For at få en idé om tidsudv. af systemet, kan vi kigge på kov. af $x_j(t)$ og $x_\ell(0)$:

$$\mathrm{Cov}(x_j(t), x_\ell(0)) = \langle k| x_j(t) x_\ell(0) |k\rangle .$$

$$x_j(t)x_\ell(0) = \left(\frac{1}{\sqrt N}\sum_{q_1} x_{q_1}(t) e^{iq_1 ja}\right)\left(\frac{1}{\sqrt N}\sum_{q_2} x_{q_2}(0) e^{iq_2 \ell a}\right)$$

$$= \frac{1}{4N}\cdot\frac{2\hbar}{m}\sum_{q_1,q_2}\left(a_{q_1}e^{-i\omega(q_1)t} + a^+_{-q_1}e^{i\omega(q_1)t}\right)\left(a_{q_2}+a^+_{-q_2}\right) e^{iq_1 ja}\cdot e^{iq_2\ell a}\cdot\frac{1}{\sqrt{\omega(q_1)\omega(q_2)}}$$

Herved:

$$\langle 0|a_k x_j(t)x_\ell(0)a_k^+|0\rangle =$$

$$\frac{\hbar}{2mN}\cdot\langle 0|a_k \sum_{q_1,q_2}\left(a_{q_1}a^+_{-q_2}e^{-i\omega(q_1)t} + a^+_{-q_1}a_{q_2}e^{i\omega(q_1)t}\right) e^{iq_1 ja}e^{iq_2\ell a}\cdot\frac{1}{\sqrt{\omega(q_1)\omega(q_2)}}\, a_k^+|0\rangle$$

$$\frac{\hbar}{2mN}\sum_{q_1,q_2}\frac{1}{\sqrt{\omega(q_1)\omega(q_2)}}\left(\left(\delta_{q_1,k}\,\delta_{k,-q_2} + \delta_{q_1,-q_2}\right)e^{-i\omega(q_1)t} + \delta_{k,-q_1}\,\delta_{k,q_2}\,e^{i\omega(q_1)t}\right)e^{iq_1ja}e^{iq_2\ell a}$$

$$= \frac{\hbar}{2mN}\left(\frac{1}{\omega(k)}e^{-i\omega(k)t}e^{ik(j-\ell)a} + \sum_{q_2}\frac{1}{\omega(q_2)}e^{-i\omega(q_2)t}e^{iq_2(\ell-j)a} + \frac{1}{\omega(k)}e^{i\omega(k)t}e^{ik(\ell-j)a}\right).$$

Desuden:

$$\langle 0|x_j(t)x_\ell(0)|0\rangle = \frac{\hbar}{2mN}\langle 0|\sum_{q_1,q_2}\frac{1}{\sqrt{\omega(q_1)\omega(q_2)}}\left(a_{q_1}a^+_{-q_2}e^{-i\omega(q_1)t} + \cancel{a^+_{-q_1}a_{q_2}e^{i\omega(q_1)t}}\right)e^{iq_1ja}e^{iq_2\ell a}|0\rangle$$

$$= \frac{\hbar}{2mN}\sum_{q_2}\frac{1}{\omega(q_2)}e^{-i\omega(q_2)t}e^{iq_2(\ell-j)a}$$

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