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Aflevering uge 3, E18 Kvantemekanik

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I et uendelig brøndpotential $V(x) = \begin{cases} 0 & 0 \le x \le a \\ \infty & \text{ellers} \end{cases}$

har vi en partikel med masse m

og til $t=0$ har vi $\Psi(x,0) = \begin{cases} A & 0 \le x \le a \\ 0 & \text{ellers.} \end{cases}$

1) Vi skal bestemme A ved normalisering

$$\int_{-\infty}^{\infty} \Psi^* \cdot \Psi \, dx = \int_0^a A^2 \, dx = A^2 [x]_0^a = A^2 a = 1$$

$$\Rightarrow A = \sqrt{\frac{1}{a}} \Rightarrow \Psi(x,0) = \begin{cases} \sqrt{\frac{1}{a}} & 0 \le x \le a \\ 0 & \text{ellers.} \end{cases}$$

2) Vi skal bestemme ss. for at partiklen har energi E_1

$$\Psi(x,0) = \sum_{n=1}^{\infty} c_i \psi_n(x)$$, ψ_n er en komplet basis derfor kan $\Psi(x,0)$ skrives på denne form

$\Downarrow$

$$c_1 = \int_0^a \psi_1(x)^* \cdot \Psi(x,0) \, dx$$ (fouriers trick)

$$= \int_0^a \sqrt{\frac{2}{a}} \sin\left(\frac{\pi}{a} x\right) \cdot \sqrt{\frac{1}{a}} \cdot dx$$ indsat ψ_1 for det uendelige brøndpotential

$$= \sqrt{2} \frac{1}{a} \int_0^a \sin\left(\frac{\pi}{a} x\right) dx$$ konstant udenfor Integraltegn

$$= \frac{\sqrt{2}}{a} \left[-\cos\left(\frac{\pi}{a} x\right) \frac{a}{\pi} \right]_0^a$$ evaluer

$$= \frac{\sqrt{2}}{a} \left(\frac{a}{\pi} - \cancel{\cos}(\pi) \right) = \frac{\sqrt{2}}{\pi} = 0.45$$

$$P(E_1) = |c_1|^2 = \underline{\underline{\frac{2}{\pi^2}}}$$, Check at $P(E_1) \in [0,1]$

3) Udregning af ss. for E_2, løses som 2)

$$P(E_2) = |c_2|^2$$

$$c_2 = \int \psi_2^*(x) \cdot \Psi(x) \, dx$$ $\longrightarrow$

$$= \int_0^a \sqrt{\frac{2}{a}} \sin\left(\frac{2\pi}{a}x\right) \cdot \sqrt{\frac{1}{a}} \, dx$$

$$= \frac{\sqrt{2}}{a} \frac{a}{2\pi} = \frac{\sqrt{2}}{2\pi}$$

$$P(E_2) = |C_2|^2 = \underline{\underline{\frac{2}{4\pi^2}}} \quad \in [0,1] \; !$$