Biophysics Homework I (2021-2022)

ICTP Quantitative Life Sciences Diploma

1. **[1pt]** Organelles in prototypical eukaryotic cells execute a plethora of different functions. Draw an analogy of each organelle with (and explain why, in terms of its function):
   (a) a job in a factory,
   (b) a relevant person or infrastructure in a country,
   (c) any type of member, fan, or infrastructure of a football club.

2. **[2pt]** Experimental results from a space mission suggest that the pH of ancient waters on Mars were acidic enough (pH $\simeq 3$) to leave a very singular mineral signature.

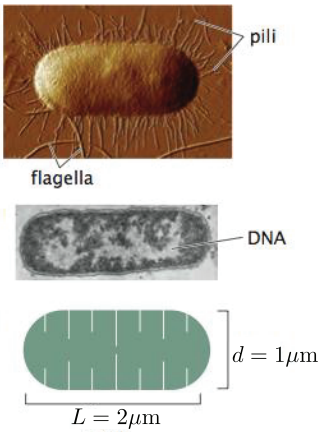

Figure 1: Anatomy of a prototypical E. Coli cell

(a) Consider a single *E. Coli* bacteria whose three-dimensional shape is sketched in the bottom panel Fig. 1. How many ions may such a bacteria have in Mars? Assume that bacteria's pH is the same as that in the ancient waters in Mars. Compare this number compared to that of the same bacteria living on Earth.
(b) For a population of bacteria whose length L is not fixed but follows the distribution

$$P(\ell) = \frac{1}{\Gamma(k)} \frac{e^{-\ell/L}}{\ell} \left(\frac{\ell}{L}\right)^k , \tag{1}$$

with $L > 0$ a characteristic length of the population, $k > 0$ a dimensionless shape parameter, and $\Gamma(x)$ denotes here the Gamma function. Check that the length distribution is normalized, compute its mean and variance. Plot the length distribution for $L = 2$ and $k = 1, 2, 3, 4, 5$. What is the distribution $P(V)$ of the volume of the bacteria?

3. **[2pt]** Information bits on the human genome.
   (a) What's the length of human's genome (in basepairs)? Assuming that the human genome is a straight line, what would be its length in kilometers? Cite the bionumbers used for these estimates.

(b) Consider a minimal polymerization model of single-stranded DNA in which each basepair (A, C, C or G) is added to the template with equal probability following an i.i.d. process. What is the total (Shannon) entropy, in bits, of the entire human genome?

4. **[2pt]** Assume that a cell is spherical capacitor as that depicted in Fig. 2. For simplicity, we will consider that the cell cytoplasm is a charged conducting sphere with total charge Q that is surrounded by a spherical lipid bilayer of thickness $\delta = 4$nm. The outer shell of the cell facing the extracellular medium has radius $R = 5\mu$m and a total charge $-Q$ uniformly distributed along its surface. The transmembrane potential of the cell is $\phi = 60$meV and its relative permitivity $\epsilon_r = 2$.

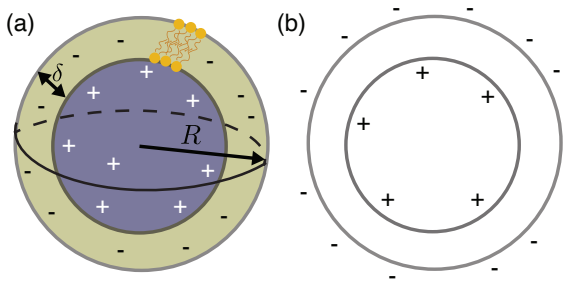

Figure 2: Illustration of the cellular spherical capacitor model: (a) 3D view. (b) 2D view.

(a) What is the value of Q in Coulombs? How many protons does it correspond to?
(b) Consider an active ion pump that can transport electrons across the membrane, towards the extracellular medium. How much energy does the pump need to transport one electron? Express this result in eV, Joules and k_BT. How much power does the pump need to develop in order to achieve a current of 1pA? Express the result in Watts, k_BT/s and in ATP/s. Cite any bionumber used for this calculation.

5. **[3pt]** For a proper control of its disease, the blood glucose levels of a patient with Type-I Diabetes should be in the range between 80 and 170 miligrams of glucose per decilitre of blood. A biotech startup is developing a set of tools to optimize the control of the disease. They ask you to solve the following problems:
(a) In US, doctors prescribe treatments using different units, namely mmol/l. What are the threshold blood-glucose values that patients should never cross, in units of mmol/l? How would you transform the time series in Fig. 3 (blood glucose concentration vs time) from mg/dl to mmol/l?

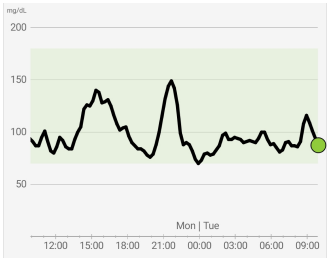

Figure 3: A typical time trace of the glucose blood levels of a patient as a function of time.

(b) Estimate how many grams of glucose are in the entire blood circulation for a human being of weight 70kg with glucose level equal to 125mg/dl. Repeat the calculation for a spherical droplet of blood of radius 1mm.

(c) A hypoglycaemia happens when the blood glucose levels fall below 70mg/dl. Estimate how many grams in glucose were consumed in the blood system of a patient initially in normal levels (125mg/dl) that suffers a hypoglycaemic event.
(d) How much energy is consumed by the organism when catabolizing the amount of glucose in the hypoglycaemia event in (c)? If this event takes 30min to happen, estimate the average power used by the organism in this process?
(e) You develop a stochastic model for the evolution of the blood glucose levels $G(t)$ as a function of time t. Your model is as follows: The initial condition is $G(0) = G_0 = 125\text{mg/dl}$. $G(t)$ is a one-dimensional Brownian diffusion in continuous time and space (a Wiener process) with diffusion coefficient D_G. The threshold values that do not need to be surpassed in order to stay in a healthy state are $G_- = 80\text{mg/dl}$ and $G_+ = 170\text{mg/dl}$. What is the value of D_G if the average time for the patient to fall below G_- or pass above G_+ is $\langle\tau\rangle = 2\text{h}$?

6. **[Extra 0.5pt]** Write a 10 sentence summary about one research topic in the talk "Computational Methods in the Discovery of Bioactive Natural Products" `https://youtu.be/LMUqQSnvFTA`. For this purpose, pick a relevant research article of those included within the references of the seminar.