

Some calculations, phonons

Kristian Knakkegaard Nielsen

2. december 2016

Remember $k_n = n \frac{2\pi}{Na}$, and $-\pi/a \leq k < \pi/a$. Hence, $\Delta k = \frac{2\pi}{Na}$. So:

$$E_0 = \frac{1}{2} \sum_k \hbar \omega(k) = \frac{1}{2} \frac{1}{\Delta k} \sum_k \hbar \omega(k) \Delta k \rightarrow \frac{1}{2} \frac{1}{\Delta k} \int_{-\pi/a}^{\pi/a} \hbar \omega(k) dk = \frac{1}{2} \frac{Na}{2\pi} \int_{-\pi/a}^{\pi/a} \hbar \omega(k) dk. \quad (1)$$

This is the standard approach of replacing a discrete sum with an integral. It is allowed to do as long as the function we integrate varies slowly over Δk . This is obtained for $N \gg 1$.