

# Einführung in Visual Computing

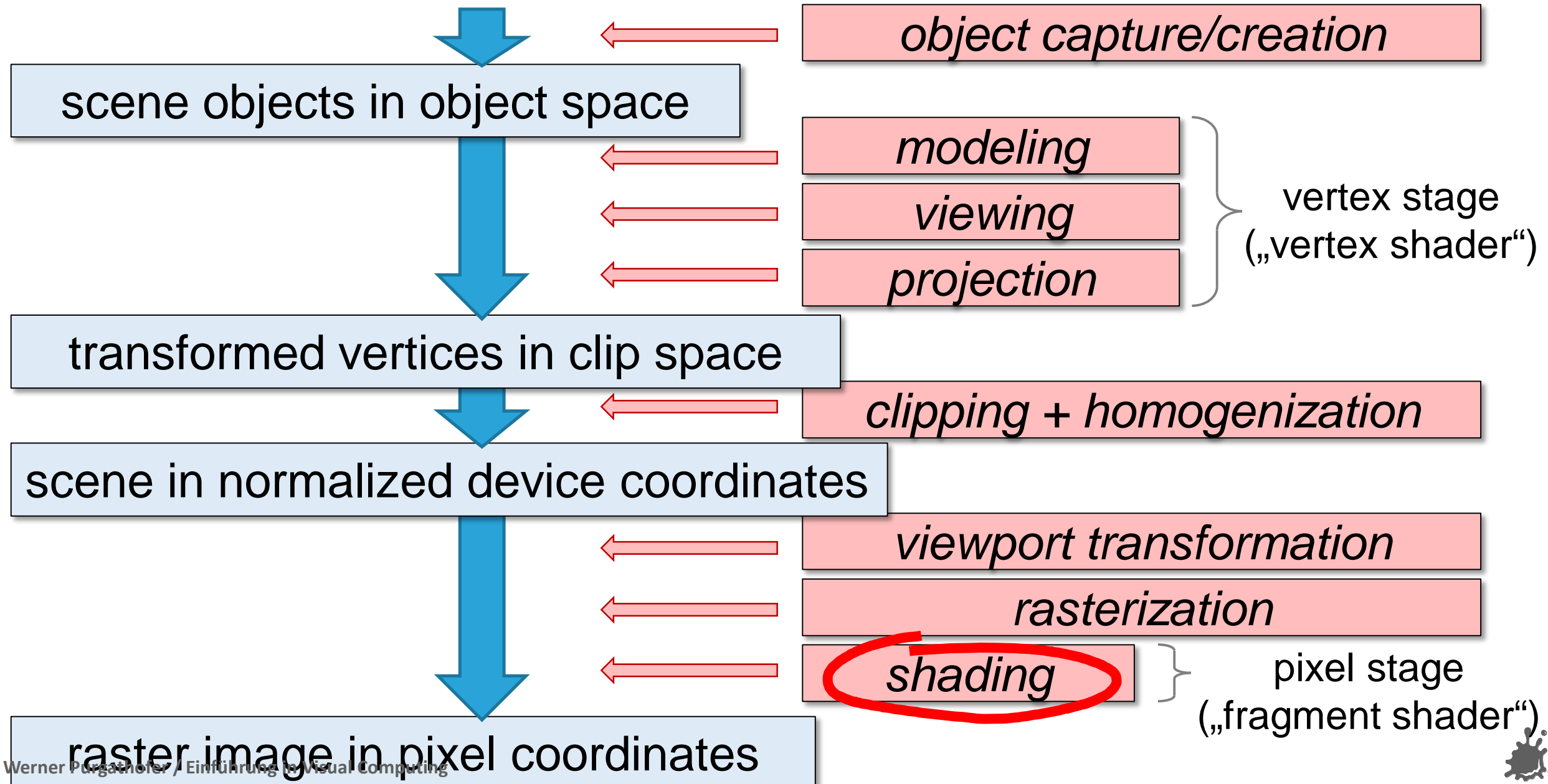
186.822

## Lighting and Shading

Werner Purgathofer



# Shading in the Rendering Pipeline



## light sources

### basic model

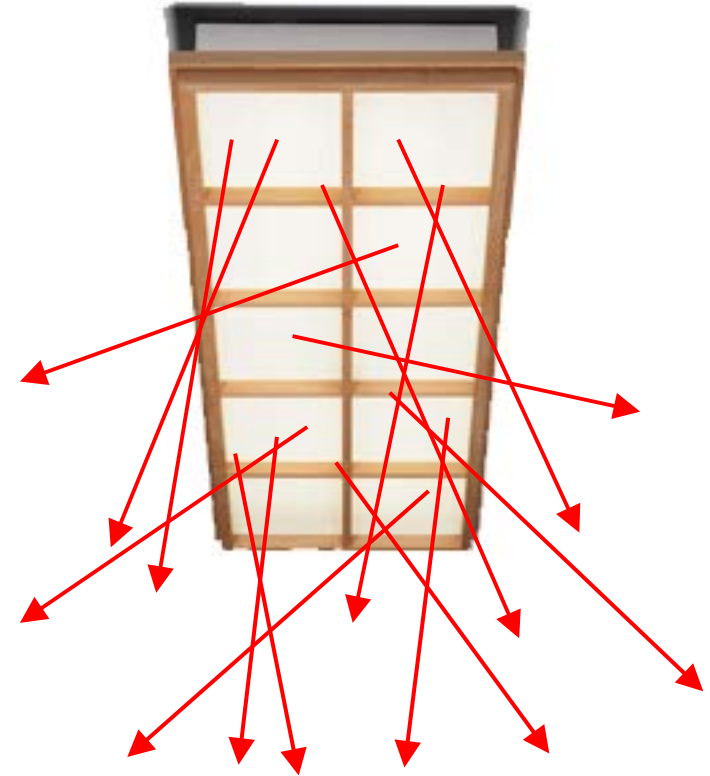
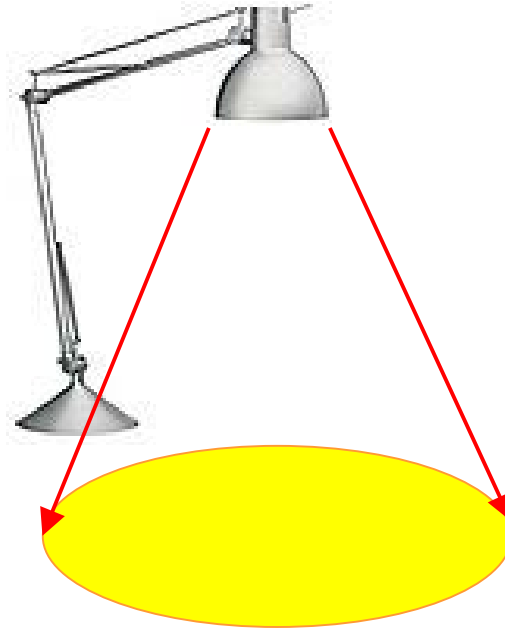
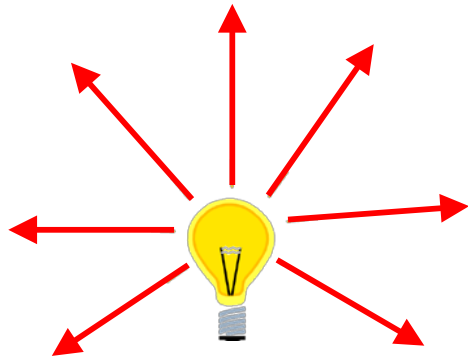
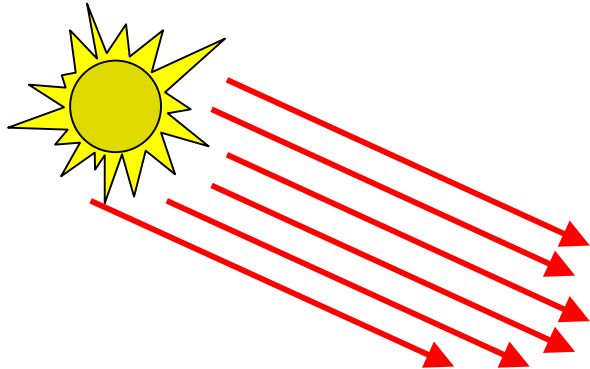
- ambient light
- diffuse shading
- specular highlights

### shading interpolation

- Gouraud Shading
- Phong Shading



- directional light
- point light source (sometimes directional)
- distributed light source (“area light source”)

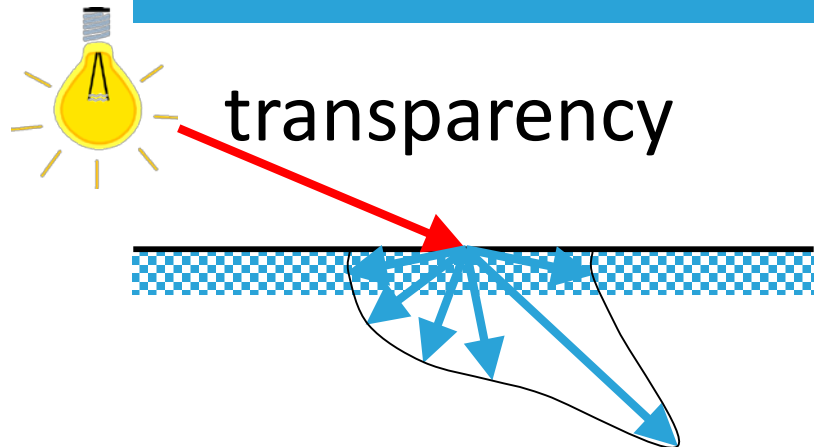




diffuse reflection

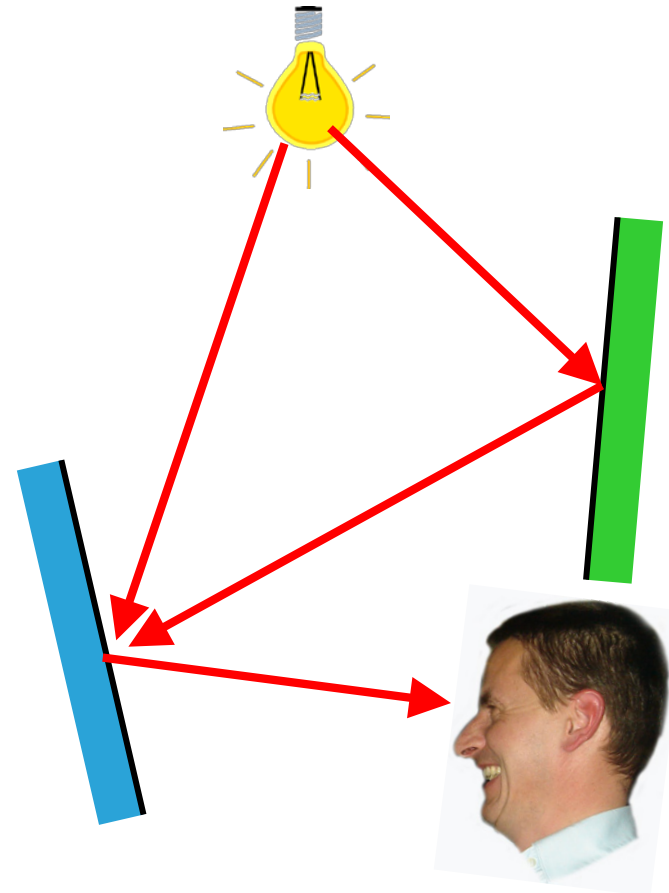


specular reflection



transparency

reflections from  
other surfaces



models are empirical

lighting calculations include

- surface properties (glossy, matte, opaque,...)
- background lighting conditions (ambient light)
- light-source specification
- reflection, absorption
- ...



- ambient light (background light)  $I_a$
- constant over a surface
- independent of viewing direction
- diffuse-reflection coefficient  $k_d$  ( $0 \leq k_d \leq 1$ )
- approximation of global diffuse lighting effects

$$L_{\text{ambdiff}} = k_d I_a$$



shaded surfaces generate a spatial impression

**the flatter light falls on a surface,  
the darker it will appear**

therefore:

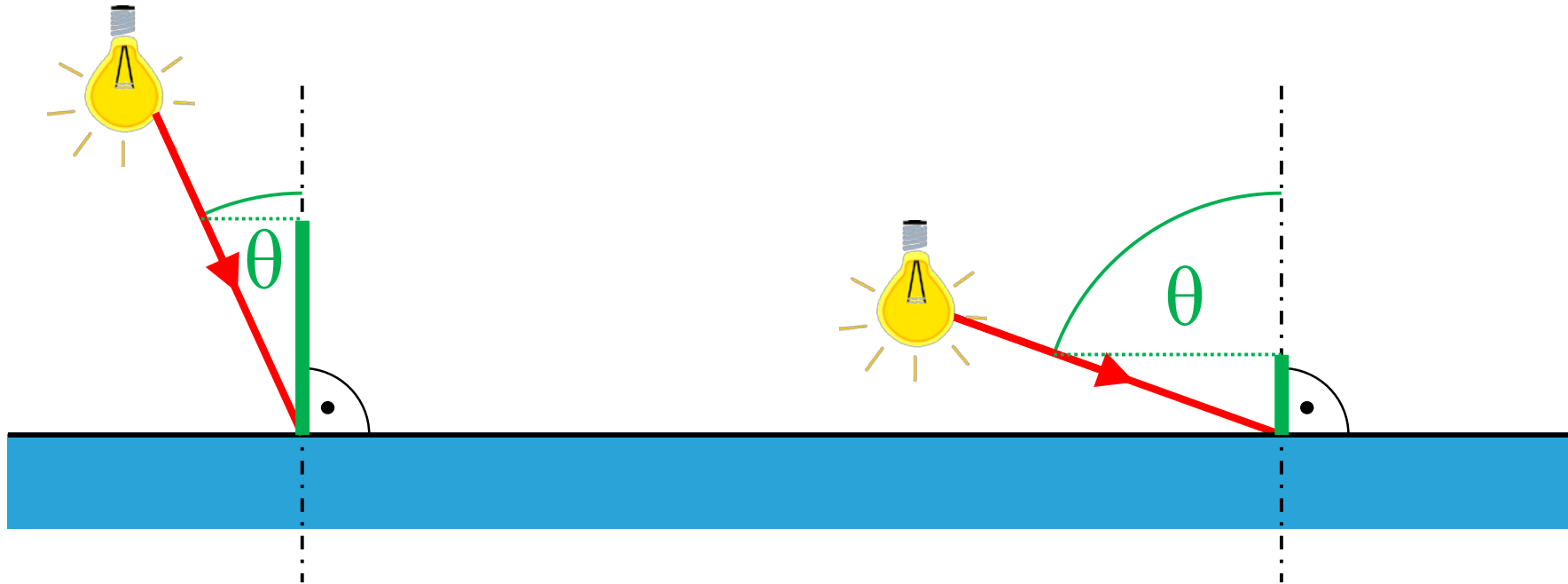
we need the **incident light direction**

or

the position of the (point) **light source**







$$L = I \cdot \cos \theta$$

when considering  
the material:

$$L = k_d \cdot I \cdot \cos \theta$$

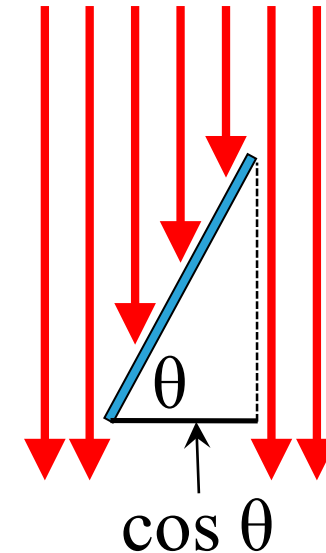
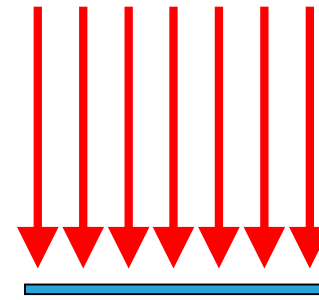
$I$  ... light source intensity

$L$  ... pixel color

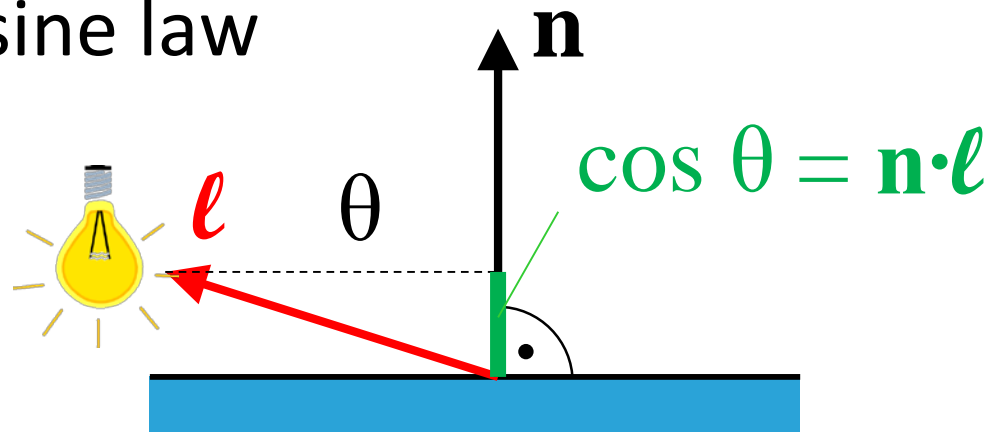
$k_d$  ... diffuse coefficient



- for ideal diffuse reflectors (Lambertian reflectors)
- brightness depends on orientation of the surface:



- Lambert's cosine law

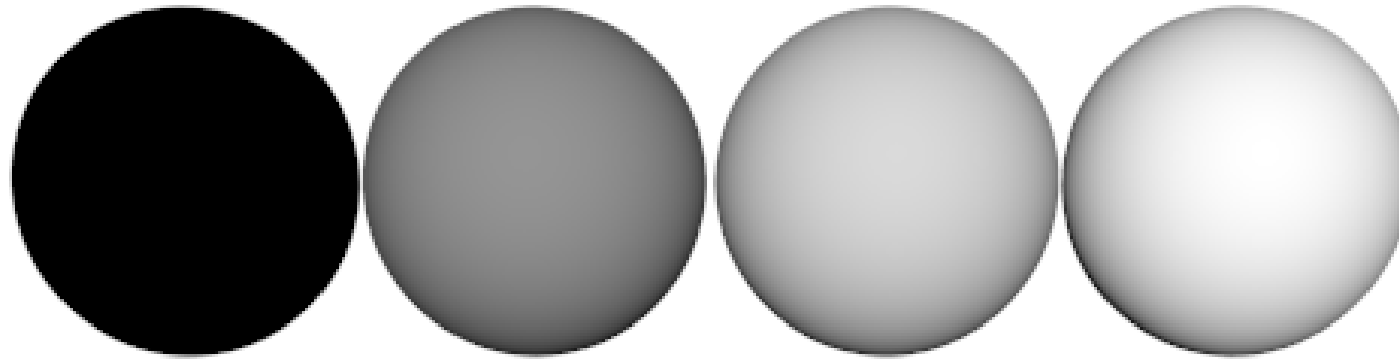


$$L_{\text{diff}} = k_d \cdot I \cdot (\mathbf{n} \cdot \boldsymbol{\ell})$$



varying  $k_d$  :

result for varying values of  $k_d$  ( $I_a = 0$ )



$k_d=0$

$k_d=0.3$

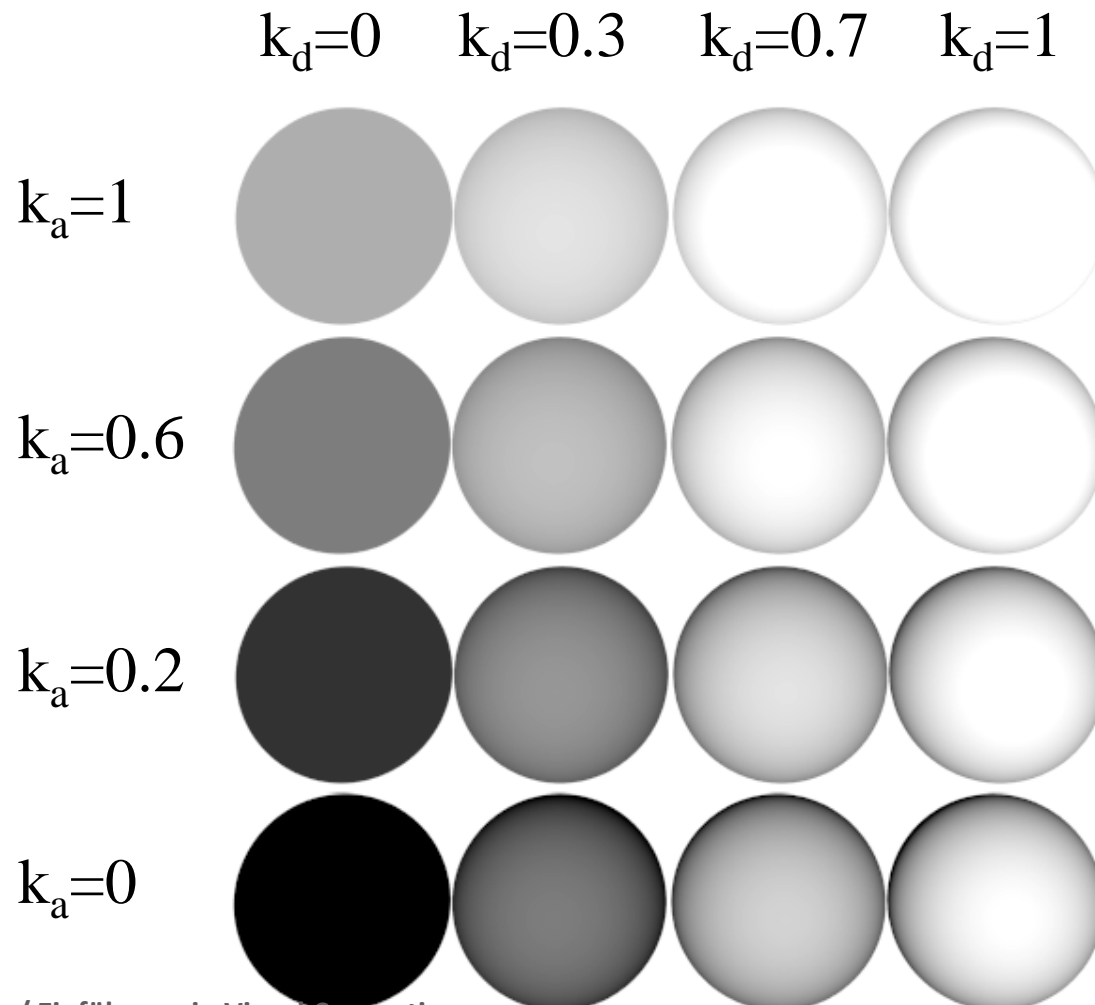
$k_d=0.7$

$k_d=1$



total diffuse reflection:

$$L_{\text{diff}} = k_a I_a + k_d I(\mathbf{n} \cdot \boldsymbol{\ell})$$

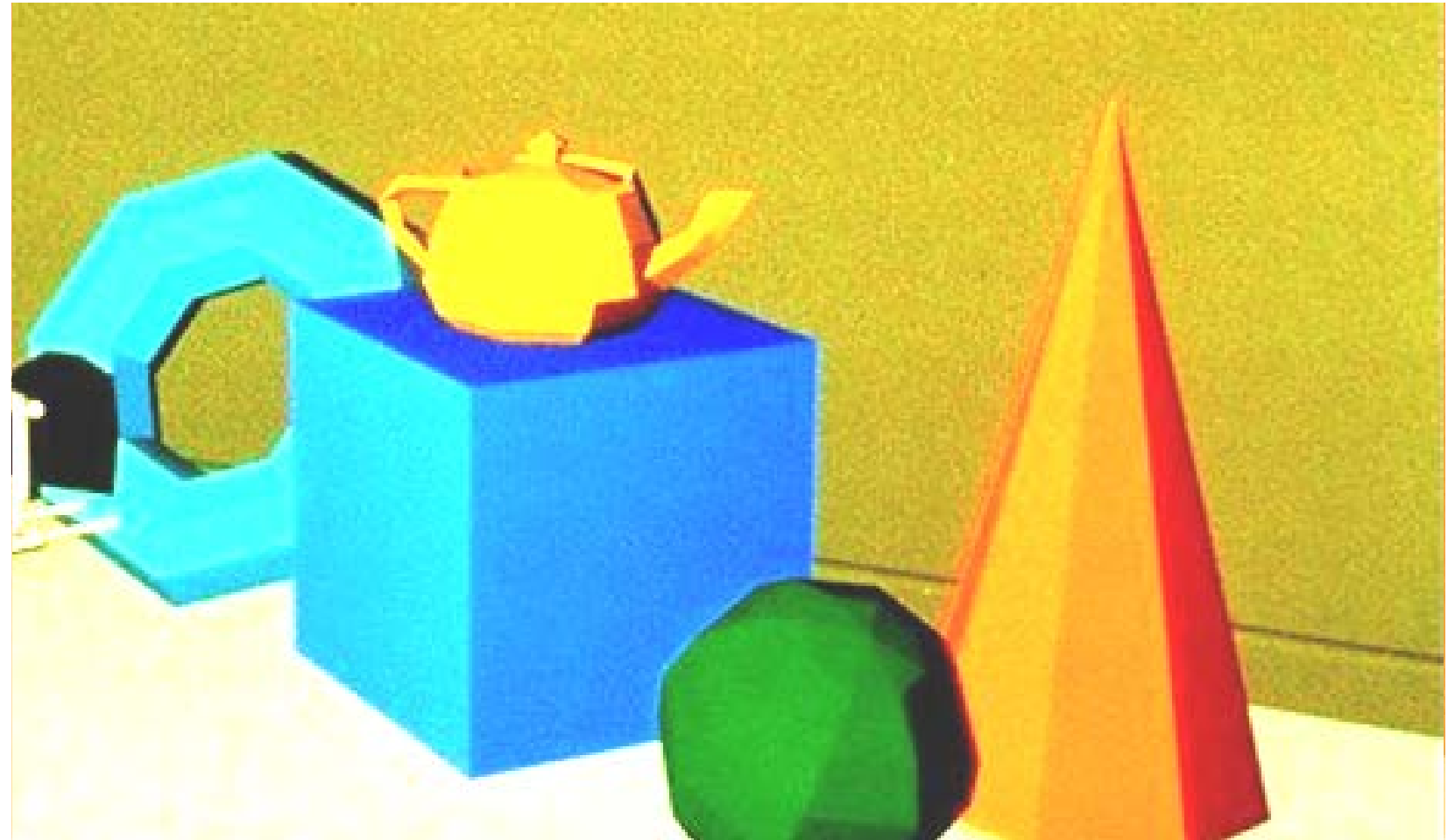


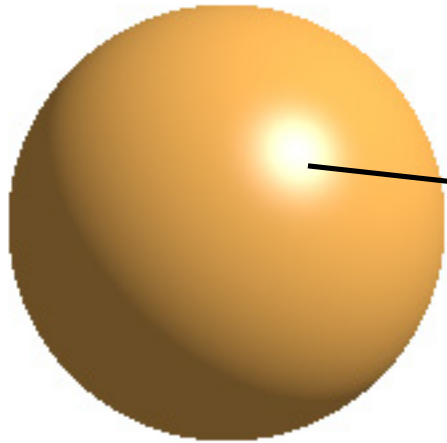
(sometimes extra  $k_a$   
for ambient light)



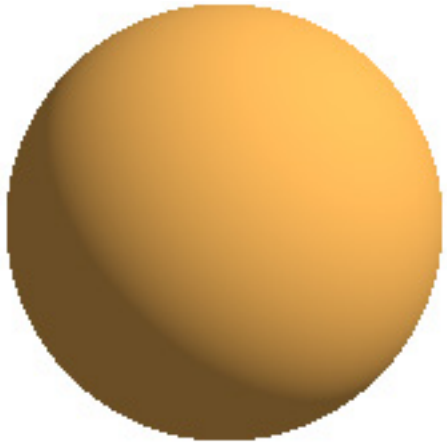
total diffuse reflection:

$$L_{\text{diff}} = k_a I_a + k_d I(\mathbf{n} \cdot \boldsymbol{\ell})$$

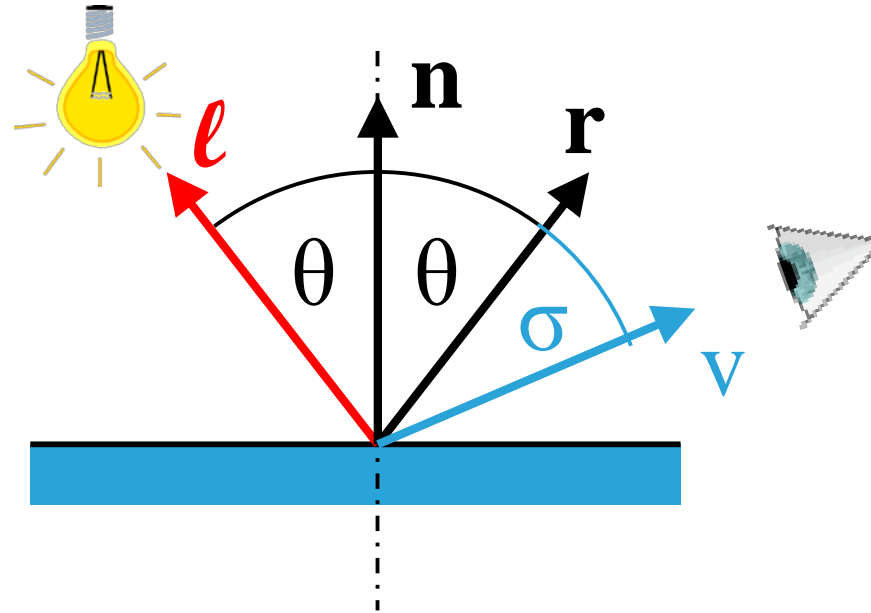
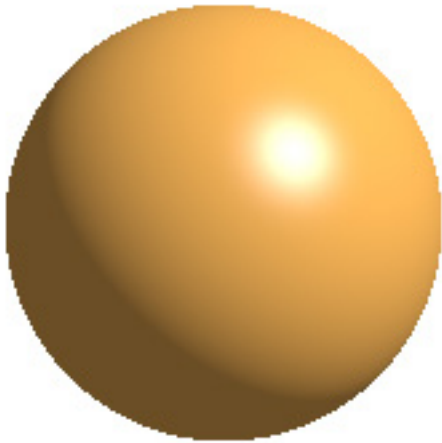




this area must be lighter than the shading model calculates, because the light source is reflected directly into the viewer's eye



reflection of incident light around specular-reflection angle:



⇒ empirical Phong model

$$L_{\text{spec}} = k_s \cdot I \cdot \cos^p \sigma$$



# Specular Reflection Coefficient $p$

$\cos^p \sigma$

$p=1$



$p=2$



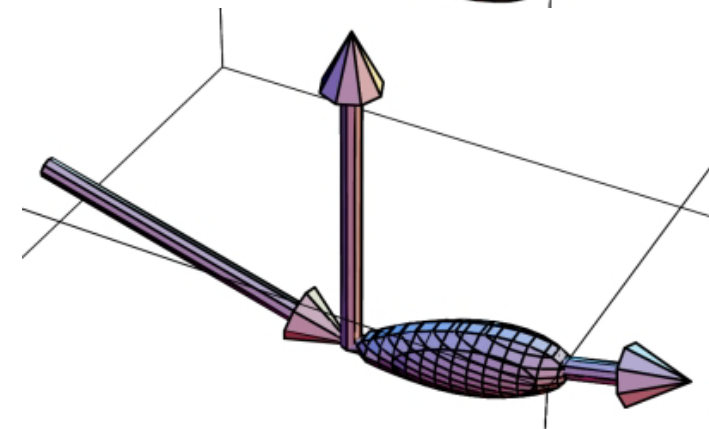
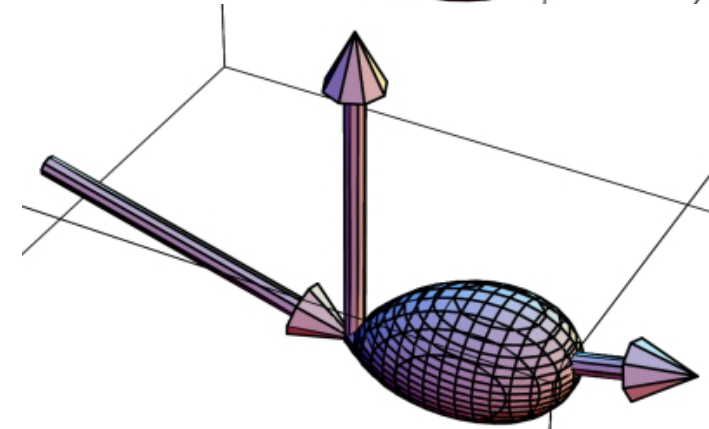
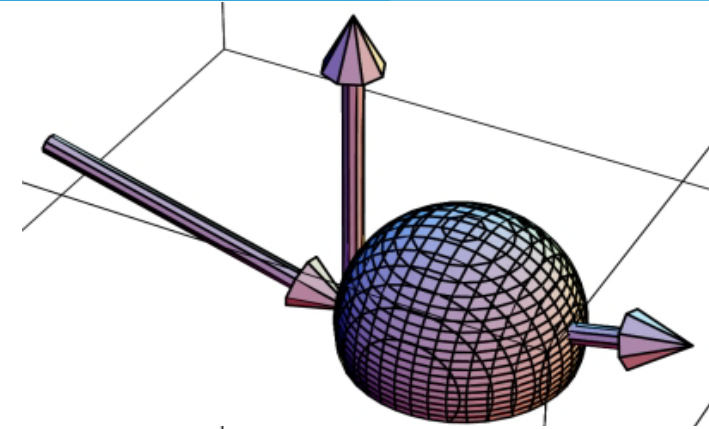
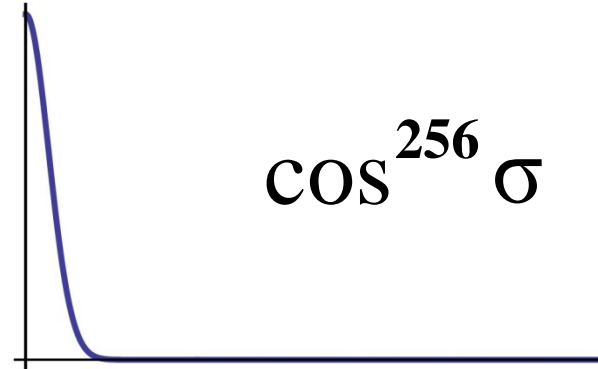
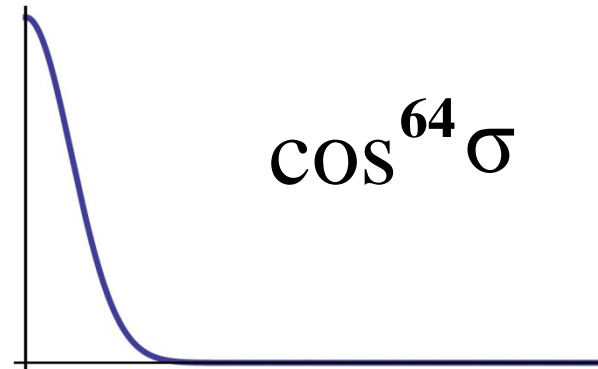
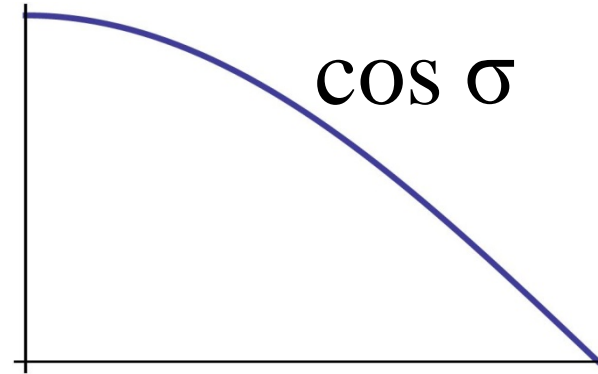
$p=4$



$p=16$



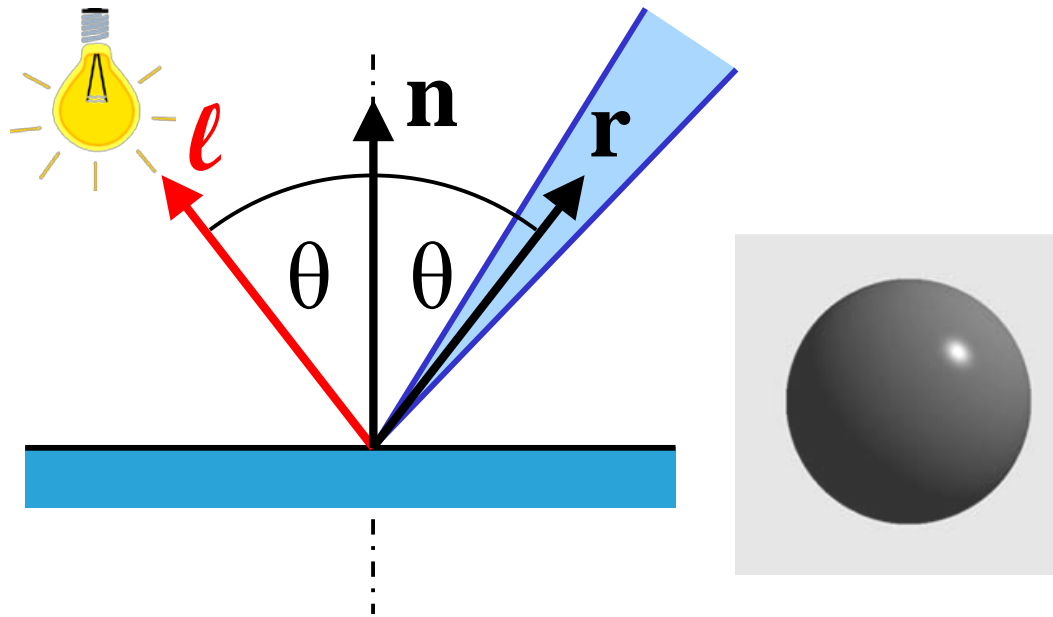
$p=64$



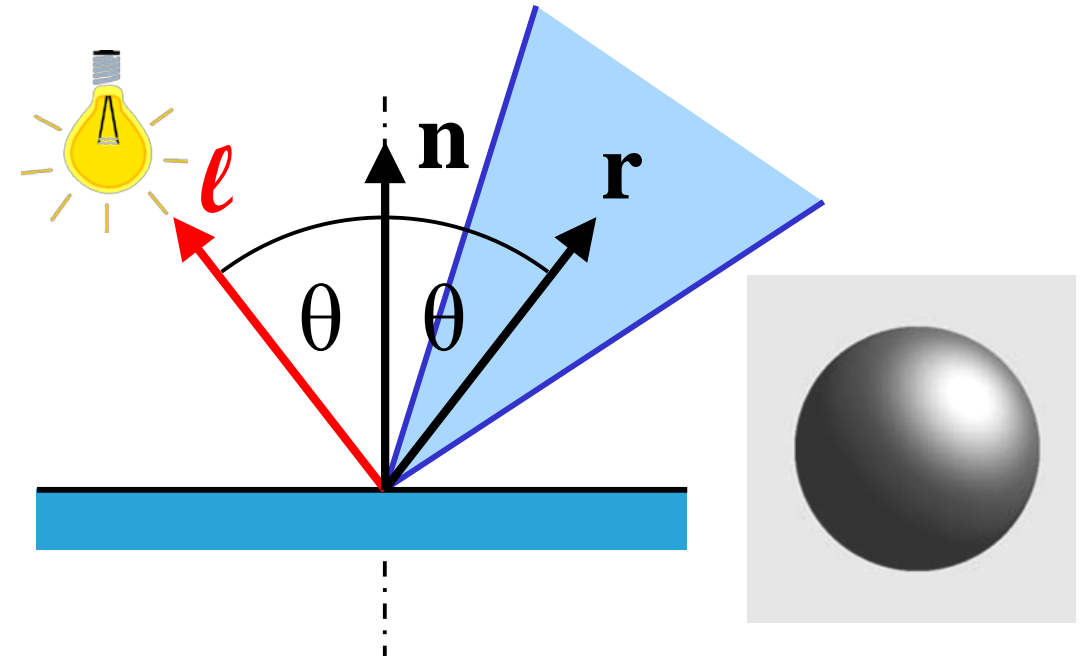


empirical Phong model  $L_{\text{spec}} = k_s \cdot I \cdot \cos^p \sigma$

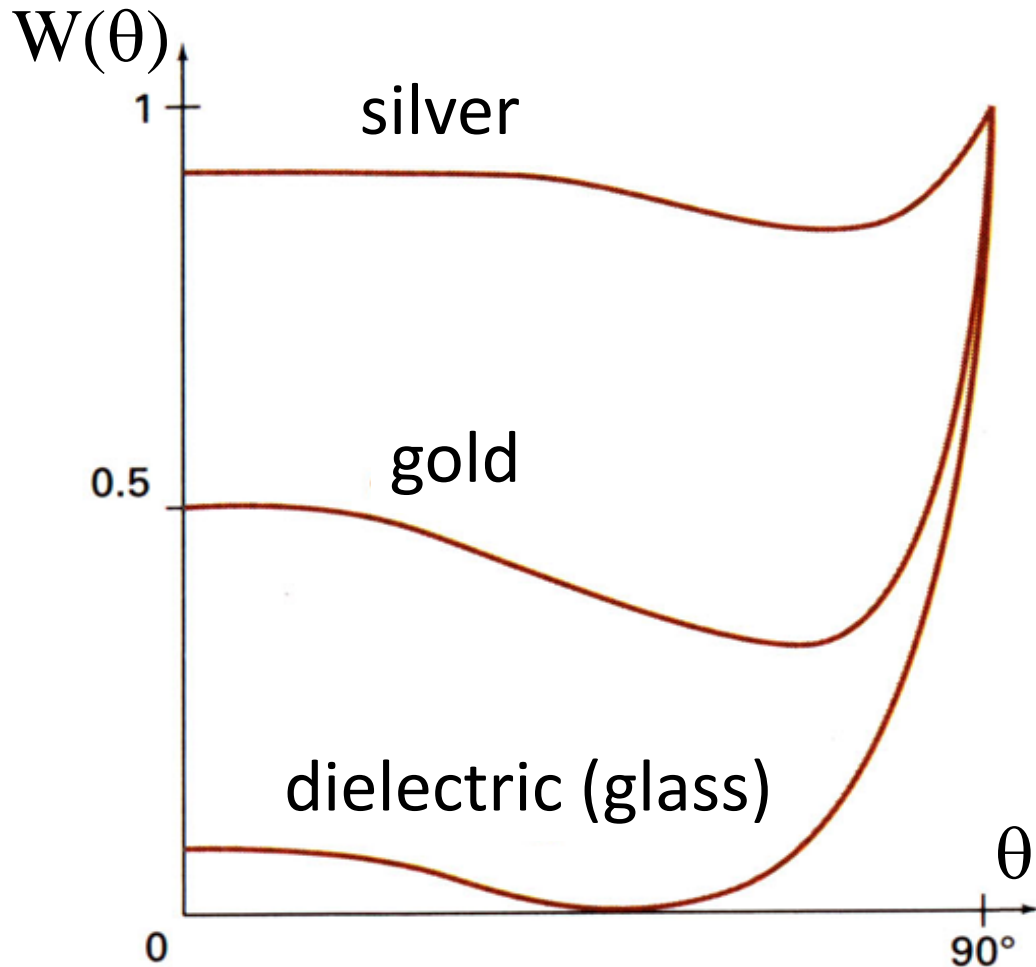
**p large**  $\Rightarrow$  shiny surface



**p small**  $\Rightarrow$  dull surface



Fresnel's laws of reflection use specular reflection coefficient  $W(\theta)$

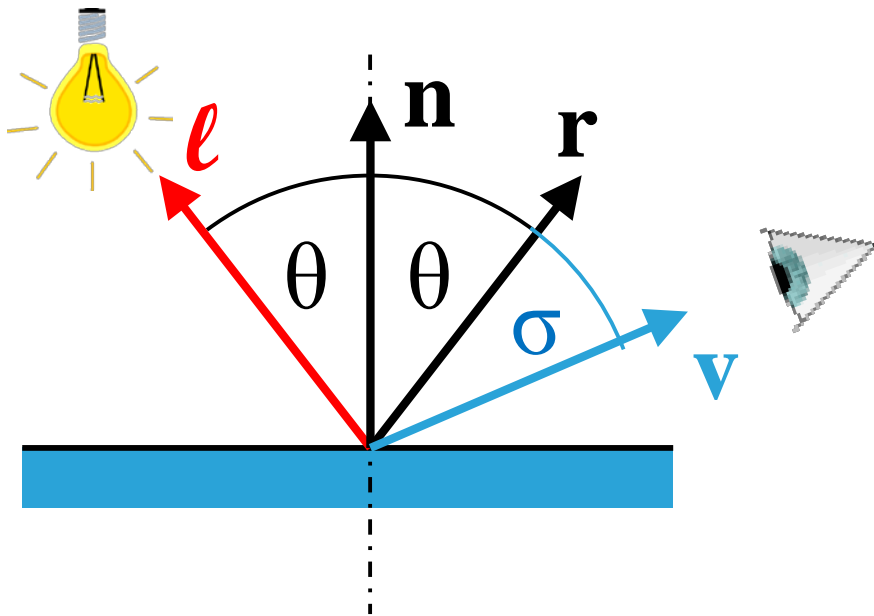


$$L_{\text{spec}} = W(\theta) I_\ell \cos^p \sigma$$

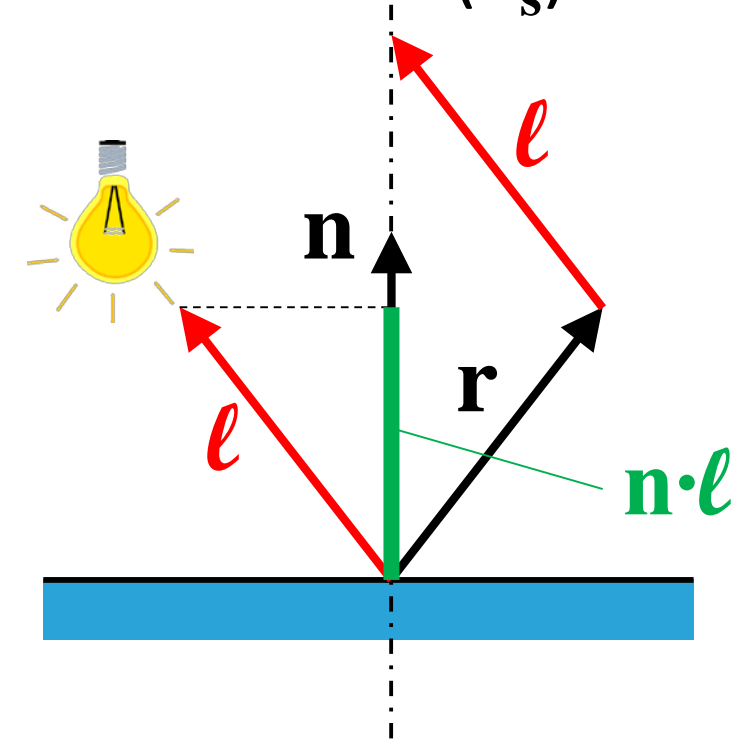
specular reflection coefficient as a function of angle of incidence for different materials



$W(\theta) \approx \text{constant}$  for many opaque materials ( $k_s$ )



$$L_{\text{spec}} = k_s \cdot I \cdot (\mathbf{v} \cdot \mathbf{r})^p$$



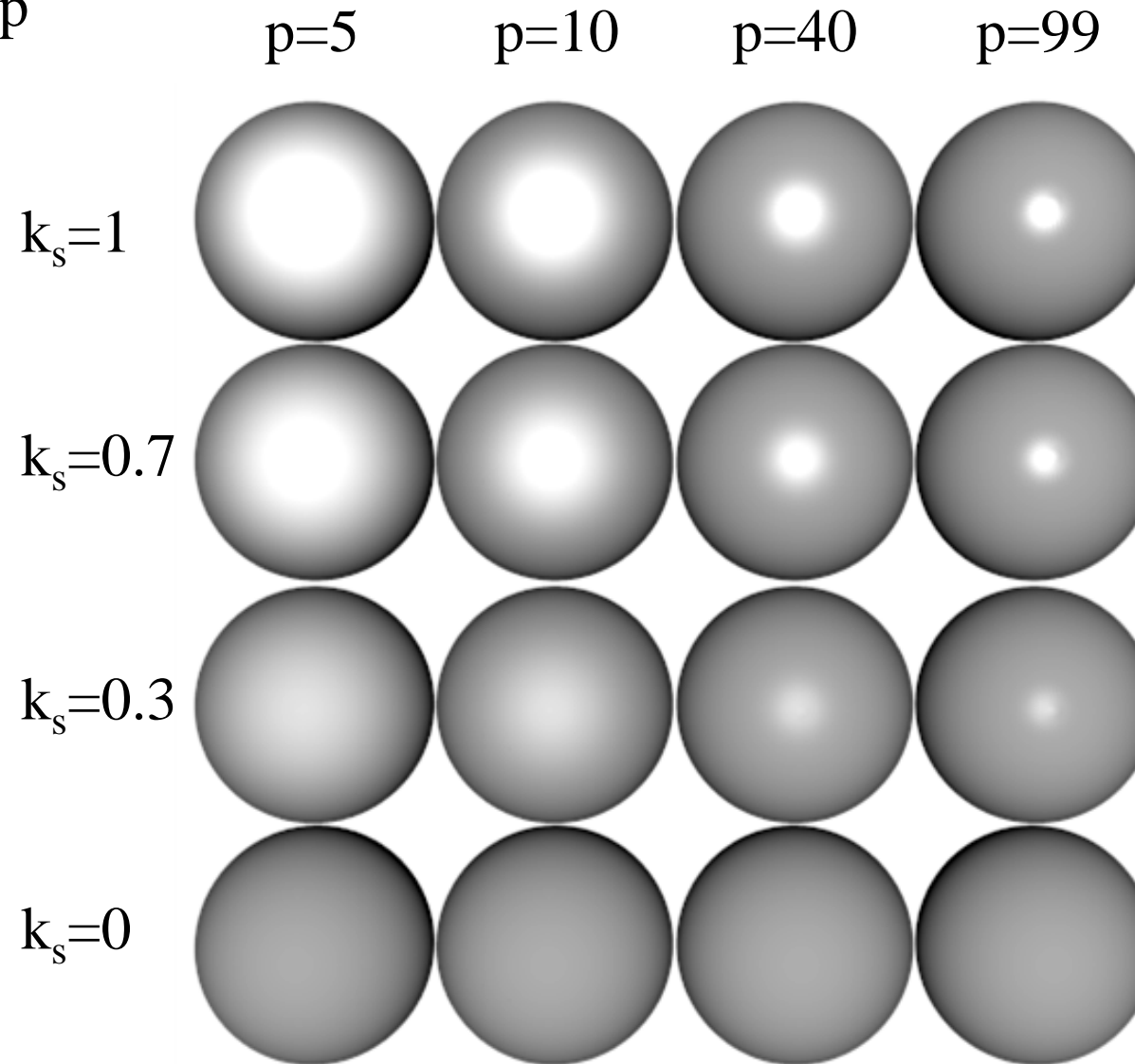
calculation of  $\mathbf{r}$ :

$$\mathbf{r} + \mathbf{l} = (2\mathbf{n} \cdot \mathbf{l})\mathbf{n}$$

$$\mathbf{r} = (2\mathbf{n} \cdot \mathbf{l})\mathbf{n} - \mathbf{l}$$

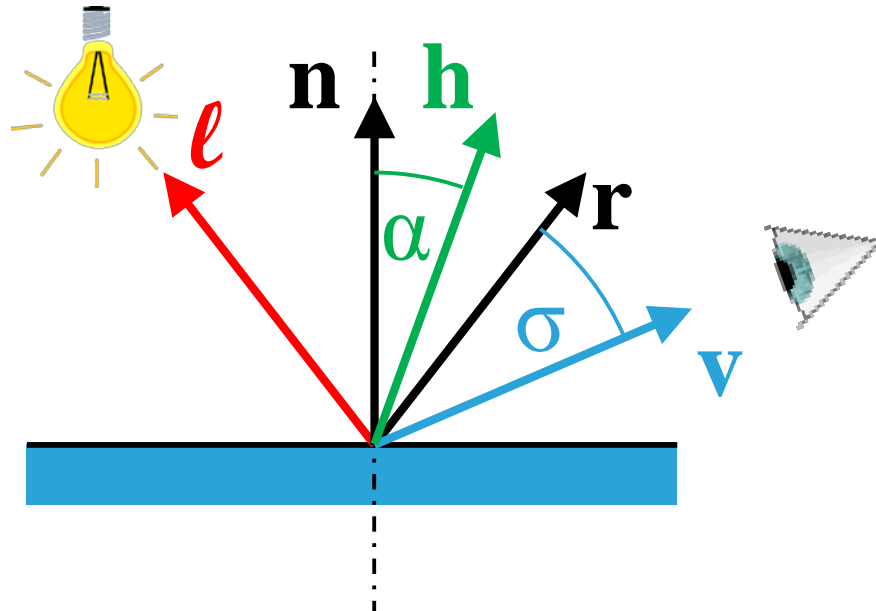


$$L_{\text{spec}} = k_s \cdot I \cdot (\mathbf{v} \cdot \mathbf{r})^p$$



simplified Phong model with halfway vector  $\mathbf{h}$

$$L_{\text{spec}} = k_s \cdot I \cdot (\mathbf{v} \cdot \mathbf{r})^p \quad \rightarrow \quad L_{\text{spec}} = k_s \cdot I \cdot (\mathbf{n} \cdot \mathbf{h})^p$$



$$\mathbf{h} = \frac{\mathbf{l} + \mathbf{v}}{\|\mathbf{l} + \mathbf{v}\|}$$

this revised model is called  
“Blinn-Phong Shading”



# Diffuse and Specular Reflection

$$L = k_a I_a + \sum_{i=1}^N I_i \left[ k_d (\mathbf{n} \cdot \boldsymbol{\ell}_i) + k_s (\mathbf{n} \cdot \mathbf{h}_i)^p \right]$$



ambient



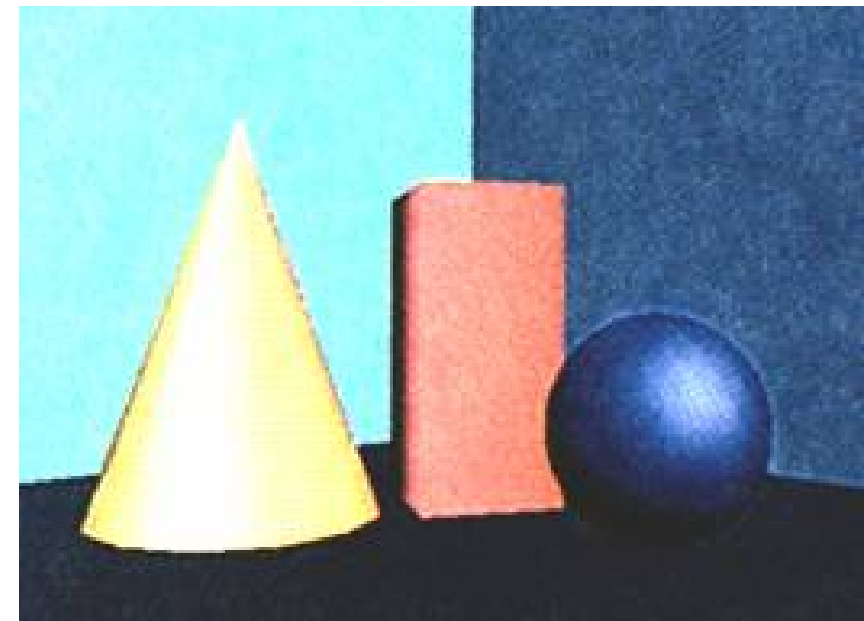
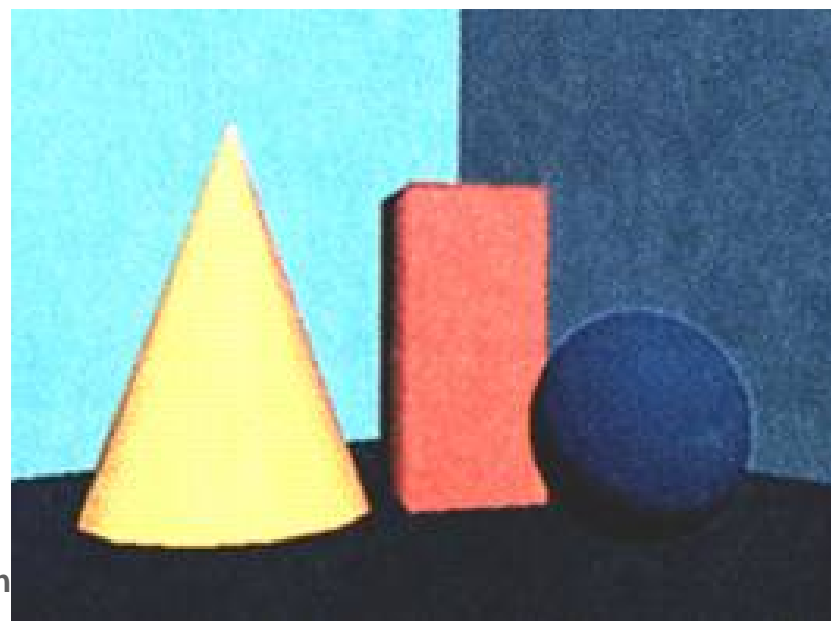
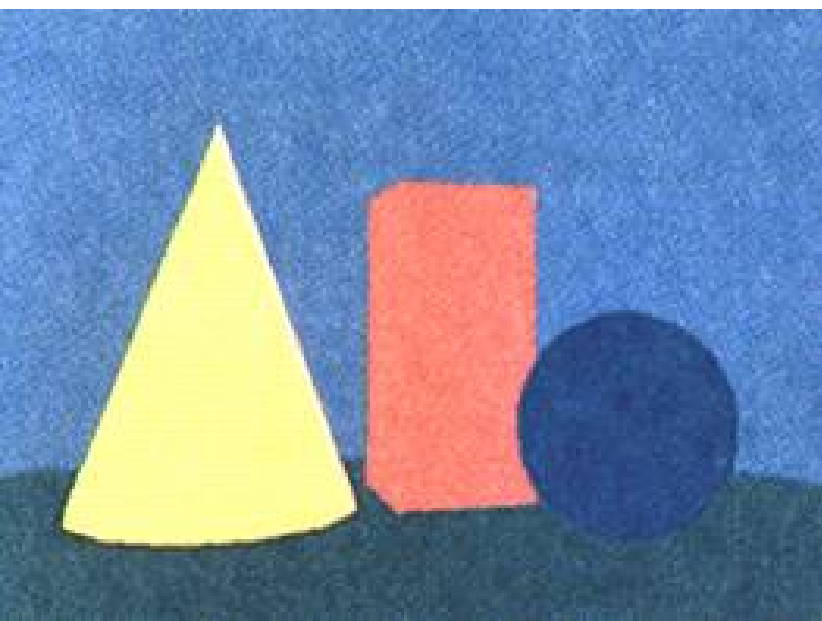
ambient + diffuse



ambient + diffuse  
+ specular

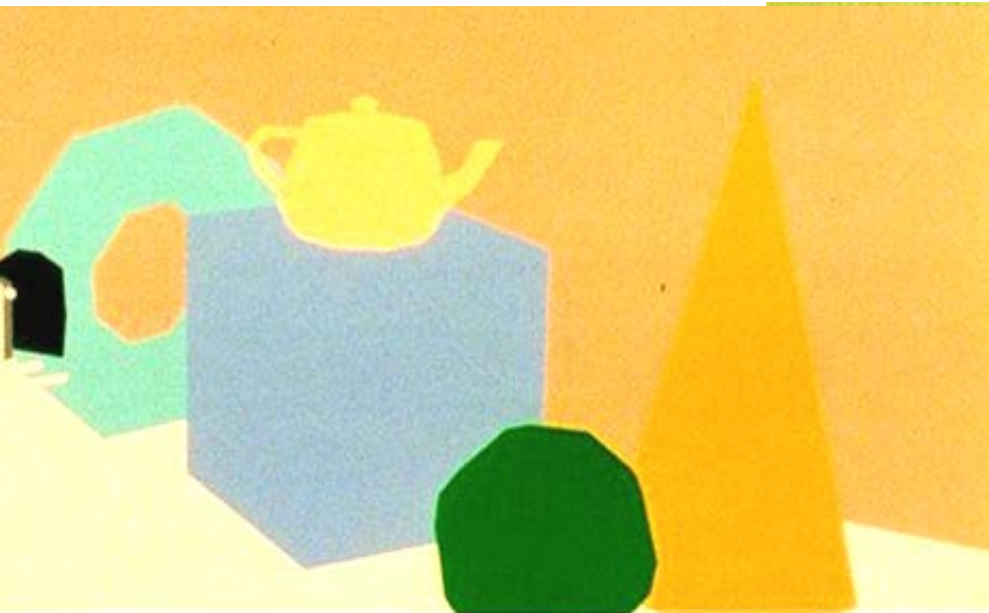
© James Tasker

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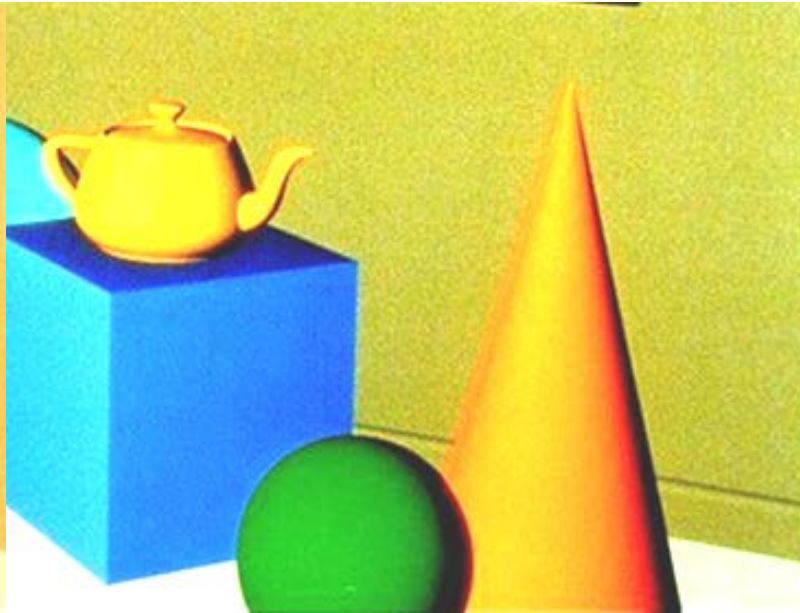


$$L = k_a I_a + \sum_{i=1}^N I_i \left[ k_d (\mathbf{n} \cdot \boldsymbol{\ell}_i) + k_s (\mathbf{n} \cdot \mathbf{h}_i)^p \right]$$

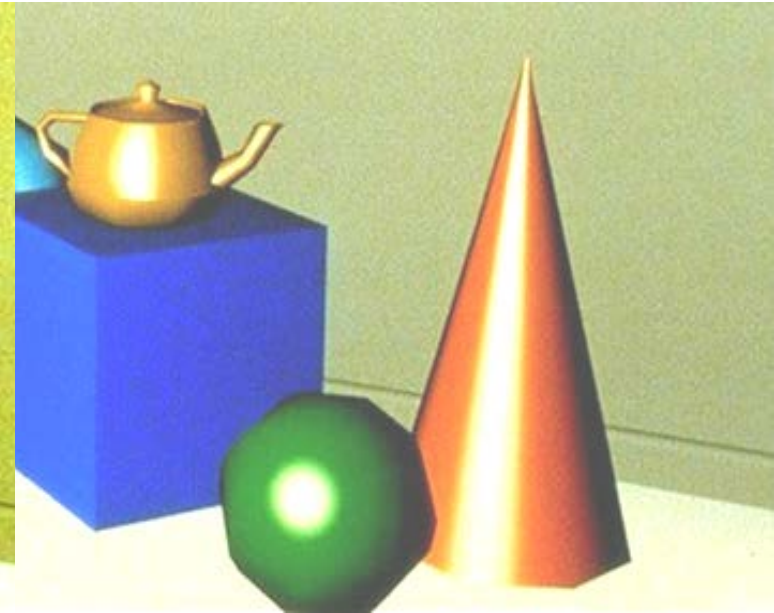
ambient



ambient + diffuse



ambient + diffuse  
+ specular



- intensity attenuation with distance
- anisotropic light sources
- transparency (Snell's law)
- atmospheric effects
- shadows
- ...





application of illumination model to polygon rendering  
constant-intensity shading (flat shading)

= single intensity for each polygon

*flat*



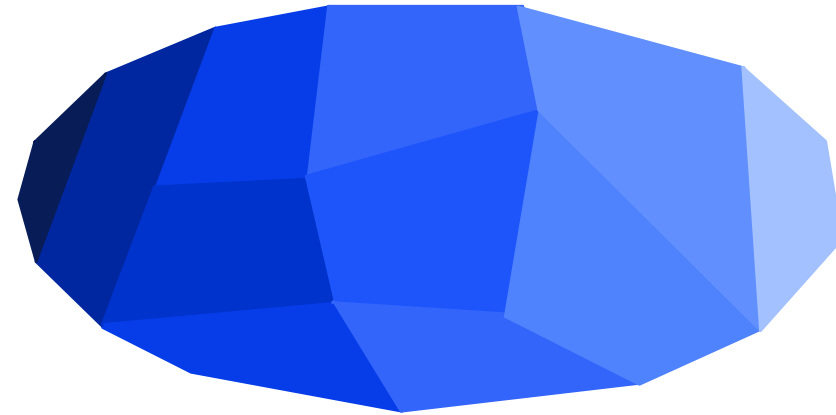
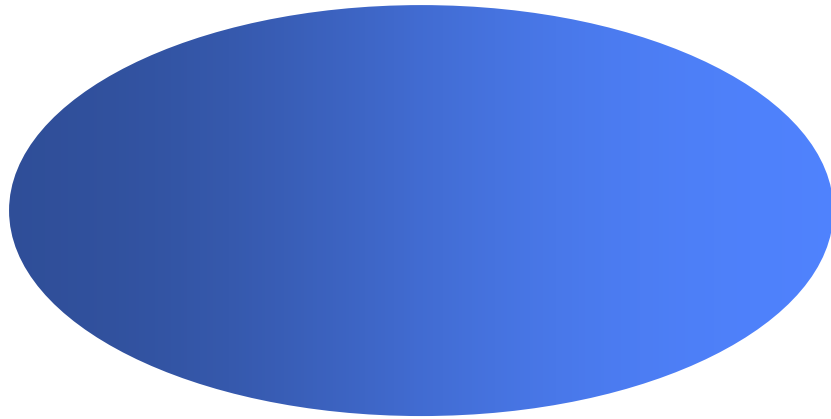
*Gouraud*



the shading of a polygon is not constant, because it normally is only an approximation of the real surface  $\Rightarrow$  **interpolation**

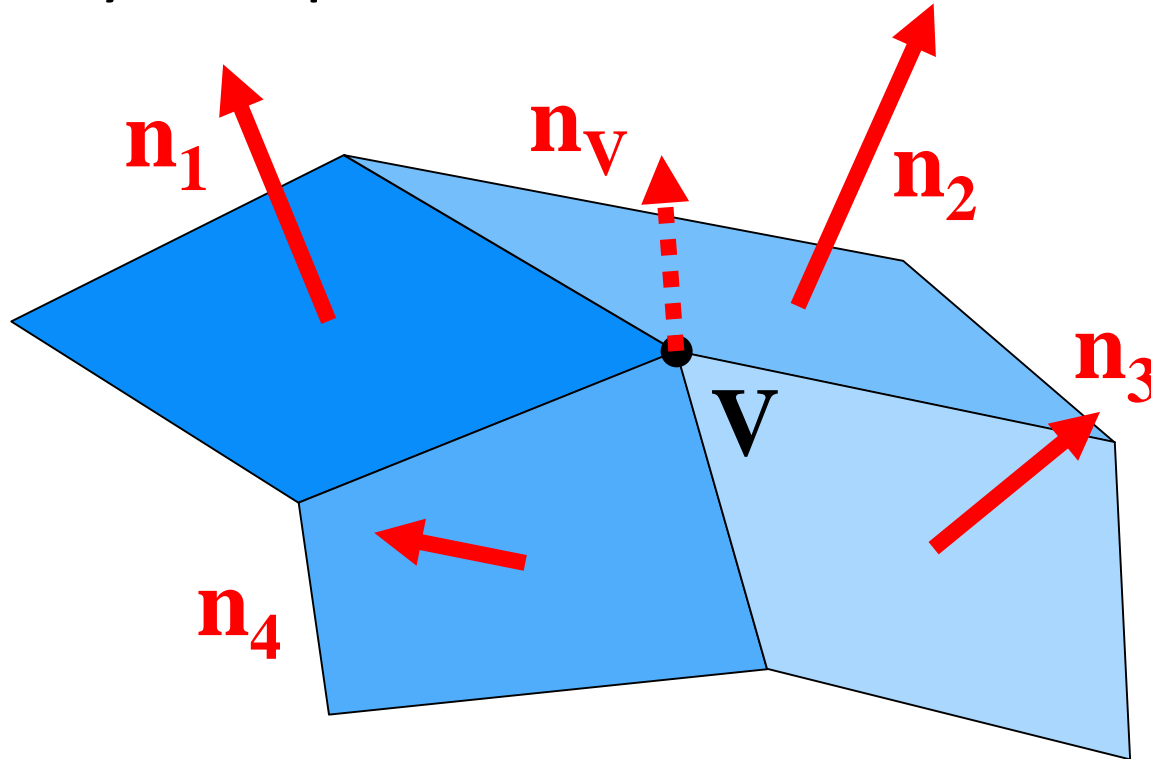
**Gouraud shading:** intensities

**Phong shading:** normal vectors



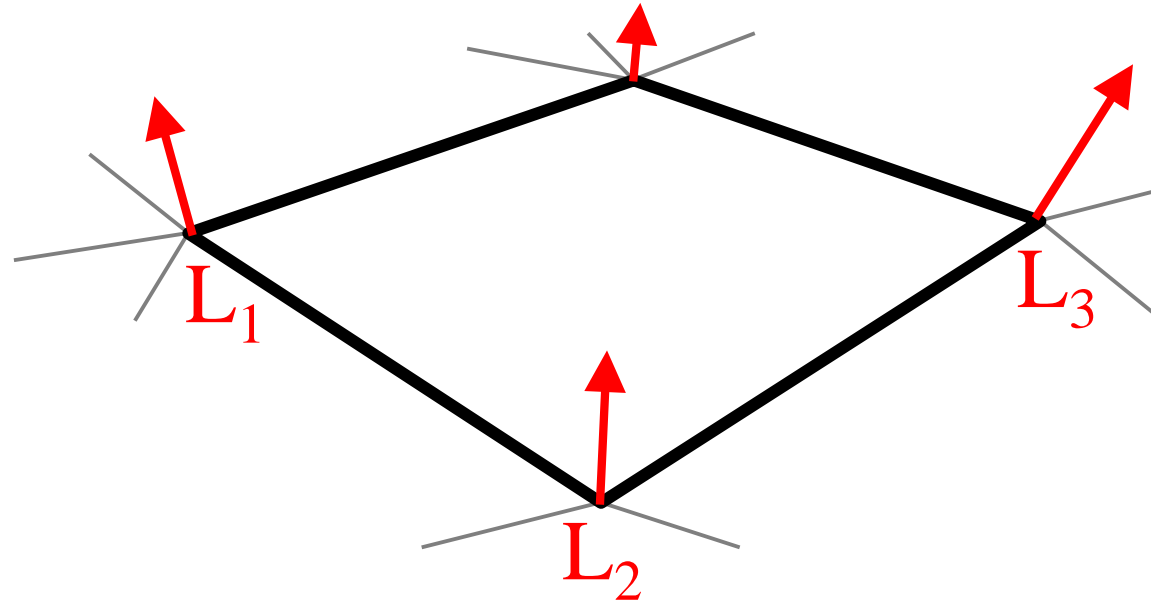
intensity-interpolation:

- determine average unit normal vector at each polygon vertex
- apply illumination model to each vertex
- linearly interpolate vertex intensities



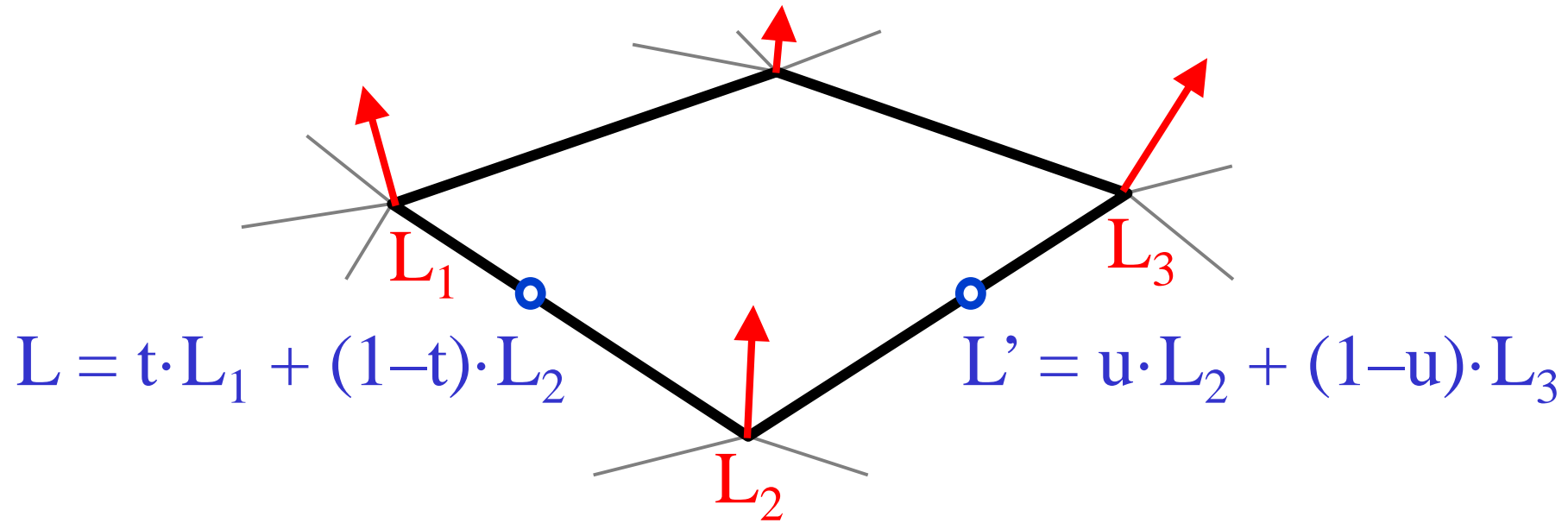
$$\mathbf{n}_v = \frac{\sum_{k=1}^N \mathbf{n}_k}{\left\| \sum_{k=1}^N \mathbf{n}_k \right\|}$$





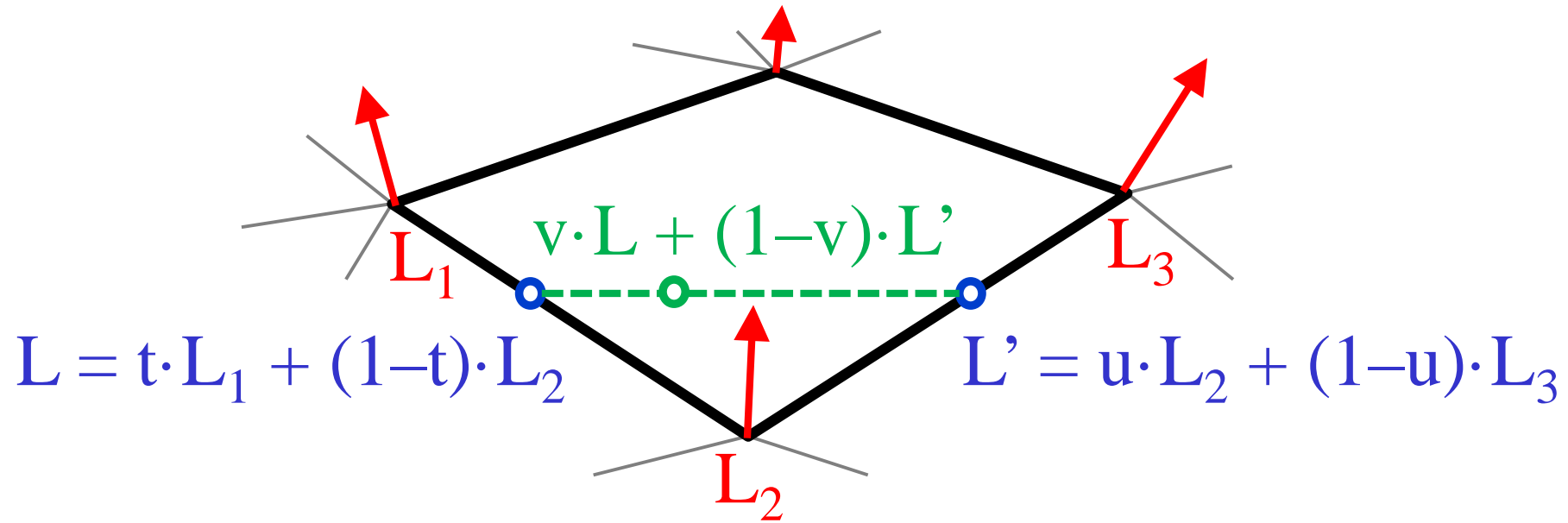
1. find normal vectors at corners and calculate shading (intensities) there:  $L_i$





1. find normal vectors at corners and calculate shading (intensities) there:  $L_i$
2. interpolate intensities along edges linearly:  $L, L'$

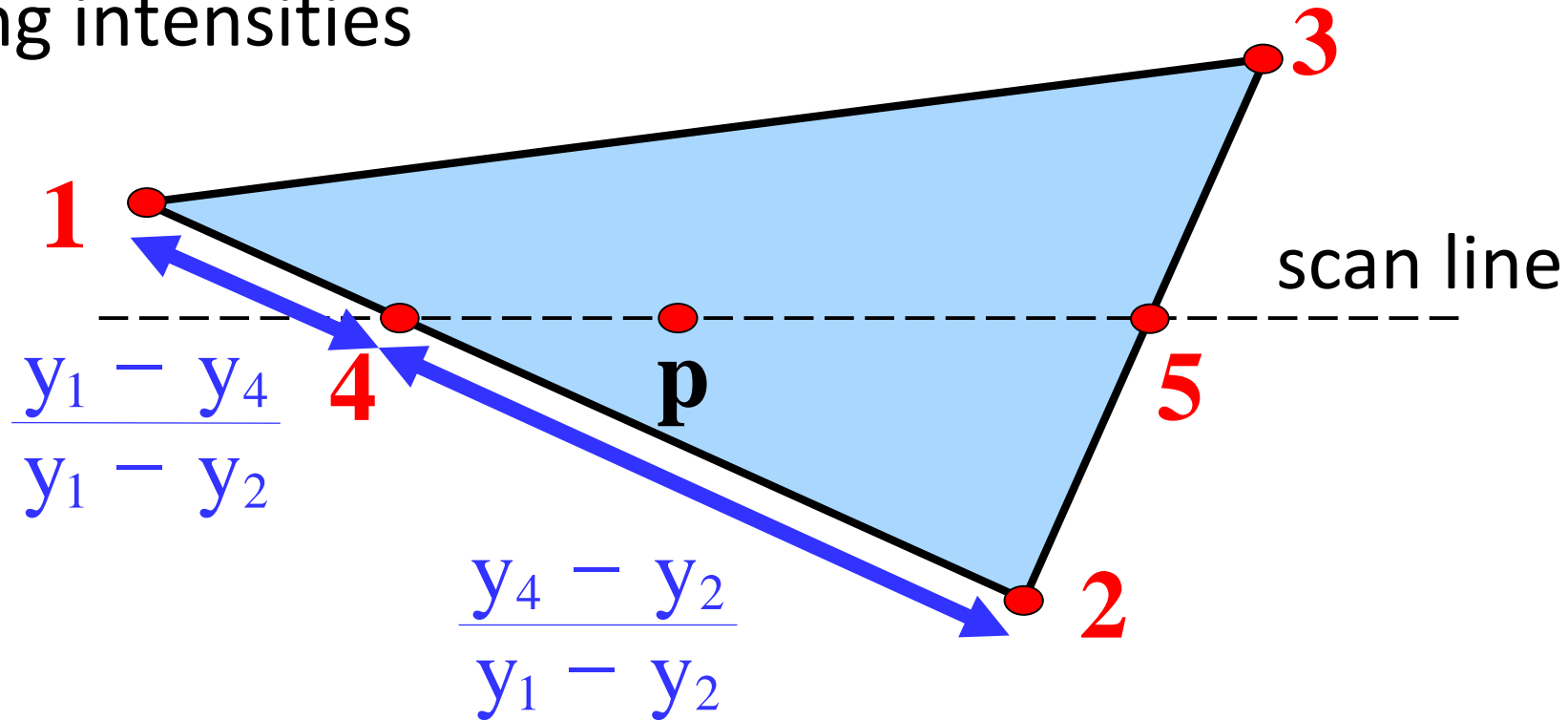




1. find normal vectors at corners and calculate shading (intensities) there:  $L_i$
2. interpolate intensities along edges linearly:  $L, L'$
3. interpolate intensities along scanlines linearly:  $L_p$



interpolating intensities

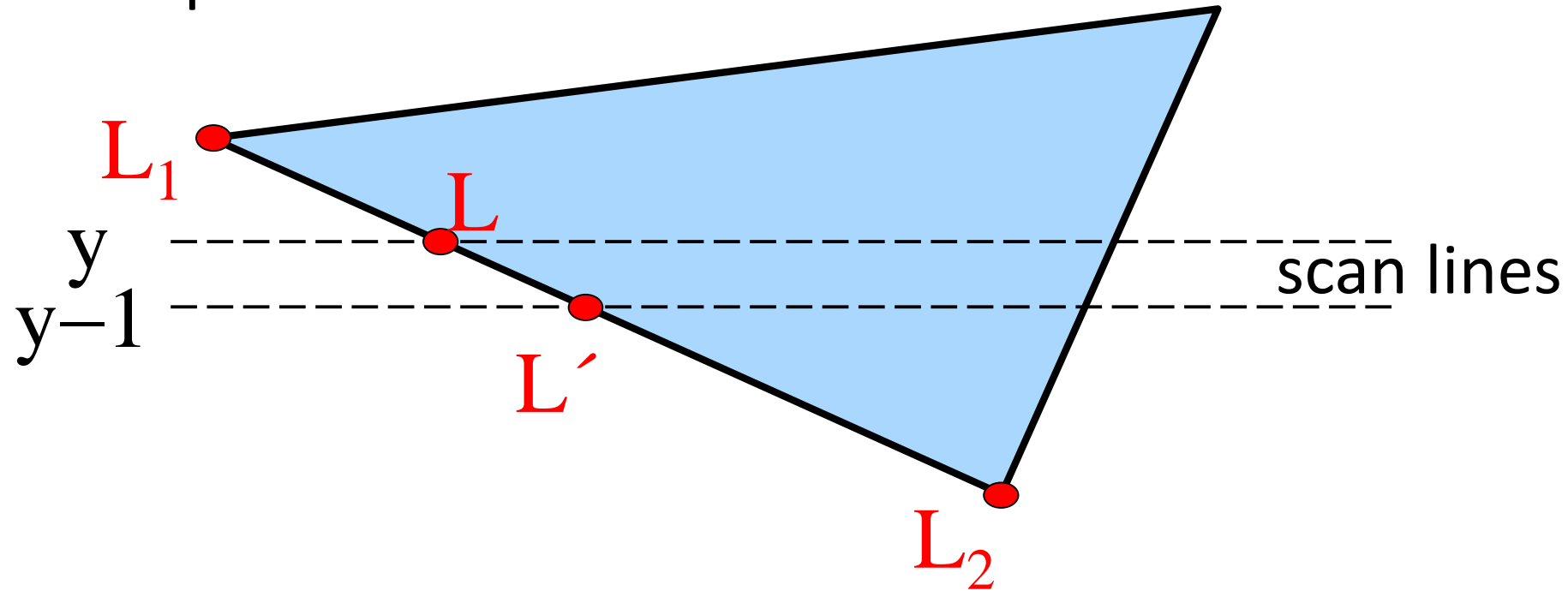


$$L_4 = \frac{y_4 - y_2}{y_1 - y_2} L_1 + \frac{y_1 - y_4}{y_1 - y_2} L_2$$

$$L_p = \frac{x_5 - x_p}{x_5 - x_4} L_4 + \frac{x_p - x_4}{x_5 - x_4} L_5$$



incremental update



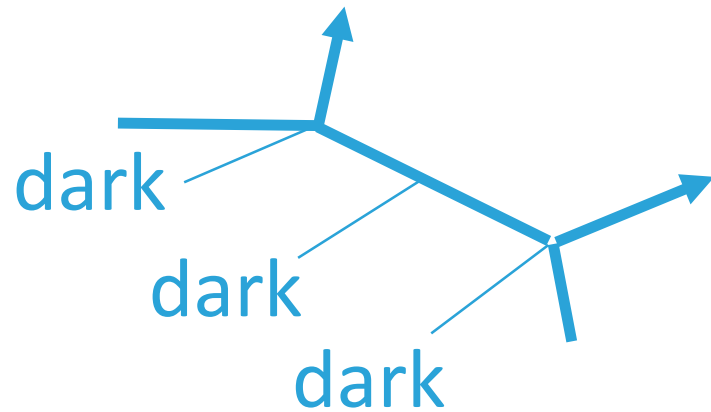
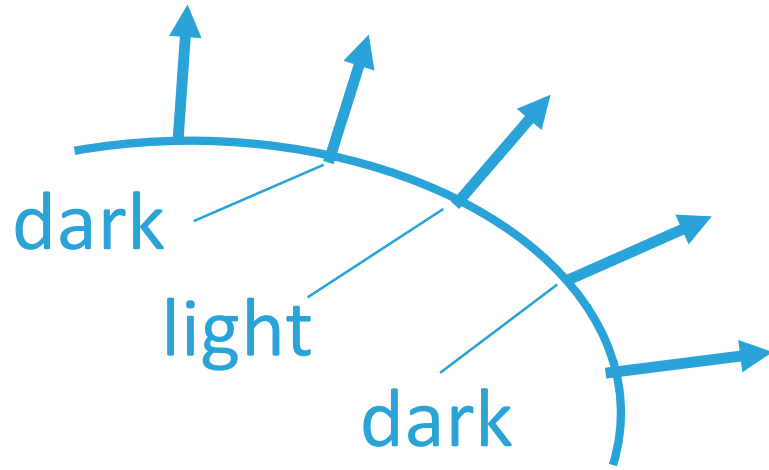
$$L = \frac{y - y_2}{y_1 - y_2} L_1 + \frac{y_1 - y}{y_1 - y_2} L_2$$

$$L' = L + \frac{L_2 - L_1}{y_1 - y_2}$$

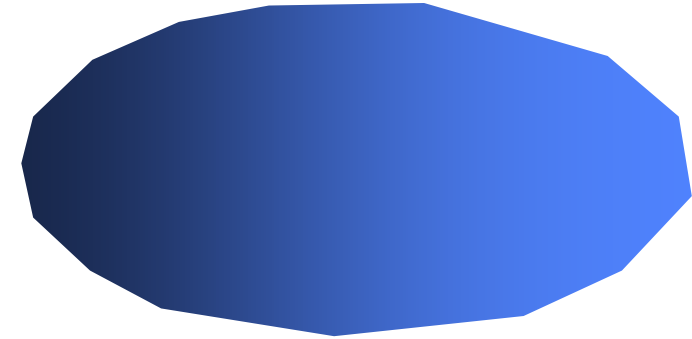




highlights can get lost or grow



corners on silhouette remain



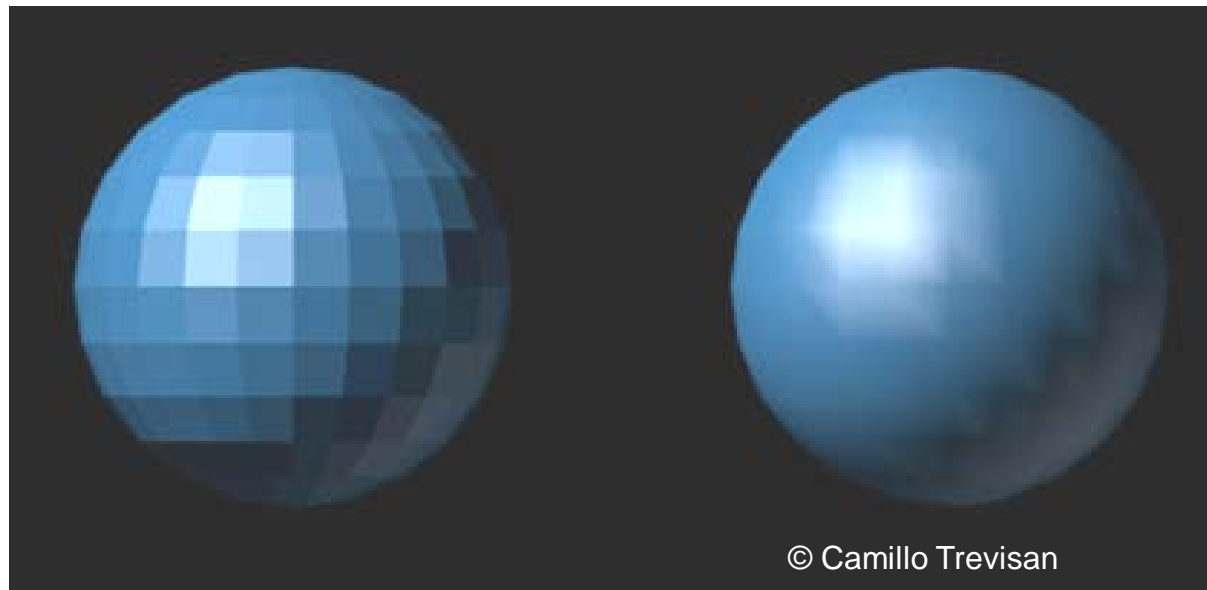
Mach band effect is visible  
at some edges



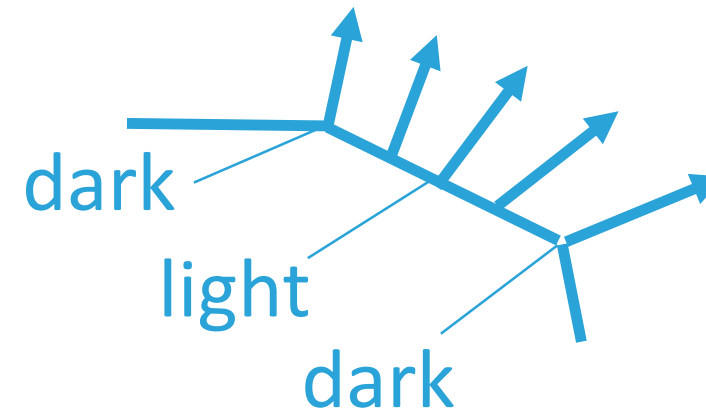
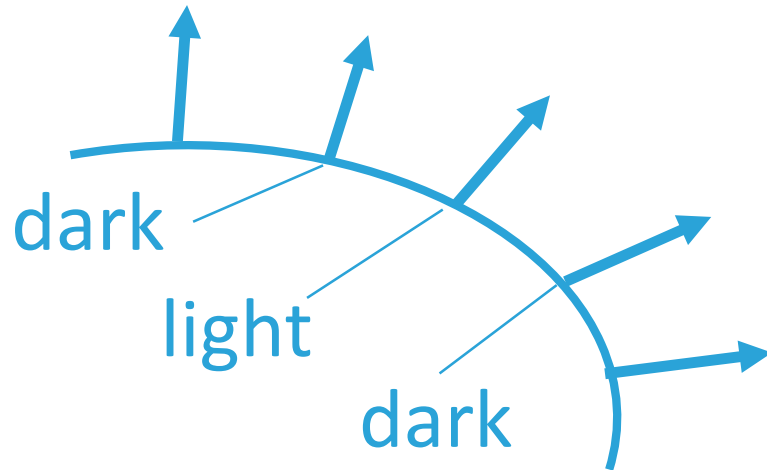
- no intensity discontinuities
- Mach bands due to linear intensity interpolation
- problems with highlights

*flat*

*Gouraud*



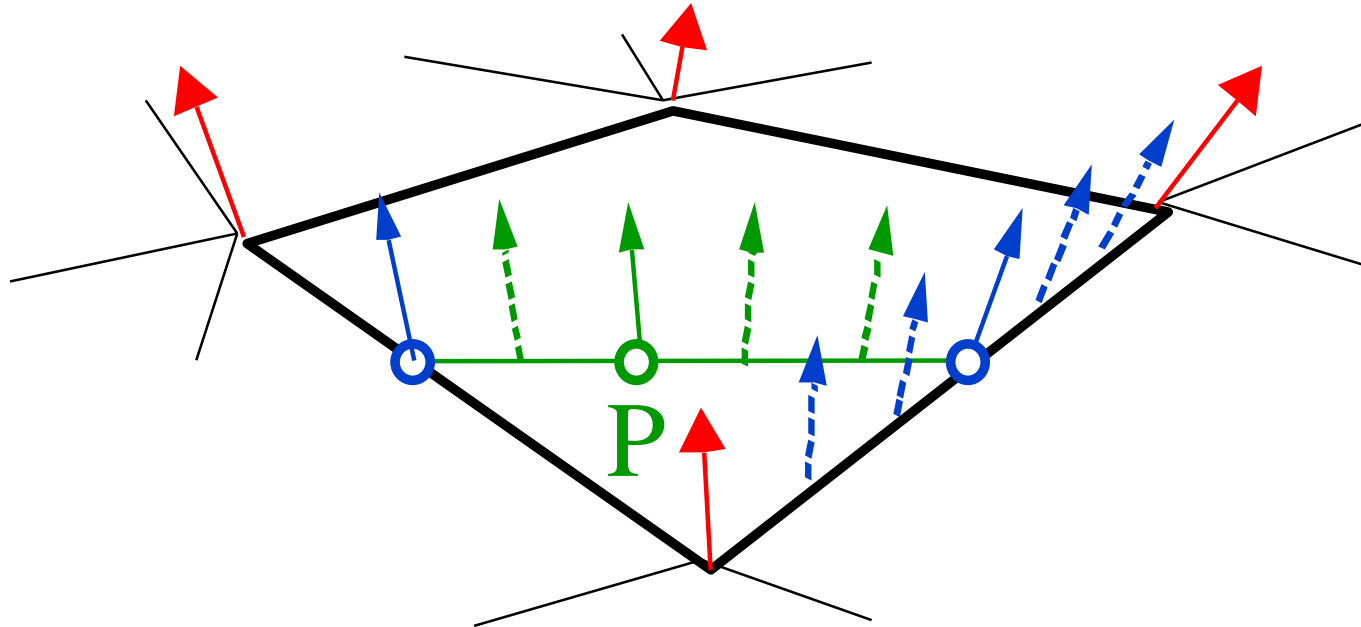
instead of intensities the normal vectors are interpolated, and for every point the shading calculation is performed separately



## normal-vector interpolation

- a. determine average unit normal vector at each polygon vertex
- b. linearly interpolate vertex normals
- c. apply illumination model along each scan line

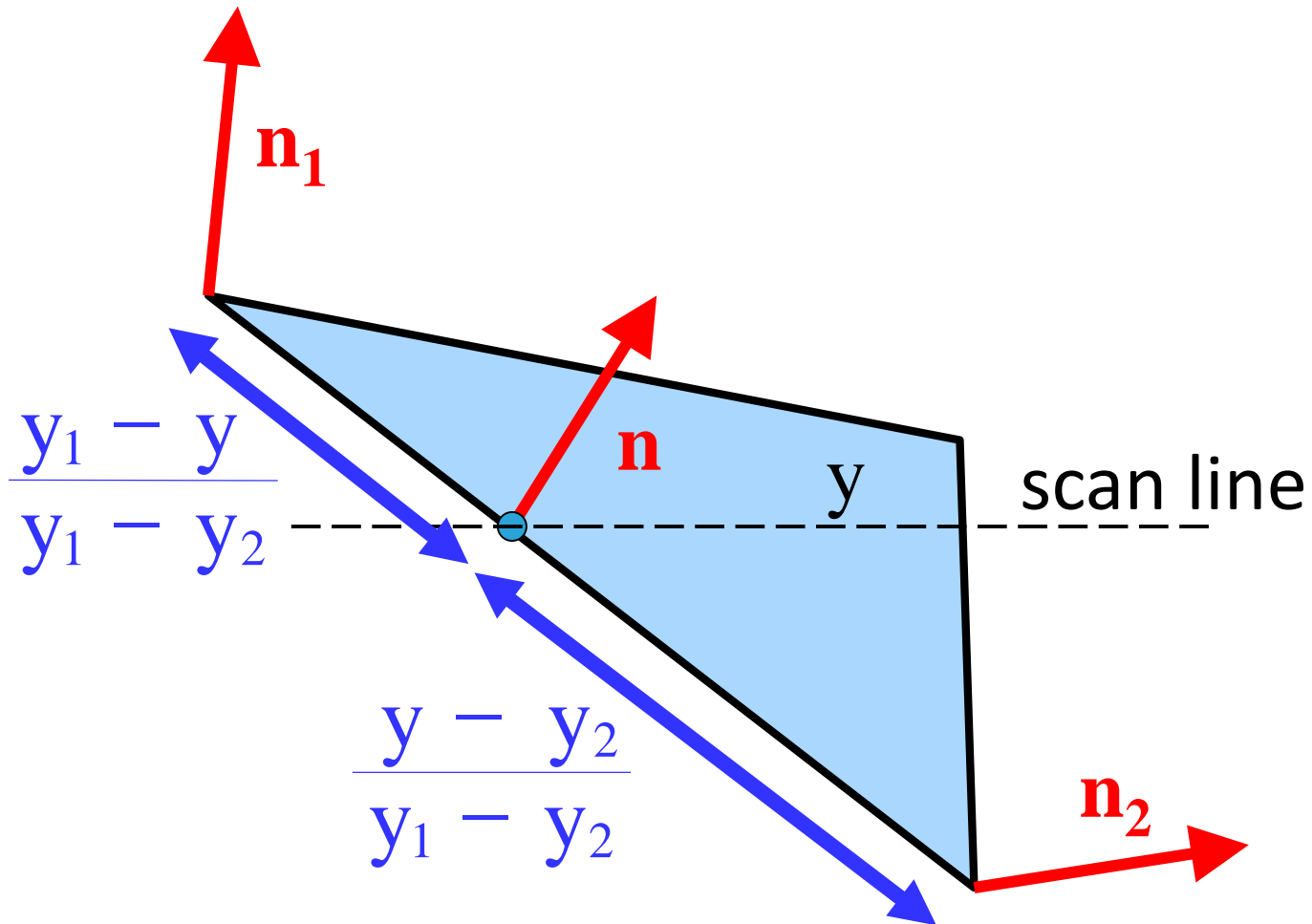




1. normal vectors at corner points
2. interpolate normal vectors along the edges
3. interpolate normal vectors along scanlines  
& calculate shading (intensities) for every pixel



## normal-vector interpolation



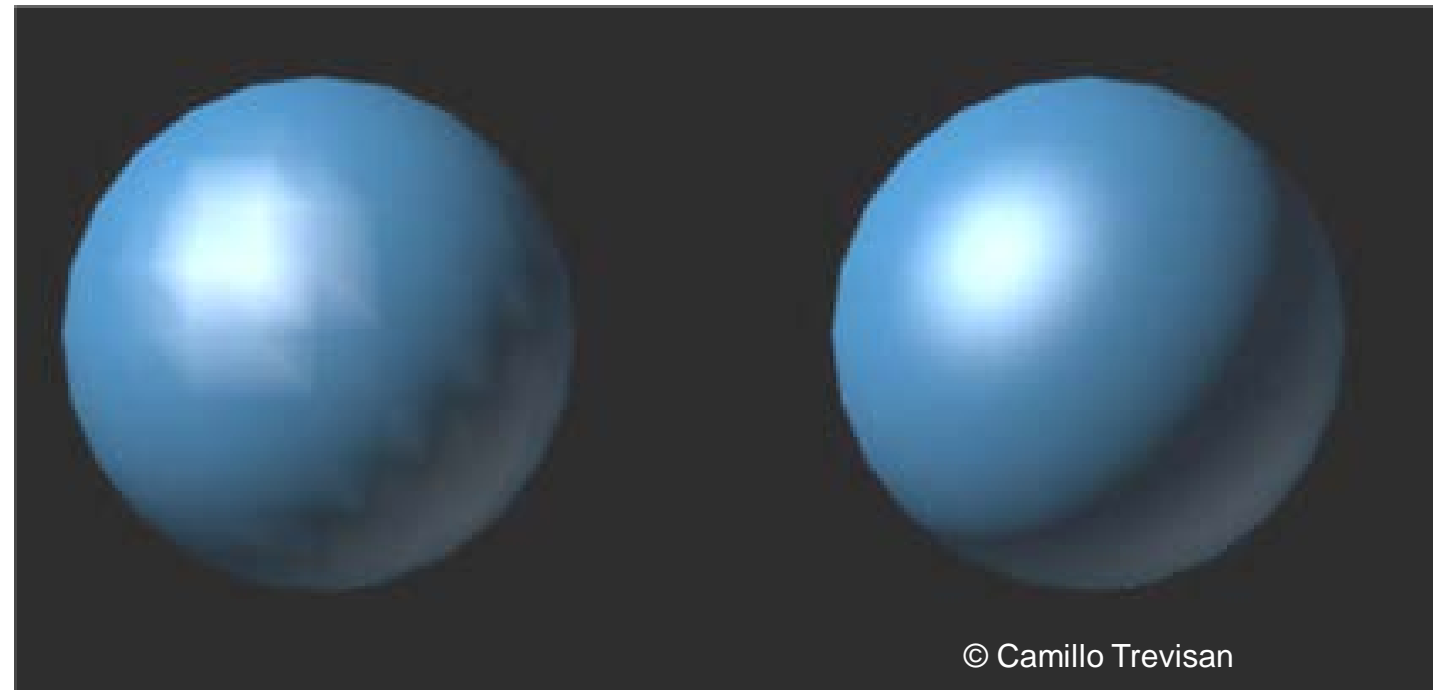
$$\mathbf{n} = \frac{y - y_2}{y_1 - y_2} \mathbf{n}_1 + \frac{y_1 - y}{y_1 - y_2} \mathbf{n}_2$$



incremental normal vector update along and between scan lines

comparison to Gouraud shading

- better highlights
- less Mach banding
- more costly
- wrong silhouette stays!



# Flat/Gouraud/Phong Comparison

