1 Lecture 12

1.1 Identities Between Operations

We have seen three operations defined by ∇.

$$\text{Gradient: } \nabla f, \quad \text{Divergence: } \nabla \cdot \vec{F}, \quad \text{Curl: } \nabla \times \vec{F}$$

There are many identities, we'll now look at one.

Proposition. For any scalar field f, we have

$$\nabla \times (\nabla f) = 0$$

Proof. We have $\nabla f = (f_x, f_y, f_z)$, then

$$\nabla \times (\nabla f) = \left| \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ f_x & f_y & f_z \end{bmatrix} \right| = (f_{zy} - f_{yz}, -f_{zx} + f_{xz}, f_{yx} - f_{xy})$$

But partial derivatives can be exchanged. Then we find that $\nabla \times (\nabla f) = 0$ □

We are going to use this when we discuss conservative fields.

1.2 Line Integrals

Consider the curve C with

$$\vec{r}\,(t) = (x\,(t)\,, y\,(t))\,, t_0 \leq t \leq t_1$$