1 Lecture 6

1.1 Substitution Method

We want to maximize / minimize $f(x, y)$ with constraint $g(x, y) = 0$. We can solve $g(x, y) = 0$ for one variable $y = y(x)$.

Example. Consider $f(x, y) = x^2 + y^2$ and $g(x, y) = xy - 1 = 0$. In this case, we have $f = \frac{1}{x}$ from $g = 0$, we get

$$h(x) = f\left(x, x^{-1}\right) = x^2 + x^{-2}$$

We have found the minima at $(1, 1)$ and $(-1, -1)$.
This method isn't always feasible, so lets look at some alternatives.

1.2 Lagrange's Method

Example. Lets look at the level curves, which are circles. We have the that $f(x, y) = x^2 + y^2 = c$.

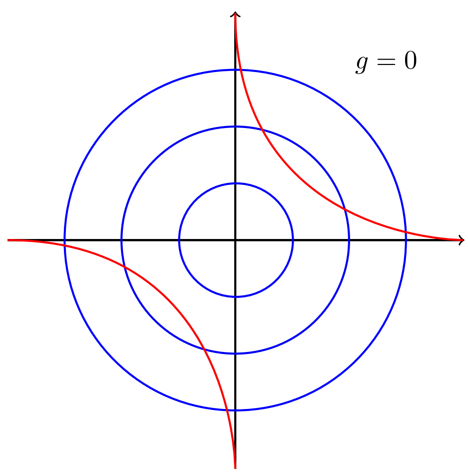
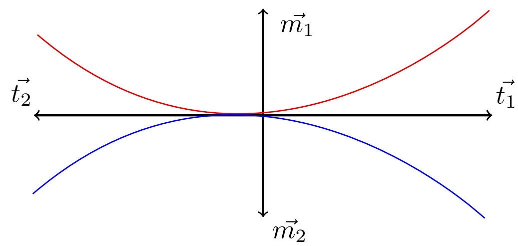


Figure 1: $f(x, y)$

Smaller circles correspont to smaller values of $f(x, y)$, but we must also satisfy $g(x, y) = 0$. In the best case, $f(x, y) = c$ is just touching $g(x, y) = 0$. If this is worked out geometrically, we get $(1, 1)$ and $(-1, -1)$.

This idea is used in Lagranges method. We want $f(x, y) = c$ to be parallell to $g(x, y) = 0$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