No.

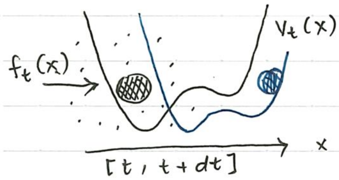
Date

Stochastic Energetics

K. Sekimoto "Langevin Eqn and thermodynamics" (1997)

P. Langevin Stochastic Dynamics (1908)

Example: particle (overdamped) in 1D, trapped in $V_t(x)$, force "external" $f_t(x)$



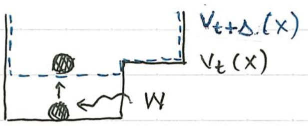
x_t = position of particle at time t

$V_t(x_t)$ = energy of particle at time t

2 energy changes : Heat and work

① Heat - change of particle's energy from environment interactions

② Work - change of potential



Langevin Equation:

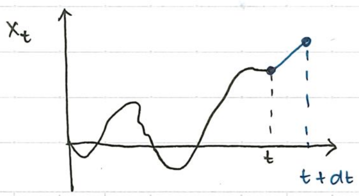
$$\cancel{m\ddot{x}_t} = -\gamma \dot{x}_t \underbrace{- V'_t(x_t) + f_t(x_t)}_{\text{total force}} + \sqrt{2k_B T \gamma}\,\xi_t$$

- neglect when mass of particle is small
- $\Delta t \gg \frac{m}{\gamma}$

Overdamped Langevin Equation:

$$(1) \quad 0 = -\gamma \dot{x}_t \underbrace{- V'_t(x_t) + f_t(x_t)}_{F_t(x_t)} + \sqrt{2k_B T\gamma}\,\xi_t$$

$\Delta t \gg m/\gamma$



In $[t, t+dt]$ (time interval) we measure $x_t \in \mathbb{R}$, $x_{t+dt} \in \mathbb{R}$,

how much δQ_t is absorbed by the system ?

$$\delta Q_t \overset{\text{definition}}{=} (\overbrace{-\gamma \dot{x}_t + \sqrt{2k_B T\gamma}\,\xi_t}^{\text{forces from environment}}) \circ dx_t$$

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Stochastic Heat

$$\delta Q_t = (-\gamma \dot{x}_t + \sqrt{2k_B T\gamma}\, \xi_t) \circ dx_t$$

cannot measure this in the lab

$\hookrightarrow dx_t = x_{t+dt} - x_t$

$$\overset{(1)}{=} \underbrace{[\, V'_t(x_t) - f_t(x_t)\,]}_{\text{we can measure this in the lab}} \circ dx_t$$

Stochastic Work Work done in $[t, t+dt]$

$$\delta W_t = \underbrace{f_t(x_t)}_{\text{external}} \circ dx_t + \underbrace{\partial_t V_t(x_t)\, dt}_{\text{control potential}}$$

First law?

$$\delta Q_t + \delta W_t = V'_t(x_t) \circ dx_t - \cancel{f_t(x_t)} \circ dx_t + \cancel{f_t(x_t)} \circ dx_t + \partial_t V_t(x_t)\, dt$$

$$= \partial_t V_t(x_t)\, dt + V'_t(x_t) \circ dx_t + f\, dx - f\, dx$$

$$= dV_t$$

$$\partial_t V_t(x_t)\, dt + V'_t(x_t) \circ dx_t + f dx - f dx$$

$$\underbrace{\partial_t V_t(x_t)\, dt + f dx}_{\text{Work}} + \underbrace{V'_t(x_t) \circ dx - f dx}_{\text{Heat}}$$

$$\left(\text{Reminder: } dV = \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial x} \circ dx \right)$$

(Stochastic) Heat along a trajectory $x_{[0,t]}$

$$Q_t = Q[x_{[0,t]}] \quad (\text{a "functional"})$$

$$= \int_0^t \delta Q_s = \int (-\gamma \dot{x}_s + \sqrt{2k_B T\gamma}\, \xi_s) \circ dx_s = -\int F_s(x_s) \circ dx_s$$

(Stratonovich integral)

not a state function

"total force"

$$F_t(x_t) = -V'_t(x_t) + f_t(x_t)$$

$$Q_t \simeq \left(\frac{F_0(x_0) + F_1(x_1)}{2} \right)(x_1 - x_0) + \left(\frac{F_1(x_1) + F_2(x_2)}{2} \right)(x_2 - x_1) + \ldots$$

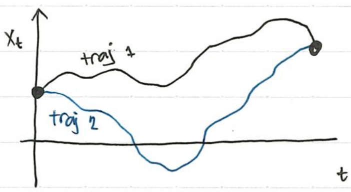
(stochastic) Work along a trajectory $x_{[0,t]}$

$$W_t = \int_0^t dW_s = \int_0^t f_s(x_s) \circ dx_s + \partial_s V_s(x_s)\, ds$$

not a state function

$$\Delta V_t = V_t(x_t) - V_0(x_0)$$

$$\boxed{Q_t + W_t = \Delta V_t} \quad \text{1st law}$$



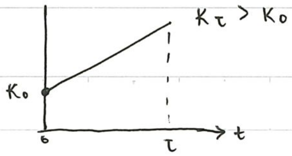
- energy is the same for both trajectories but work and heat is not

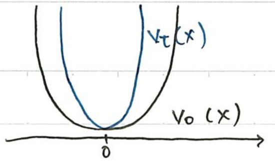
(bec. not state functions)

Examples :

① $V_t(x) = \frac{1}{2} K_t x^2$

$f_t(x) = 0$





(change trap stiffness)

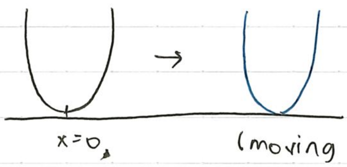
$$dV = \partial_t V\, dt + \partial_x V\, dx = \frac{\partial V}{\partial K} dK + \frac{\partial V}{\partial x} dx = \underbrace{\frac{1}{2} x^2 dK}_{\text{not stochastic}} + K x \circ dx$$

$$dQ = V_t'(X_t) \circ dX_t = \frac{\partial V}{\partial x} \circ dx = K x \circ dx \Rightarrow Q_t = \int K_s X_s \circ dX_s$$

$$dW = \frac{\partial V}{\partial t} dt = \frac{\partial V}{\partial K} dK = \frac{1}{2} x^2 \dot{K} dt \Rightarrow W_t = \int \frac{1}{2} x_s^2 \dot{K}\, ds$$

② $V_t(x) = \frac{1}{2} K (x - c_t)^2$

$f_t(x) = 0$



$c_t = v t$

$dc_t = v dt$

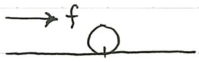
(moving a trap bec. of v_t term)

$$\delta Q = \frac{\partial V}{\partial x} \circ dx = K(x_t - c_t) \circ dx_t$$

$$\delta W = \frac{\partial V}{\partial c} \circ dc = -K(x - c_t)\, v dt$$

③ $V_t(x) = 0$

$f_t(x) = f$



$$\left.\begin{aligned} \delta Q &= -f \circ dX_t \\ \delta W &= f \circ dX_t \end{aligned}\right] \quad dV = 0$$

④ $V_t(x) = \frac{1}{2} K x^2$

$$dV = \underbrace{\partial_t V}_{0} dt + \partial_x V dx$$

$f_t(x) = F_0 \cos \omega t$

$$\left.\begin{aligned} \delta Q_t &= K x_t \circ dX_t - F_0 \cos \omega t \circ dX_t \\ \delta W_t &= F_0 \cos \omega t \circ dX_t \end{aligned}\right] \quad dV_t = K X_t \circ dX_t$$