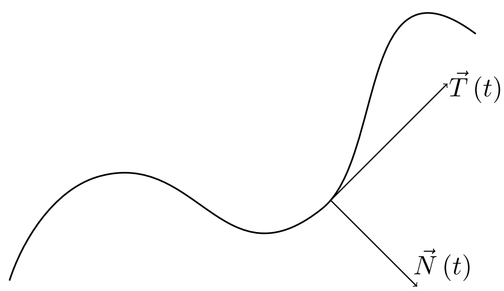
1 Lecture 9

1.1 Normal Vectors - Continued

Yesterday we saw that

$$\vec{N}(t) = \frac{\frac{d\vec{T}}{dt}}{\left|\frac{d\vec{T}}{dt}\right|}$$



asdfasdfasdfas

Example. Consider the circle

$$\vec{r}(t) = (\cos t, \sin t), 0 \le t \le 2\pi$$

The velocity is

$$\vec{v}(t) = \vec{r}' = (-\sin t, \cos t)$$

We have $|\vec{v}(t)| = 1$, since

$$\vec{v}(t) \cdot \vec{v}(t) = \left(\sin t^2 = \cos t^2\right) = 1$$

We find that $\vec{T}(t) = \vec{v}(t)$. To find $\vec{N}$, we need first

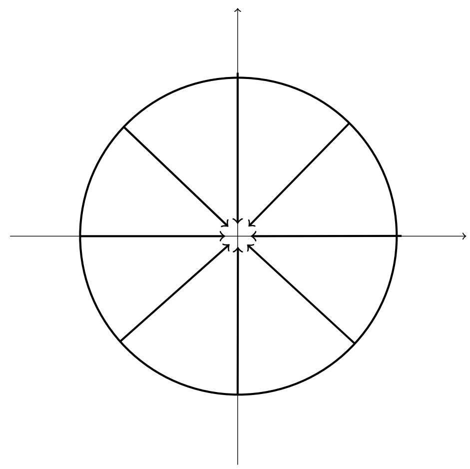
$$\frac{d\vec{T}}{dt} = \frac{d\vec{v}}{dt} = (-\cos t, -\sin t)$$

We check that $\left|\frac{d\vec{T}}{dt}\right| = 1$, then

$$\vec{N}(t) = (-\cos t, -\sin t) = -\vec{r}(t)$$

We compute more explicitly:

$$\vec{N}(t) \cdot \vec{T}(t) = (-\cos t, -\sin t) \cdot (-\sin t, \cos t) = \cos t \cdot \sin t - \sin t \cdot \cos t = 0$$



We revisit the implicit case $f(x, y) = 0$.

Proposition 1.

Proof. $\square$

$\diamond$