# Fixed Income Derivatives

Risk Management and Financial Institutions

December 11, 2020

Exam Number: S133708

## 1 Discounting Shenanigans

### 1.1 NPV of Swap Portfolio

Using the listed zero coupon forward and discount rate by applying =*fidswappv()*, I find the total mid-market present value of the swap portfolio as: $93{,}162{,}860.74$ (See figure 1 below).

**Figure 1:** Mid-market NPV of Swap Portfolio

| Swap Portfolio | | | | | |
|---|---|---|---|---|---|
| **Notional** | **Start** | **End** | **Type** | **Fixed** | **PV** |
| 8,000,000 | 1y | 10y | payer | 3.0320% | (1,318,561.78) |
| 37,000,000 | 30m | 20y | receiver | 1.2000% | (3,165,552.02) |
| 80,000,000 | 15m | 8y | payer | 1.0000% | 1,494,909.54 |
| 47,000,000 | 5y | 25y | receiver | 4.7300% | 28,429,538.40 |
| 175,000,000 | 2y | 2y | payer | 2.0000% | (3,716,158.74) |
| 12,000,000 | 10y | 10y | payer | 4.4000% | (2,561,962.96) |
| 4,000,000 | 2y | 15y | receiver | 2.1000% | 284,822.59 |
| 50,000,000 | 5y | 10y | receiver | 4.0500% | 10,708,072.56 |
| 11,000,000 | 1y | 7y | payer | 0.3300% | 610,646.90 |
| 88,000,000 | 1y | 2y | receiver | 5.5000% | 8,343,584.12 |
| 200,000,000 | 18m | 4y | payer | 1.0800% | (618,787.52) |
| 16,000,000 | 5y | 5y | payer | 4.5000% | (2,229,398.40) |
| 32,000,000 | 10y | 5y | receiver | 4.0130% | 3,023,100.13 |
| 36,000,000 | 3y | 20y | payer | 1.1600% | 3,484,809.44 |
| 500,000,000 | 3y | 1y | receiver | 0.5000% | (2,518,198.11) |
| 400,000,000 | 18m | 10y | receiver | 3.0000% | 61,968,709.41 |
| 23,000,000 | 1y | 3y | receiver | 1.2500% | 287,188.82 |
| 80,000,000 | 15m | 12y | receiver | 2.4500% | 9,275,400.91 |
| 80,000,000 | 15m | 12y | payer | 2.4000% | (8,821,835.14) |
| 30,000,000 | 10y | 5y | receiver | 3.0400% | 1,545,241.28 |
| 872,000,000 | 1y | 5y | receiver | 1.0000% | 226,408.93 |
| 100,000,000 | 5y | 5y | payer | 4.0000% | (11,569,117.59) |
| | | | | **Total Portfolio NPV** | 93,162,860.74 |

### 1.2 Model Rate Delta Vector of the Swap Portfolio

The model rate delta vector consists of the dollar value changes from a 1 bps bump in the respective forward and discount zero-coupon rates. In panel (a) of figure 2, this vector is listed. The first column refers to the rate being bumped, the second column to the forward xIBOR DV01, the third to the discounting DV01 and the final column is simply the sum of the forward and discounting DV01s.

In general the DV01 is estimated by shifting one rate or the entire curve (forward, discount or both) by 1 bps. E.g. the result of a 1 bps increase in the 6M forward xIBOR rate yields a dollar value change of: $-3{,}099.45$ and if the entire forward xIBOR curve is bumped by 1 bps, then the DV01 is $-813{,}676.09$, as listed in figure 2a.

The corresponding DV01 for the entire discounting curve is $-92{,}279.04$. This implies negative sensitivity towards discounting rates, i.e. an increase in the level of the discounting curve will lower the swap portfolio value and vice versa, this is expected a positive present value portfolio, as is the case. However, when interpreting the size of the discounting DV01, this is small compared to the forward rate DV01.

If, for example, one would position themselves towards lower discounting rates, as Goldman Sachs in the risk magazine article (Cameron, 2013), then one should focus on hedging the unwanted forward rate sensitivity and increase or maintain the negative sensitivity on the discount rates.

**Figure 2:** Model Rates Delta Vectors (DV01) for: (a) the unhedged portfolio in question 1.1 - 1.2 and, (b) the same portfolio, but with the single hedge trade from question 1.3.

| **Model Rates Delta Vector (DV01)** | | | |
|---|---|---|---|
| | | *Unhedged* | |
| | **Fwd** | **Disc** | **Sum** |
| 6M | (3,099.45) | (65.79) | (3,165.24) |
| 1Y | 106,391.35 | 357.89 | 106,749.24 |
| 2Y | (15,798.17) | (2,381.65) | (18,179.82) |
| 3Y | 114,473.50 | (3,054.40) | 111,419.09 |
| 4Y | (75,943.91) | (1,071.32) | (77,015.23) |
| 5Y | (267,686.77) | (4,177.96) | (271,864.73) |
| 7Y | (151,637.03) | (9,356.71) | (160,993.74) |
| 10Y | (165,399.18) | (24,788.47) | (190,187.65) |
| 15Y | (272,463.72) | (12,303.34) | (284,767.06) |
| 20Y | 17,456.02 | (22,972.20) | (5,516.18) |
| 30Y | (99,968.72) | (12,465.08) | (112,433.80) |
| **Total** | **(813,676.09)** | **(92,279.04)** | **(905,955.13)** |

**(a)** Unhedged Portfolio

| **Model Rates Delta Vector (DV01)** | | | |
|---|---|---|---|
| | | *Hedged* | |
| | **Fwd** | **Disc** | **Sum** |
| 6M | (3,099.45) | (65.79) | (3,165.24) |
| 1Y | 106,391.35 | 357.89 | 106,749.24 |
| 2Y | (15,574.57) | (2,350.26) | (17,924.83) |
| 3Y | 25,111.15 | (3,292.65) | 21,818.49 |
| 4Y | (75,639.97) | (556.31) | (76,196.28) |
| 5Y | (265,813.27) | (3,435.88) | (269,249.15) |
| 7Y | (180,816.70) | (9,119.92) | (189,936.63) |
| 10Y | 18,951.00 | (27,184.76) | (8,233.76) |
| 15Y | (30,540.14) | (12,390.36) | (42,930.50) |
| 20Y | (7,984.09) | (22,950.00) | (30,934.09) |
| 30Y | (99,968.72) | (12,465.08) | (112,433.80) |
| **Total** | **(528,983.42)** | **(93,453.13)** | **(622,436.55)** |

**(b)** Hedged Portfolio

## 1.3 At-market Hedge Trade

Looking at the Forward xIBOR DV01, the greatest absolute sensitivities are observed at the 5Y and 15Y maturities with $-267{,}686.77$ and $-272{,}463.72$ respectively. Being allowed to do a single trade only, I choose to target the largest of these, i.e. the 15Y rate. Further, I note a DV01 of $114{,}473.50$ at the 3Y forward rate, implying considerable positive sensitivity at this point. Fortunately, a forward starting 3Y10Y payer swap allows for hedging both the 3Y and 15Y risks. A further constraint is that the swap must be at-market, i.e. have zero NPV. Thus, I use the *=fidswaprate()* function to estimate a fair swap rate of $1.5393\%$ on this 3Y10Y swap, such that the present value of the hedge is zero and the total portfolio value is unchanged at $93{,}162{,}860.74$.

The notional of the hedge is chosen using a simple manual *trial-and-error* method, aiming at minimizing the absolute sensitivites across the delta vector. The most suitable notional is chosen to be $300{,}000{,}000$ - In conclusion a single at-market hedge trade in 300m notional of 3Y10Y payer IRS at a fair swap rate of $1.5393\%$ is made and the new model rate delta vector is included in panel (b) of figure 2.

## 1.4 Market Rate Delta Vector

To compute the market rate delta vector, I will use the Jacobian trick from Linderstrøm (2013), this is computed in excel using its built-in matrix functions to solve equation 1 below, i.e. finding the product of the inverse Jacobian and the model rate delta vector.

$$\underset{N\times 1}{\frac{\partial V(\boldsymbol{P})}{\partial \boldsymbol{B}}} = \Bigg(\underset{N\times M}{\frac{\partial \boldsymbol{B}(\boldsymbol{P})}{\partial \boldsymbol{P}}}^{\top}\Bigg)^{-1} \underset{M\times 1}{\frac{\partial V(\boldsymbol{P})}{\partial \boldsymbol{P}}^{\top}} \tag{1}$$

The computed market rate delta vector is summarized in the fourth column of figure 3.

## 1.5 At-market IRS and CCS Hedge Trades

Keeping the 300m notional 3Y10Y IRS payer from question 1.3, I will now decide on the single at-market CCS hedge to undertake. From figure 3, I see that most CCS risk is placed around the 10Y maturity, and being allowed to do one CCS hedge trade only, I target this point.

Estimating the fair CCS spread using *=fidccsspread()*, the at-market spread of a spot-starting (2B) 10Y CCS is: $-0.3479\%$. Hedging the negative sensitivity implies trading an instrument with the opposite sensitivity, i.e. a payer CCS. A notional of 28,000,000 is chosen to closely offset the sensitivity, at that maturity, after the IRS hedge from problem 1.3 is already in place.

Both hedges and their impact are summarized in column 5-6 in figure 3. Column 7 is the market rates delta vector for the hedged portfolio. As both hedges are traded at-market, i.e. zero NPV, then the portfolio value is unchanged at 93,162,860.74.

**OBS!** *I am a bit conflicted on how to approach this - In the introduction for problem 1, it is stated that I should attempt to execute a strategy similar to Goldman Sach's from the risk magazine article (Cameron, 2013), this would imply positioning the book towards decreasing discount rates. If this is the case, I would buy a 1Y at-market CCS receiver, as this is the only CCS rate with a positive sensitivity. However, in my answer above as well as for the rest of problem 1, I will assume that I should hedge any DV01 risk!*

**Figure 3:** Market Rates Delta Vector

| Market Rates Delta Vector | | | | | | |
|---|---|---|---|---|---|---|
| **Instrument** | **Start** | **Maturity** | **Unhedged Portfolio** | **IRS payer 3Y10Y** | **CCS payer 10Y** | **Hedged Portfolio** |
| IRS | 2B | 6M | (1,688.54) | (0.90) | (0.00) | (1,689.44) |
| IRS | 2B | 1Y | 105,582.58 | (218.91) | 0.00 | 105,363.68 |
| IRS | 2B | 2Y | (16,536.86) | (432.12) | 0.01 | (16,968.96) |
| IRS | 2B | 3Y | 113,644.06 | (90,247.25) | 0.01 | 23,396.82 |
| IRS | 2B | 4Y | (75,694.84) | 189.36 | 0.01 | (75,505.47) |
| IRS | 2B | 5Y | (268,231.61) | 401.93 | 0.01 | (267,829.68) |
| IRS | 2B | 7Y | (155,680.00) | (34,509.70) | (0.07) | (190,189.77) |
| IRS | 2B | 10Y | (183,695.69) | 180,036.52 | (3.82) | (3,662.98) |
| IRS | 2B | 15Y | (300,980.26) | 258,262.36 | 0.00 | (42,717.91) |
| IRS | 2B | 20Y | 6,366.53 | (27,832.97) | (0.00) | (21,466.45) |
| IRS | 2B | 30Y | (131,947.44) | (14.34) | (0.00) | (131,961.78) |
| CCS | 2B | 6M | (47.20) | 0.02 | (0.00) | (47.18) |
| CCS | 2B | 1Y | 958.88 | (222.48) | 0.00 | 736.40 |
| CCS | 2B | 2Y | (1,439.29) | (440.37) | 0.01 | (1,879.65) |
| CCS | 2B | 3Y | (2,581.73) | (660.83) | 0.01 | (3,242.55) |
| CCS | 2B | 4Y | (1,154.94) | 189.81 | 0.01 | (965.12) |
| CCS | 2B | 5Y | (4,160.02) | 404.78 | 0.01 | (3,755.24) |
| CCS | 2B | 7Y | (10,402.74) | 1,690.72 | (0.07) | (8,712.09) |
| CCS | 2B | 10Y | (29,365.32) | 1,656.35 | 27,562.50 | (146.46) |
| CCS | 2B | 15Y | (14,773.94) | 1,960.30 | 0.00 | (12,813.64) |
| CCS | 2B | 20Y | (26,625.30) | (393.34) | (0.00) | (27,018.64) |
| CCS | 2B | 30Y | (15,110.53) | 7.80 | (0.00) | (15,102.73) |
| **Total** | | | **(1,013,564.19)** | **289,826.73** | **27,558.62** | **(696,178.84)** |

## 1.6 Bid IRS Portfolio

Assuming that the senior trader wants a bid quote on the original portfolio from problem 1.1, then the bid would be 88,171,689.90, i.e. the original PV of 93,162,860.74 minus the unhedged forward xIBOR DV01$\times$5 bps and minus the unhedged discounting DV01$\times$10bps. The calculations are summarized in figure 4 below.

**Figure 4:** Pricing the Portfolio

| Pricing the Portfolio | | | |
|---|---|---|---|
| | ***DV01*** | ***x bps*** | |
| Mid-market Present value | | | 93,162,860.74 |
| Forward xIBOR Risk | (813,676.09) | 5 | (4,068,380.46) |
| Discounting Risk | (92,279.04) | 10 | (922,790.38) |
| **Total:** | | | **88,171,689.90** |

## 1.7 QE Announcement Impact

The upwards pressure on the cross-currency spread from the expanded US FED QE-program, will directly affect the zero-coupon discount curve by moving it higher. As the initial portfolio have negative sensitivity towards discount rates, it is effectively positioned towards lower discount rates and this scenario will then lead to a negative P&L on the portfolio, assuming only the first order effect, i.e. assuming that xIBOR forward rates are unchanged and not affected by the QE program.

This is a somewhat simplified result, as excess liquidity from QE typically affects the shorter end of the yield curve more, thus effectively flattening the underlying cross-currency swap curve and thereby flattening the discount curve. However, as most of the IRS portfolio's negative discount rate DV01 is at the long end of the curve, then the total loss from the QE announcement will be moderate relative to a scenario with an outright increase in the level of the cross-currency spread.

## 1.8 Cherry-picked Swap Portfolio

When selecting which swaps from the original portfolio to choose, the expanded model delta vector, i.e. the model delta vectors for each individual swap is a very helpful tool. Picking the swaps that have positive sensitivity to discount rates, while simultaneously having the least amount of sensitivity towards forward rates, from the initial portfolio. In figure 5 below, I have chosen 5 swaps from the original portfolio, that fulfill these objectives.

**Figure 5:** The Cherry-picked Swap Portfolio

| Cherry-picked Swap Portfolio | | | | | |
|---|---|---|---|---|---|
| **Notional** | **Start** | **End** | **Type** | **Fixed** | **Mid-market PV** |
| 8,000,000 | 1y | 10y | payer | 3.0320% | (1,318,561.78) |
| 37,000,000 | 30m | 20y | receiver | 1.2000% | (3,165,552.02) |
| 12,000,000 | 10y | 10y | payer | 4.4000% | (2,561,962.96) |
| 16,000,000 | 5y | 5y | payer | 4.5000% | (2,229,398.40) |
| 100,000,000 | 5y | 5y | payer | 4.0000% | (11,569,117.59) |
| | | | | **Total:** | **(20,844,592.76)** |

In general, when positioning towards a tighter cross-currency basis spread, longer duration out-of-the-money swaps will benefit the most, as these are most sensitive towards increases in the discounting rate. Intuitively, they are long maturity liabilities, and outright changes in the level of the discounting curve, will decrease the size of these liabilities, thus providing positive P&L. The five swaps I have cherry-picked in figure 5 are all OTM and is all quite long-maturity contracts.

When quoting the cherry-picked (CP) portfolio, I need to re-estimate the model rate delta vector*. This confirms that the CP portfolio is indeed positioned for increased discounting rates, while keeping the forward rate DV01 quite moderate. The total forward xIBOR rate and discount rate DV01 are $7{,}563.77$ and $20{,}558.52$ respectively. Looking at the CP portfolio market rate delta vector in figure A.3, I also note that the portfolio is correctly positioned towards increases in the cross-currency spread.

As the mid-market present value of the swaps are known from question 1.1, the mid-market PV of the CP portfolio is trivially found as $-20{,}844{,}592.76$. Adjusting for DV01 hedging costs of 5-10 bps, the final bid quote for the portfolio is $-21{,}087{,}996.85$. With a high conviction on a tightening CCS, the 10 bps hedging costs on the discounting risk DV01 can be left out in a competitive bid to the counterparty.

**Figure 6:** Pricing the Cherry-picked Portfolio

| Pricing the Cherry-picked Portfolio | | | |
|---|---|---|---|
| | ***DV01*** | ***x bps*** | |
| Mid-market Present value | | | (20,844,592.76) |
| Forward xIBOR Risk | 7,563.77 | 5 | (37,818.86) |
| Discounting Risk | 20,558.52 | 10 | (205,585.24) |
| **Total:** | | | **(21,087,996.85)** |

*I have included both the model rate and market rate delta vectors of the cherry-picked portfolio in the appendix of this paper as figure A.3 and A.2 respectively.

# 2 Risk Views

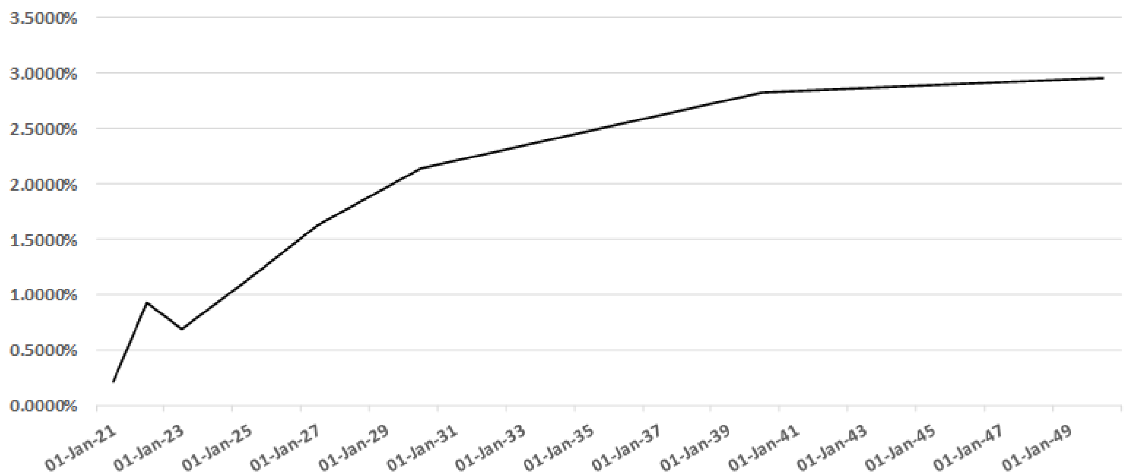
## 2.1 Single Curve Calibration

Using the EUR IRS conventions from table 3 in Linderstrøm (2013) and using the approach from class, i.e. estimating the model swap rate using the zero rates price vector with *=fidswaprate()* and then solve for the zero rates that minimizes the formal calibration problem below:

$$\min_{\boldsymbol{P}} \|\boldsymbol{B}(\boldsymbol{P}) - \boldsymbol{A}\|^2 \tag{2}$$

Where $\boldsymbol{P}$ is the zero rates vector, $\boldsymbol{A}$ is the market quote vector and $\boldsymbol{B}(\boldsymbol{P})$ is the model quote vector. The resulting zero rates from the calibration are listed and illustrated in figure 7 below.

**Figure 7:** Fitted Zero Coupon Curve




| | 11-Dec-21 | 11-Dec-22 | 11-Dec-23 | 11-Dec-25 | 11-Dec-27 | 11-Dec-30 | 11-Dec-40 | 11-Dec-50 |
|---|---|---|---|---|---|---|---|---|
| Z Rates | 0.2069% | 0.9210% | 0.6908% | 1.1449% | 1.6243% | 2.1375% | 2.8225% | 2.9500% |

## 2.2 Forward-Starting Jacobian Matrix

The forward-starting Jacobian matrix is computed by bumping each element in the zero curve vector by 1 bps and then estimating the percentage point difference between model swap rate estimated from the original and the bumped zero rates respectively. The Jacobian matrix for the forward-starting IRS portfolio, is included below as figure 8. The last row is the model rate delta vector.

The Jacobian matrix for the usual spot-starting IRS portfolio is included in the appendix (figure A.4).

**Figure 8:** Forward-Starting Jacobian Matrix

| Forward-Starting Swaps | | | JACOBIAN MATRIX dB(P)/dP | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Instrument | Start | Maturity | 1Y | 2Y | 3Y | 5Y | 7Y | 10Y | 20Y | 30Y |
| IRS | 2B | 1Y | 0.010% | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% |
| IRS | 1Y | 1Y | -0.010% | 0.020% | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% |
| IRS | 2Y | 1Y | 0.000% | -0.020% | 0.030% | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% |
| IRS | 3Y | 2Y | 0.000% | 0.000% | -0.015% | 0.025% | 0.000% | 0.000% | 0.000% | 0.000% |
| IRS | 5Y | 2Y | 0.000% | 0.000% | 0.000% | -0.025% | 0.036% | 0.000% | 0.000% | 0.000% |
| IRS | 7Y | 3Y | 0.000% | 0.000% | 0.000% | 0.000% | -0.024% | 0.034% | 0.000% | 0.000% |
| IRS | 10Y | 10Y | 0.000% | 0.000% | 0.000% | 0.000% | -0.001% | -0.009% | 0.020% | 0.000% |
| IRS | 20Y | 10Y | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | -0.001% | -0.019% | 0.030% |
| **Model Rate Delta Vector** | | | 155.50 | 278.88 | 1,100.65 | 1,797.51 | 3,776.34 | 83,878.33 | (23.62) | - |

## 2.3 Market Rates Delta Vectors

Having computed the Jacobian for both spot- and forward-starting swap portfolios, I will use the Jacobian trick, I accounted for in problem 1.4, again. To estimate the market rates delta vector I need two components: the Jacobian Matrix - which I already have from the previous problem - and secondly the model rates delta vector. The model rates delta vector is computed similarly to the Jacobian, but instead of percentage point differences, one would compare the initial present value relative to the present value of the same portfolio computed using the bumped Z-rates.

The problem description states that I should value the market rates delta vector for a 100m at-market 10Y payer swap. As the swap should be at-market, i.e. zero initial NPV, I start by estimating the fair swap rate using $= fidswaprate()$ in excel. The corresponding swap rate is: $2.0960\%$. Keeping the swap rate fixed, I estimate the model rate delta vector - this vector is not influenced by whether the market risk is expressed as spot-starting or forward-starting rates, as it is directly estimated on the z-rates. The model rate delta vector is summarized in the last row of figure 8.

Having computed the two components, i.e. the Jacobian and the model rate delta vector, the market rate delta vectors for both the spot- and forward-starting IRS can be computed trivially in Excel. Both model rate delta vectors are listed in column 3 and 4 respectively in figure 9 below.

**Figure 9:** Market Rate Delta Vectors

| Market Rate Delta Vectors | | | |
|---|---|---|---|
| **Maturity** | **Spot Starting** | **Forward Starting** | **diff** |
| 1Y | 0.02 | 9,950.83 | 9,950.81 |
| 2Y | 0.04 | 9,577.80 | 9,577.76 |
| 3Y | 0.11 | 9,626.79 | 9,626.67 |
| 5Y | 0.13 | 18,196.32 | 18,196.19 |
| 7Y | (0.14) | 17,197.93 | 17,198.07 |
| 10Y | 91,888.89 | 24,551.45 | (67,337.44) |
| 20Y | (0.00) | 11.98 | 11.98 |
| 30Y | (0.00) | 0.09 | 0.09 |
| **Sum** | **91,889.05** | **89,113.19** | (2,775.86) |

Significant differences between the market rate delta vectors can be seen from figure 9. A major determinant of these differences is that the spot-starting market rate delta vector have most of its risk in the 10Y bucket, whereas the forward-starting market rate delta vector have about the same level of total risk, but more uniformly distributed throughout the curve. This is consistent with the Jacobians (figure 8 and A.4), where the spot-starting Jacobian is sensitive across the diagonal only, i.e. at the maturities of the swaps, whereas the forward-starting Jacobian, have some offsetting sensitivity in the starting period bucket.

# 3 Bonds, Carry and Swaptions

## 3.1 Cash Flow Schedule

To compute the cash flow schedule and corresponding cash flows, I begin by using the *=fidgenerateschedule()* function on the bond data in the excel sheet. The output is summarized in column 1-5 of figure 10. Next, I estimate the semiannually coupon rates and add 100% to the end date, as the bullet bond pay back the entire principal at maturity. I compute this in both percentage terms and notional amounts (column 6-7).

To calculate the present value of cash flows I compute the discount factors using *=fiddiscfactor()* and multiply these factors with the notional cash flows. See column 8-9 in figure 10. The discount factors are based on the stated zero-coupon rates plus the 102.5 bps Z-spread.

**Figure 10:** Cash Flow Schedule and NPV Calculation

| Unadjusted | | Unadjusted | | Cvg | %CF | $CF | ZC Bond | Disc. $CF | Obj. |
|---|---|---|---|---|---|---|---|---|---|
| *Start Date* | *End* | *Start Date* | *End* | | *Cvg × Coup.* | *%CF × Not.* | P(0,T) | P(0,T) × $CF | $CF × (1+YTM)^(-Σ Cvg) |
| 11-12-20 | 09-06-21 | 11-12-20 | 09-06-21 | 0.494 | 0.803% | 401,736.11 | 0.994269 | 399,433.65 | 398,380.86 |
| 09-06-21 | 09-12-21 | 09-06-21 | 09-12-21 | 0.500 | 0.813% | 406,250.00 | 0.988539 | 401,593.99 | 399,454.79 |
| 09-12-21 | 09-06-22 | 09-12-21 | 09-06-22 | 0.500 | 0.813% | 406,250.00 | 0.981701 | 398,815.95 | 396,081.26 |
| 09-06-22 | 09-12-22 | 09-06-22 | 09-12-22 | 0.500 | 0.813% | 406,250.00 | 0.974481 | 395,882.82 | 392,736.22 |
| 09-12-22 | 09-06-23 | 09-12-22 | 09-06-23 | 0.500 | 0.813% | 406,250.00 | 0.967387 | 393,000.95 | 389,419.44 |
| 09-06-23 | 09-12-23 | 09-06-23 | 11-12-23 | 0.506 | 0.822% | 410,763.89 | 0.960148 | 394,394.11 | 390,384.21 |
| 09-12-23 | 09-06-24 | 11-12-23 | 10-06-24 | 0.497 | 0.808% | 403,993.06 | 0.952540 | 384,819.36 | 380,724.67 |
| 09-06-24 | 09-12-24 | 10-06-24 | 09-12-24 | 0.497 | 0.808% | 403,993.06 | 0.944666 | 381,638.46 | 377,527.11 |
| 09-12-24 | 09-06-25 | 09-12-24 | 09-06-25 | 0.500 | 0.813% | 406,250.00 | 0.936675 | 380,524.33 | 376,430.05 |
| 09-06-25 | 09-12-25 | 09-06-25 | 09-12-25 | 0.500 | 0.813% | 406,250.00 | 0.928506 | 377,205.59 | 373,250.97 |
| 09-12-25 | 09-06-26 | 09-12-25 | 09-06-26 | 0.500 | 0.813% | 406,250.00 | 0.920273 | 373,860.91 | 370,098.75 |
| 09-06-26 | 09-12-26 | 09-06-26 | 09-12-26 | 0.500 | 0.813% | 406,250.00 | 0.911916 | 370,466.02 | 366,973.14 |
| 09-12-26 | 09-06-27 | 09-12-26 | 09-06-27 | 0.500 | 0.813% | 406,250.00 | 0.903526 | 367,057.45 | 363,873.93 |
| 09-06-27 | 09-12-27 | 09-06-27 | 09-12-27 | 0.500 | 0.813% | 406,250.00 | 0.894999 | 363,593.54 | 360,800.90 |
| 09-12-27 | 09-06-28 | 09-12-27 | 09-06-28 | 0.500 | 0.813% | 406,250.00 | 0.886392 | 360,096.71 | 357,753.82 |
| 09-06-28 | 09-12-28 | 09-06-28 | 11-12-28 | 0.506 | 0.822% | 410,763.89 | 0.877640 | 360,502.79 | 358,640.14 |
| 09-12-28 | 09-06-29 | 11-12-28 | 11-06-29 | 0.500 | 0.813% | 406,250.00 | 0.868996 | 353,029.81 | 351,703.49 |
| 09-06-29 | 09-12-29 | 11-06-29 | 10-12-29 | 0.497 | 0.808% | 403,993.06 | 0.860335 | 347,569.45 | 346,812.17 |
| 09-12-29 | 09-06-30 | 10-12-29 | 10-06-30 | 0.500 | 0.813% | 406,250.00 | 0.851671 | 345,991.30 | 345,804.37 |
| 09-06-30 | 09-12-30 | 10-06-30 | 09-12-30 | 0.497 | 100.808% | 50,403,993.06 | 0.843018 | 42,491,456.43 | 42,544,083.32 |
| | | | | | | | **NPV =** | **49,640,933.62** | **49,640,933.62** |

## 3.2 Bond Price and Yield

To calculate the price of bond XYZ, I simply add the present values of all cash flows. This net present value is: $49{,}640{,}933.62$. Having computed the price, finding the yield-to-maturity (YTM) is a straightforward internal rate of return-type calculation. I start by setting up the last column of figure 10, by multiplying the notional cash flows by the compounded YTM discount factors, next I use solver to minimize the difference between the price of the bond and the model price from the YTM-calculations, by changing the YTM. The YTM that minimizes this difference is: $1.7107\%$, slightly above the fixed rate of the bond.

Another way to approach the bond pricing, which is useful for the following problems, is by acknowledging that the bullet bond can be decomposed into a floating rate note (FRN) and a IRS swap with identical maturity and coupon frequencies/tenors. Thus, these instruments can be priced individually using the methods from the course.

Using *=fidSwapPv()* to compute the PV of the receiver swap, I find this value to be: $-355{,}908.37$ adding this to the notional amount $49{,}996{,}841.98$ (50m discounted by two business days using *=fiddiscfactor()*), yields the same bond price as I found in figure 10, i.e. $49{,}640{,}933.62$.

## 3.3 Carry and Roll-Down

For this question, I have combined the definitions of the carry and roll-down with the mathematical relation from Koijen et al. (2018). I define the excess carry return $r_t^{carry}$ of a funded position in the bond. The formal definition of this excess carry return is:

$$r_t^{carry} = \underbrace{y_t^T - r_t^f}_{\text{carry}} + \underbrace{\frac{P_{t+1}^{T-1}(y_t^{T-1}) - P_{t+1}^{T-1}(y_t^T)}{P_t^T}}_{\text{roll-down}} \tag{3}$$

Where $y_t^T$ is the yield (YTM) in the initial period $t$ until maturity $T$. $r_t^f$ is the funding rate (in this example $0.40\%$) and $P_t^T(y)$ is the price of a bond in period $t$ with maturity $T$, given the yield curve $y$ - This definition of the carry return is expressed in percentages, but multiplying with the initial bond price, it can be adjusted to a notional amount as well.

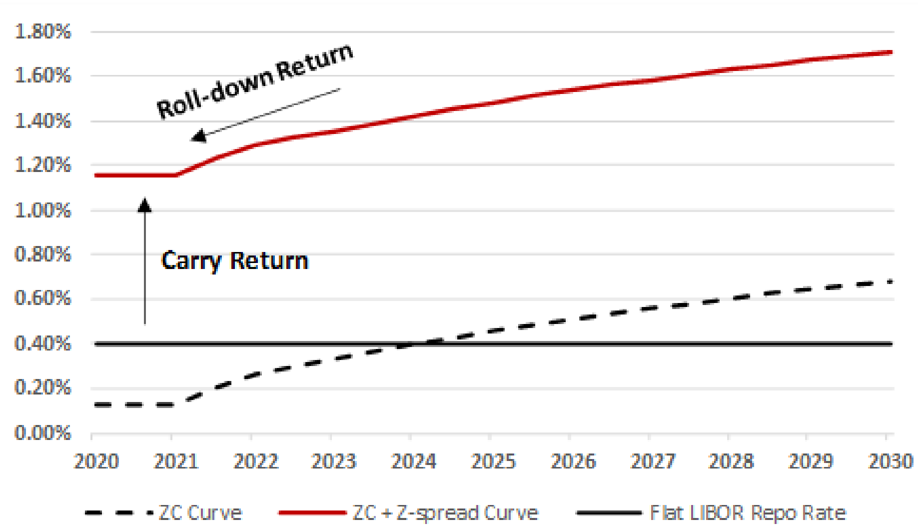
To estimate the excess carry return, I start by estimating the different bond prices, i.e. the one-year forward price of a 9Y maturity bond and the one-year forward price of a 10Y bond. The underlying assumption here, is a constant zero-coupon yield curve, i.e. the excess carry return is the return, if the entire yield curve is unchanged in a year. Using the approach described in problem 3.2 to estimate the bond prices[†], I can compute the excess carry return as:

$$r_t^{carry} = (1.7107\% - 0.40\%) + \frac{49{,}019{,}239.79 - 48{,}839{,}905.98}{49{,}640{,}933.62} = \underbrace{1.3107\%}_{\text{carry}} + \underbrace{0.3613\%}_{\text{roll-down}} = 1.6720\% \quad (4)$$

Multiplying this with the initial bond price, yields a nominal carry return of $650{,}646.871$ and a roll-down return of $179{,}333.80$, for a total excess carry return of $829{,}980.81$.

These values doesn't seem unreasonable if the yield curve stays the same. In figure 11 below, I have tried to illustrate the two effects. First the carry is simply the yield over the funding cost, while the roll-down is the return if the entire term structure of zero-coupon rates stay constant - in this case I note that the 10Y bond, will be the a 9Y bond in one year, and thus, the bond will "roll-down" the yield curve, increasing in price, when the yield curve is upward sloping, as is the case in this example.

**Figure 11:** Carry and Roll-down return




## 3.4 Bond DV01

Computing the DV01 of Bond XYZ is similar to computing the model rate delta vectors in problem 1.2 and 2.2. The process involves bumping each zero-coupon rate by 1 bps and estimating the dollar value change.

The total DV01 is: $-45{,}978.60$, so the bond is sensitive towards higher rates across the curve, i.e. decreasing in value when rates increase. This sensitivity is particularly present around maturity, which makes sense as this is when the notional is exchanged. The DV01 results are listed in figure 12 below.

**Figure 12:** Bond XYZ Model Rate Delta Vector (DV01)

| Maturities | 1Y | 2Y | 3Y | 4Y | 5Y | 7Y | 10Y |
|---|---|---|---|---|---|---|---|
| DV01 | (76.41) | (170.60) | (230.73) | (232.93) | (435.06) | (1,757.51) | (43,075.37) |
| | | | | | | **Total** | **(45,978.60)** |

[†]The estimated bond prices is included as figure A.5 in the appendix

## 3.5 Break-Even Shift

The total excess carry and roll return was $829{,}980.81$. When determining the size of the "break-even" shift in the level of the zero-coupon curve, I use the total DV01 computed in the previous problem (3.4), to find the bps increase sufficient to offset this gain. Dividing the excess carry return by the total DV01, yields a value of $18.20$, i.e. if the level of the zero-coupon curve changes by $18.20$ bps, then the gain is offset and the carry and roll strategy is P&L-neutral, when everything but the level of the ZC-curve stays the same.

As DV01 is an estimate for marginal changes in the zero rates, some precision is lost when larger changes are made. So it should be noted that there is some model risk from using the linear DV01 estimate.

## 3.6 Call Provision

I have already stated in problem 3.2 that a bullet bond can be decomposed into a FRN and a IRS. Intuitively, a call provision, i.e. an embedded call option on the bond can then be evaluated as a swaption on the IRS - Swaption terminology implies that a 2Y8Y swaption, is a 2-year option to enter into a 8Y IRS. At the 2-year expiry date, there is 8-years to bond maturity, and thus, the contract matches the maturity of the bond.

When pricing the bond in problem 3.2, the bond price was seen from the perspective of the buyer, i.e. receiving the fixed rate, that's why the bond was composed of a FRN and a receiver IRS - This also means that the bond issuer effectively holds a payer swap, and thus, when entering into a receiver swap with an identical fixed rate and conventions, the swaps exactly offsets each other.

In regards to the conventions and specifically the tenor on the floating leg, this should be made to match the bond and its coupon frequency, not follow standard IRS conventions.

## 3.7 Pricing an Embedded Option

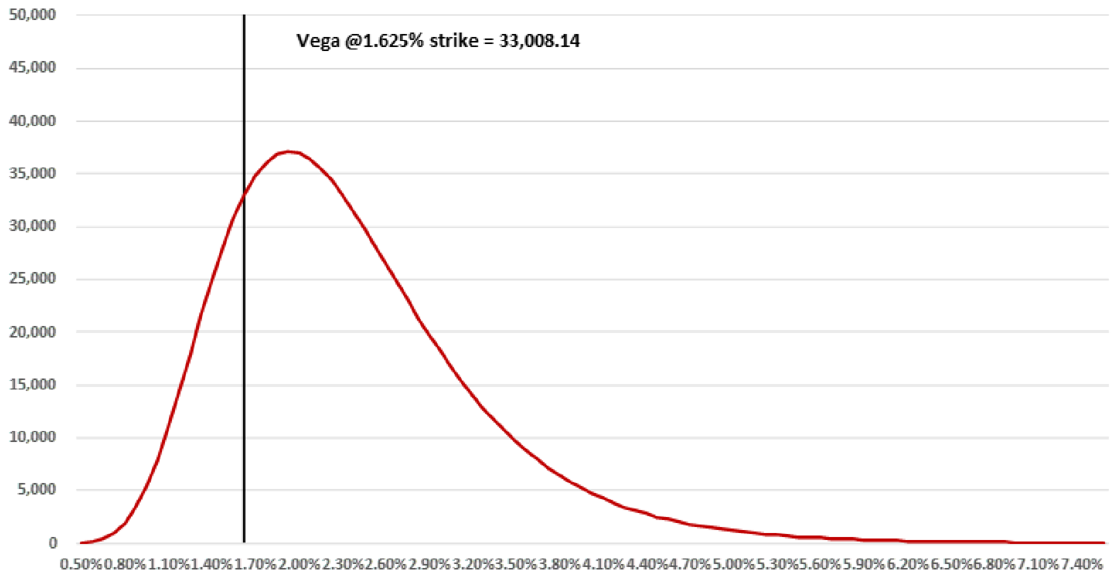
When pricing the 2Y8Y swaption, I use the *=fidswaptionpv()* function, applying Black's model at a strike price identical to the $1.6250\%$ fixed rate on the bond. Thus, the option is effectively a buyback of the remaining bond, if physical settlement is chosen. Assuming a constant Black volatility of 25%, the present value of the 2Y8Y receiver swaption is: $576{,}046.58$.

## 3.8 Bond Vega

Using the bump-and-reprice approach from class to computing the vega, i.e. the option sensitivity to volatility, involves bumping the Black volatility by 1 percentage point and computing the difference in the swaption value, from using the initial and bumped volatility.

Using volatilities of 25% and 26%, I estimate the vega of the 2Y8Y swaption, and indirectly the bond vega, to: $33{,}008.14$ at the $1.625\%$ strike. In figure 13 below, I have included the Vega for different strikes and note that as the strike moves farther from the fair swap rate, the swaption vega decreases.

**Figure 13:** Vega at Different Strikes



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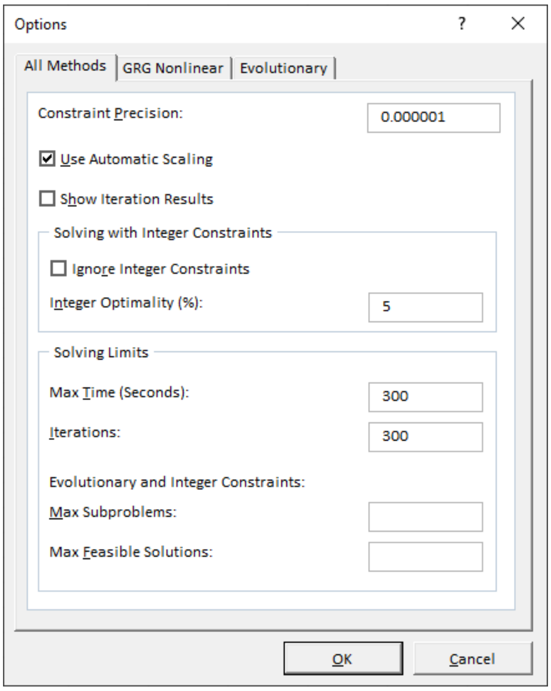
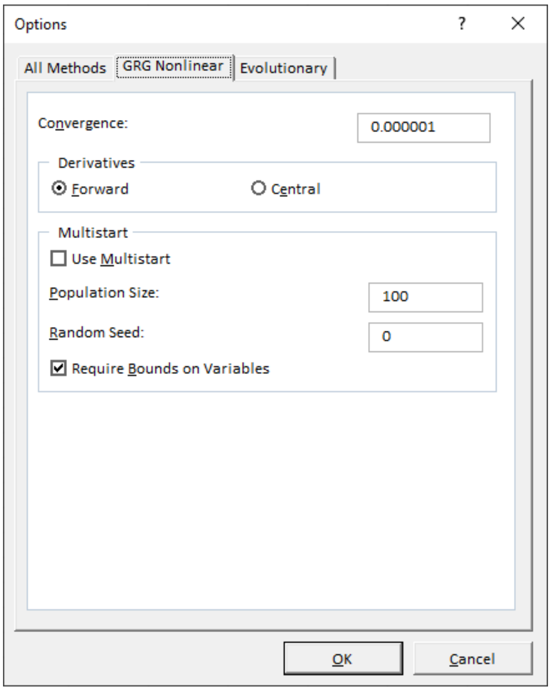
# Appendix

## Appendix 1 - Excel Solver Settings




## Appendix 2 - Cherry-picked Portfolio Delta Vectors

**Figure A.2:** Cherry-picked Portfolio Model Delta Vector

| Model Rates Delta Vector (DV01) | | | |
|---|---|---|---|
| | Fwd | Disc | Sum |
| 6M | (0.30) | 0.57 | 0.27 |
| 1Y | (1,376.73) | (1.86) | (1,378.59) |
| 2Y | 5,203.33 | 42.70 | 5,246.03 |
| 3Y | 5,120.58 | 55.70 | 5,176.29 |
| 4Y | (756.81) | (256.96) | (1,013.77) |
| 5Y | (56,588.25) | 725.06 | (55,863.19) |
| 7Y | 1,079.46 | 5,877.20 | 6,956.66 |
| 10Y | 104,938.48 | 7,076.17 | 112,014.64 |
| 15Y | 14,791.76 | 3,665.41 | 18,457.17 |
| 20Y | (56,772.84) | 3,380.79 | (53,392.05) |
| 30Y | (8,074.91) | (6.25) | (8,081.16) |
| **Total** | **7,563.77** | **20,558.52** | **28,122.30** |

**Figure A.3:** Cherry-picked Portfolio Market Delta Vector

| Market Rates Delta Vector | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Instrument | Start | Maturity | 1Y10Y payer @ 3.032% 8m Not. | 30M20Y receiver @ 1.200% 37m Not. | 10Y10Y payer @ 4.400% 12m Not. | 5Y5Y payer @ 4.500% 16m Not. | 5Y5Y payer @ 4.000% 100m Not. | Cherry-picked Portfolio |
| IRS | 2B | 6M | (10.55) | (6.30) | 1.58 | 1.37 | 7.11 | (6.79) |
| IRS | 2B | 1Y | (794.86) | (510.10) | (7.31) | (7.83) | (47.11) | (1,367.21) |
| IRS | 2B | 2Y | 30.28 | 5,270.08 | (14.11) | (15.18) | (91.59) | 5,179.48 |
| IRS | 2B | 3Y | 41.40 | 5,201.10 | (21.15) | (22.98) | (138.55) | 5,059.82 |
| IRS | 2B | 4Y | 41.04 | (561.25) | (21.59) | (109.00) | (627.85) | (1,278.65) |
| IRS | 2B | 5Y | 101.04 | 80.78 | (53.90) | (7,798.31) | (48,854.16) | (56,524.55) |
| IRS | 2B | 7Y | (836.81) | 162.01 | (304.45) | 1,058.78 | 5,648.34 | 5,727.87 |
| IRS | 2B | 10Y | 8,801.99 | 439.73 | (10,953.91) | 16,415.22 | 101,708.58 | 116,411.61 |
| IRS | 2B | 15Y | 1,326.28 | 18,731.84 | 2,511.05 | (29.85) | (164.74) | 22,374.58 |
| IRS | 2B | 20Y | (144.10) | (81,516.64) | 23,484.52 | (0.03) | (0.14) | (58,176.38) |
| IRS | 2B | 30Y | (0.07) | (9,470.67) | (41.09) | (0.00) | (0.00) | (9,511.83) |
| CCS | 2B | 6M | 0.81 | 1.58 | 1.56 | 1.35 | 7.02 | 12.32 |
| CCS | 2B | 1Y | (8.14) | 37.90 | (7.45) | (7.98) | (47.99) | (33.66) |
| CCS | 2B | 2Y | 27.54 | 69.19 | (14.50) | (15.56) | (93.78) | (27.11) |
| CCS | 2B | 3Y | 41.85 | 70.45 | (21.65) | (23.43) | (141.13) | (73.91) |
| CCS | 2B | 4Y | 41.65 | 25.95 | (21.99) | (81.77) | (456.75) | (492.91) |
| CCS | 2B | 5Y | 102.41 | 83.32 | (55.19) | 94.57 | 474.64 | 699.75 |
| CCS | 2B | 7Y | 277.88 | 171.87 | (297.44) | 1,053.10 | 5,594.94 | 6,800.36 |
| CCS | 2B | 10Y | 393.62 | 531.83 | 659.96 | 965.12 | 5,127.95 | 7,678.49 |
| CCS | 2B | 15Y | 27.65 | 500.54 | 2,550.15 | (23.82) | (126.62) | 2,927.90 |
| CCS | 2B | 20Y | (4.83) | 1,553.18 | 1,706.73 | (0.02) | (0.12) | 3,254.95 |
| CCS | 2B | 30Y | 0.04 | 42.43 | (39.86) | (0.00) | (0.00) | 2.61 |
| **Total IRS** | | | **8,555.63** | **(62,179.43)** | **14,579.66** | **9,492.20** | **57,439.88** | **27,887.95** |
| **Total CCS** | | | **900.49** | **3,088.25** | **4,460.31** | **1,961.56** | **10,338.16** | **20,748.78** |

## Appendix 3 - Spot-starting Swap Jacobian

**Figure A.4**

| Spot Starting | JACOBIAN MATRIX dB(P)/dP | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Maturity** | **1Y** | **2Y** | **3Y** | **5Y** | **7Y** | **10Y** | **20Y** | **30Y** |
| 1Y | 0.010% | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% |
| 2Y | 0.000% | 0.010% | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% |
| 3Y | 0.000% | 0.000% | 0.010% | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% |
| 5Y | 0.000% | 0.000% | 0.000% | 0.010% | 0.000% | 0.000% | 0.000% | 0.000% |
| 7Y | 0.000% | 0.000% | 0.000% | 0.000% | 0.010% | 0.000% | 0.000% | 0.000% |
| 10Y | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 0.009% | 0.000% | 0.000% |
| 20Y | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 0.001% | 0.008% | 0.000% |
| 30Y | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 0.001% | 0.002% | 0.007% |
| **Model Rate Delta Vector** | 155.50 | 278.88 | 1,100.65 | 1,797.51 | 3,776.34 | 83,878.33 | (23.62) | - |

## Appendix 4 - Bond Prices for Calculating Carry Return

**Figure A.5**

| Bond Prices | | | |
|---|---|---|---|
| **Start** | 2B | 1Y | 1Y |
| **Date** | 11-Dec-20 | 09-Dec-21 | 09-Dec-21 |
| Maturity | 10Y | 9Y | 10Y |
| **Date** | 09-Dec-30 | 10-Dec-29 | 09-Dec-30 |
| **Receiver PV** | (355,908.37) | (407,712.97) | (587,046.77) |
| **Notional PV** | 49,996,841.98 | 49,426,952.75 | 49,426,952.75 |
| | **49,640,933.62** | **49,019,239.78** | **48,839,905.98** |