} \tag{8.5}$$

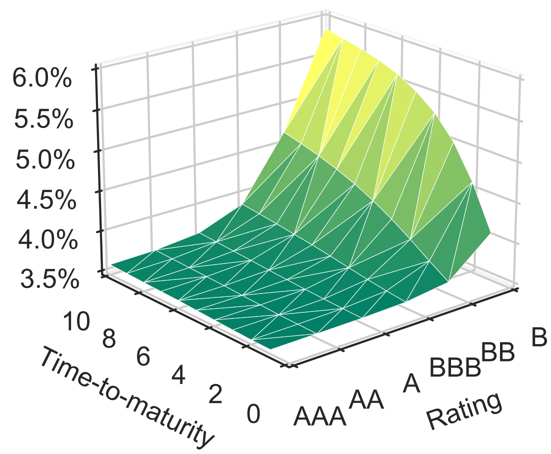
The continuously compounded yield from period t to maturity T can then be computed as:

$$y(t,T) = \frac{1}{T-t}\ln\frac{B}{D(t)} \tag{8.6}$$

and the spread is computed by subtracting the risk-free rate:

$$s(t,T) \equiv y(t,T) - r_f \tag{8.7}$$

Figure 8.1: Yields from Merton's Model



Note: Parameter assumptions are: $\mu = 0.091$ and $\sigma = 0.274$. Cumulative default probabilities for each rating and maturity is summarized in table 8.1.

8.1.2 Multiple Tranches

If the same firm choose to issue multiple classes of debt instead of funding itself with a single class of debt i.e. tranches, the face and market values are calculated by taking the first differences of the aggregate values:

$$\mathcal{B}_{i,k_i} = B_{k_i} - B_{k_{I-1}} \tag{8.8}$$

$$\mathcal{T}_{i,k_i} = D_{k_i} - D_{k_{I-1}} \tag{8.9}$$

for $i = 1, \ldots, I$ and where the first tranche is: $\mathcal{B}_{1,k_1} = B_{k_1}$ and $\mathcal{T}_{1,k_1} = D_{k_1}$. Substituting D_t, B_t with $\mathcal{T}, \mathcal{B}$ in equation 8.6 and 8.7, the equilibrium yield and spread for each tranche is found.

Using the determined tranche values, the equity value is found as the residual:

$$E_0 = V_0 - \sum_{I=1}^{I} \mathcal{B}_{k_i} \tag{8.10}$$

From this, it is not clear why and when there are benefits to tranching. If CLO lenders effectively price the increased risk from subordination, then no gains will accrue from the structuring. However, as discussed in section 3.3.4, tranches with different risk profiles, credit ratings, or other characteristics can appeal to different clienteles of investors. Also, Brennan et al. (2009) demonstrate that if investors fail to price the increased risk in structured debt securities with subordination, and debt tranches are instead traded at prices reflecting those of equivalently rated debt, then there is a gain to the issuer. This gain increases by the number of tranches and decreasing with the rating of the lowest rated tranche. Assuming that subordination risk is not completely priced, this result is consistent with the empirical observation that the number of tranches in European securitizations have been increasing over time (Cuchra and Jenkinson, 2005; S&P Global Market Intelligence, 2021a).

8.1.3 Pricing Prepayment Optionality

Acknowledging that leveraged loans have embedded redemption features, we now return to the single tranche corporate issuer from section 8.1.1. In this section, we adjust the price of the zero-coupon debt to include the embedded option and using this to find the fair market value of a callable loan. For this stylized model, we introduce a Bermudian call option. Herein the corporate borrower has the right, but not the obligation, to repay, i.e. buy back, its outstanding loan at a prespecified price at one or more prespecified dates prior to its maturity.

In perfect markets with no transaction costs, the borrowers' decision to repay its debt at a call date prior to maturity, depends on the benefit from refinancing the existing loan with a less expensive loan with identical remaining duration. For zero-coupon debt, this occurs when the current equilibrium value of debt, $D(t)$, exceeds the face value discounted at the promised yield, y_0, in addition to any transaction costs. This decision is represented by the following inequality:

$$D(t) > Be^{-y_0(T-t)} + \mathcal{P} \tag{8.11}$$

where $\mathcal{P}$ is an optional penalty to refinancing, e.g. a fixed or percentage cost of the amount refinanced. For our baseline scenario, we assume: $\mathcal{P}(B) = 0$.

Payoffs to the Lender

Letting τ be the first time $D(t) > Be^{-y_0(T-t)} + \mathcal{P}$, i.e.:

$$\tau = \inf\left\{t \in [0, T] : D(t) > Be^{-y_0(T-t)} + \mathcal{P}\right\} \tag{8.12}$$

then the cash flows, received by the lender, at the call date τ is: $CF(\tau) = Be^{-y_0(T-\tau)}$. If the loan is not repaid prematurely, the lender will receive $CF(T) = \min\left(V(T), B\right)$, at maturity T.

Pricing the Loan

Next, the fair price of a callable loan D^c can be estimated numerically by averaging the discounted cash flows under the risk-neutral measure, from N simulations:

$$D_0^c = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=0}^{T} CF_i\left(t\right)\ e^{-r_f\, t} \tag{8.13}$$

The equilibrium option price is then $\mathcal{C} \equiv D_0^c - D_0$ and by substituting $D\left(t\right)$ with D_0^c in equation 8.6 and 8.7, the equilibrium yield, y^c, and spread, s^c, of the callable loan can be estimated and then compared with the non-callable yield and spread, to investigate the relative size of the call option.

8.2 Numerical Example: A Corporate Issuer

In the following section, we apply the framework described throughout section 8.1 on a theoretical example. The purpose of the example is to examine, how the introduction of prepayment risk affect a single corporate issuer. The parameters used for the theoretical corporate issuer is: $T = 5$, $V_0 = 100$, $r_f = 3.5\%$, $r_m = 10.5\%$, $\sigma_m = 0.14$, $v = 0.25$ and $\beta = 0.8$, such that: $\mu = 0.091$ and $\sigma = 0.274$.

We begin the numerical example by generating six sample paths for $V(t)$ under the risk-neutral measure:

Table 8.2: Sample Paths for $V(t)$

Path	0	1	2	3	4	5
1	100.0	88.68	89.72	87.18	139.95	143.73
2	100.0	99.03	64.38	71.82	53.93	61.83
3	100.0	107.69	87.26	116.63	125.07	184.20
4	100.0	83.69	87.31	107.76	108.61	122.38
5	100.0	118.14	146.20	145.76	143.89	133.17
6	100.0	152.47	97.51	96.27	143.60	104.37

We assume that the firm issues a single class of zero-coupon B-rated debt at time 0 and with maturity in five years $T = 5$. To find the face value and the equilibrium market value, we use equation 8.2 and 8.3, and apply the five-year cumulative default probability from S&P: $\pi_B(5) = 17.4\%$. Using these equations and the cumulative default probability, we find that the face value is $B_B = 73.44$ and the market value is $D_{B,0} = 55.92$. This corresponds to a promised annual yield of $y_B(0,5) = 5.45\%$ and a spread of $s_B(0,5) = 5.45\% - 3.50\% = 1.95\%$.

Given the simulated asset paths in table 8.3, we can determine the corresponding market values of debt in each period. As the lenders upside is limited by the option-like payoff on debt, the terminal value of debt is found as $D(t) \equiv \min(V(t), B)$. The intermediate values $D(t)$ for $t = 1, \ldots, T-1$ are found from equation 8.3. Using the simulated asset paths and the above equations, we find the estimated market values of debt, $D(t)$ in table 8.3.

Table 8.3: Estimated Values for $D(t)$

Path	0	1	2	3	4	5
1	55.92	57.03	60.47	63.67	70.86	73.44
2	55.92	58.74	53.00	59.29	57.00	61.83
3	55.92	59.82	59.98	67.15	70.74	73.44
4	55.92	56.00	59.99	66.52	70.30	73.44
5	55.92	60.81	65.23	68.11	70.87	73.44
6	55.92	62.59	61.76	65.23	70.87	73.44

From the table, we find that the terminal value, $V(T)$, is insufficient to repay face value, B, in one of the six cases. In this case, the debt holder will receive the remaining asset value in its entirety and equity will receive nothing.

8.2.1 Pricing the Bermudian Call Option

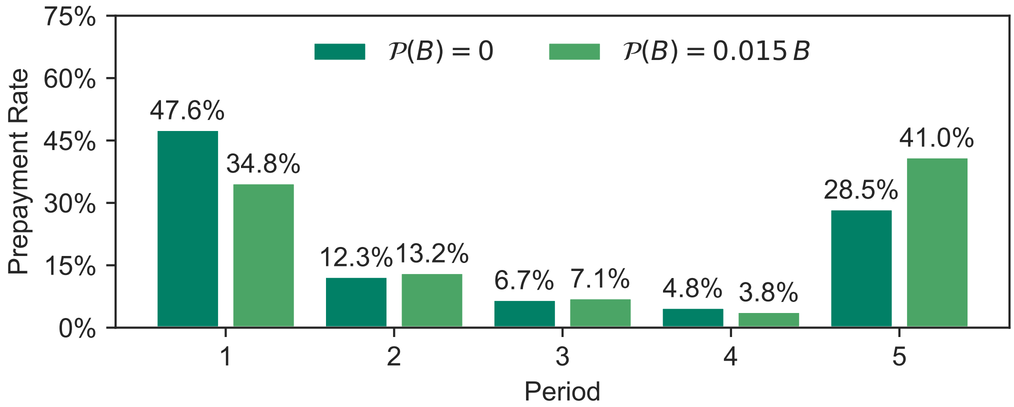
We now assume that the firm has the right, but not the obligation, to repurchase, or *call*, the loan at times $t = 1, 2, 3, 4$. Using this assumption we can find the value of prepayment option. The firm's decision at each time is given by the inequality 8.11 and if we assume no penalty $\mathcal{P}(B) = 0$, the firm decides to call in the following periods:

Table 8.4: Calls and Repayments

Path	0	1	2	3	4	5
1	-	-	-	-	Call	-
2	-	-	-	-	-	Repayment
3	-	Call	-	-	-	-
4	-	-	-	Call	-	-
5	-	Call	-	-	-	-
6	-	Call	-	-	-	-

Thus, for sample 1 and 3 to 6, the loan is repaid prior to maturity, while for sample 2, the loan is not repaid before maturity. The prepayment rates for each period is summarized in 8.2 for $N = 250{,}000$ simulations and for both the base case $\mathcal{P} = 0$ scenario and the $\mathcal{P} = 0.015B$, where a 1.5% "fixed fee" is introduced on the face value of the loan.

Figure 8.2: Simulated Prepayment Rates



Notes: The mean prepayment rates at each call date. Estimated from $N = 250{,}000$ simulations, both without and with a penalty to refinancing. For the latter case a penalty of 1.5% of the face value is assumed

The figure illustrates, that when there is no penalty around 50 % of the loans are prepaid in the first period. By introducing the penalty, the percentage of loans prepaid in the first period falls to 34.8 %. This is due to the fact that the market value of the loan has to be higher than in the case with no penalty to trigger the borrower to prepay the loan. The figure illustrates that the introduction of penalty decreases the number of prepayments. This becomes apparent as 41 % instead of 28.5 % of the loans are first repaid at maturity in period 5.

Finally, we price the callable loan and indirectly the Bermudian option using equation 8.13 for N simulations. Running $250{,}000$ simulations returns an equilibrium value of $D_0^c = 54.69$ on the zero-coupon loan, which implies a yield of: $y^c\,(0,5) = 5.9\%$. This is a difference of $D_0^c - D_0 = 1.23$ and $y^c\,(0,5) - y\,(0,5) = 45\,$bps.

8.2.2 Cash Flows to the Lender

Next, we examine the cash flows to the lender. The strictly higher yield on a callable loan: $y^c > y$, we found in the previous section is a compensation to lenders for the "prepayment-risk", e.g. missing out on returns from the firms that are the most likely to repay their obligations in full. Thus, the market value of debt will have to be adjusted by the new callable yield. This is found by substituting y by y^c in the payoff function, and as the time of calls are unchanged, the cash flows to lenders are: $CF(\tau) = Be^{-y_0^c(T-\tau)}$ at each intermediate call date τ or $\min(V(T), B)$ at maturity. In table 8.5, the adjusted cash flows to lenders are presented for the paths presented in table 8.4

Table 8.5: Cash Flows to the Lender

Path	0	1	2	3	4	5
1	-	-	-	-	69.24	
2	-	-	-	-	-	61.83
3	-	58.04	-	-	-	-
4	-	-	-	65.29	-	-
5	-	58.04	-	-	-	-
6	-	58.04	-	-	-	-

8.2.3 Option Spread and Option Value

Finally, we investigate how the introduction of callable debt in the case of a corporate issuer affects the option spread given different ratings for the loan, and the option value for a B-rated loan with six tranches of debt. The results are presented in table 8.6. Panel A illustrates the results for a callable loan with different ratings. In Panel B, the results from a single issuer of tranched debt with an aggregate face value equal to the B-rated loan is investigated. All results are equilibrium values hence no benefits come from tranching.

From panel A, we find that the option spread, found as the difference between the loan spread and the callable spread, is increasing with the rating of the loans and is highest for the loan with rating B. If we instead issue several tranches for the B-rated loans, the results in panel B shows, that the option value is increasing with the rating of the tranches. For the tranche AAA to A, the option value is close to 0, while for the B rated tranche is 0.79. The results presented in table 8.6 allow us to compare the results of our SPV simulations in section 8.3 with a tranched corporate issuer.

Table 8.6: Valuation and Pricing of Callable Debt for a Corporate Issuer

Panel A: Ratings based Results for a Corporate Issuer with a Single Class of Debt

Rating -	S&P rating k	Probability of default $\pi(k,T)$	Face value B_k	Loan value D_k	Loan spread s_k	Callable Value D_k^c	Callable spread s_k^c	Option spread $s_k^c - s_k$
1	AAA	0.27%	23.77	19.92	0.03%	19.90	0.05%	1.6 bps
2	AA	0.30%	24.27	20.34	0.04%	20.32	0.06%	1.8 bps
3	A	0.46%	26.50	22.18	0.06%	22.15	0.08%	2.5 bps
4	BBB	1.54%	34.80	28.95	0.18%	28.85	0.25%	6.8 bps
5	BB	6.43%	51.51	41.71	0.72%	41.27	0.93%	21.0 bps
6	B	17.35%	73.44	55.92	1.95%	54.69	2.39%	44.5 bps

Panel B: Ratings based Results for a Corporate Issuer with Six Tranches of Debt

Tranche i	S&P rating k	Probability of default $\pi(k,T)$	Face value $\mathcal{B}_{k_i}$	Tranche value $\mathcal{T}_{k_i}$	Tranche spread s_{k_i}	Callable value $\mathcal{T}_{k_i}^c$	Callable spread $s_{k_i}^c$	Option value $\mathcal{T}_{k_i}^c - \mathcal{T}_{k_j}$
1	AAA	0.27%	23.77	19.92	0.03%	19.90	0.05%	0.02
2	AA	0.30%	0.51	0.42	0.21%	0.42	0.30%	0.00
3	A	0.46%	2.23	1.84	0.27%	1.83	0.38%	0.01
4	BBB	1.54%	8.30	6.77	0.58%	6.70	0.79%	0.07
5	BB	6.43%	16.70	12.76	2.09%	12.42	2.43%	0.34
6	B	17.35%	21.94	14.22	5.18%	13.42	6.32%	0.79
–	Equity			44.08		45.31		1.23

Notes: In Panel A the values are obtained from the framework described throughout section 8.1 for the single-class debt issuer. Where B_k is calibrated from the S&P default probability $\pi(k,T)$ for each rating k AAA to B. This provides a useful reference, when estimating the tranche values in Panel B, for a single corporate issuer, with outstanding debt totalling that of a B-rated firm from Panel A. Equilibrium equity value is total debt subtracted from initial firm value V_0. Call option values are found numerically using $N = 250{,}000$ simulations.
Parameter Assumptions: $T = 5$, $V_0 = 100$, $r_f = 3.5\%$, $r_m = 10.5\%$, $\sigma_m = 0.14$, $v = 0.25$ and $\beta = 0.8$ in both panels. Default probabilities in column two is S&P five-year cumulative default probabilities for each respective ratings, these are summarised in table 6.7.

8.3 Dynamics of a Loan Securitization

Having described the dynamics of a corporate loan issue with and without prepayments in the previous section, this section expands on this, and outlines the dynamics of a securitization of loans.

8.3.1 Collateral Dynamics

Generalizing equation 8.1 to be applicable for the assets of corporate issuer j, for $j = 1, \ldots, J$, we use the following representation for the evolution of the asset value:

$$V_j(t) = V_{j,0} \exp\left[\left(\mu_j - 0.5\sigma_j^2\right) t + \beta_j \sigma_m Z_m(t) + v_j Z_j(t)\right] \tag{8.14}$$

where $Z_m(t), Z_j(t) \overset{iid}{\sim} \mathcal{N}(0, t)$ are two Brownian motions describing the market and idiosyncratic factors respectively. β_j is the CAPM beta-coefficient, v_j and σ_m are the firm j and market variation parameters, and lastly, the firm specific drift and volatility constants are found as:

$$\begin{aligned} \mu_j &= r_f + \beta_j(r_m - r_f) \\ \sigma_j^2 &= \beta_j^2 \sigma_m^2 + v_j^2 \end{aligned}$$

Analogously, we find the asset value under the risk-neutral measure, $V_j^Q(t)$, by substituting μ_j with r_f.

If the corporate issuers is homogeneous, then the correlation between the returns on any two firms is: $\rho \equiv \beta^2\sigma_m^2/(\beta^2\sigma_m^2 + vj)$. If each firm issues a single-class of debt with a given rating, k, and maturity, τ, and hence an implied default probability of $\pi(k, \tau)$, then the face value of debt is $B_j(\pi)$ with an initial equilibrium value of $D_{j,0}$. This is computed using equation 8.2 and 8.3. In this case the initial market value of a SPV holding a portfolio of J loans, must be:

$$\bar{D}_0 = \sum_{j=1}^{J} D_{j,0} \tag{8.15}$$

Assuming that the loans and the securitization have identical maturities, i.e., $T = \tau$, where T is the maturity of the SPV, then the payoff at maturity is:

$$CF_{SPV}(\tau) = \sum_{j=1}^{J} \min\Big(V_j(\tau), B_j\Big) \tag{8.16}$$

And, under the risk-neutral measure:

$$CF^{Q}_{SPV}(\tau) = \sum_{j=1}^{J} \min\left(V^{Q}_{j}(\tau), B_j\right) \tag{8.17}$$

In both equation, we clearly see an upper bound on the cash flows of: $\sum_{j=1}^{J} B_j$.

8.3.2 Simulation Approach

To obtain results on the SPV payoffs, we follow the simulation approach outlined in Appendix B of the Brennan et al. (2009) paper closely:

Step 1) Determination of Debt Face Value

Using the same approach as in section 8.1, we can determine the face value of debt B_j, for the corporate issuers, $j = 1, \ldots, J$, using an implied default probability, for any given rating, k, and time to maturity, τ, from the adjusted historical default rates in table 8.1. We assume that all the issuers are identical and have a single-class of debt outstanding. This base case assumption can be generalized by introducing heterogeneity among the issuers. However, when looking at aggregate-level changes in the parameters of the corporate issuers, homogeneity is an attractive feature.

Step 2) Simulation of SPV Value

For each corporate issuer, we can simulate its asset paths using equation 8.14, as described in section 8.3. When the corporate issuers have no prepayment option, we are only interested in the terminal value $V_j(\tau)$, as the cash flows to the SPV is defined by equation 8.16 and 8.17 for the physical and risk-neutral scenarios, respectively.

This process is repeated and applying Monte Carlo methods, as described in section 7.5 to simulate N outcomes for the asset values of all J issuers in the portfolio. We can then estimate $CF_{SPV,n}$ and $CF^{Q}_{SPV,n}$ for each simulation $n = 1, \ldots, N$ and hence obtain approximations of the Cumulative Distribution Function (CDF) for both the risk-neutral and physical distribution of SPV cash flows, $F^{Q}_{SPV}(x)$ and $F_{SPV}(x)$ respectively.

Lastly, we can estimate the equilibrium market value of the SPV portfolio at time t, by discounting the risk-neutral expectation of the portfolio value:

$$D_{SPV}(t) = \mathbb{E}^{Q}\left[CF^{Q}_{SPV}(\tau)\right] \, e^{-r_f(\tau - t)} \tag{8.18}$$

Which should equal the sum of the J loan market values, from equation 8.15.

Step 3) Tranche Valuation

Having approximated the cumulative distribution functions using Monte Carlo simulation, we can apply this function to determine the tranche sizes. If F_{SPV} denotes the cumulative distribution function of the physical distribution and is strictly increasing and continuous, and $Q_{SPV}(p),\ p \in [0,1]$, is the unique real number, x, such that $F_{SPV}(x) = p$, then Q_{SPV} denotes the inverse CDF (or quantile function) for the same distribution.

This is useful as the attachment point of the SPV tranches is estimated using the historical default probabilities of an equivalently rated corporate bond, $\pi(k, T)$, where k is the rating and T is the maturity of the SPV tranche or bond. If a SPV issues I tranches with ratings k_i, for $i = 1, \ldots, I$, to fund the portfolio of J loans, then the aggregate face value of debt $\bar{B}_{k_i}$ is found as:

$$\bar{B}_{k_i} = Q_{SPV}\Big(\pi\big(k_i, T\big)\Big) \tag{8.19}$$

Given the aggregate face value $\bar{B}_{k_i}$, we can estimate the equilibrium market value under the risk-neutral measure, analogously to equation 8.18:

$$D_{k_i}(t) = \mathbb{E}^Q\left[\min\Big(CF^Q_{SPV}(T),\ \bar{B}_{k_i}\Big)\right]\ e^{-r_f(T-t)} \tag{8.20}$$

Similarly to the corporate issuer, the face and market values of the tranches is calculated as the first differences of the aggregate values:

$$\mathcal{B}_{i,k_i} = B_{k_i} - B_{k_{i-1}} \tag{8.21}$$

$$\mathcal{T}_{i,k_i} = D_{k_i} - D_{k_{i-1}} \tag{8.22}$$

For $i = 1, \ldots, I$ and where, for the first tranche: $\mathcal{B}_{1,k_1} = B_{k_1}$ and $\mathcal{T}_{1,k_1} = D_{k_1}$.

Similarly, the equity value is calculated as the difference between the equilibrium value of the SPV, and the aggregate tranche value:

$$E_{SPV}(t) = D_{SPV}(t) - \sum_{i=1}^{I} \mathcal{T}_{i,k_i} \tag{8.23}$$

Finally, we can use the found face value $\mathcal{B}_{i,k_i}$ and the market value $\mathcal{T}_t$ to calcalute the promised yield on each tranche as:

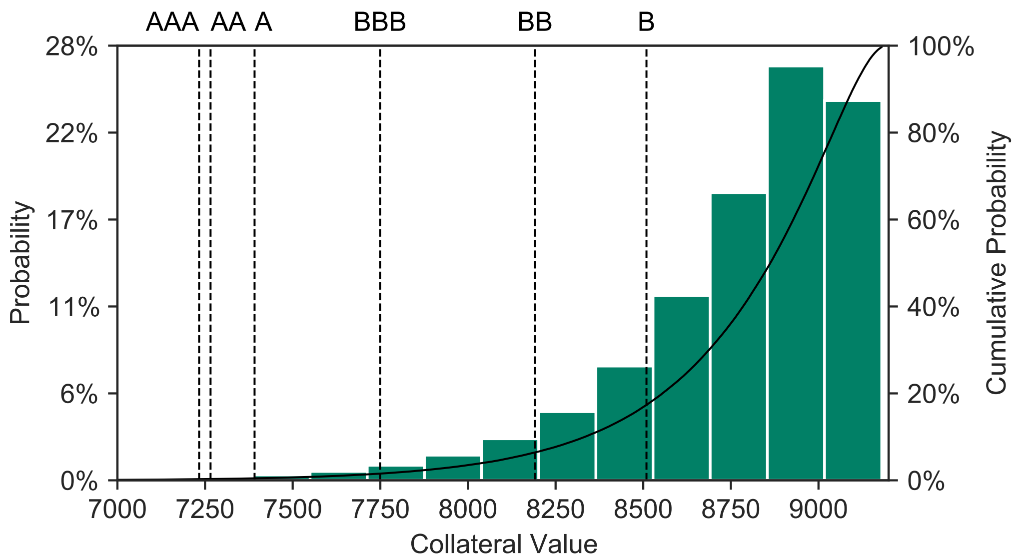
$$y_{i,k_i}(t,T) = \frac{1}{T-t}\ln\frac{\mathcal{B}_{i,k_i}}{\mathcal{T}(t)} \tag{8.24}$$

The spreads on the tranches is the found by subtracting the risk-free rate, as in equation 8.7.

8.4 Base Case: No Collateral Prepayments

The following section presents the results of simulating a SPV with $J = 125$ loans using the dynamics the dynamics of a loan securitization presented above. By assuming that the 125 loans are issued by identical corporate issuers with the same parameter assumptions as in section 8.1 and – for now – no prepayment option on the loans, we can scale the simulations from the section to determine the equilibrium value of the aggregate portfolio and the terminal payoffs: $CF_{SPV}(\tau)$ and $CF^{Q}_{SPV}(\tau)$. Additionally, we assume that the maturity of the securitization matches the maturity of the loans, i.e. $T = \tau$. By doing this, the aggregate terminal payoff on the loans is the payoffs to the SPV and hence, to the claim holders. Under these assumptions, we generate $N \times J \Rightarrow 250{,}000 \times 125 = 31{,}250{,}000$ outcomes for the corporate issuer and then divide the outcomes by the number of portfolio simulations, N. As the issuers in each cohort is conditional on the same market factor, Z_m, as formulated in equation 8.14, we have to do the N SPV simulations individually. The simulated collateral values is illustrated in 8.3.

Figure 8.3: Simulated Collateral Values



Notes: Frequencies of the simulated collateral values, under the physical measure ($N = 250{,}000$). The attachment points of the tranches is illustrated as well as the approximated cumulative distribution function $F_{SPV}(x)$.

We note that the maximal payoff of any simulation is the aggregate face value $B \times J \Rightarrow 73.44 \times 125 = 9{,}180$. The minimal payoff must be strictly greater than zero, but other than this, we cannot state anything meaningful about the bounds on the SPV cash flows, prior to the simulations.

8.4.1 Computing the Attachment Points

Running the simulations for $CF_{SPV}(T)$, we find the physical payoff distribution. Next, the attachment point of each tranche is found using the inverse CDF, $Q_{SPV}(p)$, as in equation 8.19 and computing the $\pi(k,T)$-quantile, where $\pi(k,T)$ is the default probability for the equivalent rating from table 8.1. The physical distribution, the cumulative distribution function, $F_{SPV}(x)$, and the attachment points of each tranche are illustrated in figure 8.3, while the estimated attachment points are summarized in table 8.7 below:

Table 8.7: Tranche Attachment Points

Tranche i	Rating k_i	Probability of Default	Attachment Point	Percent of Maximum
1	AAA	0.27%	7,233	78.8%
2	AA	0.30%	7,266	79.1%
3	A	0.46%	7,391	80.5%
4	BBB	1.54%	7,749	84.4%
5	BB	6.43%	8,191	89.2%
6	B	17.35%	8,509	92.7%
–	Maximum	–	9,180	100.0%

8.4.2 Computing the Equilibrium Value of the SPV

In order to compute the equilibrium value of the SPV, we have to repeat the simulations. By replacing μ_j with the risk-free rate r_f in the asset dynamics from equation 8.14, we can generate the asset paths under the risk-neutral measure, $V_j^Q(T)$ and hence find the payoff on each loan as: $\min\left(V_j^Q(T), B_j\right)$. Then by discounting the risk-neutral expectation of the loan payoffs, we can calculate the total equilibrium value of the SPV, D_{SPV}. As a sanity test this should equal the sum of the loan market values, i.e. $J \times B \Rightarrow 125 \times 55.92 = 6{,}990$. This is the case for our $N = 250{,}000$ simulations.

If we relate this to the mean payoff under the physical measure, and compute the yield using equation 8.24, we find a total promised yield of 4.55%. For reference the maximum yield is 5.45%, which is computed by replacing the mean payoff with the maximum payoff under the physical measure.

8.4.3 Pricing the Tranches

The attachment points from table 8.7 represents the aggregate face value, $\bar{B}_{k_i}$, issued for each tranche, i, and in order to compute the aggregate market values at each tranche, $D_{k_i,0}$, we use equation 8.20, with $\bar{B}_{k_i}$ as input.

As with the case with the corporate issuer, we take the first differences as outlined in equation 8.21 and 8.22, of the aggregate face and market values for all subordinated tranches to compute the tranche values. The results from this procedure is outlined in table 8.8, where the values are normalized by the equilibrium SPV value $D_{SPV} = 6{,}990$. These results are used as a base case, in chapter 9, to estimate the effect of the prepayment risk.

Table 8.8: Numerical Results – Base Case, without Prepayments

Tranche i	S&P Rating k	Probability of default $\pi(k,T)$	Cash Flows $Q_{SPV}(\pi)$	Face value $\mathcal{B}_{i,k_i}$	Tranche value $\mathcal{T}_{i,k_i}$	Tranche spread s_{i,k_i}
1	AAA	0.27%	103.5	103.47	86.64	0.05%
2	AA	0.30%	103.9	0.47	0.37	0.98%
3	A	0.46%	105.7	1.80	1.42	1.19%
4	BBB	1.54%	110.9	5.12	3.87	2.11%
5	BB	6.43%	117.2	6.32	4.11	5.13%
6	B	17.35%	121.7	4.55	2.15	11.50%
–	Equity		–		1.44	–
	Total		131.3		100.00	0.72%

Parameter Assumptions: $T = 5$, $V_0 = 100$, $r_f = 3.5\%$, $r_m = 10.5\%$, $\sigma_m = 0.14$, $v_j = 0.25$, $\beta_j = 0.8$, and an implied correlation between any two firms of $\rho = 0.17$, for all corporate issuers, $j = 1, \ldots, 125$. The results in column 4-6 are normalized by the equilibrium SPV value: $D_{SPV} = 6{,}990$.

8.5 Introducing Collateral Prepayments

In this section, we extend the framework of our model from collateral dynamics to a securitization holding a portfolio of callable loans. This approach holds similarities with the case, we outlined in section 8.1. Herein, we presented the dynamics of a corporate issuer with the ability to prematurely repay its debt, and we showed how the introduction of prepayment affect the cash flows received by the lender.

We begin the introduction of the collateral prepayment by presenting our assumptions on the SPV dynamics in the following. First, we exclusively consider *organic* substitution of loans in the collateral portfolio, i.e. changes from loan prepayments, and not changes triggered by active management of the collateral. Furthermore, we assume that loans, callable or *non*-callable are priced at their equilibrium prices, as we are interested in the mispricing of the securitization tranches, and not the collateral assets. Additionally, we make some practical assumptions to improve the computational simplicity of our model. These assumption concerns how cash flows from loan prepayments are reinvested. We assume that if a callable loan is repaid, then the SPV reinvests in a similarly rated non-callable loan, with a maturity matching the remaining life of the redeemed loan, such that the life of the collateral assets matches the life of the SPV. Using identically rated loans, we argue, that the collateral will always consist of J loans with the same rating implied risk characteristics and hence infer a stable rating on the loan portfolio. This assumption will have some effects on how the cash flows in the SPV behave.

8.5.1 Cash Flows and Equity Distributions

Following from the loan dynamics from section 8.1.3, we note that the yield on a callable loan is strictly greater than the yield on a non-callable with otherwise identical characteristics. Hence, receiving the proceeds from prepayment on a callable loan will lead to the yield on the lenders invested capital dropping. However, this effect is offset by the higher compensation on callable loans in the period prior to their redemption. Consequently, the total effect of the assumption that proceeds from prepayments are reinvested in non-callable loans is unclear, compared to the base case with no collateral prepayments.

For the zero-coupon universe in our model, we have to make some considerations, as there is no carry component from coupon or interest payments, and hence no intermediate cash flows to the SPV in the base case model. If our model including prepayments should have the same property, we could calibrate the market values of newly issued callable loans to match the proceeds from prepayments at each call date τ. Hereby no intermediate cash flows would occur and all proceeds are accrued. However, for numerical simplicity in the simulations, we allow mismatches between the prepayment proceeds and the amount reinvested. We assume that the newly bought loans are issued by firms with identical parameters as the original issuers, namely an identical initial asset value V_0. Furthermore, the new issuers similarly determine the principal issued by equation 8.2 for a given default probability $\pi(k, T)$, with the same rating k as the original issuers, although with a shorter time to maturity and no prepayment feature.

In table 8.9 below, we have accounted for the face and market values of newly issued debt with time to maturity, $T-t$, and the cash flows at time $t = \tau$ from a callable loan repaid prematurely, with the same issuer parameters as in the previous sections.

Table 8.9: Face and Market Values of B-rated Loans

Time to Maturity $T-t$	Probability of Default $\pi_B(t)$	Face Value B_B	Market Value D_B	Market Value $y_B(t,T)$	Cash Flows $CF(\tau)$	Equity Distributions $CF(\tau) - D_B$
5	17.35%	73.44	55.92	5.45%	–	–
4	14.89%	70.01	56.72	5.26%	58.04	1.32
3	11.75%	66.83	57.54	4.99%	61.55	4.01
2	7.80%	64.23	58.61	4.58%	65.29	6.68
1	3.34%	63.85	61.32	4.04%	69.24	7.92
0	–	–	–	–	73.44	–

Note: The cash flows in column 6 is whose from section 8.1.3: $CF(\tau) = Be^{-y_0^c(T-\tau)}$, for loans repaid at time $t = \tau$. Column 2-4 is computed for a non-callable loans issued at time t with maturity at time T.
Parameter Assumptions: $T = 5$, $r_f = 3.5\%$, $r_m = 10.5\%$, $\sigma_m = 0.14$, $v = 0.25$ and $\beta = 0.8$, for both the callable and non-callable issuers. The asset value is $V_0 = 100$ for the callable issuer in the initial portfolio, whereas for the non-callable issuers the asset values are: $V_0(\tau) = 100$, when their loans are included in the portfolio at time τ.

From the table, we find that prepayments in any period result in an excess cash flow and we need to make assumptions about what happens to these cash flows. As discussed in chapters 3 and 4, real-world CLO's can make distributions to equity conditional on passing specific coverage tests. However, the zero-coupon framework, we have outlined this far, is not suited for implementing coverage tests. The Interest Coverage (IC) test would require interest payments and for the Over-Collateralization (OC) test the total tranche face value is a fraction of the maximum payoffs and consequently a fraction of the face value of collateral and hence passed by default. The equity distributions in real-world is an argument for paying out the intermediate cash flow to equity. We will use this approach and therefore we need to incorporate this into our model.

8.5.2 Extending the Simulation Approach

In the process of simulating the SPV with collateral prepayments, we need to adapt step 2 and 3 of our simulation approach to incorporate both prepayments and equity distributions. The first step involves determining the call dates for the initial portfolio of callable loans. This entails the same asset value simulations as in step 1 in section 8.3.2. Using these asset simulations, we can then compute the first call date using equation 8.11.

Next, we need to determine the callable loan yield. We have already done this in section 8.1, and we can rely on this yield for computing the cash flows to the SPV, as in table 8.9.

Finally, we need to model the non-callable loan. We use the same approach as for the loans in the base case outlined in section 8.3. However, for the non-callable loan, we need to adjust the time to maturity, and consequently the face value to reflect the new default probability $\pi(k, T-t)$, from table 8.1. Thus, we end up with the same expression for the terminal payoff on the new loan j^*: $\min\left(V_{j^*}(T), B_{j^*}\right)$.

The intermediate cash flows from prepayments are distributed to equity, and the total payoff to equity in the terminal period is:

$$CF_{Equity}(T) = \max\left(CF_{SPV}(T) - \bar{B}_{k_I}, 0\right) + \sum_{t=1}^{T} M(t)\, e^{r_f(T-t)} \tag{8.25}$$

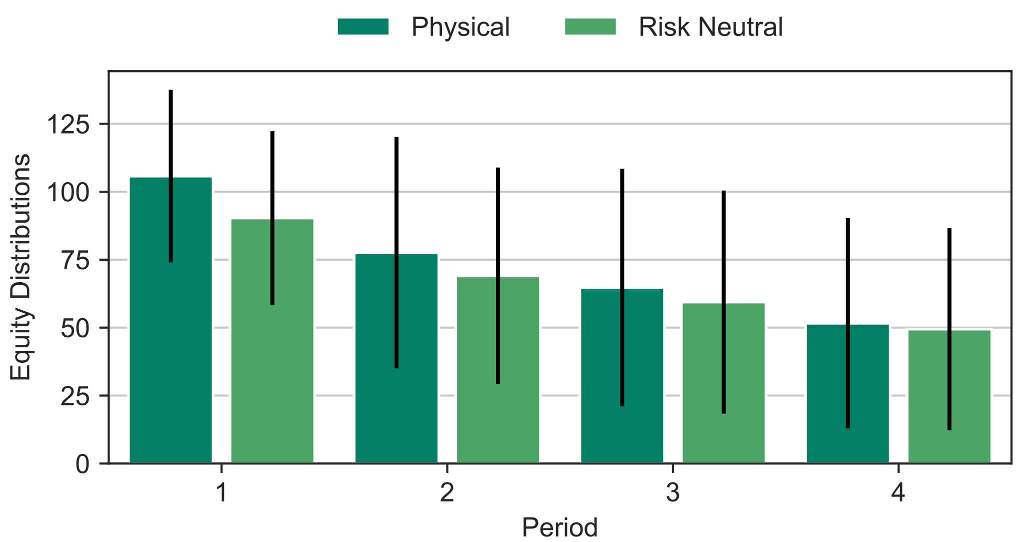
where $\bar{B}_{k_I}$ is the aggregate face value of the SPV tranches and $M(t)$ is the total equity distributions in period t.

8.6 SPV with Collateral Prepayments

The following section use the extension to the simulation approach to introduce prepayment to the collateral in the SPV. The procedure for the simulation is similar to the base case, and therefore the following section focus on the results from the simulation and the average equity distribution.

By introducing the early redemption feature on all initial loans in the securitization from section 8.4, we can determine the call date and consequently the equity distribution. To do this, we use the same parameter assumptions from previous: a callable loan yield of $y_0^c(0,5) = 5.89\%$, and the original non-callable yield of $y_0(0,5) = 5.45\%$. The results are shown in figure 8.4, in which we presents the average equity distributions at all intermediate call dates, $\tau = 1,2,3,4$.

Figure 8.4: Mean Equity Distributions



Notes: The mean equity distributions in each time period, based on $N = 250{,}000$ simulations of $J = 125$ loans from corporate issuers, whose dynamics are simulated under both the physical and risk-neutral measures. Error bars indicate +/– 1 standard deviation from the mean.

From the figure, we clearly observe that the largest distributions occur in the first period. This is consistent with the observation we made in section 8.1.3, where close to 50 % of the loans were repaid in the first period.

We can then approximate both the physical and risk-neutral distributions of the terminal payoffs using the same procedure as in section 8.4. This is done by compounding the equity distributions and determining the terminal payoffs for all $J = 125$ loans for $N = 250{,}000$ simulations. Finally, we can determine the tranche sizes and equilibrium values. The results from these simulations are summarized in table 8.10. We will use these values in chapter 9 to estimate the effect of the prepayment risk.

Table 8.10: Numerical Results – Including Collateral Prepayments

Tranche i	S&P Rating k	Probability of default $\pi(k,T)$	Cash Flows $Q^{pp}_{SPV}(\pi)$	Face value $\mathcal{B}^{pp}_{i,k_i}$	Tranche value $\mathcal{T}^{pp}_{i,k_i}$	Tranche spread s^{pp}_{i,k_i}
1	AAA	0.27%	96.6	96.60	80.90	0.05%
2	AA	0.30%	97.0	0.45	0.36	0.93%
3	A	0.46%	98.7	1.65	1.31	1.13%
4	BBB	1.54%	103.5	4.81	3.65	2.04%
5	BB	6.43%	109.4	5.88	3.84	5.03%
6	B	17.35%	113.6	4.24	2.02	11.30%
–	Equity		–		5.77	–
	Total		125.1		97.85	0.71%

Parameter Assumptions: $T = 5$, $V_0 = 100$, $r_f = 3.5\%$, $r_m = 10.5\%$, $\sigma_m = 0.14$, $v_j = 0.25$, $\beta_j = 0.8$, $\mathcal{P} = 0$, and an implied correlation between any two firms of $\rho = 0.17$, for all corporate issuers, $j = 1, \ldots, 125$. The results in column 4-6 are normalized by the equilibrium SPV value, from the base case: $D_{SPV} = 6{,}990$.

9 Results

In the previous chapter, we developed a structural model to determine the tranche prices and spreads on a SPV in a base case without collateral prepayment and in a case with collateral prepayments. In this chapter, we use these results to analyze the effects of collateral prepayments.

The chapter is divided into four sections: The first section presents a summary of the numerical results from the previous chapter and elaborates on the findings. Secondly, we analyze the equity returns in the two cases. Next, we investigate the mispricing of the prepayment risk and use the mispricing results as a base case for the fourth section, where we test the sensitivity and robustness of these results. This involves adjusting parameters from the base case.

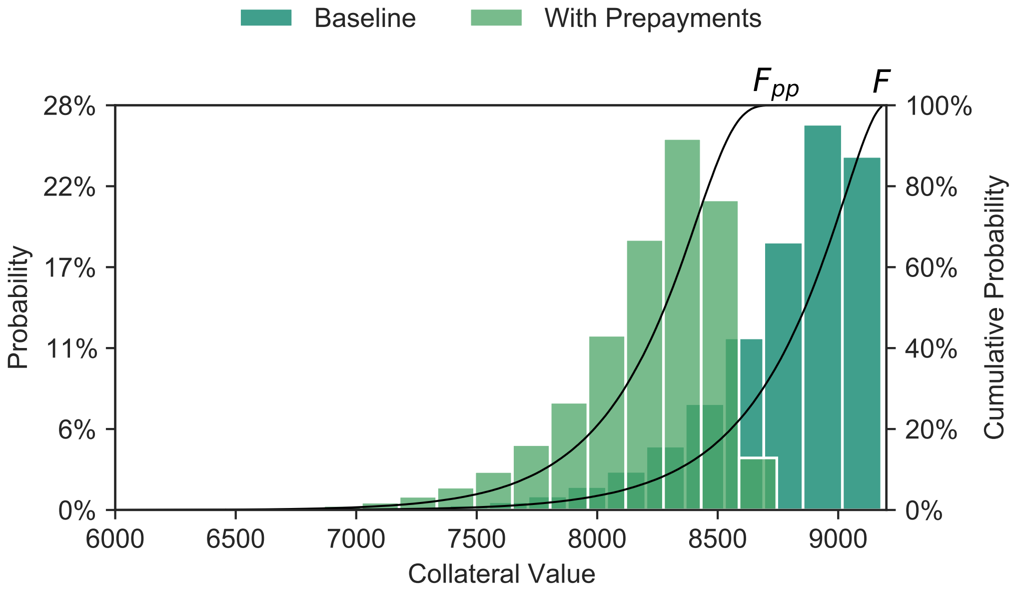
9.1 Summary of the Numerical Results

In the previous chapter, we obtained results on both the base case (i.e. without collateral prepayments) and the collateral prepayment case. These results are summarized in table 8.8 and 8.10. In both tables the reported values are normalized by the equilibrium SPV value from the base case, D_{SPV}, and we find that the case with prepayments has lower spreads across the tranches. However, when adjusting for the tranche sizes and computing the weighted average cost of debt, we find that the WACD is roughly identical for the two cases: After subtracting the risk-free rate, the weighted average *risk premia* are 0.72% and 0.71% respectively.

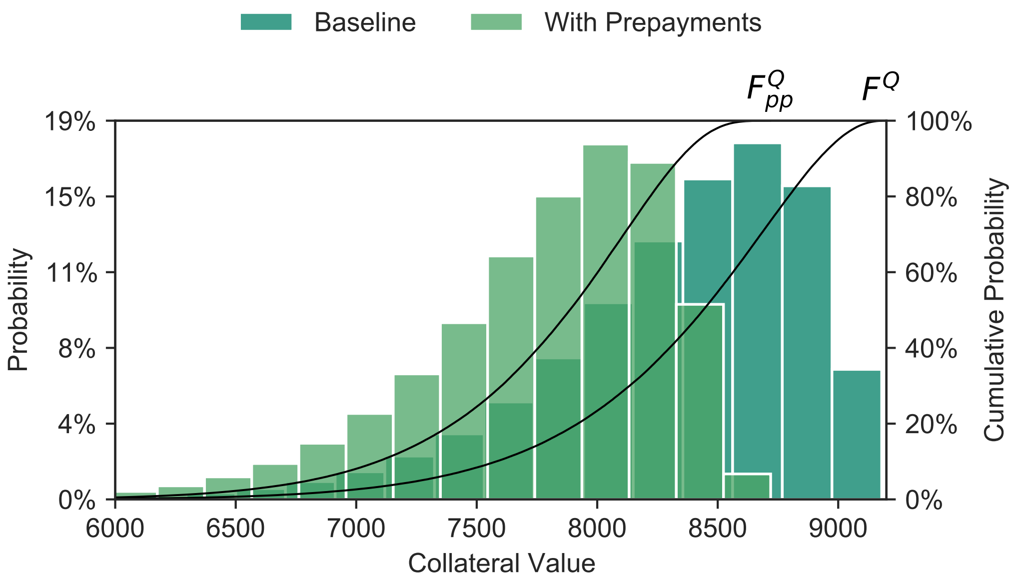
This finding is consistent with the simulated payoffs illustrated in figure 9.1, where both the physical P and risk-neutral Q distributions for both cases is illustrated. Looking at panel (a), which illustrates the physical distribution, we observe a steeper and more negative skew for the case with prepayments relative to the base case. This indicates that there are fewer good scenarios

Figure 9.1: Simulated Collateral Values

(a) Simulated Values and CDF under P-measure



(b) Simulated Values and CDF under Q-measure



Notes: In panel (a), the distributions of the simulated collateral values at maturity, T, under the physical measure is illustrated. For panel (b), its the distributions under the risk-neutral measure.
Parameter assumptions are identical to those described in the previous chapter, we refer to this chapter for a more elaborate description of the simulations For all four cases, the number of simulations are: $N = 250{,}000$.

compared to the base case. The same attributes hold for the risk-neutral distribution in panel (b). From the figure, we find that the total risk premia is almost identical for the two cases, meaning that the offset in the physical distribution, and hence the tranche sizes, is adjusted for in the risk-neutral case and hereby also in the equilibrium values.

9.2 Analysis of Equity Returns

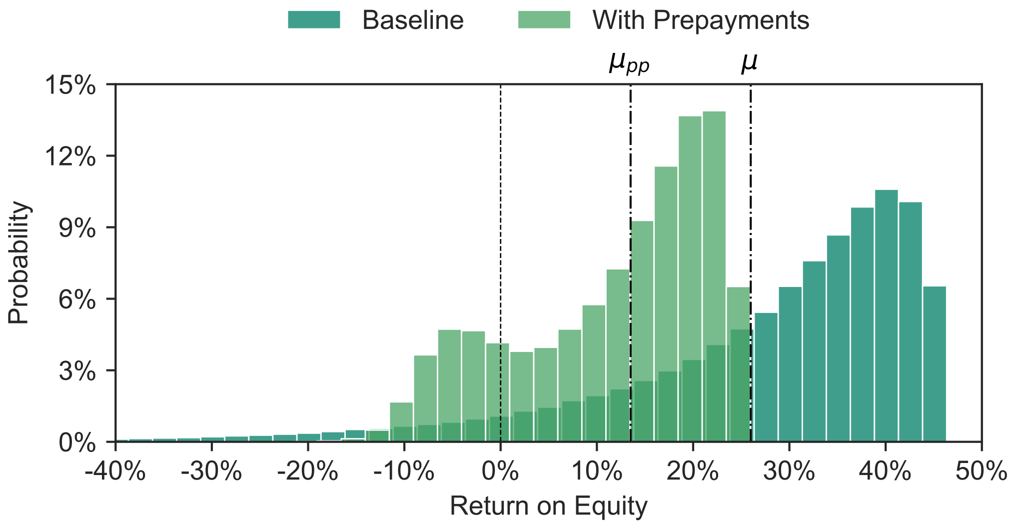
In this section, we begin by examining the equity payoffs and how the payoffs are affected by the introduction of prepayments. To do this, we use the numerical results for the SPV with and without prepayment, which we found in chapter 8.

As described in section 8.6, the introduction of collateral prepayments changes the cash flows to equity. In the case without prepayments, the equity tranche receives the residual cash flow at the terminal period after all debt tranches have been paid in full with respect to their seniority.

However, for the case with collateral prepayments, the payoffs to equity are more complicated. In this case, equity holders do not only receive the residual cash flow at the terminal period, but also any residual cash flows from the reinvestments related to callable the loan prepayments as described in section 8.5.2.

If we compound all intermediate cash flows, and include these in the payoffs to equity, we find the distributions of annualized return on equity. These are illustrated in figure 9.2.

Figure 9.2: Annualized Return on Equity



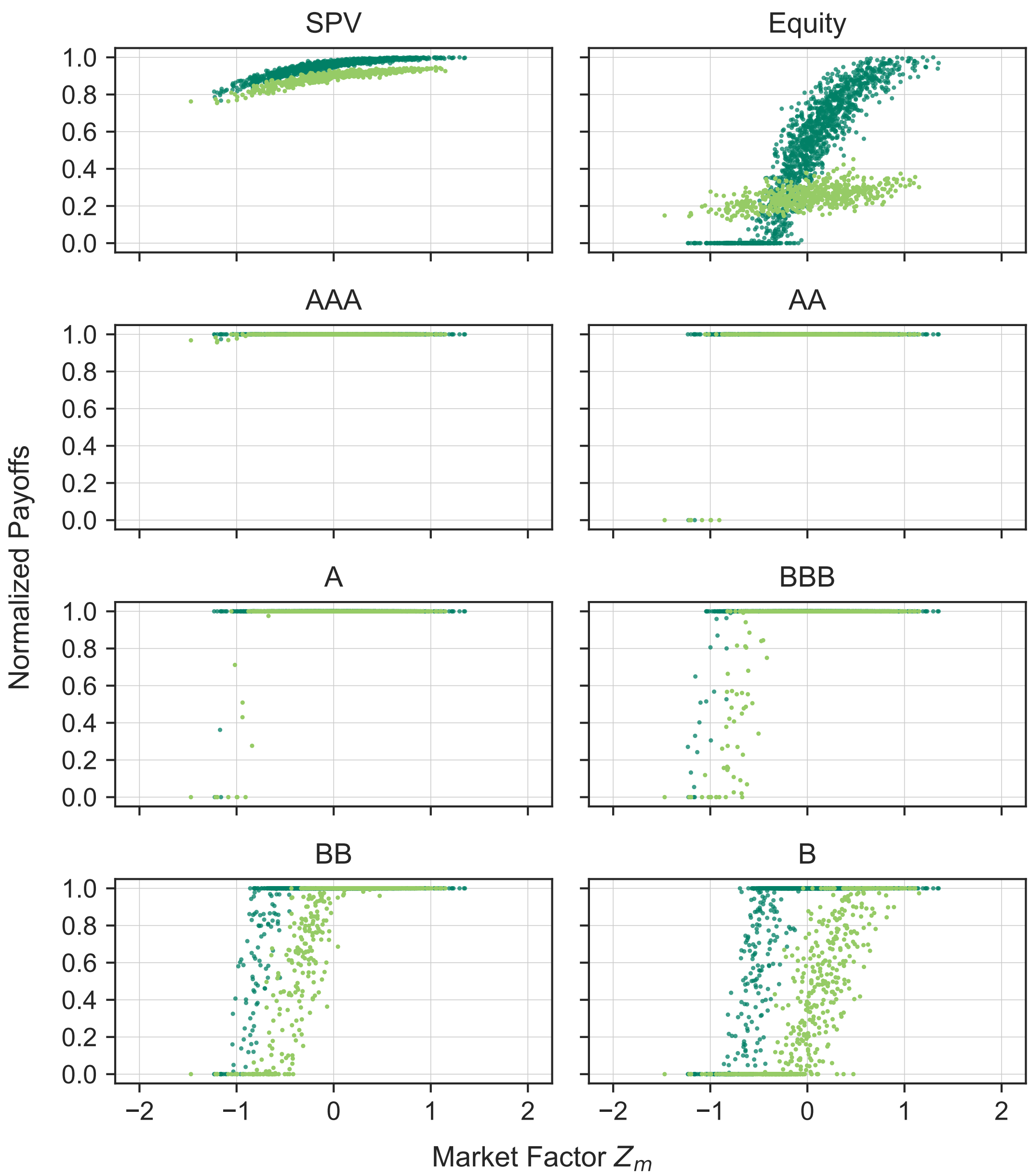
Notes: The simulated distribution of annualized Return on Equity (ROE), using $N = 250{,}000$, for both the base case and the case with prepayments, and hence intermediate cash flows. For the base case, equity loses its entire investment in 17.35% of the cases, which is expected as this percentage is also the default rate on the B-tranche, this is not included in the illustration, as total losses cannot be annualized. μ and μ_{pp} is the mean of ROE on the base case and the case including collateral prepayments, respectively.

Figure 9.2 shows a much tighter distribution of equity returns when we introduce collateral prepayments to the model. Furthermore, we find that in the case with prepayment there is no cases, in which the equity is entirely wiped. This is a big change compared to the base case, where the equity is wiped in 17.35% of cases. Hence, the introduction of prepayments shifts the mean of equity returns towards zero, but with a much lower standard deviation. The mean values are: $\mu = 0.26$ and $\mu_{pp} = 0.135$, and the standard deviations are $\sigma = 0.96$ and $\sigma = 0.34$. Using these values, we can compute the Sharpe ratios, using equation 9.1.

$$\lambda_i = \frac{(\mu_i - r_f)}{\sigma_i} \tag{9.1}$$

Using equation 9.1, we find $\lambda = 0.24$ for the base case and $\lambda_{pp} = 0.31$ for the case with collateral prepayments. Hence the latter case would be more attractive on a risk-adjusted basis for equity investors. However, the market portfolio is even more attractive at a Sharpe ratio of: $\lambda_m = (0.105 - 0.035)/0.14 = 0.5$.

Figure 9.3: Payoffs to SPV, Equity and Tranches



Notes: The simulated payoffs on the total SPV portfolio, equity, and tranches. The darker bullets (•) represents the base case, whereas the lighter (•) represent the mispricing case, where the face value is determined as in the base case, but the ”true” outcomes include prepayment risk.
The payoffs are normalized with respect to the face value for each tranche, so each payoff lies strictly in the interval $[0, 1]$. This is also true for equity, which is normalized by the maximum attainable payoff. In the mispricing case, the compounded intermediate cash flows are included, hence the distribution is flat and not as affected by the market factor.

These results are consistent with the normalized payoffs illustrated in figure 9.3, where we see that in the base case, the payoff as a function of the market factor is upward sloping and concave, whereas for the case with collateral prepayments, the payoffs are almost independent of the market factor (although slightly upward sloping).

9.3 Prepayment Risk Mispricing

In this section, we investigate the effects from mispriced collateral prepayment risk on the structure of the securitization and its tranche spreads. Our approach builds on the assumption that the rating agencies evaluate the original portfolio of loans without any assumptions about prepayments. Thus, the rating agencies do not properly account for the risk of the loans getting repaid prematurely and substituted by new loans, whose yields might be different even for the same default probability. This is not an unfair assumption, as the rating agencies do not explicitly make assumptions about collateral prepayments. As an example Moody's write the following in its "Global Approach to Rating Collateralized Loan Obligations" (2020):

> *The lives of the individual assets are based on scheduled principal payments, without any assumptions regarding prepayments.*

Under the assumption that credit rating agencies fail to incorporate prepayment risk, the rating agencies will allow too large tranche sizes to finance the collateral. Hence, using the face values from the base case, from table 8.8 and at the same time, assuming that CLO lenders are aware and correctly price the prepayment risk, we can relate the face values with the market values from the case with prepayments (table 8.10).

In table 9.1, the face and market values are combined, meaning that the face value in column 4 is from the case without prepayment, while the market value in column 5 is from the case with prepayment. In the three last columns, we have the spread from the base case (table 8.8), the adjusted spread, computing using the face and market values in column 4-5, and lastly the difference between these spreads, i.e. the mispricing.

Table 9.1: Spread Mispricing

Tranche i	S&P Rating k	Probability of default $\pi(k,T)$	Face value $\mathcal{B}_{i,k_i}$	Market value $\mathcal{T}^{pp}_{i,k_i}$	Spread s_{i,k_i}	Adj. Spread s^*_{i,k_i}	Mispricing ω_i
1	AAA	0.27%	103.47	80.90	0.05%	1.42%	1.37%
2	AA	0.30%	0.47	0.36	0.98%	1.77%	0.79%
3	A	0.46%	1.80	1.31	1.19%	2.82%	1.64%
4	BBB	1.54%	5.12	3.65	2.11%	3.31%	1.20%
5	BB	6.43%	6.32	3.84	5.13%	6.47%	1.34%
6	B	17.35%	4.55	2.02	11.50%	12.68%	1.19%
–	Equity			5.77	–	–	–
	Total			97.85	0.72%	2.08 %	1.36%

Notes: The face values and spread s_{i,k_i} are from the base case (table 8.8), the tranche market values are from the case with prepayments (table 8.10), and the adjusted tranche spreads are computed by subtracting the risk-free rate from the yields calculated on these values, using equation 8.24. The tranche mispricing is computed as: $\omega_i = s^*_{i,k_i} - s_{i,k_i}$, and the total mispricing is computed likewise, but using the aggregate face values from the base case and the total market value for the tranches (excluding equity), from the prepayment case and subtracting the WACD from the base case: $\Omega = (1/T)(\ln[\bar{\mathcal{B}}_I/\bar{\mathcal{T}}_I{}^{pp}] - \ln[\bar{\mathcal{B}}_I/\bar{\mathcal{T}}_I])$.

The main findings from table 9.1 are that adjusted spreads are consistently higher than for the base case and the mispricing is also fairly consistent, but highest for the A tranche and lowest for the AA tranche. Interestingly, the magnitude of mispricing on the AAA tranche is quite large, which explains a large part of the increase in weighted average risk premia, from 0.72% in the base case to 2.08% in the mispriced case. The total mispricing is 1.36%, which is the difference between these metrics.

Comparison with Empirical Spreads

Assuming CLO lenders correctly price the increased risk from prepayments, and the rating agencies does not, then this model of mispricing provides an explanation for the higher observed spreads on CLO tranches compared to equivalently rated corporate bonds.

But how well does the model actually explain the excess spreads between equivalently rated tranches and bonds, and is the magnitude of mispricing realistic? In figure 6.5, we illustrated the developments in the average spreads on AAA tranches in European CLOs, which fluctuate between 80 and 150. Comparing the observed spreads from the figure and the adjusted spreads from table 9.1, we find that the spreads on the spread on the AAA tranche at 1.42 % or 142 bps are close to the observed spreads.

9.4 Sensitivity Analysis

As a final reflection on our results, we test the sensitivity and robustness of our results by altering some of the parameters used in our model. In table 9.2, we have displayed the results on WACD, adjusted WACD, and hereby the mispricing, using different parameter assumptions. The variations in the table include changes in the number of loans in the collateral pool, the rating of these loans, volatility of firm assets, and the market risk premium, etc. The first row in the table shows the base case, which is described in section 9.3.

From the table, we see that the results are the most sensitive to changes in the ratings of the loans (vii) and the risk-free rate (iii). When the rating of the loans change from B to BB, while the rating of the SPV is unchanged, the WACD decreases to 3.84%, although the mispricing is unchanged. This happens as the default probability on the loans falls. Hence, total portfolio risk is strictly lower. If we change the risk-free rate from 3.5% to 5.0%, keeping all other parameters constant, the WACD and the adjusted WACD increases to 5.53 % and 6.96 % respectively. When looking at the penalty parameter, which directly affects the number of prepayment on the loans, we find that as the penalty increases the adjusted WACD and the mispricing decrease. This is quite intuitive, as the mispricing is a result of the prepayments, and the penalty parameter infer fewer prepayments. Finally, if we change the time to maturity of the SPV, T, we find that the mispricing increase, relative to the base case, when the maturity is increased, however, this is a result of the effect that the longer maturity on the loan, the more likely rating migration is and thus, the hazard rate decreases over time, after a certain period.

Table 9.2: Sensitivity Analysis

	Variation		WACD %	Adj. WACD %	Mispricing %
–	Base Case		4.22	5.58	1.60
(i)	β	1.0	4.51	6.10	1.59
		0.8	**4.22**	**5.58**	**1.36**
		0.7	4.09	5.32	1.23
(ii)	r_m	12.0	4.45	6.07	1.62
		10.5	**4.22**	**5.58**	**1.36**
		9.0	4.04	5.12	1.09
(iii)	r_f.	5.0	5.53	6.96	1.43
		3.5	**4.22**	**5.58**	**1.36**
		2.0	2.95	4.24	1.29
(iv)	σ_i	0.30	4.13	5.18	1.06
		0.25	**4.22**	**5.58**	**1.36**
		0.20	4.35	5.92	1.58
(v)	σ_m	0.18	4.26	5.54	1.28
		0.14	**4.22**	**5.58**	**1.36**
		0.10	4.22	5.63	1.42
(vi)	No. of	200	4.22	5.58	1.36
	Loans	**125**	**4.22**	**5.58**	**1.36**
		50	4.22	5.59	1.37
(vii)	Rating	BB	3.84	4.35	0.51
	of Loans	**B**	**4.22**	**5.58**	**1.36**
(viii)	Penalty	2.0	4.22	5.41	1.19
		1.0	4.22	5.51	1.30
		0.0	**4.22**	**5.58**	**1.36**
(ix)	T	7	4.44	5.75	1.31
		5	**4.22**	**5.58**	**1.36**
		3	3.92	5.04	1.12

Note: This table displays the headline results from our sensitivity analysis on a portfolio of corporate loans. The method for estimating the mispricing is described in section 9.3.

Base Case Parameters: $T = 5$, $V_0 = 100$, $r_f = 3.5\%$, $r_m = 10.5\%$, $\sigma_m = 0.14$, $v = 0.25$, $\beta = 0.8$, $\mathcal{P} = 0$, $J = 125$ and $N = 50,000$. Base case parameters are market in **bold**.

Part IV

Concluding Remarks

10 Discussion

Based on our model, the following chapter discusses the assumption we made throughout this thesis, as well as the validity of obtained results. The discussion is divided into three parts: The first part discusses the contributions of our research and how it fits into the existing literature. Next, we consider the assumptions, we have made in our models and the limitations it imposes on our results. Finally, we provide some suggestions for future research and examples of how our model can be expanded.

10.1 Contribution of our Research

In chapters 8 and 9, we have developed a structural model to price CLO debt tranches and examine their spreads. The model builds on CLO characteristics, including dynamics of the collateral assets, and the behaviour in the reinvestment period of the CLO. Our model is an expansion to Merton's Model for a single corporate issuer and provides a framework to assess how a portfolio of corporate issuer evolves, how claims on the portfolio pool can be priced, and how collateral prepayments affect the pricing of these claims.

In the first part of the model, we analyzed the dynamics of a corporate issuer and the differences between a callable and non-callable loan issued by a single corporate issuer with one or more debt tranches. The analysis showed that the introduction of a callable loan with a Bermudian call option resulted in a change of the zero-coupon loan equilibrium value from 54.59 to 55.92. Introducing this prepayment option, we find that in 71.5% of the simulated outcomes, the loan would be repaid before maturity. Additionally, we found that the yield spread between a non-callable and a callable loan is 45 bps. In table 8.6, we showed that the total option value, for a corporate issuer with six tranches of debt, is 1.23.

Using the dynamics from the corporate issuer case, we expand our model to cover the dynamics of a securitization of corporate loans. In this process, we adapt the simulation approach, introduced by Brennan et al. (2009), to obtain the payoffs on a securitization of 125 loans. This is simulated for a securitization both with and without collateral prepayments. By introducing prepayments in the model, we compute the face and tranche values both with and without collateral prepayments. When analyzing the effect of prepayments on the tranche valuation and spreads, we make the key assumption that rating agencies do not take prepayments into account when rating the securitization. Under this assumption, the CLO issues tranches with an aggregate face value without considering the prepayment risk. Concretely, the aggregate face is identical to that presented in table 8.8. However, as the CLO tranche investors are aware of the prepayment risk, they price the tranches accordingly, and are therefore only willing to pay the tranche value presented in table 8.10. Using these face and tranche values, we determine the adjusted spreads and estimate the rating agency-mispricing.

The estimates on mispricing is presented in table 9.1. An important finding is the adjusted spread estimate for the AAA tranche of 142 bps. For the lowest seniority B-tranche, we estimate an adjusted spread of 12.68%. If we compare the weighted-average cost of debt (WACD) for the SPV using the adjusted spreads, we find a WACD of 5.58%. For the base case without prepayments the WACD is 4.22%. Consequently, the total mispricing of the SPV is 1.36%. This is reasonably in line with the ”mispricing” or excess spread between CLO tranches and equivalently rated corporate bonds, observed in empirical data.

10.2 Model Assumptions and Limitations

When discussing the limitations of our model, we need to emphasize that a CLO is a quite complex financial asset with many important characteristics and risk factors. Consequently, it is difficult to incorporate all variable in a comprehensive model. When developing our model, we have had to make some assumptions, which entails some uncertainty about the validity of our results. Having stated this, our focus has been to create a model for investigating our research question and thus analyse the effects from collateral prepayments in relation to CLO tranche spreads. This involved making simplifying assumptions about the lifecycle of the securitization, the dynamics of the corporate issuers, and hence, the dynamics on the outstanding loans from those issuers.

General Assumptions

One of the critical assumptions we have made, concerns the lifecycle of the CLO. By exclusively focusing on the reinvestment period, and assuming no active management of the collateral pool, we pose substantial limits on the empirical validity of the model. This means that we do not incorporate the warehousing or amortization period. Instead of incorporating the warehousing period, we have assumed that the securitization, starts at the closing date with a portfolio of homogeneous loans. For the amortization period, we fix the maturities of all collateral loans to the terminal period of the lifetime of the securitization. Ideally, we should have included the amortization period in the model to catch the effects and risks from this period. However, this would increase complexity requiring assumptions about the amortization period. As our focus in this thesis is on the prepayment risk, which is most severe during the reinvestment period, and proceeds are only distributed in the amortization period, we find this to be a fair assumption.

Assumptions on Corporate Issuer Dynamics

We have used Merton's Model (1974) to model the dynamics of the corporate issuers and their loans, as the foundation to our model of a CLO. Therefore, we implicitly require Merton's underlying assumptions to hold for our model. An important assumption in Merton's model regards defaults. Here we assume that defaults only occur, when the firm's asset values are lower than its liabilities in the terminal period. This is an unrealistic assumption, but for our case with zero-coupon debt it is a very compelling assumption, as it simplifies the framework considerably. Furthermore, this assumption is used in numerous papers on this topic. Similarly, in regards to zero-coupon debt, it would have been more realistic and appropriate to incorporate coupon paying debt. However, for the same reasons, the tractability of modeling zero-coupon bonds is highly attractive.

Another key assumption in our model, concerns the prepayment decision facing the corporate loan issuer. Here we assume discrete call date by assuming that the call option is a Bermudian type option. Additionally, we incorporate a simplistic decision for the borrower to exercise this option, where the borrower decides to repay the debt, if the market value of the debt exceeds the discounted face value. This means that the borrowers do not evaluate alternative call date at later stages, and hence, the option value is not found as an optimal stopping problem, as would be more realistic. If we had used a more sophisticated method of pricing the prepayment option, e.g. using the Longstaff and Schwartz (2001) regression based method, we could have obtained more realistic prices of the option. This is due to the fact that actual prepayment options on leveraged loans are typically American-type options. However, our interest has been to explain the effects of the prepayment risk at a theoretical level – not to price an American-option in the most sophisticated way or analyze the optimal decision for the repayments.

Assumptions about the SPV Dynamics

When we introduce the collateral prepayments in our model of a securitization, we make some assumptions, which we have already accounted for in section 8.3, but will briefly repeat here: First, we assume that collateral loans are repaid at the discretion of the corporate issuer, as described in the collateral dynamics sections. Hence, we do not take into account active management of the loan pool, triggered by a collateral manager. This would have been difficult to incorporate, as the manager incentives are complex themselves and would have been an entire study. Furthermore, we assume for numerical simplicity that loans can only be repaid once. Hence, newly bought loans are non-callable. This can potentially affect the size of our findings, but we would expect even higher mispricing, if the amount of total portfolio changes increased.

Additionally, we assume that all newly bought loans are issued by firms with identical parameters as the original issuers. Due to the non-callable feature of the replaced loan, the proceeds received from the prepayment of the callable loan are higher than the market value of the new non-callable loans. Hereby, there will be a residual cash flow, which we assume is distributed to equity. This assumption is in line with the waterfall structure of the proceeds, described in section 5.4, where distribution can be made, when IC and OC tests are passed and fees are paid. An interesting study would be to examine the effects of replacing repaid loans with identically-rated loans with a higher market loading, which would be an effect of active management (or a limited universe of corporate issuers to buy loans from).

Other Explanations for High Tranche Spreads

Since the onset of the global financial crisis, there has been a field of study in collateralized debt obligation, and specifically in CLOs, and the risks and performance of these asset-backed securities. These studies has mainly focused on risks associated with securitization of assets with the objective of examining how they are affected from systematic risk that might be incompletely accounted for by the rating agencies.

In this thesis, we have chosen not to focus on those same ”mainstream” aspects, but rather focus on other explanations of the risks implied by the high spreads on CLO debt tranches relative to equivalently-rated corporate bonds.

10.3 Suggestions for Future Research

In the process of developing a structural model to investigate our hypothesis about the prepayment risk, we have made several assumptions, which we discussed in the previous section. These assumptions impose some limitations on the applicability of our findings, but we argue that despite these limits our research is relevant. During the process, we have made some interesting findings, which could lead to topics for future research and future improvements of our framework.

Acknowledging that optionality plays a role, when pricing financial securities, an interesting expansion on our model involves pricing the call optionality of the CLO – As explained in part two of the thesis, CLO 2.0 structures have similar prepayment options as the leveraged loans, and this would also affect the pricing of CLO spreads. As shown in the results, the equity tranche is affected by collateral prepayments, and will on average get lower returns compared to without prepayment. As prepayment of the CLO is initiated by the CLO manager, this could be a tool to increase equity returns. As CLO valuations are very complex and their payoffs more so, then an advanced method for pricing CLO callability would be necessary. Here the regression-based approach developed by Longstaff and Schwartz (2001) would be attractive.

Another interesting expansion would be to acknowledge that the CLO manager has the ability to actively change the portfolio composition, subject to rating criteria. We have not included this in our project, but this is an important element when pricing CLOs.

Lastly, we note that in this thesis, we have applied an entirely theoretical model to investigate the effects of the prepayment risk on the spreads of debt tranches. Another interesting study could be to test these effects on empirical data, and investigate in more detail, how these results compare to actual empirical findings.

11 Conclusion

The purpose of this thesis has been to investigate Collateralized Loan Obligations and how the introduction of collateral prepayment risk affect the pricing of CLO tranches.

A CLO is a separate legal entity, i.e. a SPV, where the assets is an underlying pool of mainly floating-rate, speculative-grade loans from companies, referred to as leveraged loans. Through securitization, the CLO manager sell claims on the payment stream from the asset pool to investors with different risk and return characteristics. The underlying leveraged loans have embedded prepayment features, meaning that the corporate borrowers have the possibility to repay their obligations prematurely – This is a risk for CLO investors. To manage this risk, the CLO has a reinvestment period incorporated as part of its lifecycle, between the warehousing and amortization period. In this period, proceeds from prepaid loans are reinvested.

Since its emergence in the 1980s, the CLO market has had ups and downs, where the demand almost vanished and caused a collapse during the Financial Crisis around 2007-2009. This led to the introduction of a new type of structure, referred to as CLO 2.0, which were accompanied by new requirements and regulation. Since, the CLO markets has reached new heights and is still a very popular investment among institutional investors due to the combination of attractive yields and lower capital requirements. This *reach for yield* has been accelerated by the very low interest rates on safe assets worldwide.

To investigate the effects of collateral prepayment risk, we have developed a structural model to price CLO tranches. This model builds on the well-known structural model, Merton's Model, where we have used an extension of the original framework to model the dynamics of the collateral assets. The modeling approach we have taken can be divided into three steps: First, we model a corporate issuer to examine the changes in option value and spread from introducing a call option on its loan. Secondly, using these dynamics, we model a base case, in which the dynamics of a portfolio of loans from an identical corporate issuer without the option is analyzed numerically – The framework for this Monte Carlo approach builds on Brennan et al. (2009). For the final part, we expanded the model to include loan prepayments, initiated by the corporate issuer, to include changes in the loan portfolio.

In our analysis, we have used the numerical results to analyze how the return on CLO equity is affected and if the tranches are mispriced. From this, we found that collateral prepayments affect the return to equity, and on average receive a lower payoff, than in the base case without prepayments. Also, we found that the CLO tranche spreads, in general, are mispriced as, if rating agencies does not incorporate assumptions about collateral prepayments and hence, inconsistencies between the rating on the CLO tranches and the perceived risk by the investors, arises. In total, for our example, we find a total mispricing of 1.37% on the entire CLO structure. Finally, we tested the sensitivity of our results to changes in parameter assumptions and found that the existence of a mispricing is consistent, although it changes with parameter variations. However, in general, these changes do not change the findings of the thesis.

Online Appendix

An online appendix is found on GitHub: here. The appendix consists of the followings files:

Contents of the Online Appendix

	File	Type	Description
1	Collateral Dynamics	.ipynb	A Jupyter Notebook including all calculations for the Collateral section and results
2	CumulativeDefaultTable	.xlsx	The adjusted default table
3	Figures	.ipynb	A Jupyter Notebook including code for generating most figures in this thesis
4	LICENSE	–	–
5	LoanPortfolioTool	.py	Custom Python "package" including all functions related specifically to SPV simulations.
6	OeconToolbox	.py	Custom Python "package" including various custom functions.
7	Parameter Script	.py	A custom script for generating the sensitivity table
8	Readme	.md	–
9	RatingAgencies	.xlsx	Data for the distribution of rating agencies
10	SPV Dynamics	.ipynb	A Jupyter Notebook including all calculations for the SPV dynamics section and results

Acronyms

ABS Asset-Backed Securities. 2, 11–13, 36

BDC Business Development Companies. 9

BSM Black-Scholes-Merton. 55, 56, 58, 60

CAPM Capital Asset Pricing Model. 60, 65

CBO Collateralized Bond Obligation. 13

CDF Cumulative Distribution Function. 66, 75, 76

CDO Collateralized Debt Obligation. 2, 13

CLO Collateralized Loan Obligation. 2, 4, 11, 28

CMO Collateralized Mortgage Obligation. 13

DM Discount Margin. 42, 43

EL Expected Loss. 48, 49

ELLI European Leveraged Loan Index. 27

FRN Floating Rate Note. 25

GBM Geometric Brownian Motion. 56, 58, 62, 63, 65

IC Interest Coverage. 33, 82

IRR Internal Rate of Return. 3

LGD Loss Given Default. 26

LLN Law of Large Numbers. 62

MBS Mortgage-Backed Security. 2, 13, 36

OC Over-Collateralization. 33, 82

PD Probability of Default. 53, 54

ROE Return on Equity. 88

RR Recovery Rate. 27, 53, 54

S&P Standard & Poors. 47–49, 64

SDE Stochastic Differential Equation. 58

SPV Special Purpose Vehicle. 2, 11, 13, 14

WACD Weighted Average Cost of Debt. iv, 19, 31, 39, 44, 85, 96

WAL Weighted Average Life. 38, 48

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