# Einführung in Visual Computing

186.822

# Global Illumination

Werner Purgathofer

# Surface-Rendering Methods

- polygon rendering methods
- ray tracing
- global illumination
- environment mapping
- texture mapping
- bump mapping

# **Global Illumination** in the Rendering Pipeline

*object capture/creation*

scene objects in object space

*modeling*

*viewing*

*projection*

vertex stage („vertex shader“)

transformed vertices in clip space

*clipping + homogenization*

scene in normalized device coordinates

*viewport transformation*

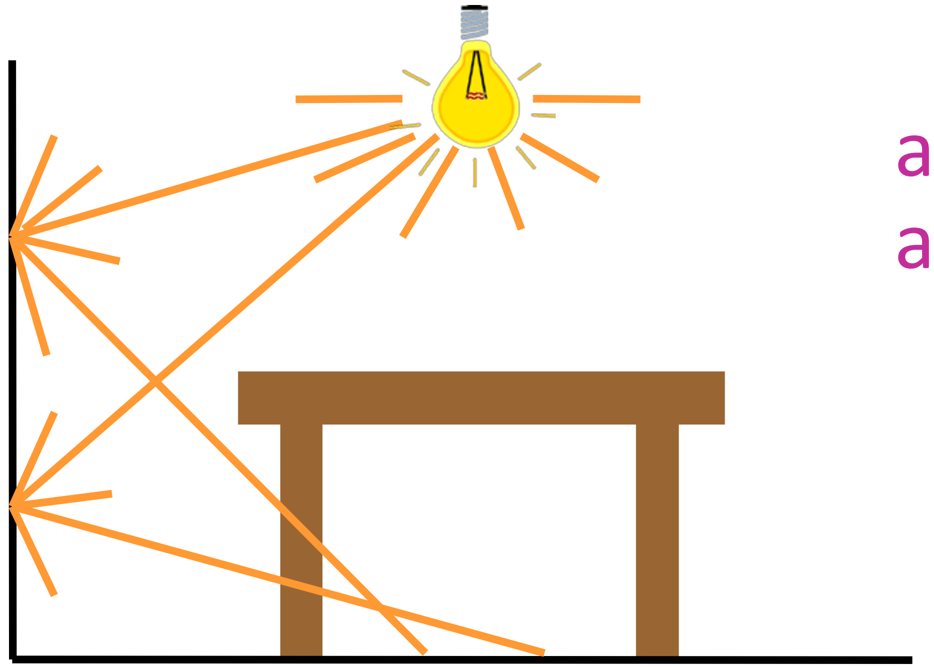
*rasterization*

*shading*

pixel stage („fragment shader“)

raster image in pixel coordinates

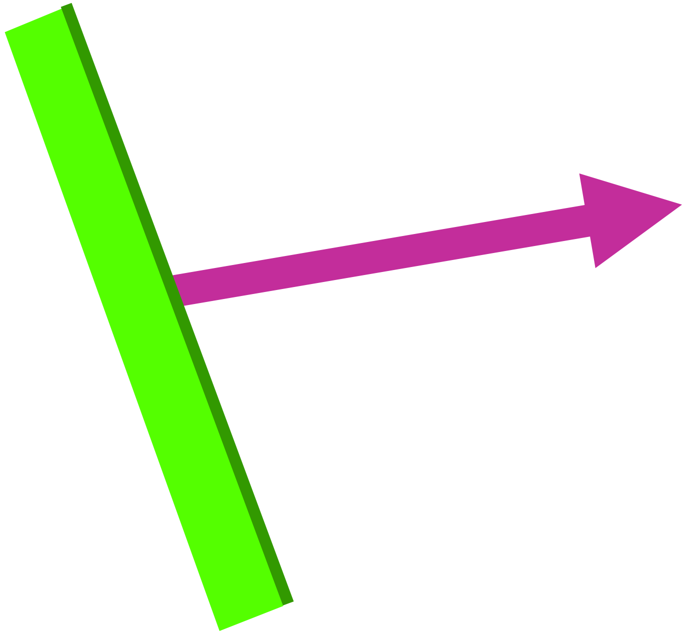
describes the physical process of light distribution in a diffuse reflecting environment



areas that are not illuminated directly are also not completely dark

every object acts as a secondary light source

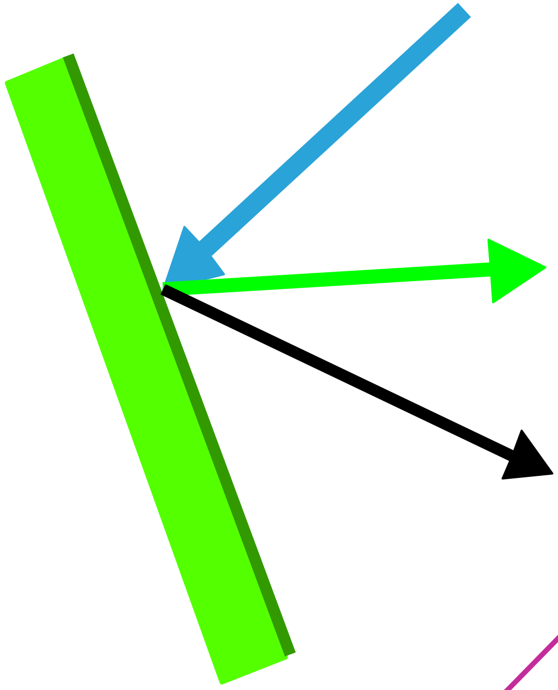
Radiosity **B** is the „radiant flux per unit area“ that is leaving a surface



# Radiosity Equation

incoming light from the environment

$$\int_{hemi} I(x)\,dx = \int_{hemi} dB$$



self emission (only for light sources) $\quad E$

reflected light from environment $\quad \rho \cdot \int_{hemi} dB$

$$B = E + \rho \cdot \int_{hemi} dB$$

**radiosity** of the point

to calculate the light influence between surfaces

***Radiosity* = total light leaving a surface point**

$$B = E + \rho \cdot \int_{\mathrm{hemi}} dB$$

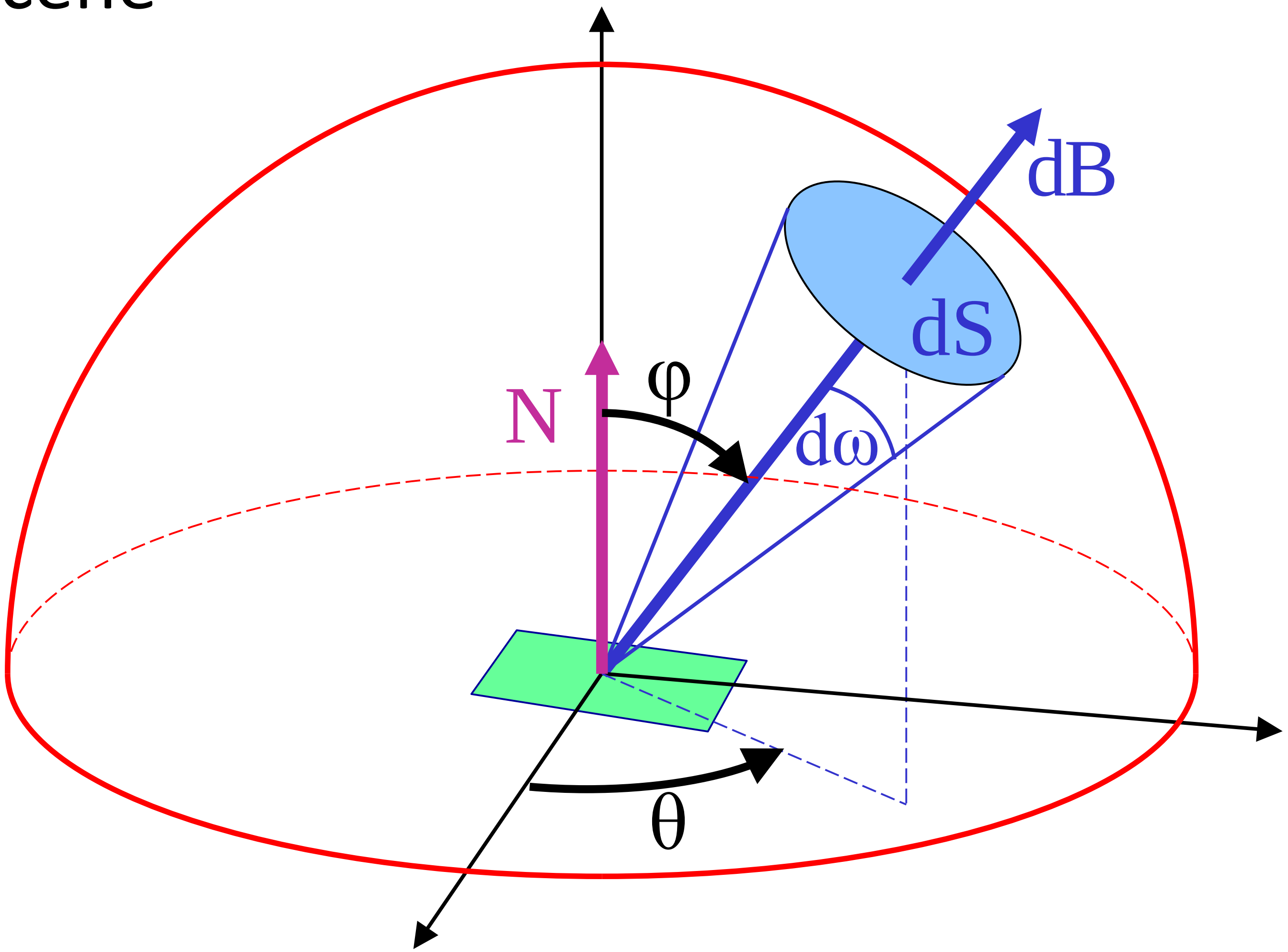
| | |
|---|---|
| B … radiosity | hemi … half space over point |
| E … self emission | ρ … reflection coefficient |

"radiosity = self emission + reflection property · sum of all incoming light"

# Radiosity Properties

- diffuse interreflections in a scene
- radiant energy transfers
- conservation of energy, closed environments
- subdivision of scene into patches with constant radiosity $B_i$

$$B = E + \rho \cdot \int_{hemi} dB$$



# Radiosity: Subdivision into Patches

the scene is discretized into $n$ **"patches"** (plane polygons) $P_i$ , for each of these patches a constant radiosity $B_i$ is assumed:

$$B = E + \rho \cdot \int_{hemi} d\,B \quad \Longrightarrow \quad B_i = E_i + \rho_i \cdot \sum_{j=1}^{n} B_j \cdot F_{ij}$$

| | |
|---|---|
| $\rho_i$ | diffuse reflection coefficient of patch $i$ |
| $F_{ij}$ | "form factor": describes what % of the influence on patch $i$ comes from patch $j$; = *geometric size !* |

# Radiosity Model

$$B_i = E_i + \rho_i \cdot \sum_{j=1}^{n} B_j \cdot F_{ij}$$

$B_i$ … radiosity of patch i

$E_i$ … self-emission of patch i

$\Sigma B_j F_{ij}$ … contribution of other patches

$F_{ij}$ … form factor, defines

- contribution of $B_i$ on patch j - which is equal to
- contribution of patch j to $B_i$

$\rho_i$ … reflectivity coefficient of patch i (“*albedo*”)

# Solving the Radiosity Equation

$$B_i = E_i + \rho_i \sum_{j \neq i} B_j F_{ij}$$

$$B_i - \rho_i \sum_{j \neq i} B_j F_{ij} = E_i$$

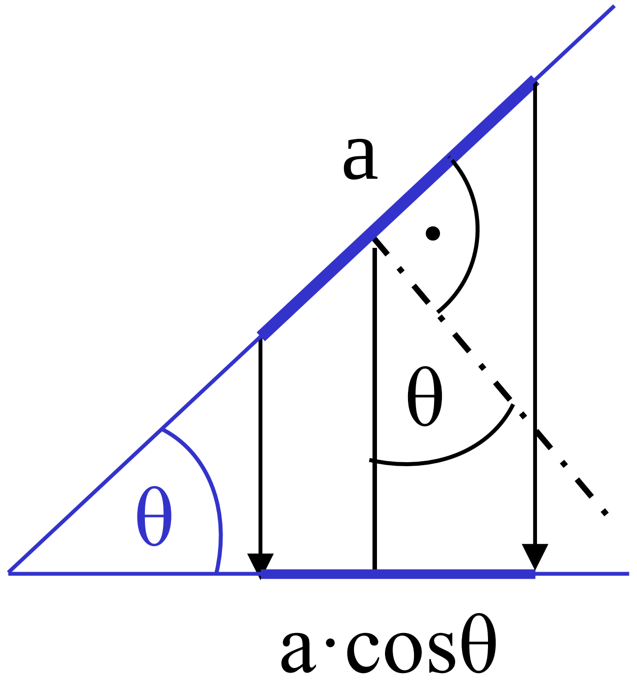
$$\begin{bmatrix} 1 & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1 & \cdots & -\rho_2 F_{2n} \\ \vdots & \vdots & & \vdots \\ -\rho_n F_{n1} & -\rho_n F_{n2} & \cdots & 1 \end{bmatrix} \cdot \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

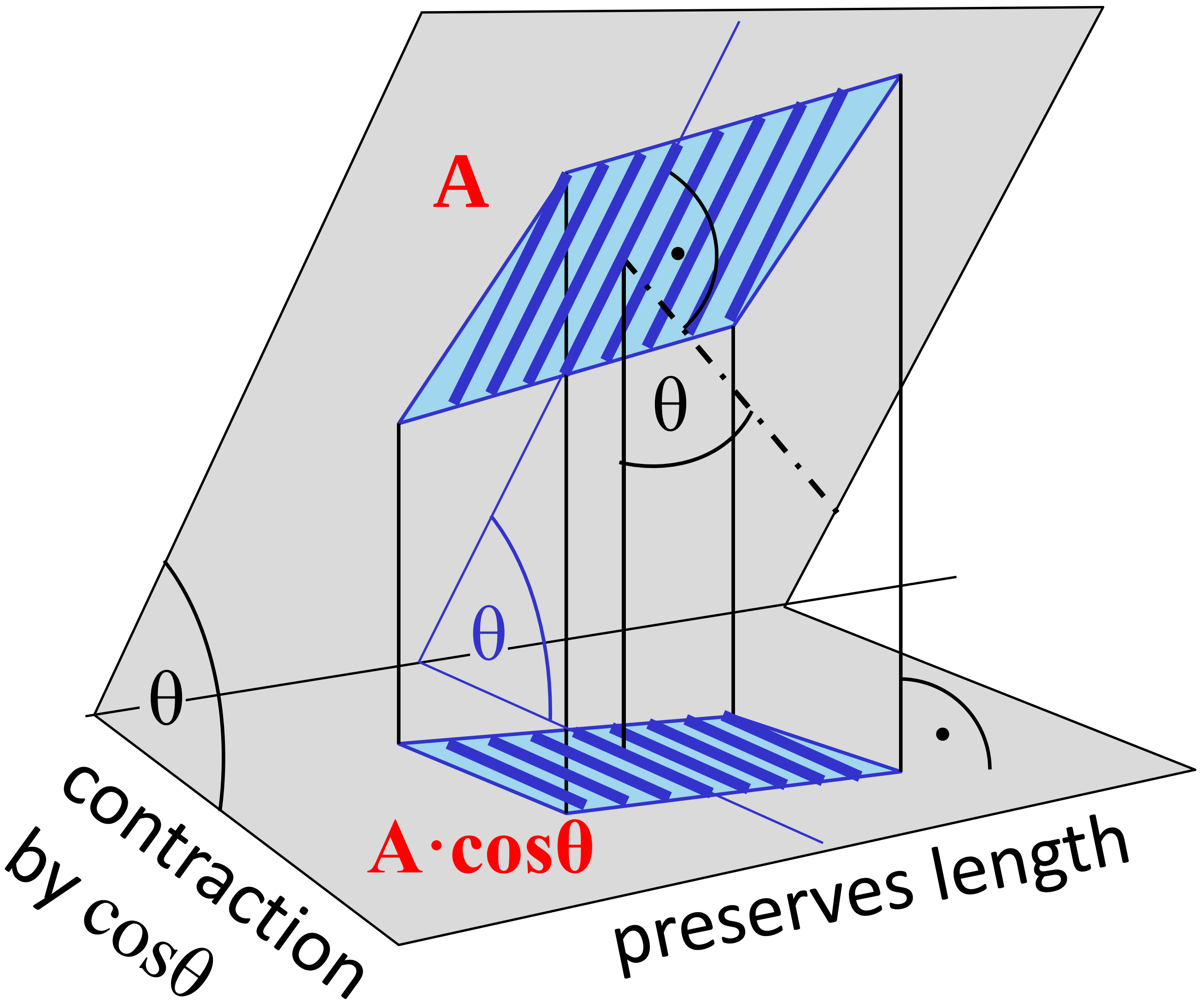
# Radiosity Equation

surface properties | form factors (constants) | **radiosities (unknowns)** | surface properties

$$\begin{bmatrix} 1 & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1 & \cdots & -\rho_2 F_{2n} \\ \vdots & \vdots & & \vdots \\ -\rho_n F_{n1} & -\rho_n F_{n2} & \cdots & 1 \end{bmatrix} \cdot \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

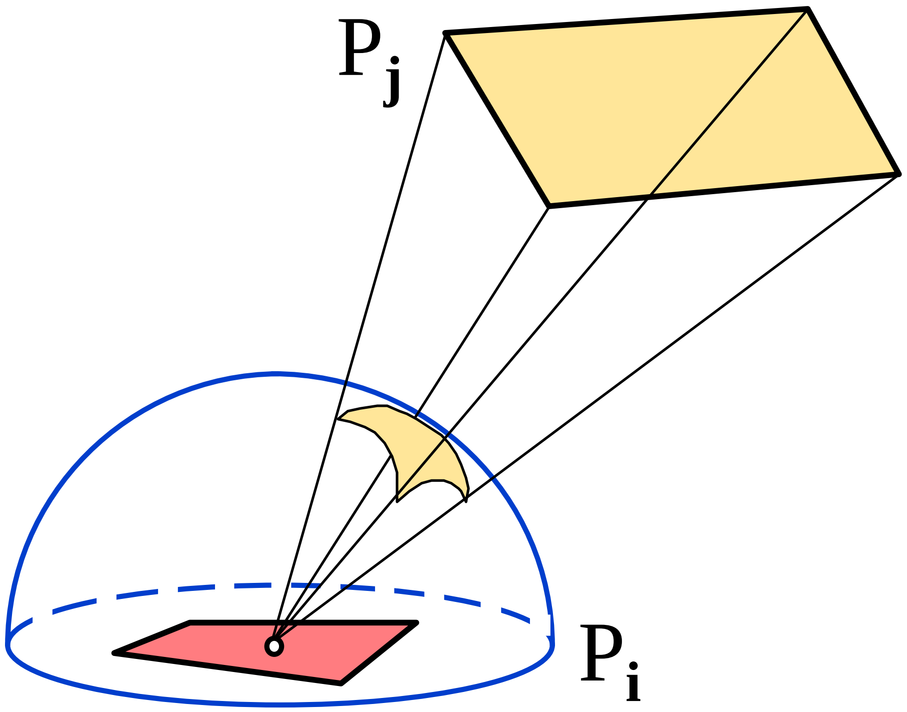
# Reminder: Projection of a Polygon



# Radiosity: Form Factors $F_{ij}$

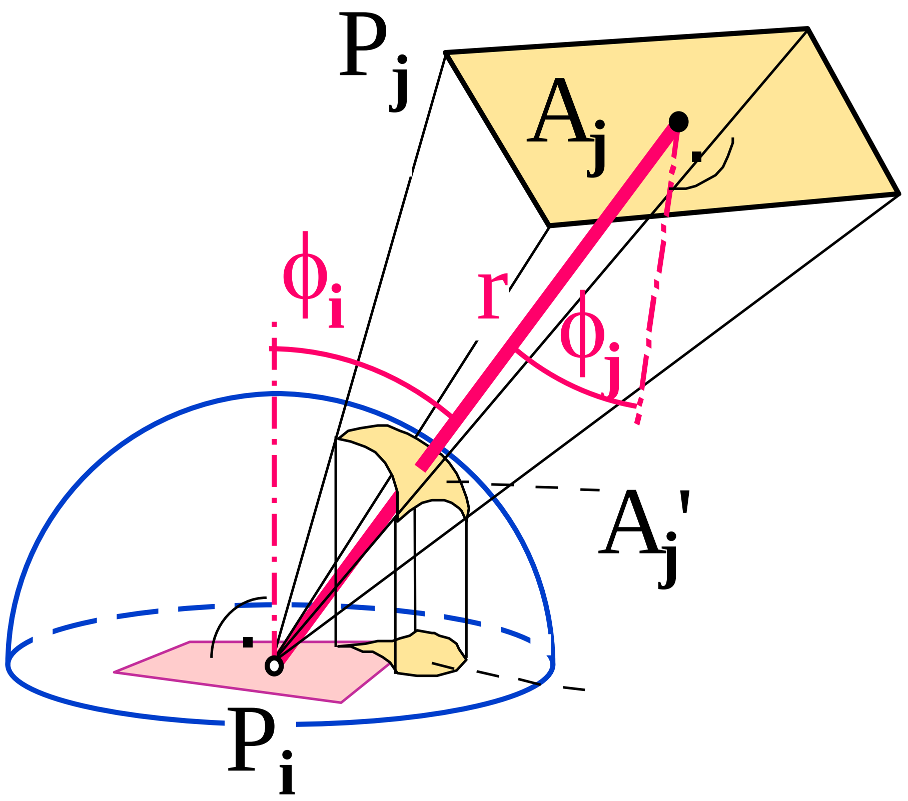
form factor $F_{ij}$: contribution of patch $P_j$ to $B_i$
= contribution of $B_i$ to patch $P_j$

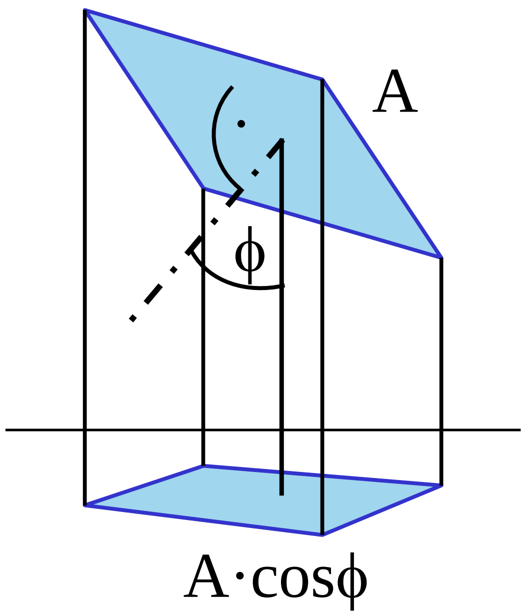



$$\frac{\text{energy reaching patch j from patch i}}{\text{total energy leaving patch i}}$$

form factor $F_{ij}$: contribution of patch $P_j$ to $B_i$

= contribution of $B_i$ to patch $P_j$



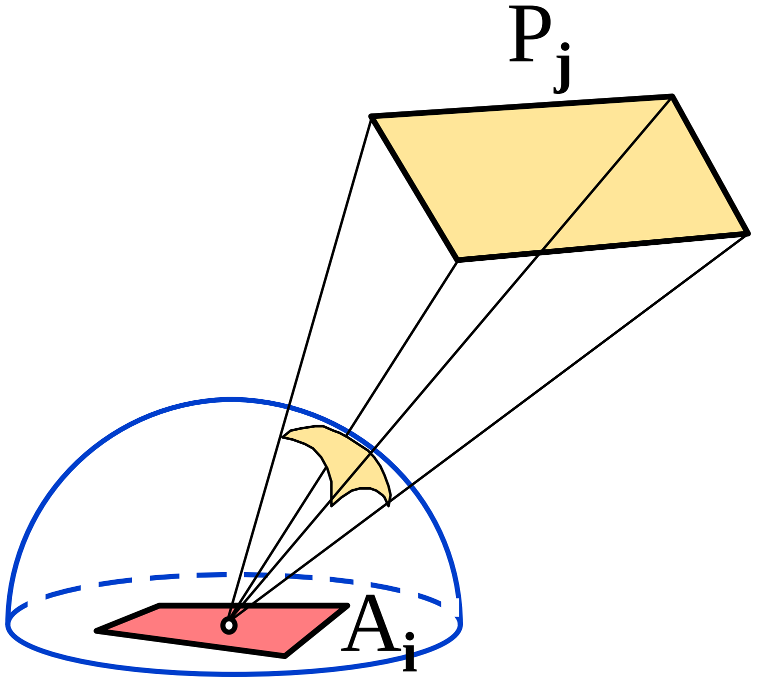




$$F_{ij} = \frac{\cos\phi_i \cos\phi_j A_j}{\pi r^2}$$

and because $\sum_{j=1}^{n} F_{ij} = 1$

form factor $F_{ij}$: contribution of patch $P_j$ to $B_i$
= contribution of $B_i$ to patch $P_j$

$$\boxed{F_{ij} = \frac{\cos\phi_i \cos\phi_j A_j}{\pi r^2}}$$




more precisely: form factor is sum over contributions from $P_j$ averaged over area $A_i$

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\phi_i \cos\phi_j}{\pi r^2} dA_j \, dA_i$$

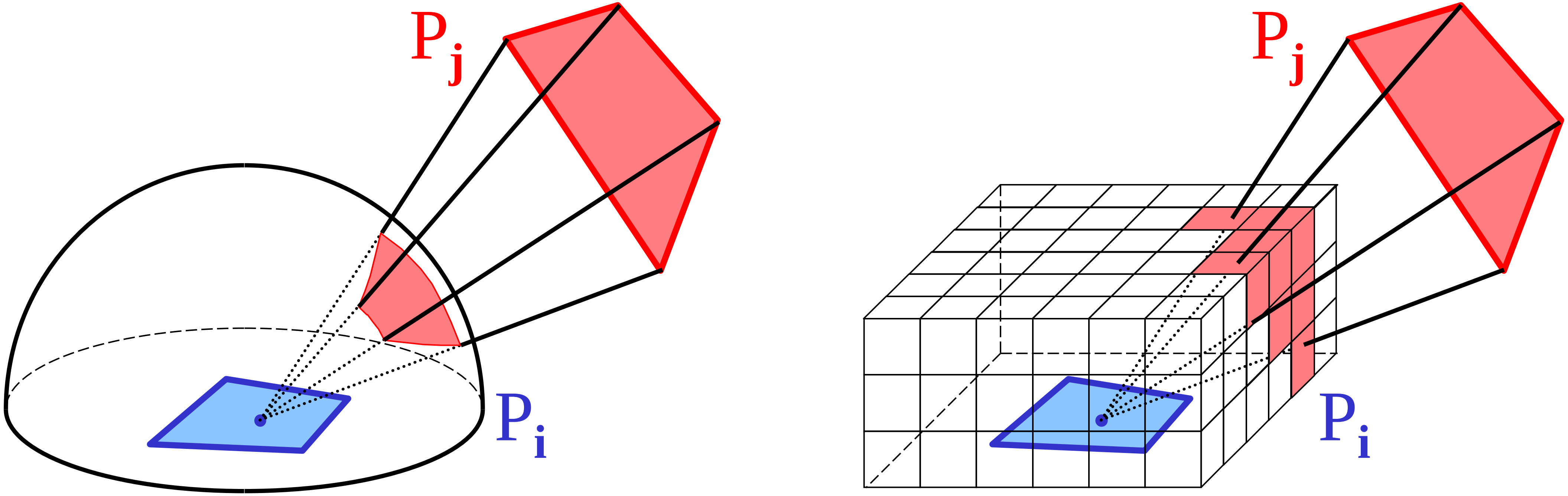
# Radiosity: Form Factors

form factor properties

- conservation of energy $$\sum_{j=1}^{n} F_{ij} = 1$$
- uniform light reflection $$A_i F_{ij} = A_j F_{ji}$$
- no self-incidence $$F_{ii} = 0$$

form factor calculation

- most expensive step in radiosity calculation
- numerical integration (Monte Carlo methods)
- *hemicube* approach (replaces hemisphere)



# Radiosity Equation

solving the radiosity equation

- Gaussian elimination
- Gauss-Seidel iteration

$$\begin{bmatrix} 1 & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1 & \cdots & -\rho_2 F_{2n} \\ \vdots & \vdots & & \vdots \\ -\rho_n F_{n1} & -\rho_n F_{n2} & \cdots & 1 \end{bmatrix} \cdot \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

very time and storage intensive

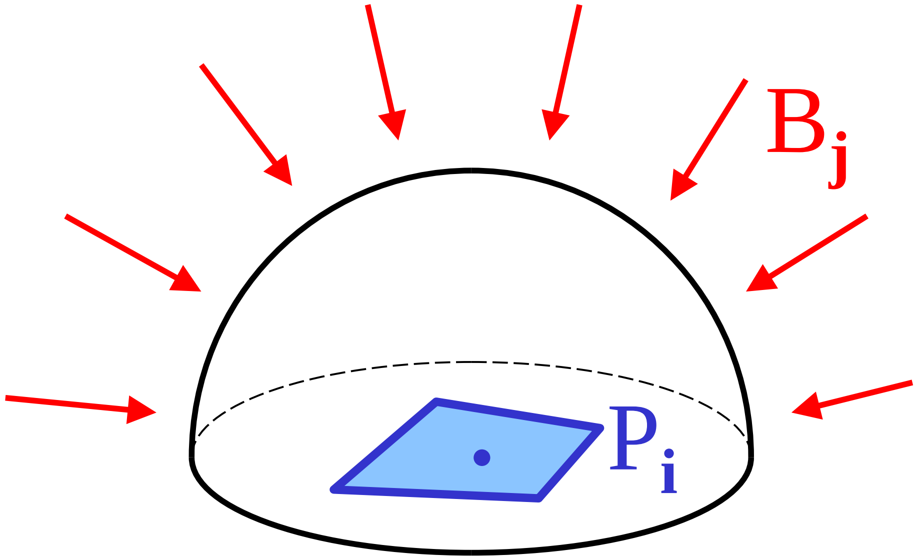
# Radiosity Equation

solving the radiosity equation

- Gauss-Seidel iteration

$$B_i^{k+1} = E_i + \rho_i \sum_{j \neq i} B_j^k F_{ij}$$

$$\begin{pmatrix} B \\ B \\ B \\ B \\ B \end{pmatrix} = \begin{pmatrix} E \\ E \\ E \\ E \\ E \end{pmatrix} + \begin{pmatrix} x & x & x & x & x \\ x & x & x & x & x \\ x & x & x & x & x \\ x & x & x & x & x \\ x & x & x & x & x \end{pmatrix} \cdot \begin{pmatrix} B \\ B \\ B \\ B \\ B \end{pmatrix}$$

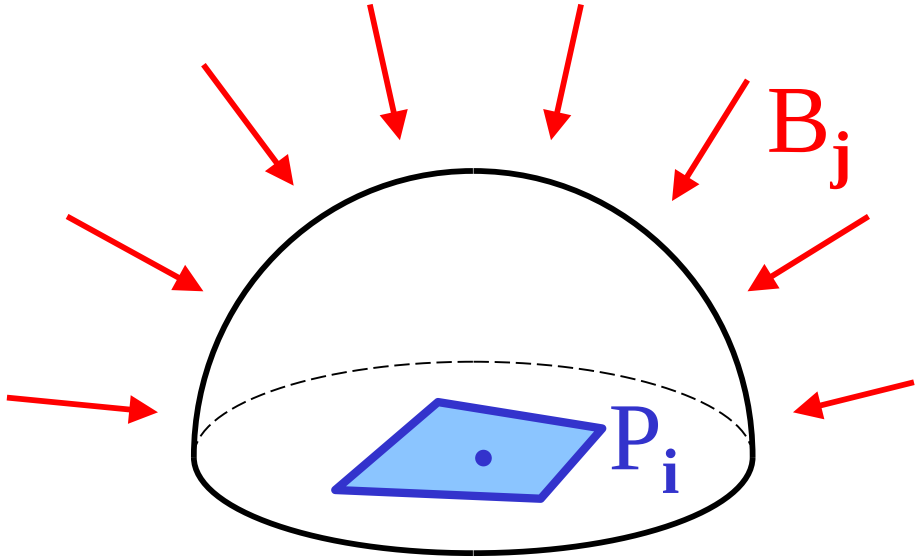


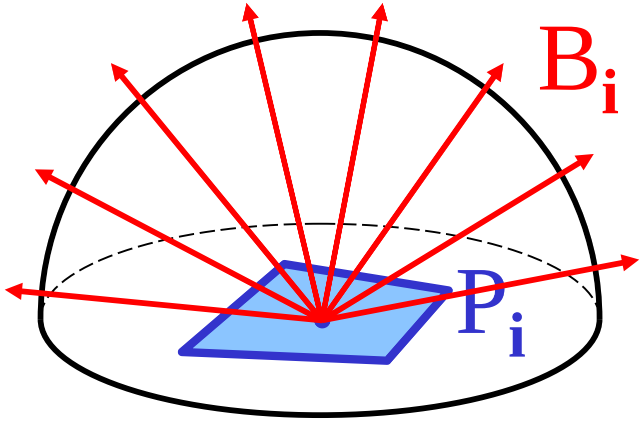

*"gathering"*

# Radiosity Equation

“gathering” vs.“*shooting*”

$$B_i^{k+1} = E_i + \rho_i \sum_{j \neq i} B_j^k F_{ij}$$




$$\begin{pmatrix} \\ \\ \mathbf{B} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} \\ \\ \mathbf{E} \\ \\ \\ \end{pmatrix} + \begin{pmatrix} & & & & \\ & & & & \\ \mathbf{x} & \mathbf{x} & \mathbf{x} & \mathbf{x} & \mathbf{x} \\ & & & & \\ & & & & \end{pmatrix} \cdot \begin{pmatrix} \mathbf{B} \\ \mathbf{B} \\ \mathbf{B} \\ \mathbf{B} \\ \mathbf{B} \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{B} \\ \mathbf{B} \\ \mathbf{B} \\ \mathbf{B} \\ \mathbf{B} \end{pmatrix} = \begin{pmatrix} \mathbf{E} \\ \mathbf{E} \\ \mathbf{E} \\ \mathbf{E} \\ \mathbf{E} \end{pmatrix} + \begin{pmatrix} & & \mathbf{x} & & \\ & & \mathbf{x} & & \\ & & \mathbf{x} & & \\ & & \mathbf{x} & & \\ & & \mathbf{x} & & \end{pmatrix} \cdot \begin{pmatrix} \\ \\ \mathbf{B} \\ \\ \\ \end{pmatrix}$$

# Progressive Refinement Radiosity (1)

"shooting" → select brightest patch $P_i$ and distribute its radiosity $B_i$

$$B_i = E_i + \rho_i \sum_{j \neq i} B_j F_{ij} \Rightarrow \begin{aligned} B_{i \text{ due to } B_j} &= \rho_i B_j F_{ij} \\ B_{j \text{ due to } B_i} &= \rho_j B_i F_{ji} \end{aligned} \Rightarrow$$

$\Rightarrow$ because of

$$A_i F_{ij} = A_j F_{ji}$$

$$\Rightarrow \boxed{B_{j \text{ due to } B_i} = \rho_j B_i F_{ij} \frac{A_i}{A_j}}$$

[one refinement step]

select patch i with highest $A_i * \Delta B_i$
<u>for</u> selected patch i {
    set up hemicube
    calculate form factors $F_{ij}$
}
<u>for</u> each patch j {
    $\Delta rad := \rho_j * \Delta B_i * F_{ij} * A_i / A_j$
    $\Delta B_j := \Delta B_j + \Delta rad$
    $B_j := B_j + \Delta rad$
}
$\Delta B_i := 0$

$$\begin{pmatrix}B\\B\\B\\B\\B\end{pmatrix} = \begin{pmatrix}E\\E\\E\\E\\E\end{pmatrix} + \begin{pmatrix} & & x & & \\ & & x & & \\ & & x & & \\ & & x & & \\ & & x & & \end{pmatrix} \cdot \begin{pmatrix} \\ \\ B \\ \\ \\ \end{pmatrix}$$

$$\boxed{B_{j \text{ due to } B_i} = \rho_j B_i F_{ij} \frac{A_i}{A_j}}$$

# Progressive Refinement Radiosity (3)

- initialize $\Delta B_i = B_i = E_i$
- select patch with highest $\Delta B_i A_i$

*cathedral rendered with progressive refinement radiosity*

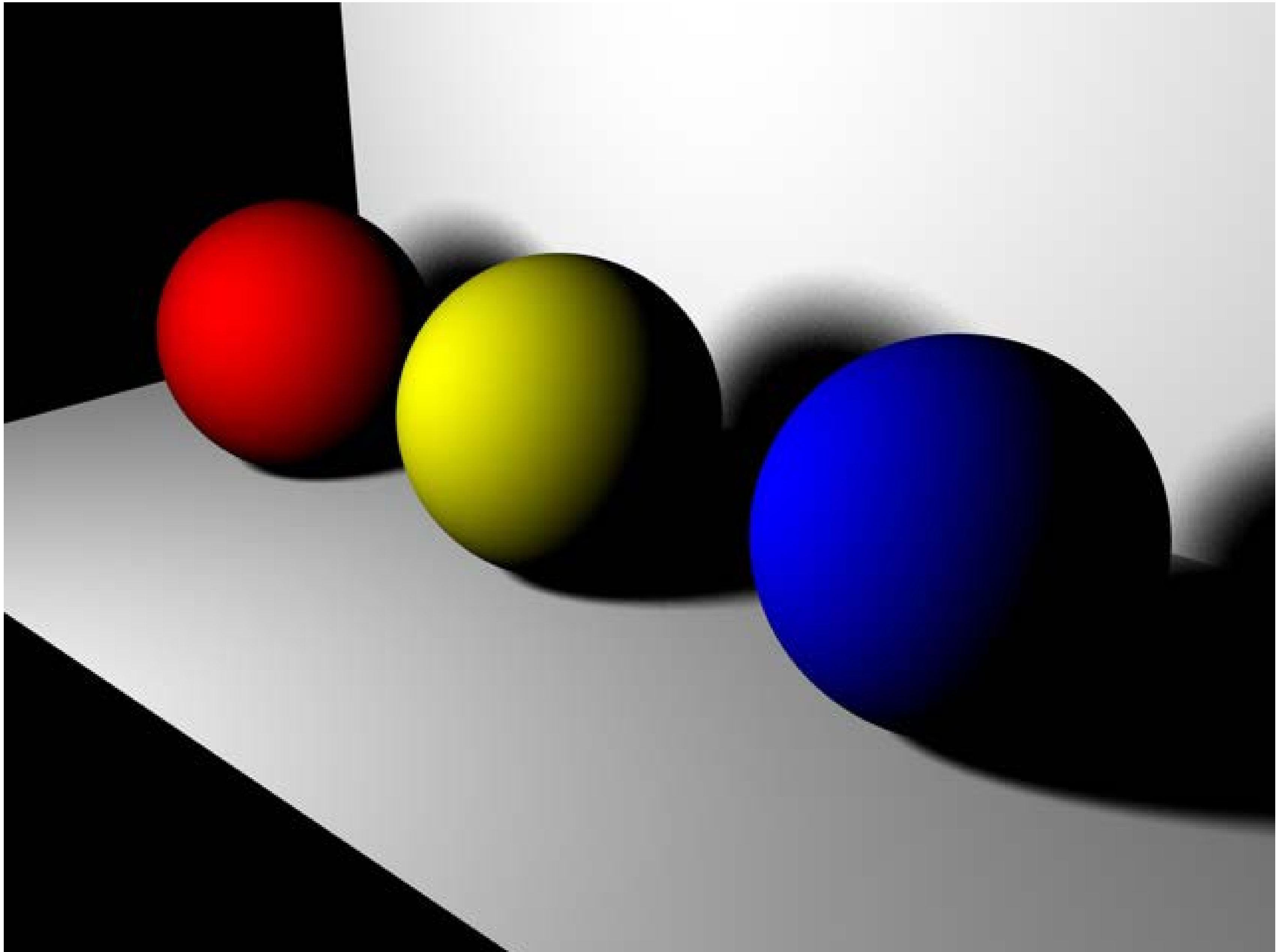
*form factors computed with ray-tracing methods*

by John Wallace & John Lin

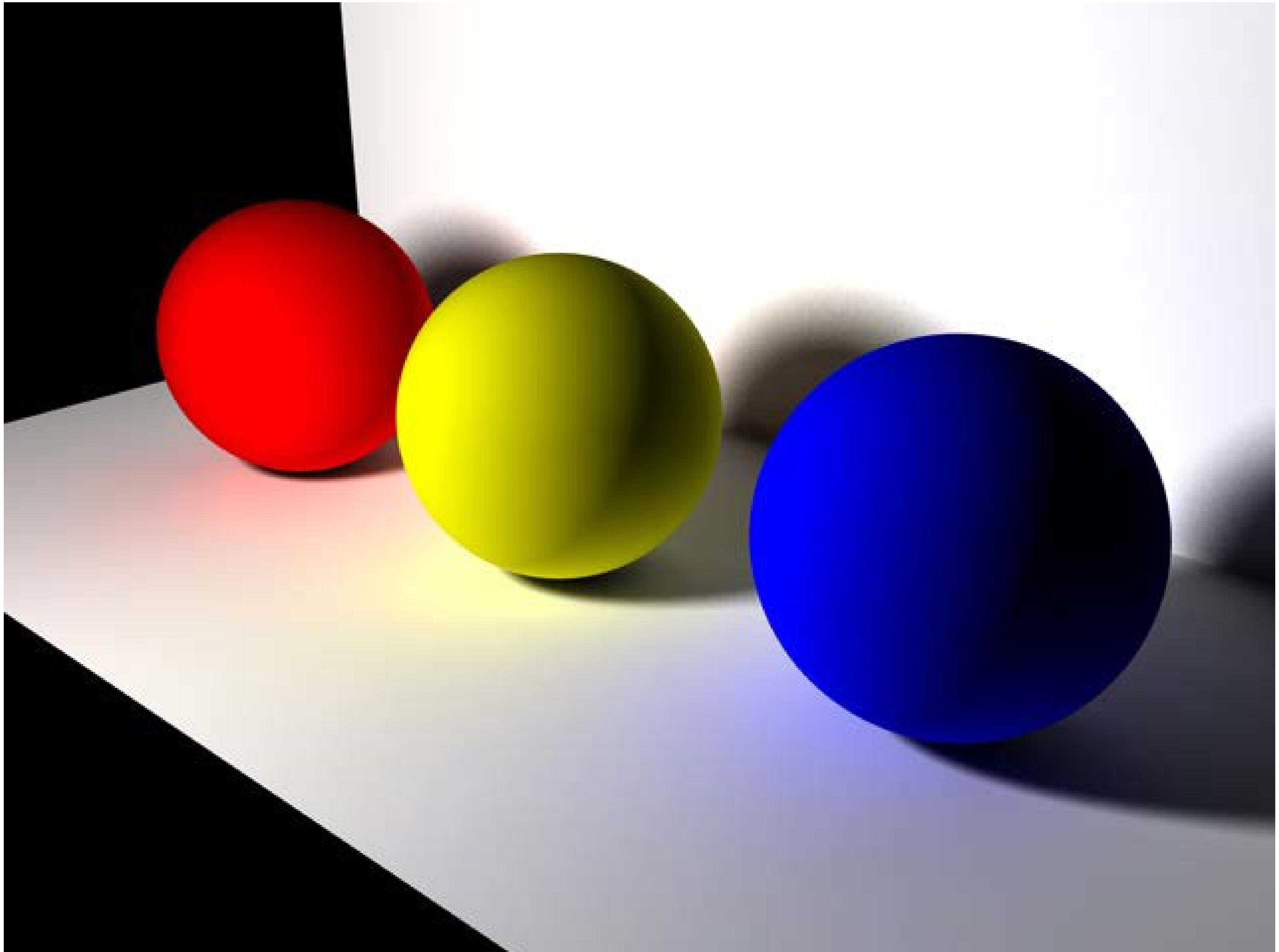
# Radiosity Example Images (1)


*image of a constructivist museum rendered with progressive refinement radiosity*

# Radiosity Results



# Radiosity Results



# Radiosity Results


# Radiosity Aspects

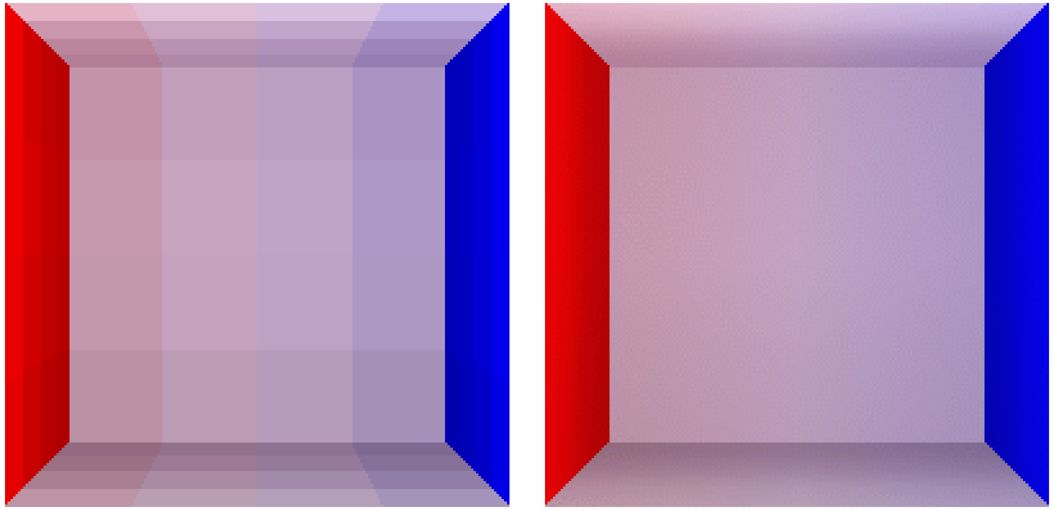
radiosity is viewpoint-independent

needs a rendering step to display

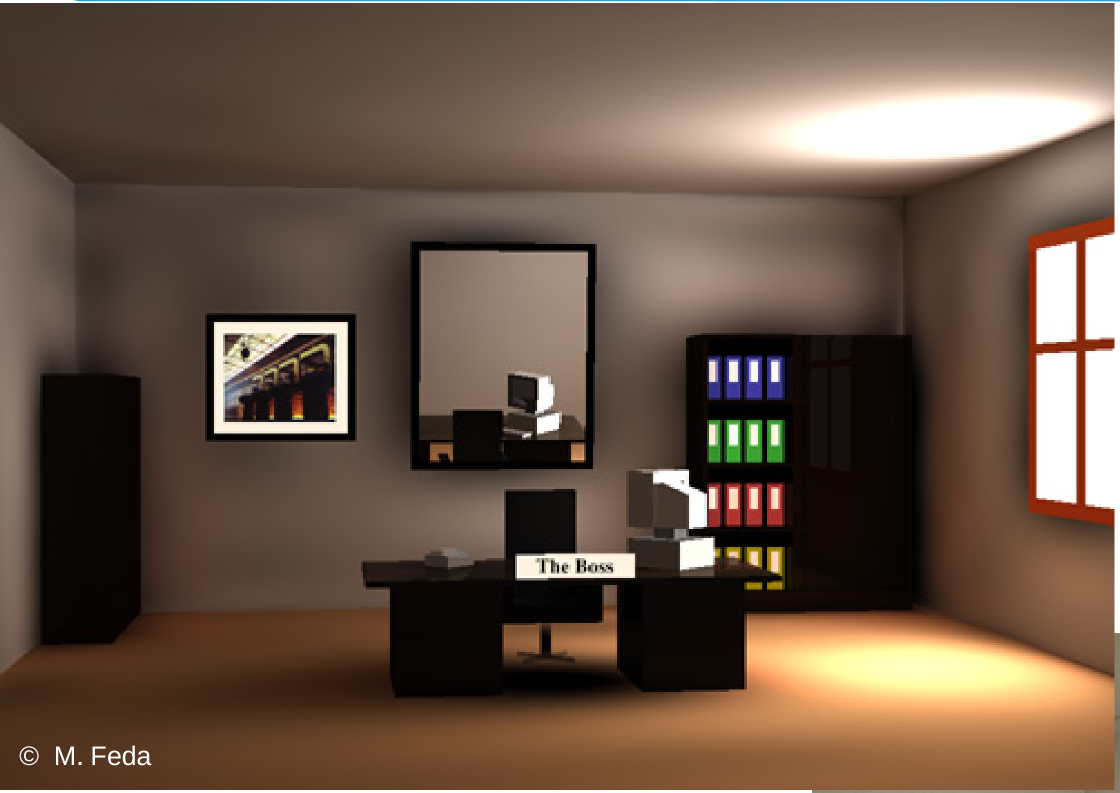
- polygon rendering
- Gouraud shading
- ray-tracing
- …

combination with ray-tracing enables

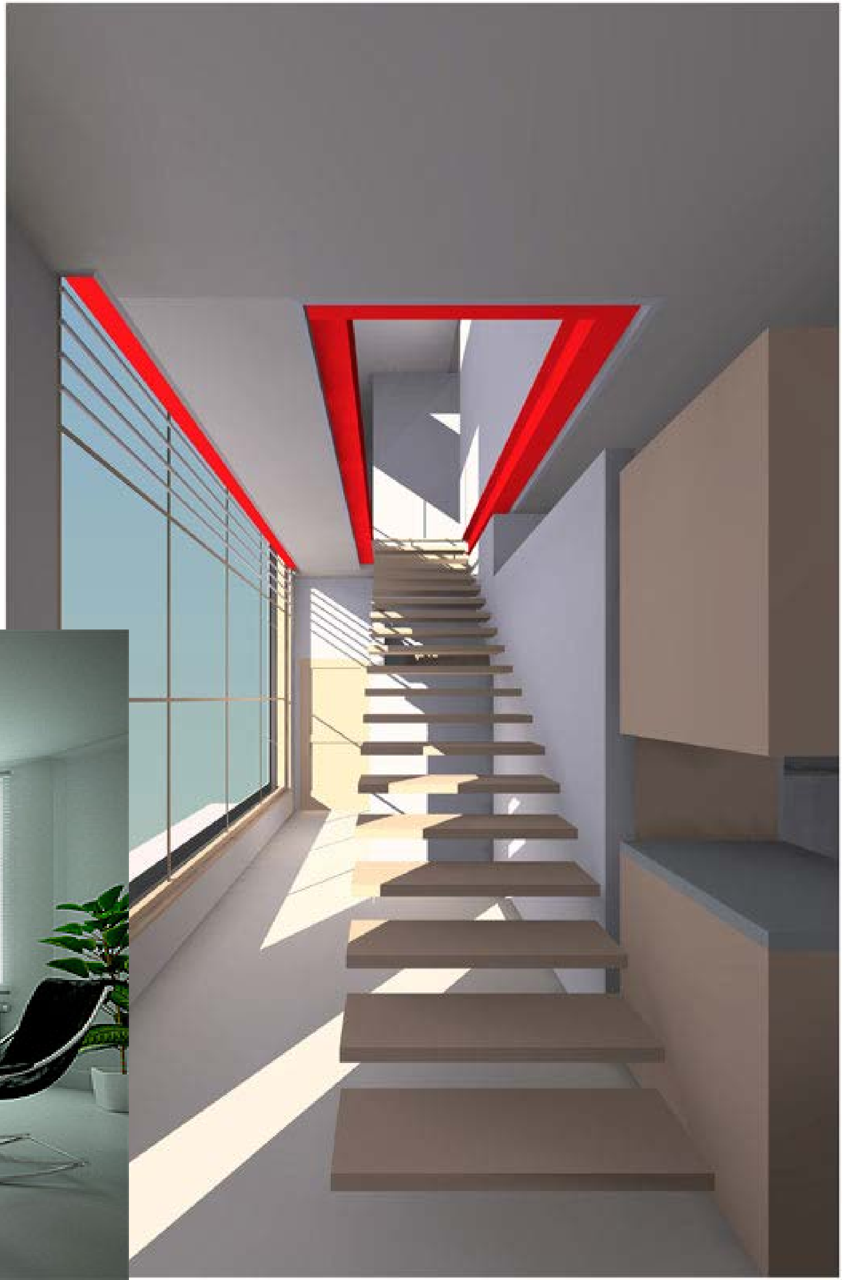
- reflections
- shadows
- …



# Radiosity Results

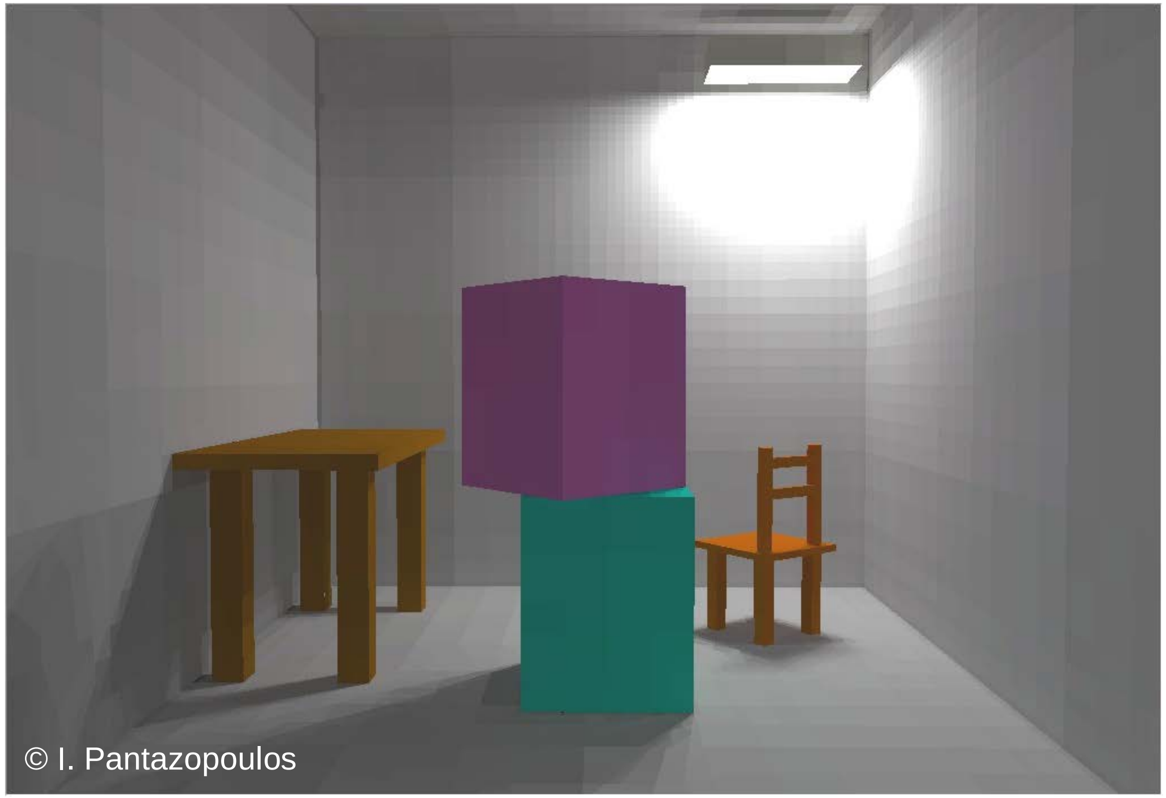


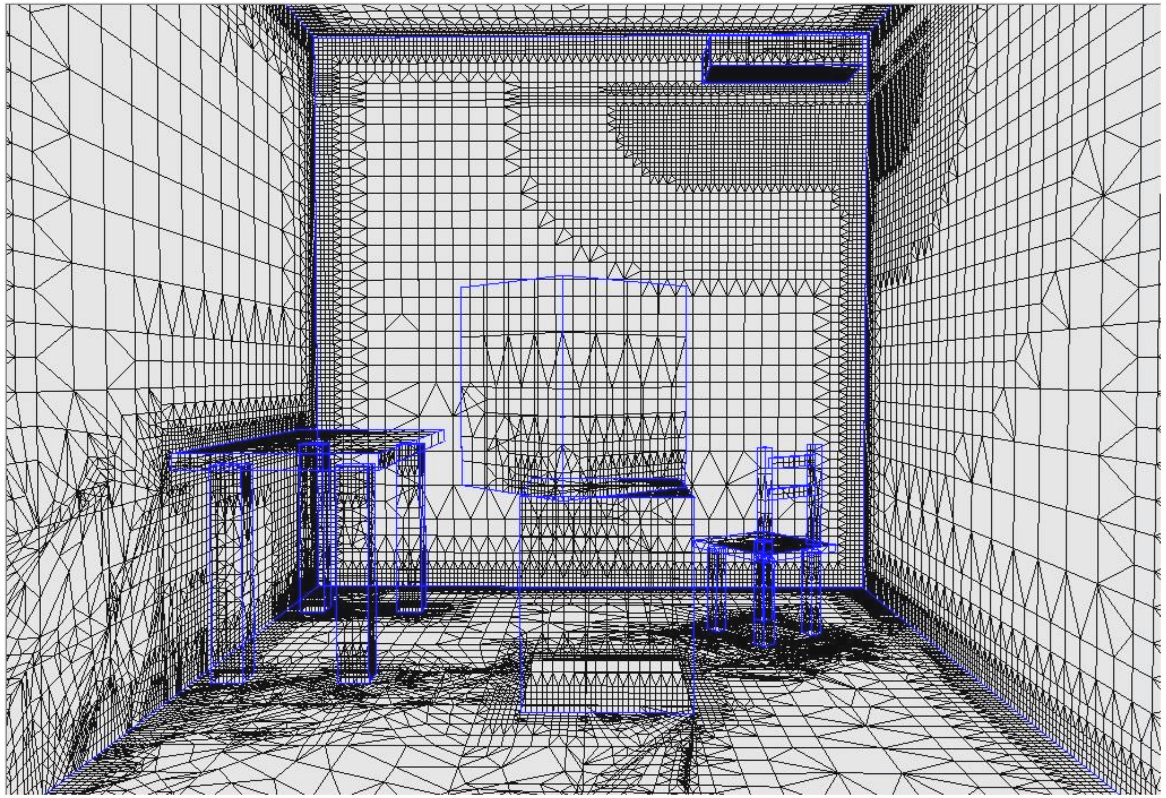




# Global Illumination Improvements/Variants

**hierarchical radiosity** → reduces number of form factors

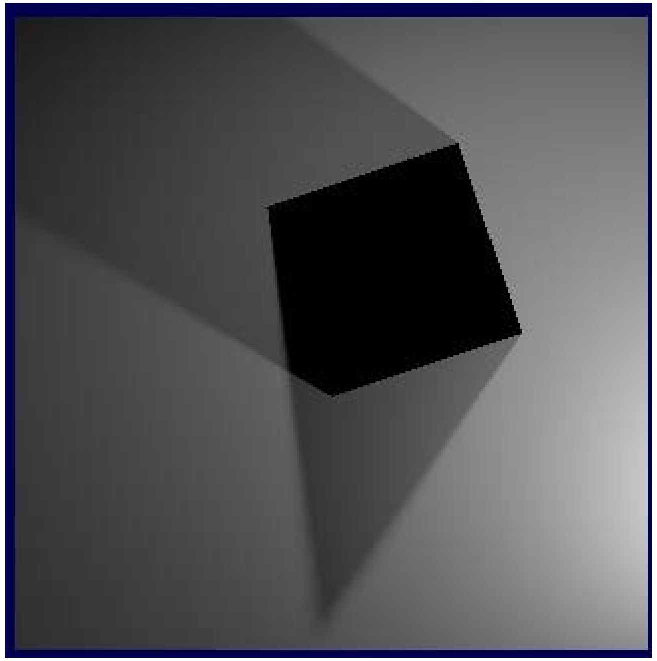


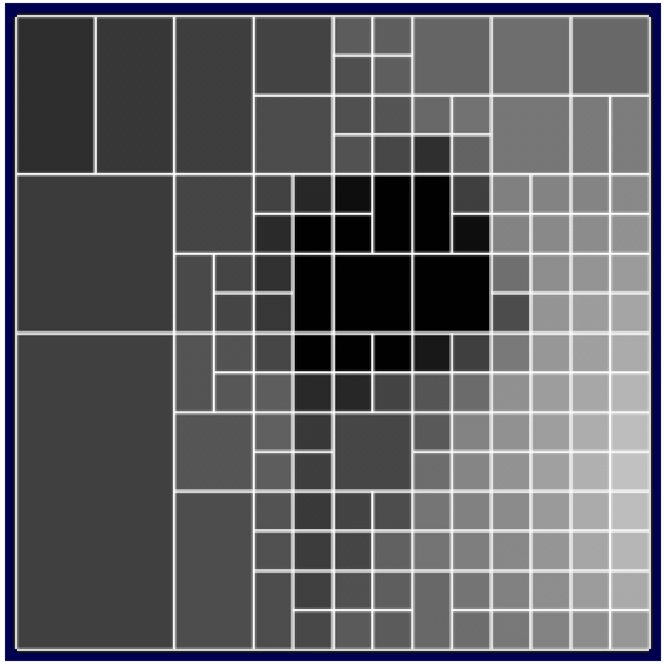
mesh density varies with importance

# Global Illumination Improvements/Variants

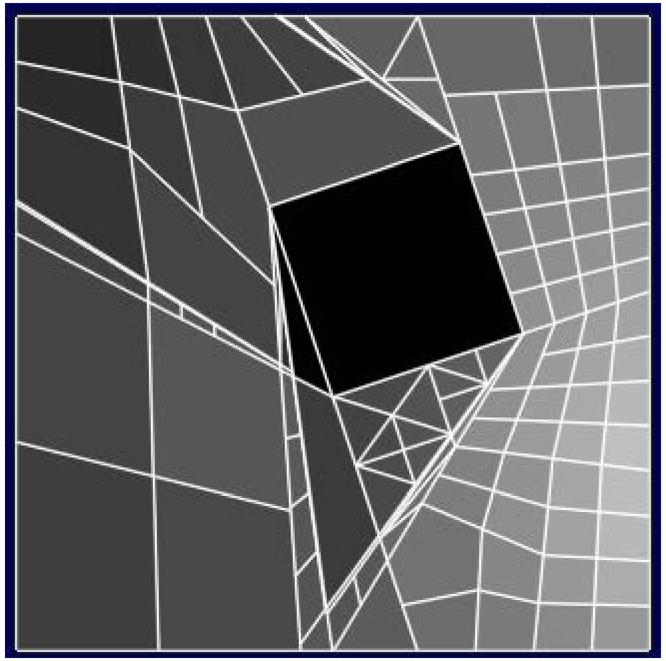
**discontinuity meshing** → improves shadow boundaries



sharp shadow boundaries …



… get blurred by arbitrary meshing, …



… stay correct with discontinuity meshing

© D. Lischinski

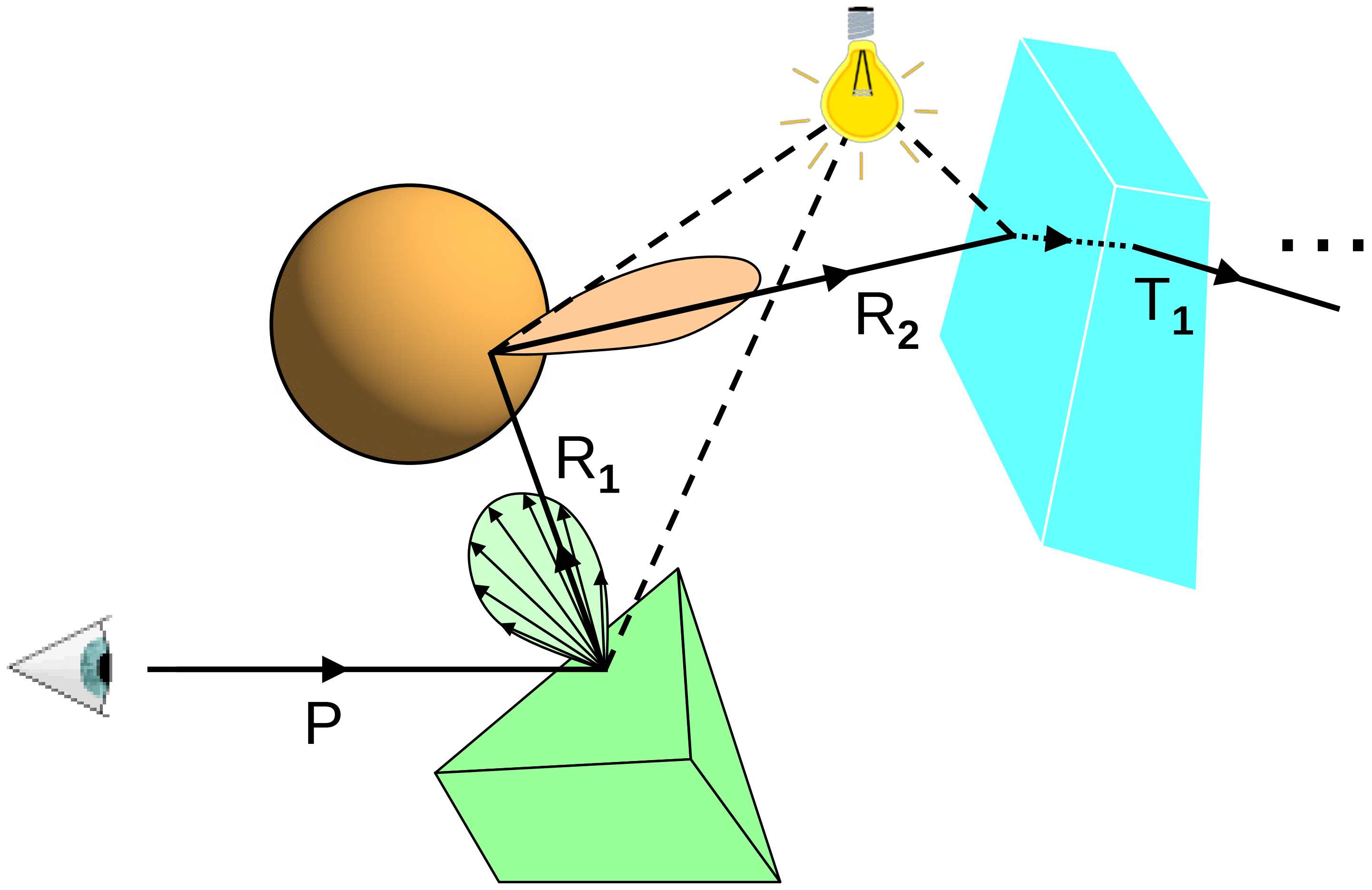
# Discontinuity Meshing Example

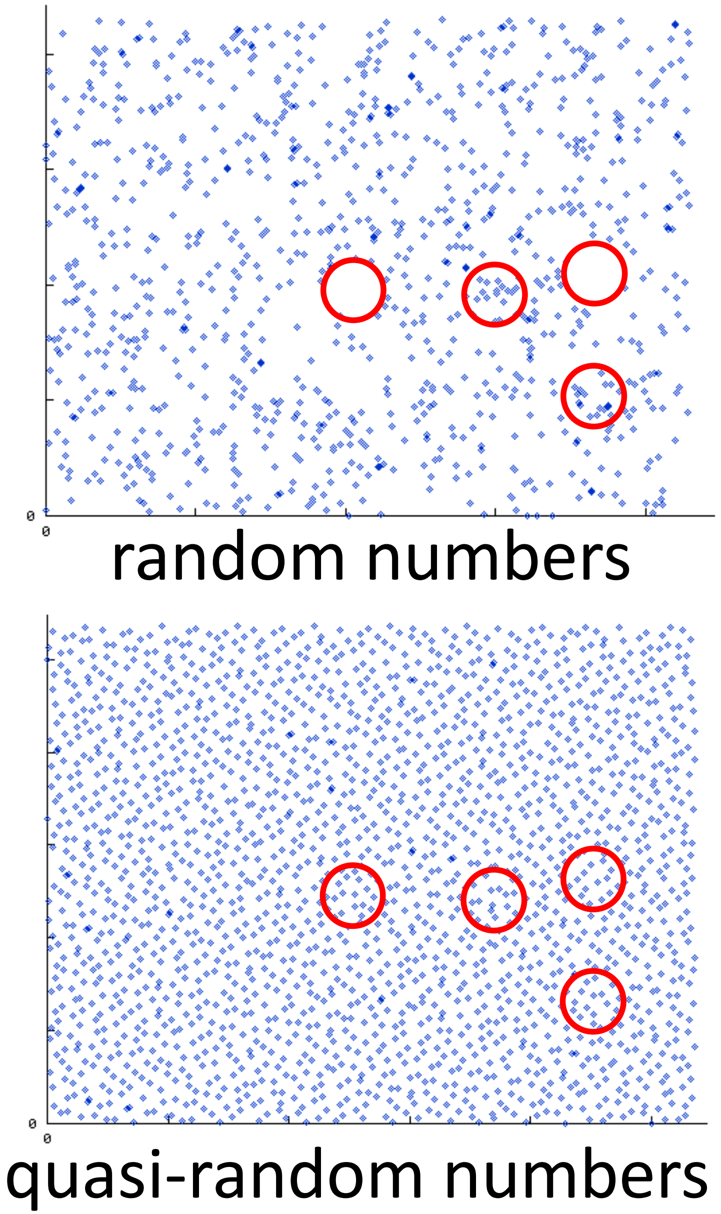

*discontinuity meshing allows for sharp shadows also*

# Radiosity Example Image

*stair tower of a building at Cornell University rendered with progressive refinement radiosity*

## path tracing

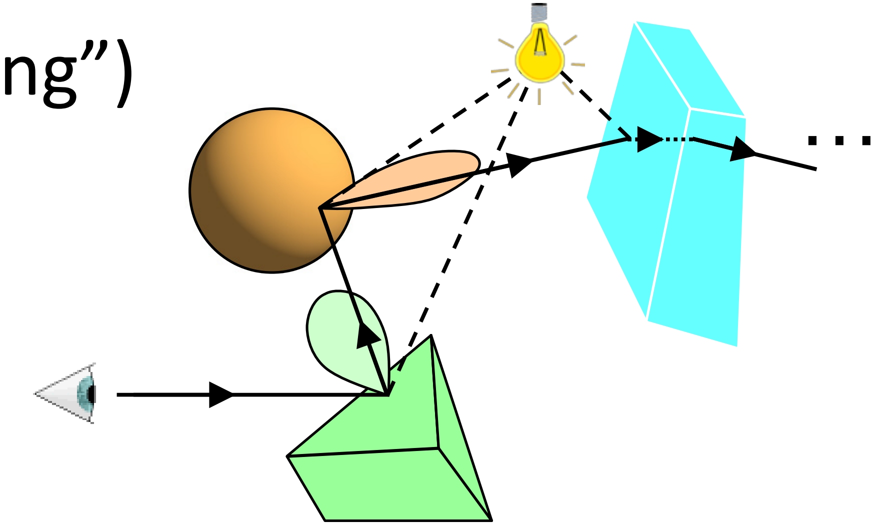


## path tracing

- also called Monte Carlo ray tracing
- randomly selects ray directions
- distribution functions ("importance sampling")
- uses Monte Carlo integration to solve

$$B = E + \rho \cdot \int_{hemi} dB$$

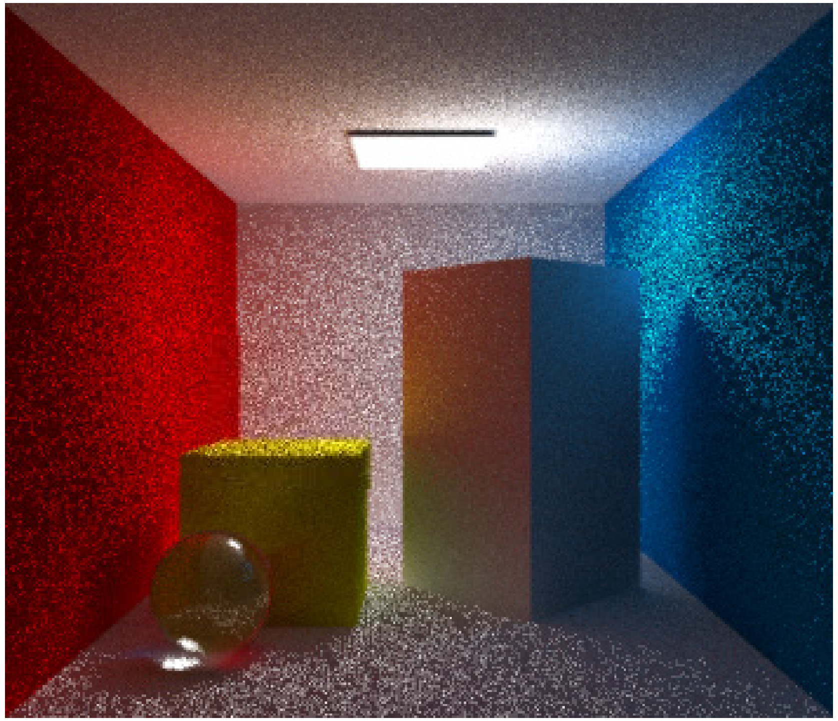



| | |
|---|---|
| B ... radiosity | hemi ... half space over point |
| E ... self emission | ρ ... reflection coefficient |

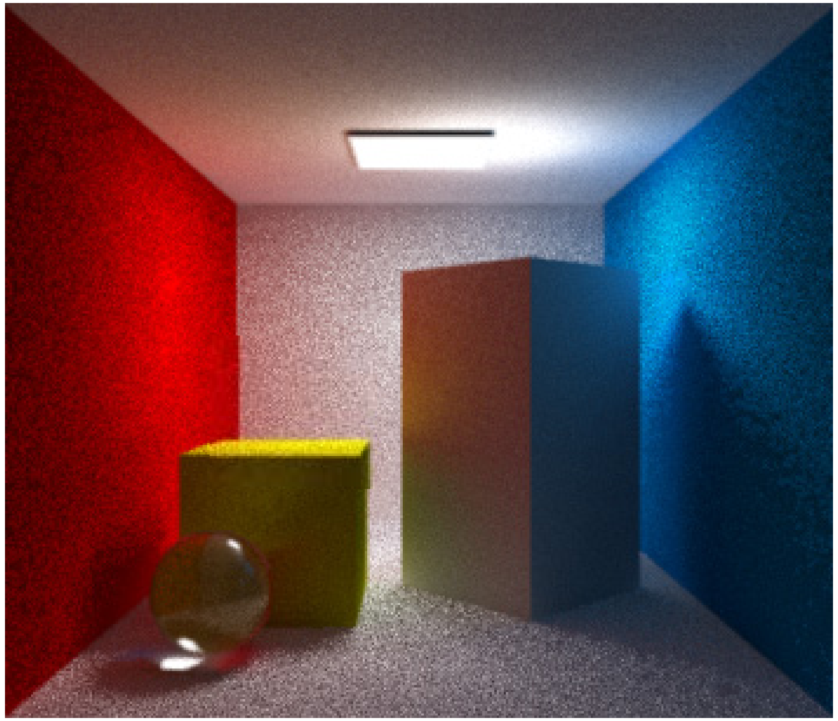
# Global Illumination Improvements / Variants

## photon mapping

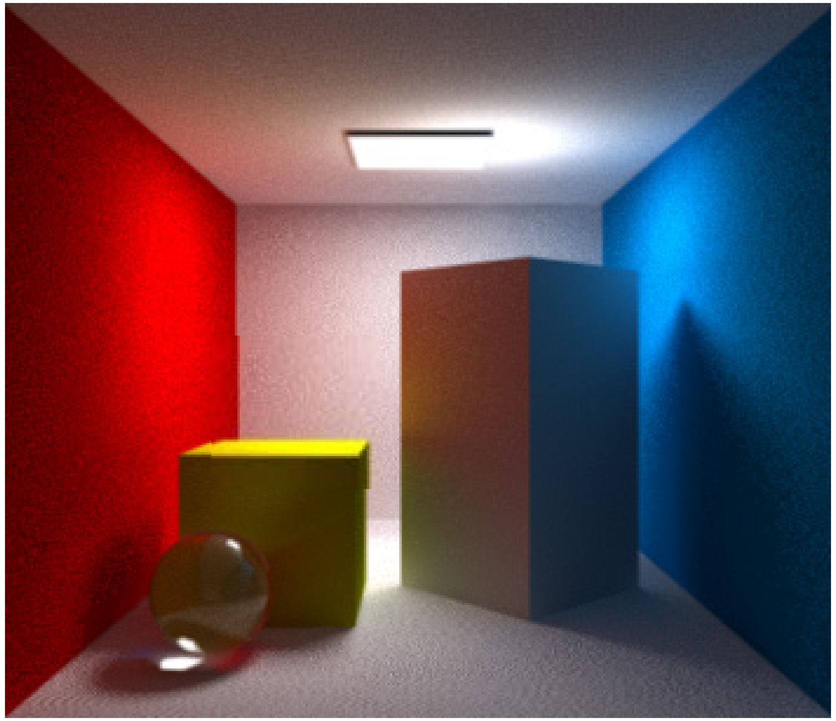
→ trace light rays from light source(s) and store illumination on objects



25 samples/pixel

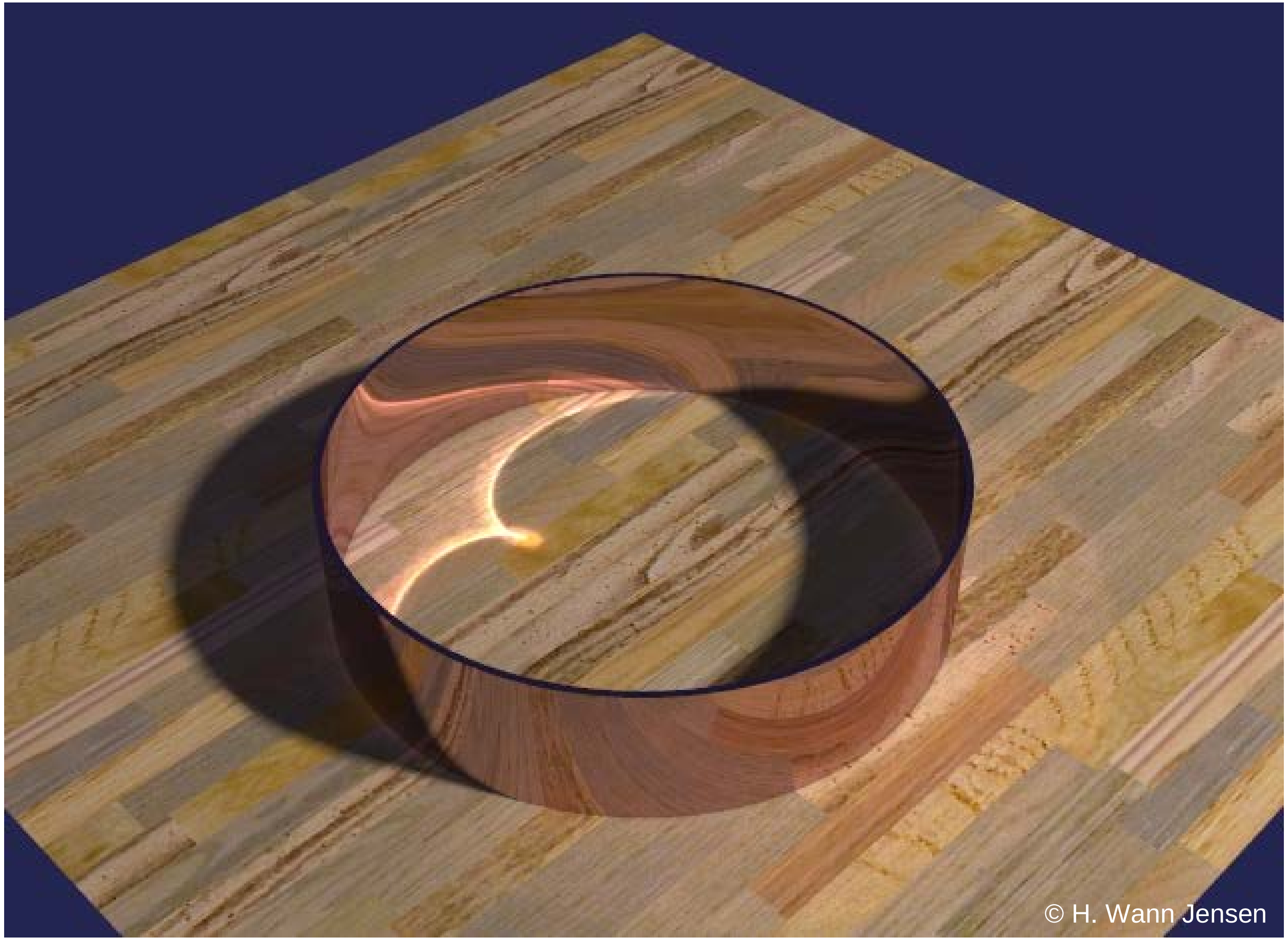


125 samples/pixel



625 samples/pixel

# Photon Mapping Example




*caustics from the inner surface of a ring created with photon mapping*

# Global Illumination Improvements / Variants

path tracing + photon mapping combined enable

- all surface properties
- area light sources / penumbras
- indirect lighting
- caustics
- antialiasing
- depth of field
- motion blur
- …

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