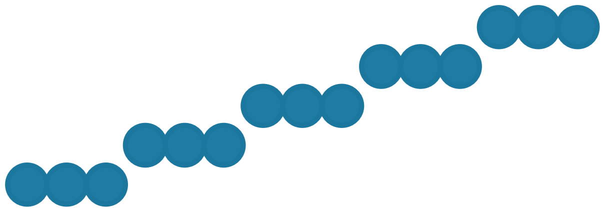
Einführung in Visual Computing

186.822



Rasterization

Werner Purgathofer

Rasterization in the Rendering Pipeline

object capture/creation

scene objects in object space

modeling

viewing

projection

vertex stage („vertex shader“)

transformed vertices in clip space

clipping + homogenization

scene in normalized device coordinates

viewport transformation

rasterization

shading

pixel stage („fragment shader“)

raster image in pixel coordinates

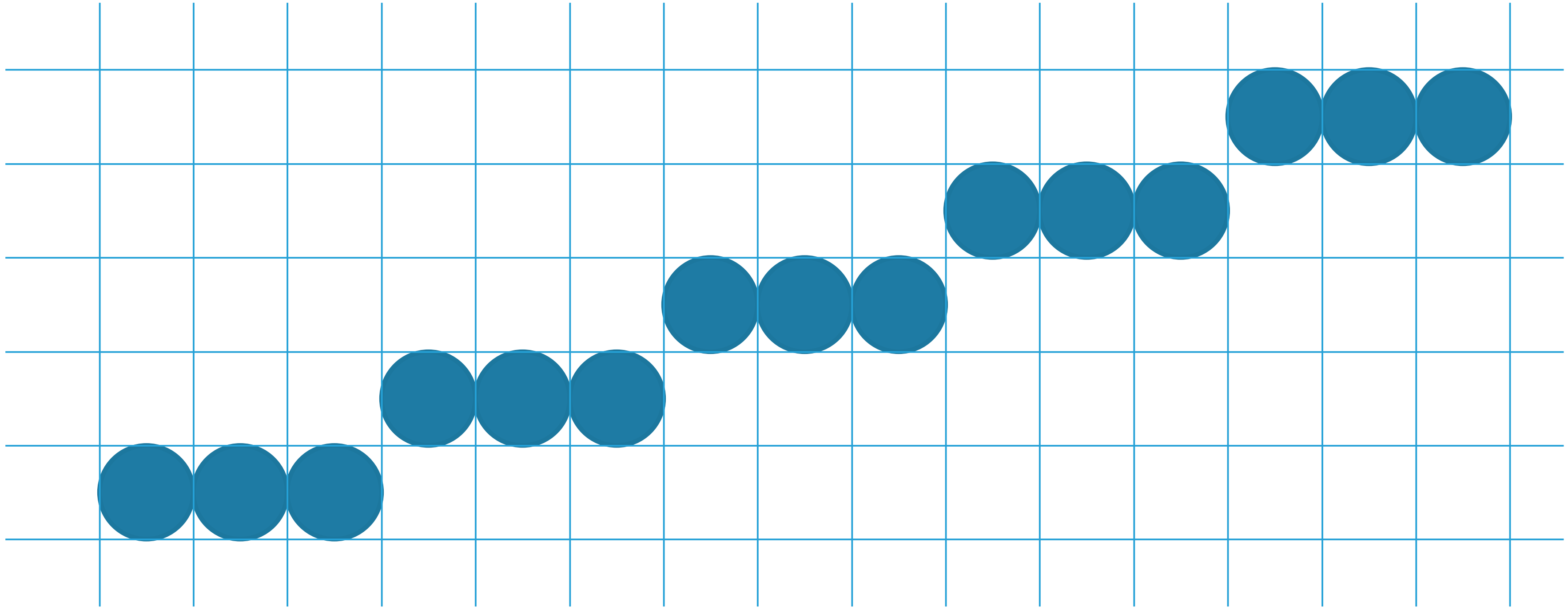
Important Graphics Output Primitives

- in 2D
  - points, lines
  - polygons, circles, ellipses & other curves (also filled)
  - pixel array operations
  - characters
- in 3D
  - triangles & other polygons
  - free form surfaces
- + commands for properties: color, texture, …

Points and Lines

- point plotting
  - instruction in display list (random scan)
  - entry in frame buffer (raster scan)
- line drawing
  - instruction in display list (random scan)
  - intermediate discrete pixel positions calculated (raster scan)
    - “jaggies”, aliasing

Lines: Staircase Effect

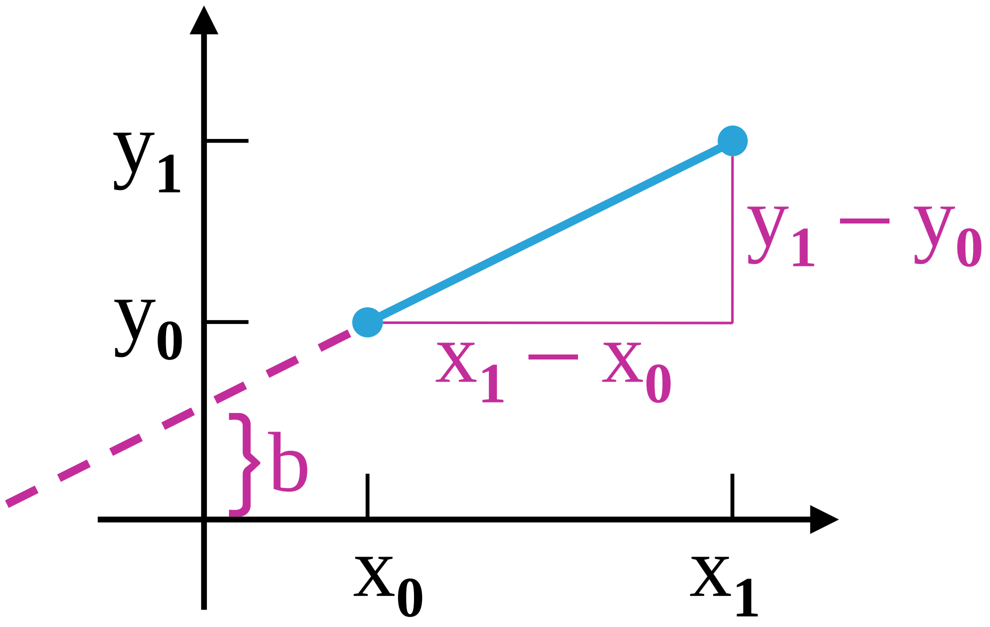


stairstep effect (jaggies) produced when a line is generated as a series of pixel positions line equation: $y = m \cdot x + b$

line path between two points:

$$m = \frac{y_1 - y_0}{x_1 - x_0}$$

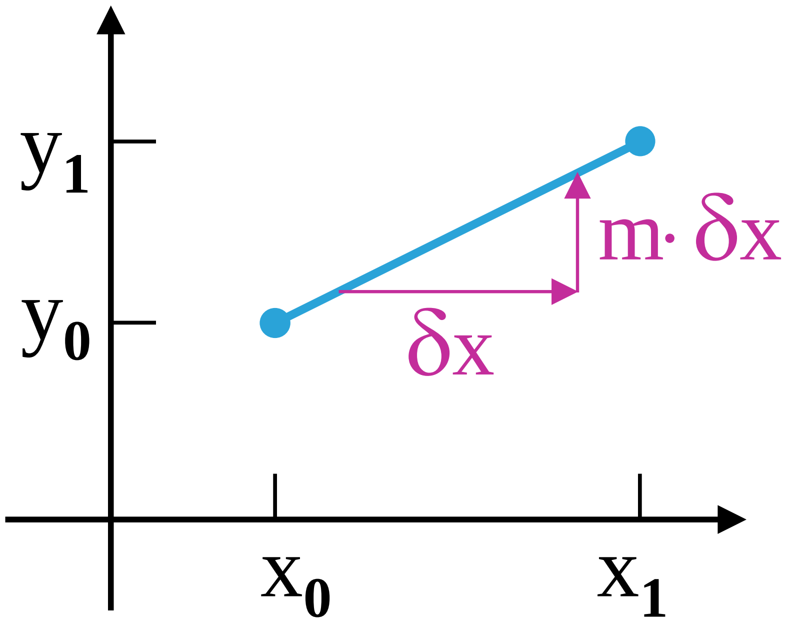
$$b = y_0 - m \cdot x_0$$



DDA Line-Drawing Algorithm

line equation: $y = m{\cdot}x + b$

$\delta y = m \cdot \delta x \quad \text{for} \quad |m|<1$

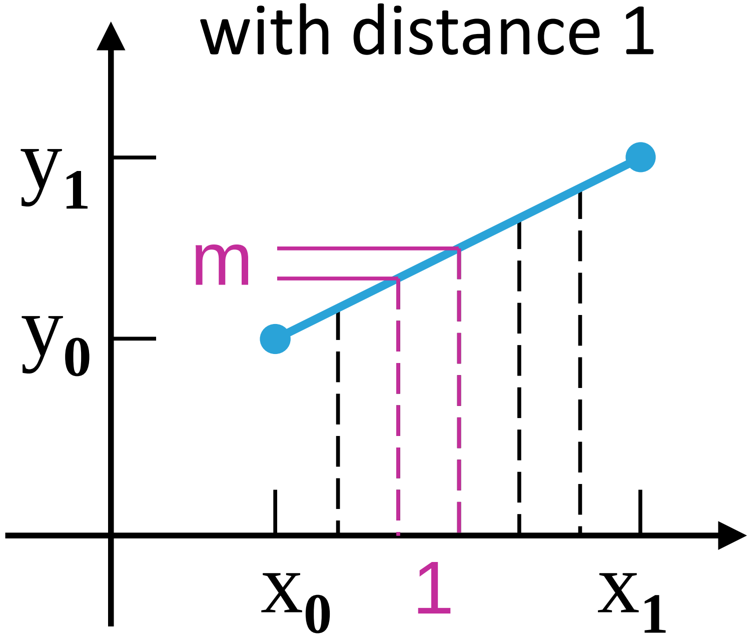


DDA Line-Drawing Algorithm

line equation: $y = m \cdot x + b$

$$\delta y = m \cdot \delta x \quad \text{for} \quad |m|<1$$

$$\left(\delta x = \frac{\delta y}{m} \quad \text{for} \quad |m|>1 \right)$$



- DDA (digital differential analyzer)

$$\boxed{\text{for } \delta x=1, \quad |m|<1 : \quad \mathbf{y_{k+1}} = \mathbf{y_k} + m}$$

DDA – Algorithm Principle

```
dx = x1 - x0; dy = y1 - y0;
m = dy / dx;

x = x0;  y = y0;
drawPixel (round(x), round(y));

for (k = 0; k < dx; k++)
  { x += 1; y += m;
    drawPixel (round(x), round(y)}
```

$$m = \frac{y_1 - y_0}{x_1 - x_0}$$

$$y_{k+1} = y_k + m$$

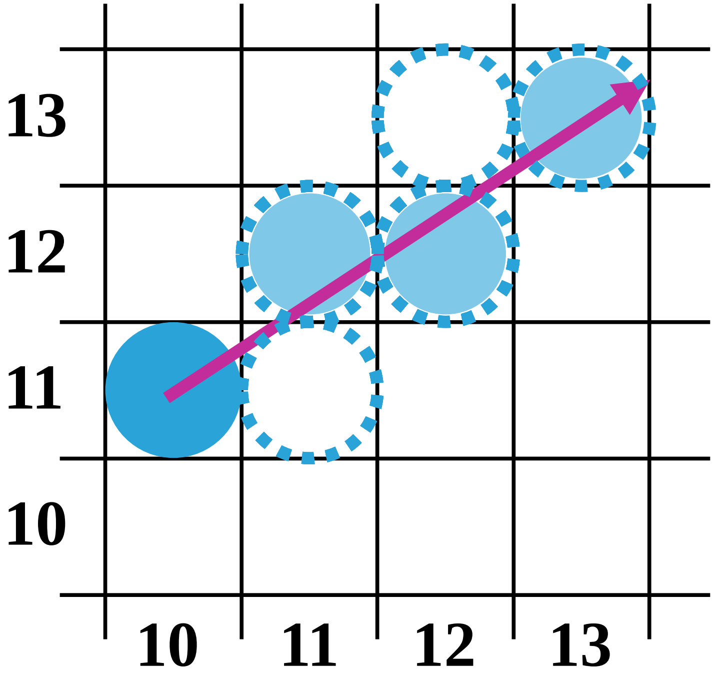
$\boxed{\text{for } \delta x = 1 , \ |m| < 1}$

extension to other cases simple

Bresenham's Line Algorithm

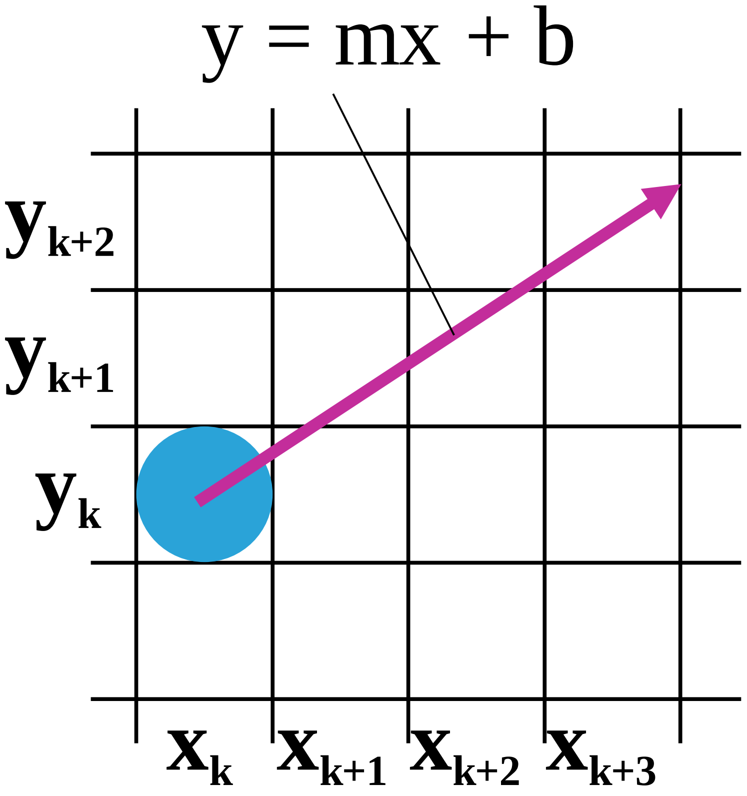
- faster than simple DDA
  - incremental integer calculations
  - adaptable to circles, other curves

$$y = m \cdot (x_{\mathbf{k}} + 1) + b$$



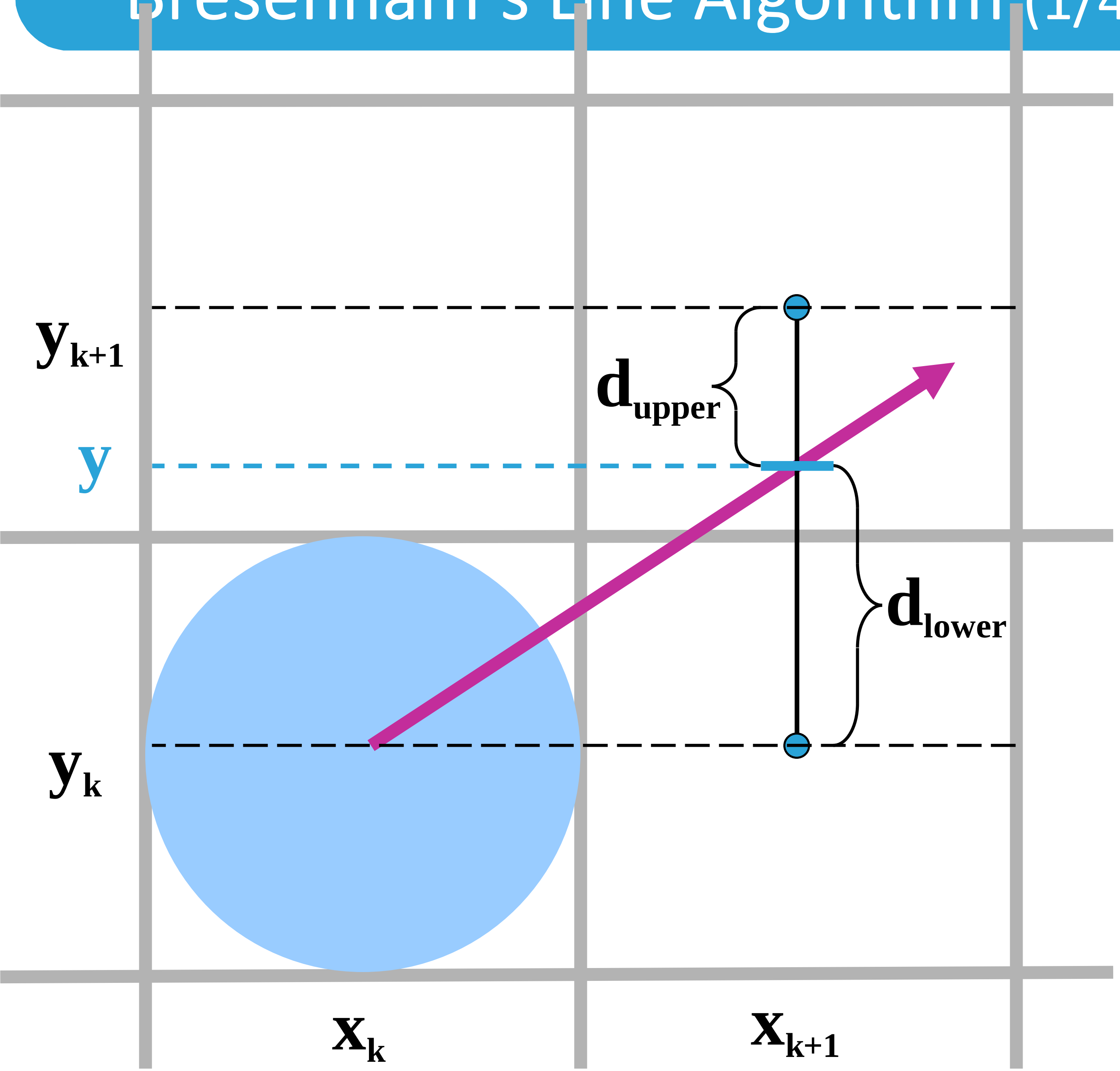
decision for every column which of the two candidate pixels is selected

Bresenham's Line Algorithm

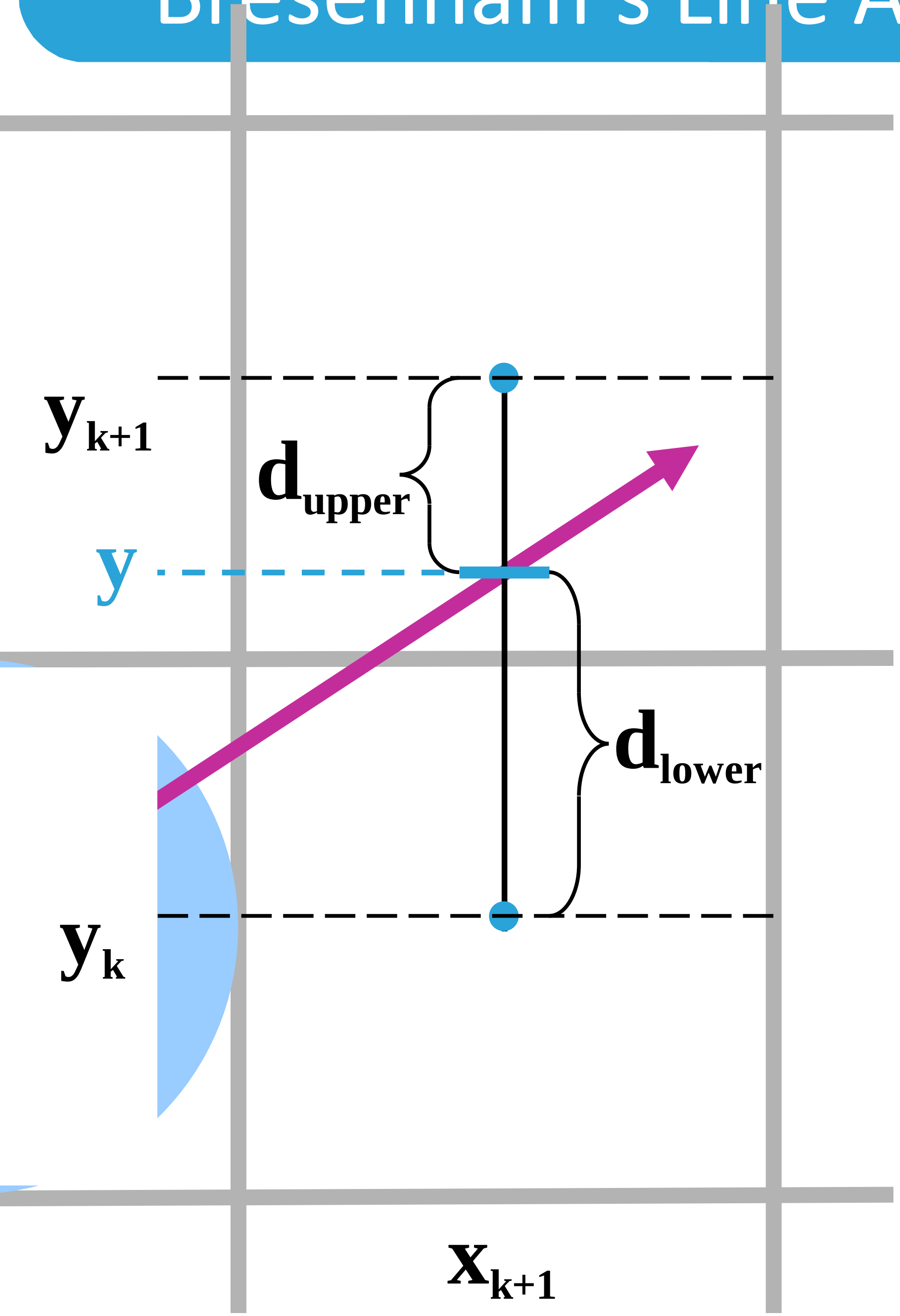


section of the screen grid showing a pixel in column x_k on scan line y_k that is to be plotted along the path of a line segment with slope $0 < m < 1$

Bresenham's Line Algorithm (1/4)



Bresenham's Line Algorithm (1/4)



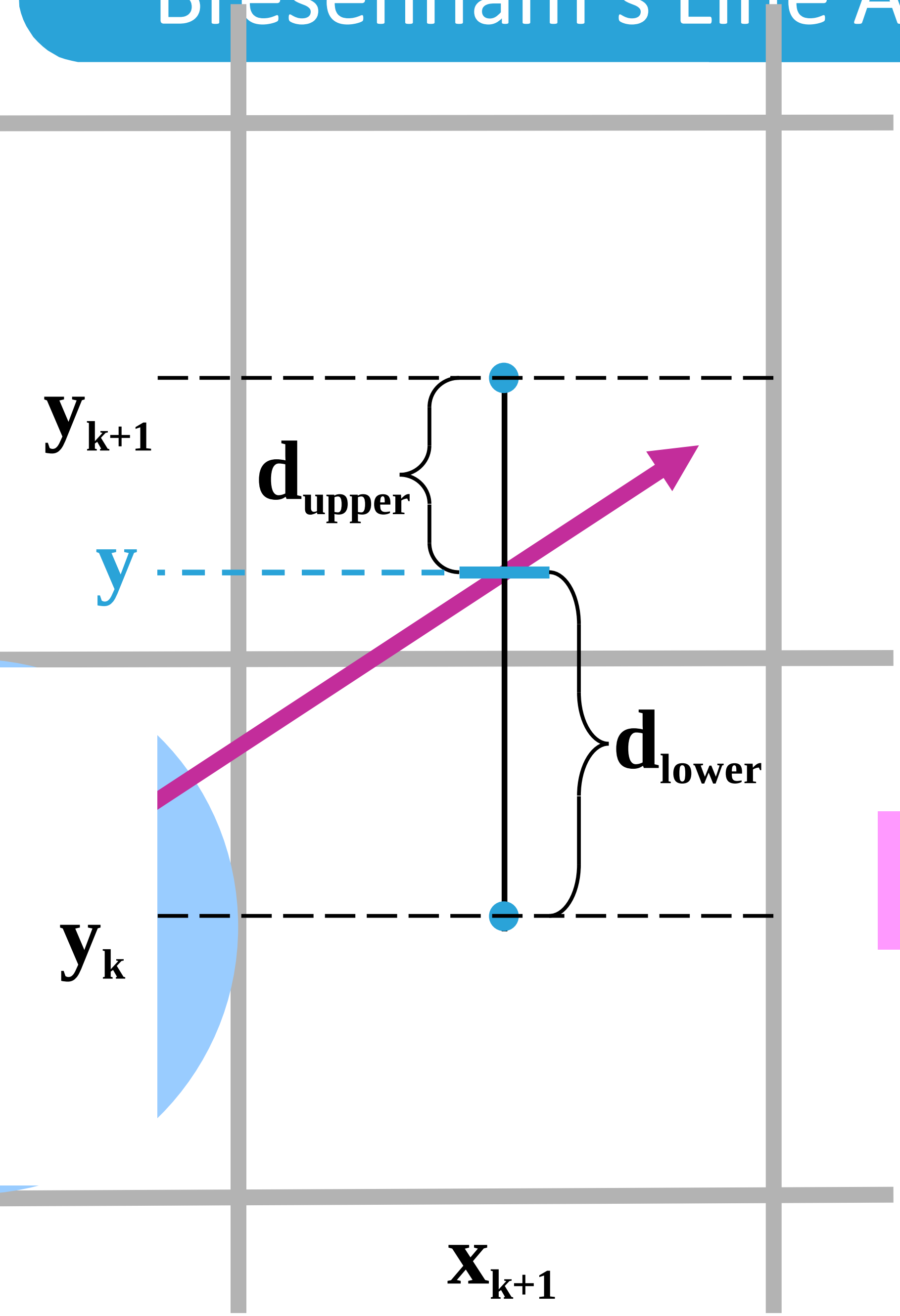
$$y = m \cdot x_{k+1} + b \quad = m \cdot (x_k + 1) + b$$

$$d_{lower} = y - y_k =$$
$$= m \cdot (x_k + 1) + b - y_k$$

$$d_{upper} = (y_k + 1) - y =$$
$$= y_k + 1 - m \cdot (x_k + 1) - b$$

$$d_{lower} - d_{upper} =$$
$$= 2m \cdot (x_k + 1) - 2y_k + 2b - 1$$

Bresenham's Line Algorithm (2/4)



$$d_{lower} - d_{upper} =$$
$$= 2m\cdot(x_k + 1) - 2y_k + 2b - 1$$
$$m = \Delta y/\Delta x$$
$$(\Delta x = x_1 - x_0,\ \Delta y = y_1 - y_0)$$

decision parameter:

$$p_k = \Delta x\cdot(d_{lower} - d_{upper}) = 2\Delta y\cdot x_k - 2\Delta x\cdot y_k + c$$

→ same sign as $(d_{lower} - d_{upper})$

Bresenham's Line Algorithm (3/4)

current decision value:

$$p_k = \Delta x \cdot (d_{\mathbf{lower}} - d_{\mathbf{upper}}) = 2\Delta y \cdot x_{\mathbf{k}} - 2\Delta x \cdot y_{\mathbf{k}} + c$$

next decision value:

$$p_{\mathbf{k+1}} = 2\Delta y \cdot x_{\mathbf{k+1}} - 2\Delta x \cdot y_{\mathbf{k+1}} + c \mathbf{+ 0}$$

$$\mathbf{+ p_k - 2\Delta y \cdot x_k + 2\Delta x \cdot y_k - c =}$$

$$= p_{\mathbf{k}} + 2\Delta y - 2\Delta x \cdot (y_{\mathbf{k+1}} - y_{\mathbf{k}})$$

starting decision value:

$$p_{\mathbf{0}} = 2\Delta y - \Delta x$$

Bresenham's Line Algorithm (4/4)

1. store left line endpoint in (x_0,y_0)
2. draw pixel (x_0,y_0)
3. calculate constants Δx, Δy, $2\Delta y$, $2\Delta y - 2\Delta x$,

   and obtain $p_0 = 2\Delta y - \Delta x$
4. at each x_k along the line, perform test:

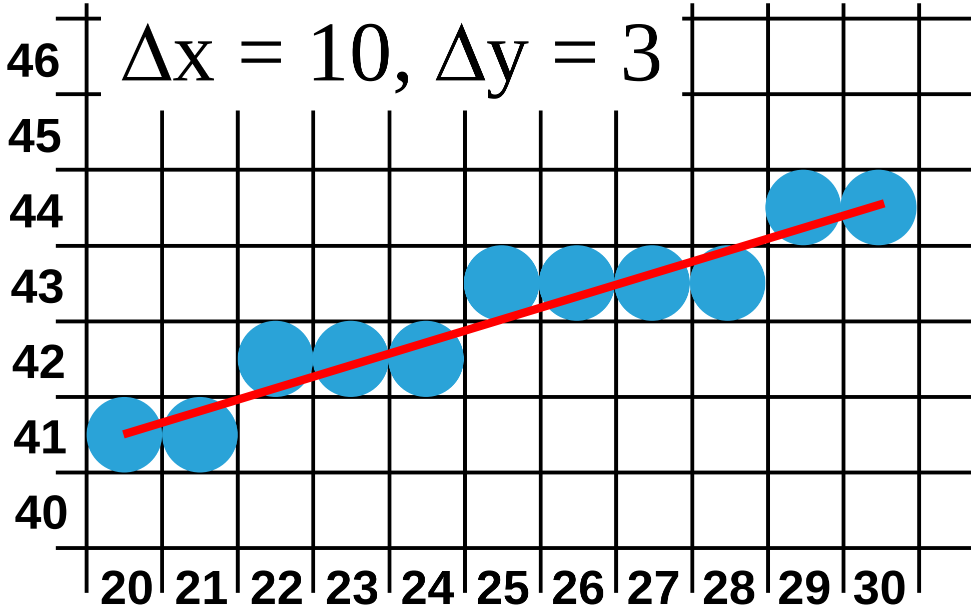
   if $p_k<0$

   then draw pixel (x_k+1,y_k); $p_{k+1} = p_k + 2\Delta y$

   else draw pixel (x_k+1,y_k+1); $p_{k+1} = p_k + 2\Delta y - 2\Delta x$
5. perform "step 4" $(\Delta x - 1)$ times.

Bresenham: Example

k	p_k	(x_{k+1},y_{k+1})
		(20,41)
0	-4	(21,41)
1	2	(22,42)
2	-12	(23,42)
3	-6	(24,42)
4	0	(25,43)
5	-14	(26,43)
6	-8	(27,43)
7	-2	(28,43)
8	4	(29,44)
9	-10	(30,44)



$p_0 = 2\Delta y - \Delta x$

if $p_k < 0$

then draw pixel (x_k+1, y_k); $p_{k+1} = p_k + 2\Delta y$

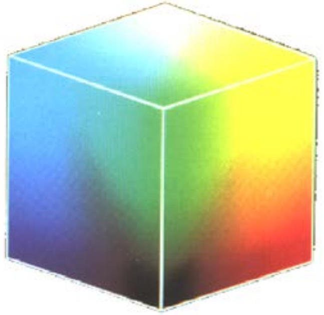
else draw pixel (x_k+1, y_k+1); $p_{k+1} = p_k + 2\Delta y - 2\Delta x$

Attributes of Graphics Output Primitives

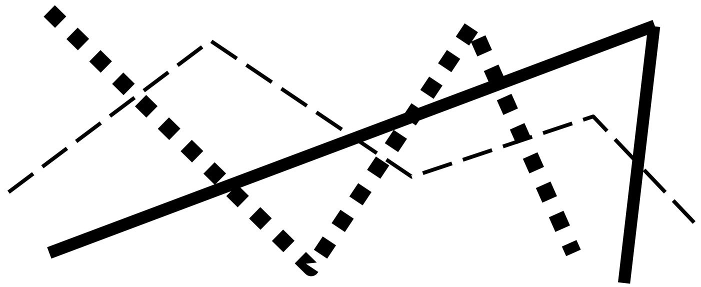
- in 2D
  - points, lines
  - characters
- in 2D and 3D
  - triangles
  - other polygons
  - ((filled) ellipses and other curves)

Points and Line Attributes

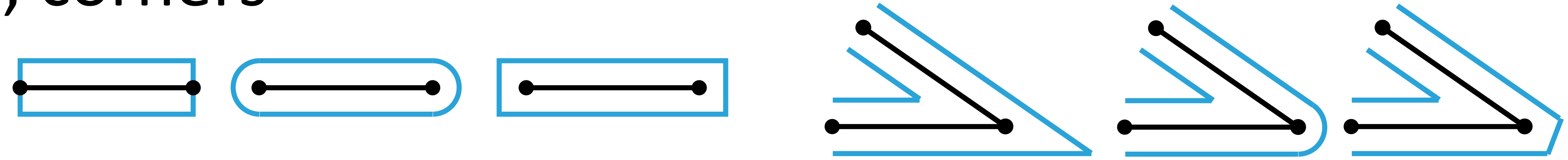
- color



- type: solid, dashed, dotted,…



- width, line caps, corners



- pen and brush options

- …

Character Attributes

- text attributes
  - font (e.g. Courier, Arial, Times, Roman, ...)
    - proportionally sized vs. fixed space fonts
  - styles (regular, **bold**, *italic*, underline,...)
  - size (32 point, 1 point = 1/72 inch)
- string attributes
  - orientation
  - alignment (left, center, right, justify)

horizontal

slanted

vertical

Displayed primitives generated by the raster algorithms discussed in Chapter 3 have a jagged, or stairstep, appearance.

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Character Primitives

- font (typeface)
  - design style for (family of) characters
- `Courier`, Times, Arial, …
  - *serif* (better readable),
  - *sans serif* (better legible)
- definition model
  - *bitmap font* (simple to define and display), needs more space (font cache)
  - *outline font* (more costly, less space, geometric transformations)

Example: Newspaper

Panorama

Nach 28 Jahren jetzt ein Fü

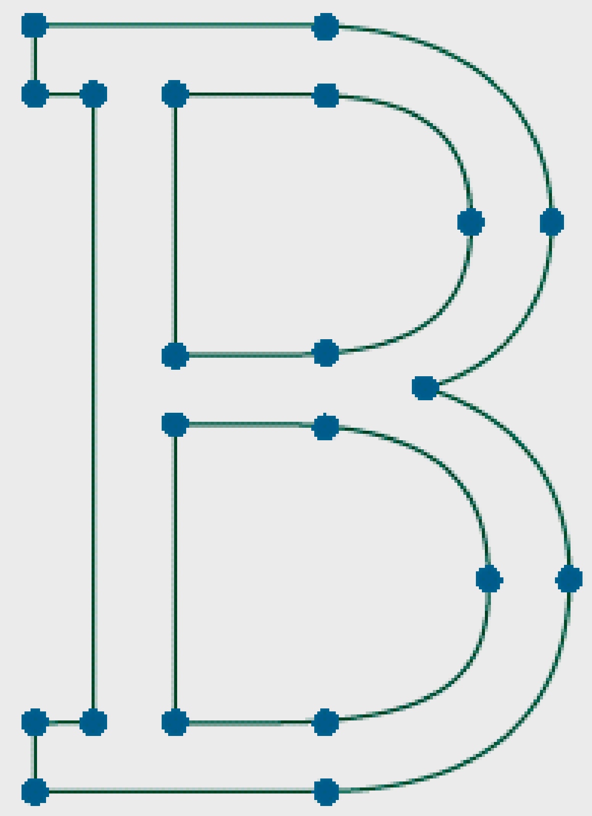
Neuer Direktor
Werkschulheim

Hans Bigenzahl ist in den verdienten Ruhestand getreten, nachdem er 28 Jahre die Geschicke des Werkschulheims Felbertal in Ebenau geleitet hatte. Sein Nachfolger ist Winfried Kogelnik, seit

ger über
ebensovi
Zukunft
fried Ko
Jahre in
tätig und
schulhei

Character Generation Examples

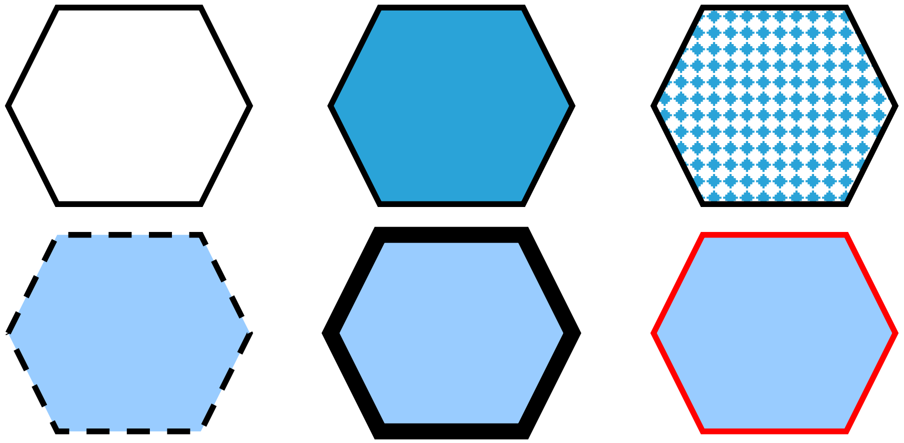
1	1	1	1	1	1	0	0
0	1	1	0	0	1	1	0
0	1	1	0	0	1	1	0
0	1	1	1	1	1	0	0
0	1	1	0	0	1	1	0
0	1	1	0	0	1	1	0
1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0

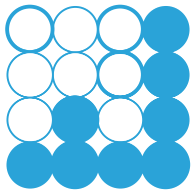


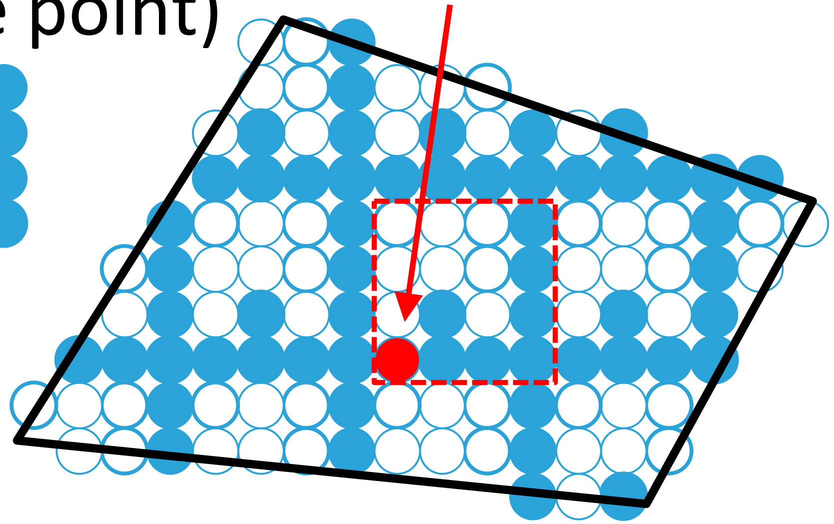
the letter **B** represented with an 8x8 bilevel bitmap pattern and with an outline shape defined with straight line and curve segments

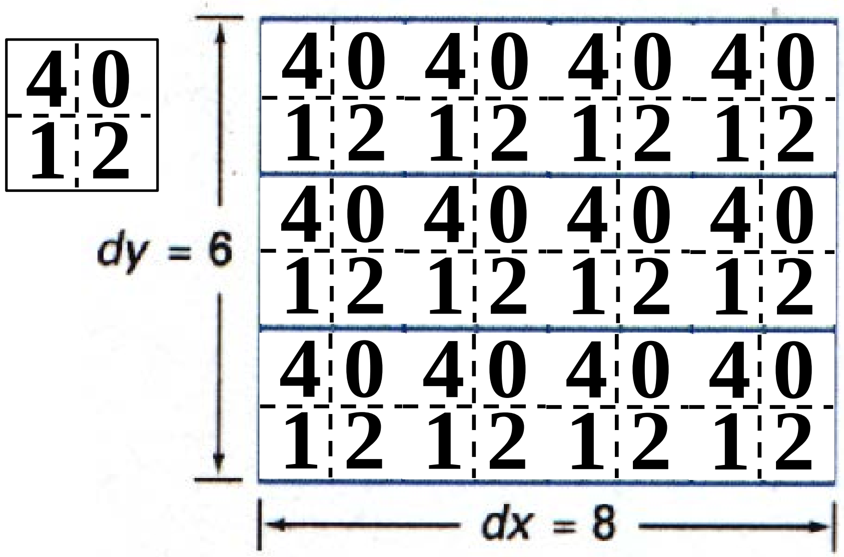
Area-Fill Attributes (1)

- fill styles
  - hollow, solid fill, pattern fill
- fill options
  - edge type, width, color
- pattern specification
  - through pattern tables
  - tiling (reference point)



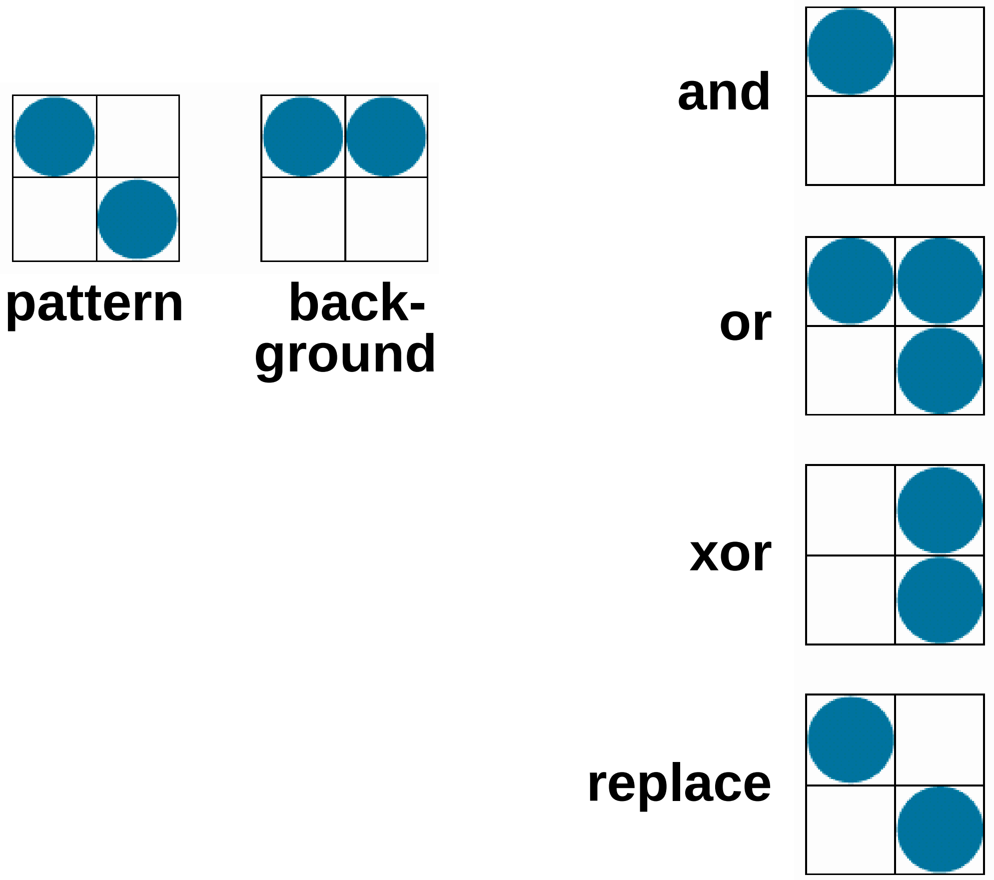






Area-Fill Attributes (2)

- combination of fill pattern with background colors
- soft fill
  - combination of colors
  - antialiasing at object borders
  - semitransparent brush simulation
  - example: linear soft-fill



F... foreground color
B...background color

$$P = t \cdot F + (1 - t) \cdot B$$

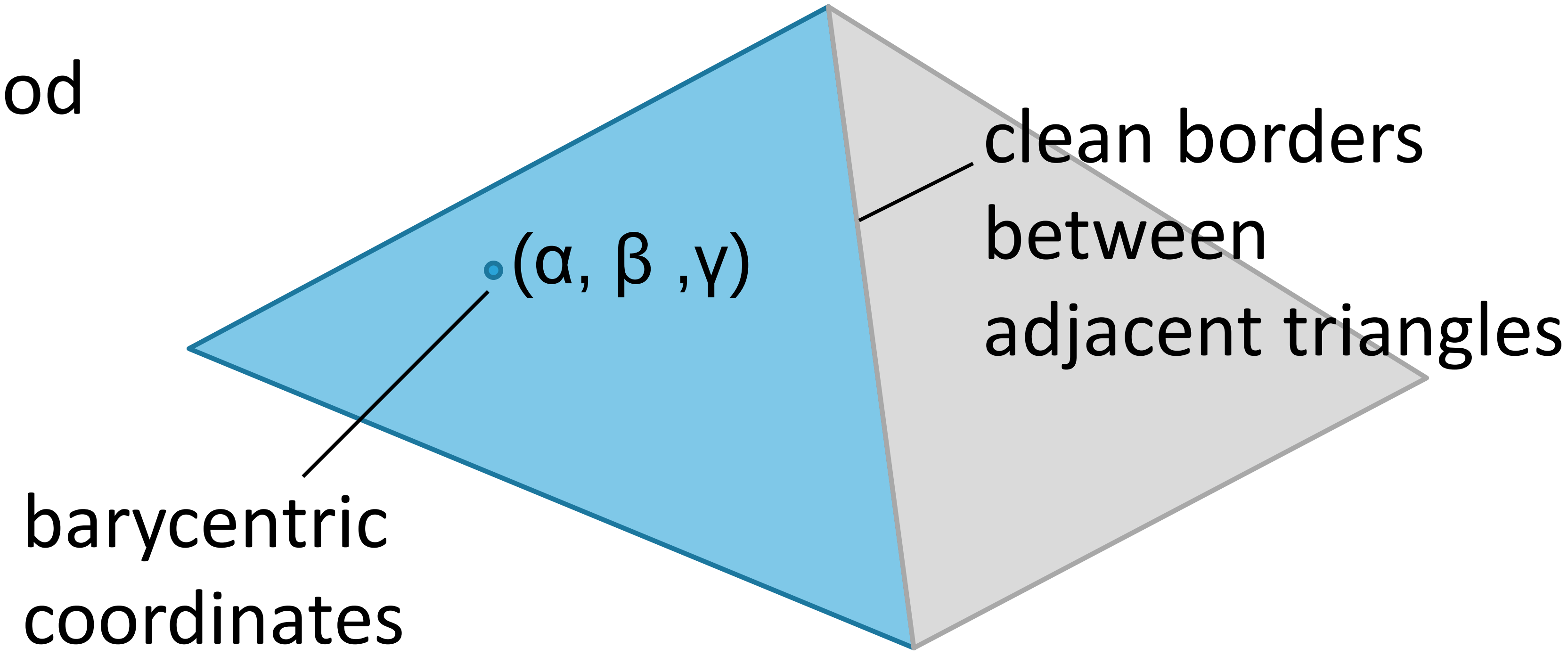
Triangle and Polygon Attributes

- color
- material
- transparency
- texture
- surface details
- reflexion properties, …

→ defined illumination produces effects

General Polygon Fill Algorithms

- ***triangle rasterization***
- other polygons: what is inside?
- scan-line fill method
- flood fill method



General Polygon Fill Algorithms

- triangle rasterization
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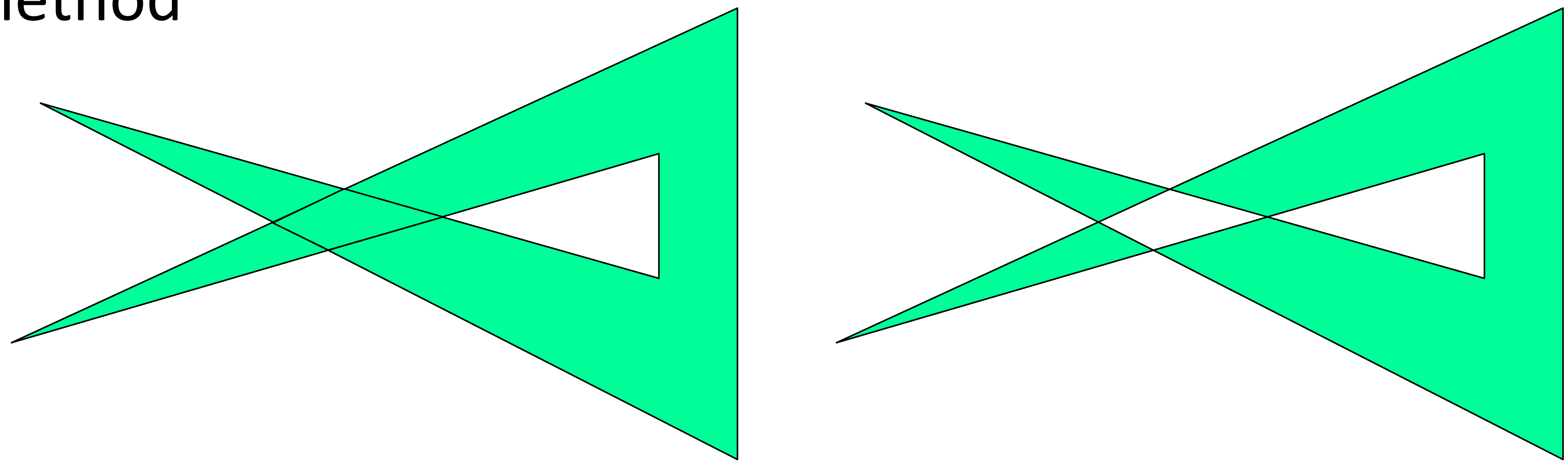
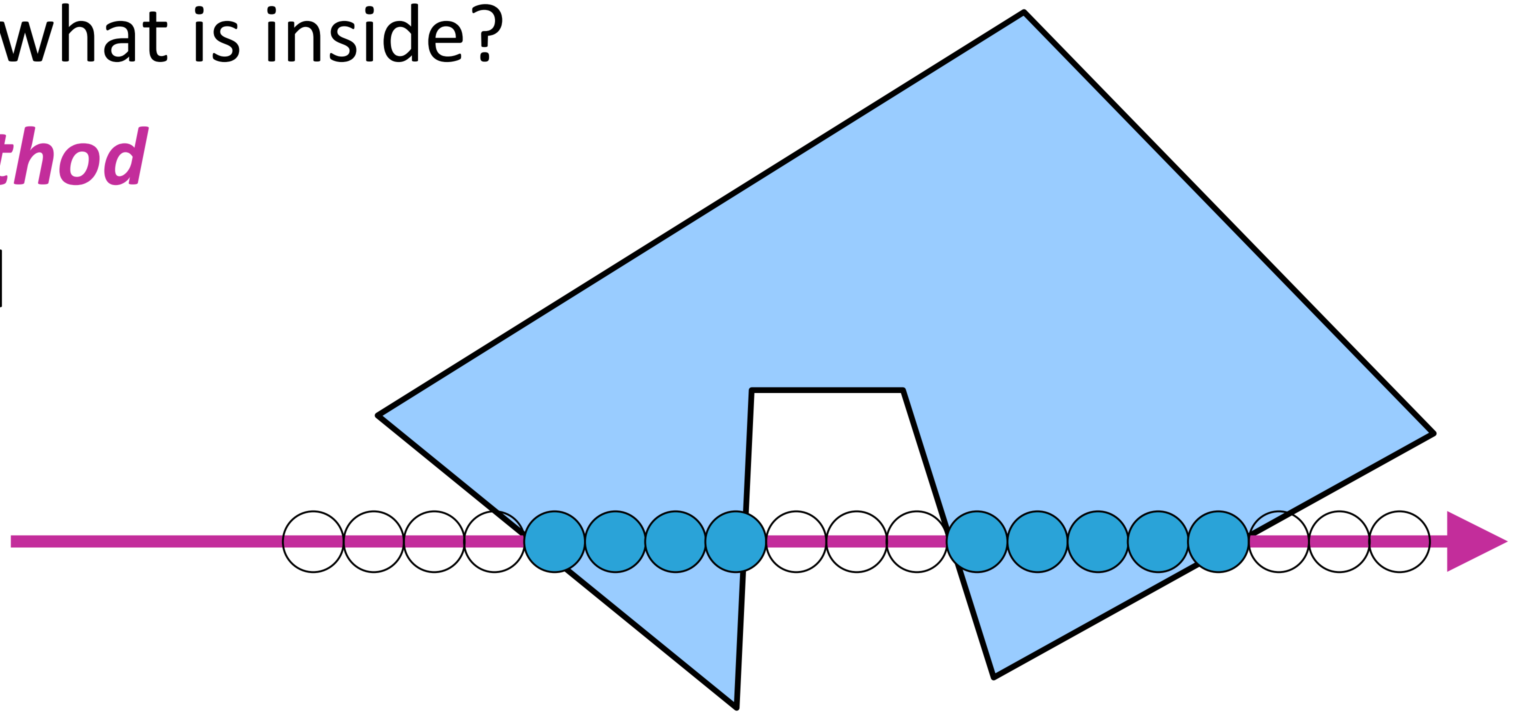


“interior”, “exterior” for self-intersecting polygons?

General Polygon Fill Algorithms

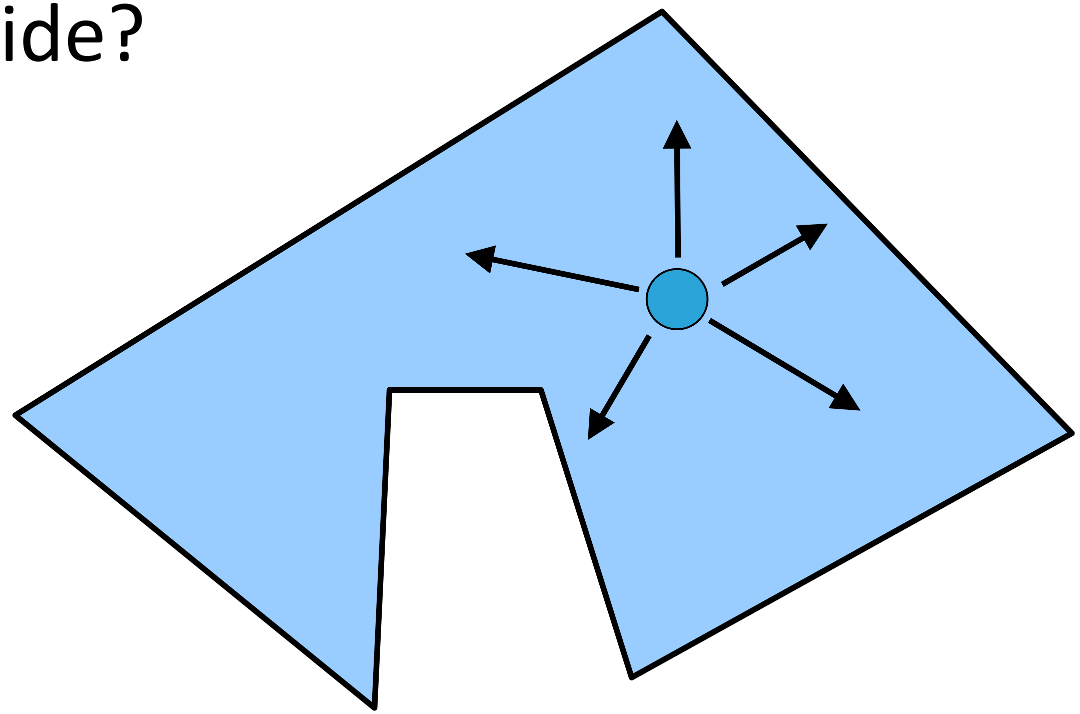
- triangle rasterization
- other polygons: what is inside?
- *scan-line fill method*
- flood fill method



interior pixels along a scan line passing through a polygon area

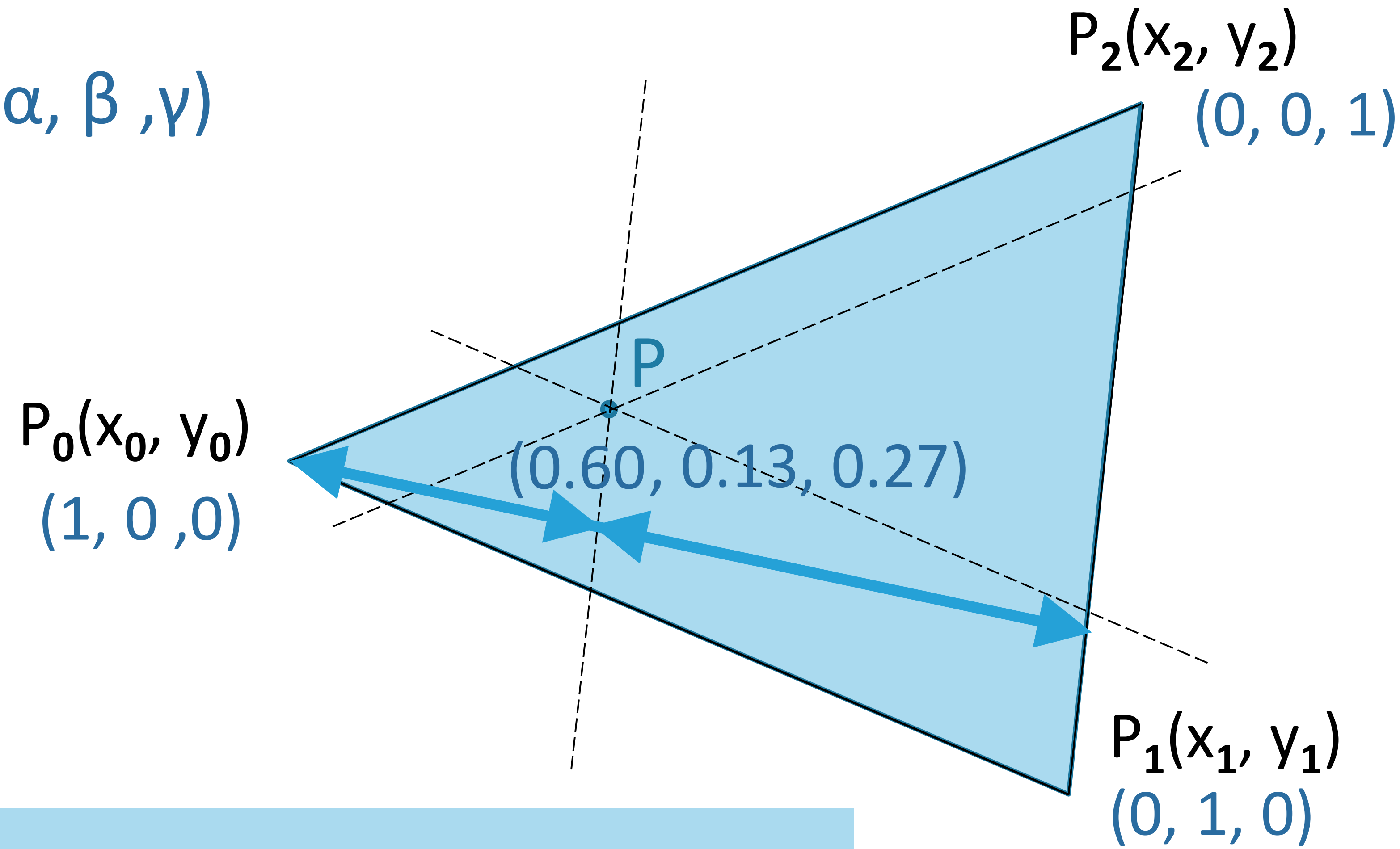
General Polygon Fill Algorithms

- triangle rasterization
- other polygons: what is inside?
- scan-line fill method
- ***flood fill method***



starting from a seed point: fill until you reach a border

- notation: (α, β, γ)

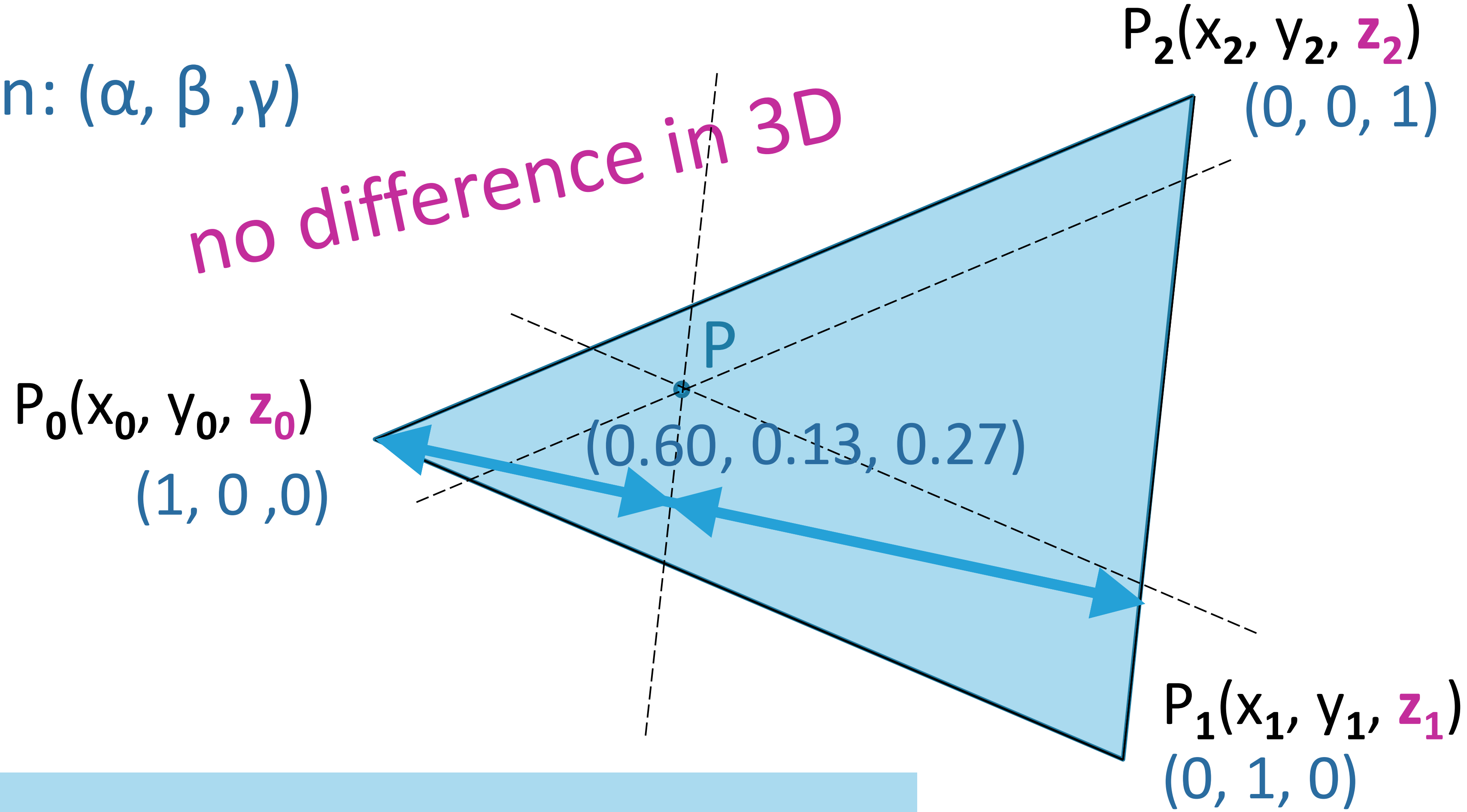


$P = \alpha P_0 + \beta P_1 + \gamma P_2$

triangle = $\{P \mid \alpha+\beta+\gamma=1,\ 0<\alpha<1,\ 0<\beta<1,\ 0<\gamma<1\}$

Barycentric Coordinates in 3D

- notation: (α, β, γ)



$P = \alpha P_0 + \beta P_1 + \gamma P_2$

triangle = $\{P \mid \alpha+\beta+\gamma=1,\ 0<\alpha<1,\ 0<\beta<1,\ 0<\gamma<1\}$

Triangle Rasterization Algorithm

```
for all x
   for all y               /* use a bounding box!*/
      {compute (α, β ,γ) for (x,y) ;
      if (0<α<1) and (0<β<1) and (0<γ<1)
      { c = αc0 + βc1 + γc2 ;
         draw pixel (x,y)  with color c
      }}
```

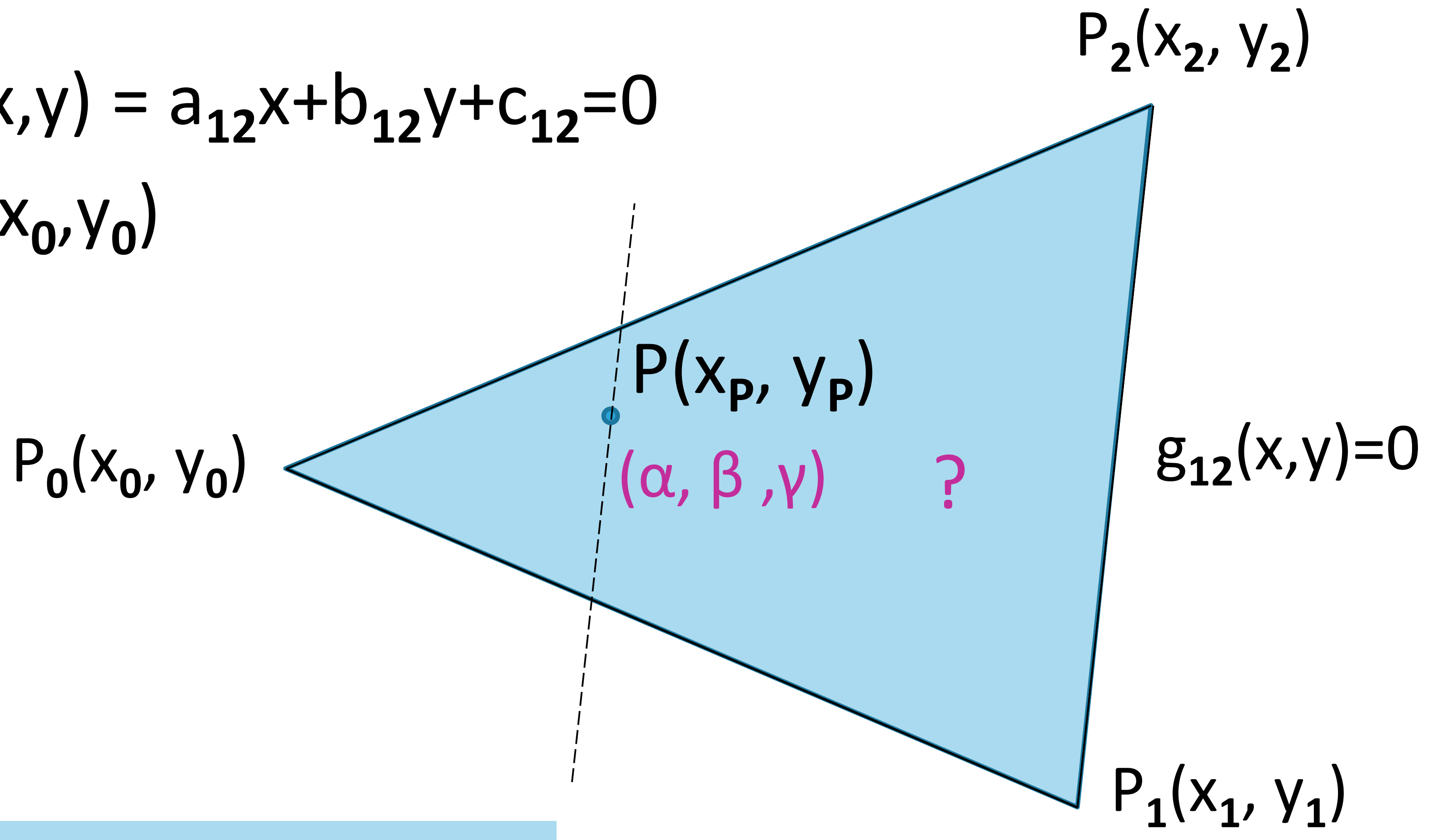
interpolates corner values (vertices) linearly inside the triangle (and along edges)

Computing (α, β ,γ) for P(x,y)

line through P_1, P_2: $g_{12}(x,y) = a_{12}x+b_{12}y+c_{12}=0$

then $\alpha = g_{12}(x_P,y_P) / g_{12}(x_0,y_0)$

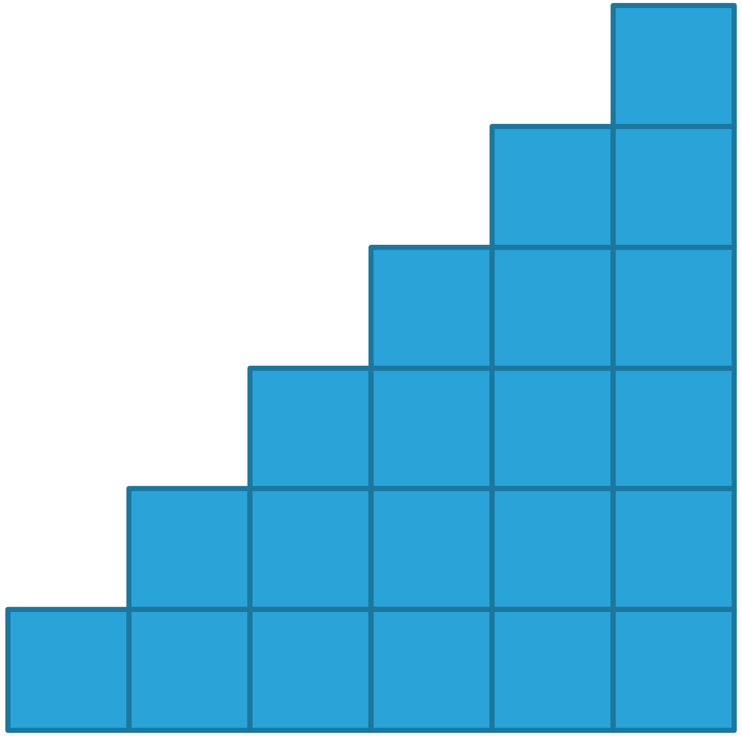
β,γ analogous



$P = \alpha P_0 + \beta P_1 + \gamma P_2$

triangle = $\{P \mid \alpha+\beta+\gamma=1,\ 0<\alpha<1,\ 0<\beta<1,\ 0<\gamma<1\}$

Barycentric Coordinates Example



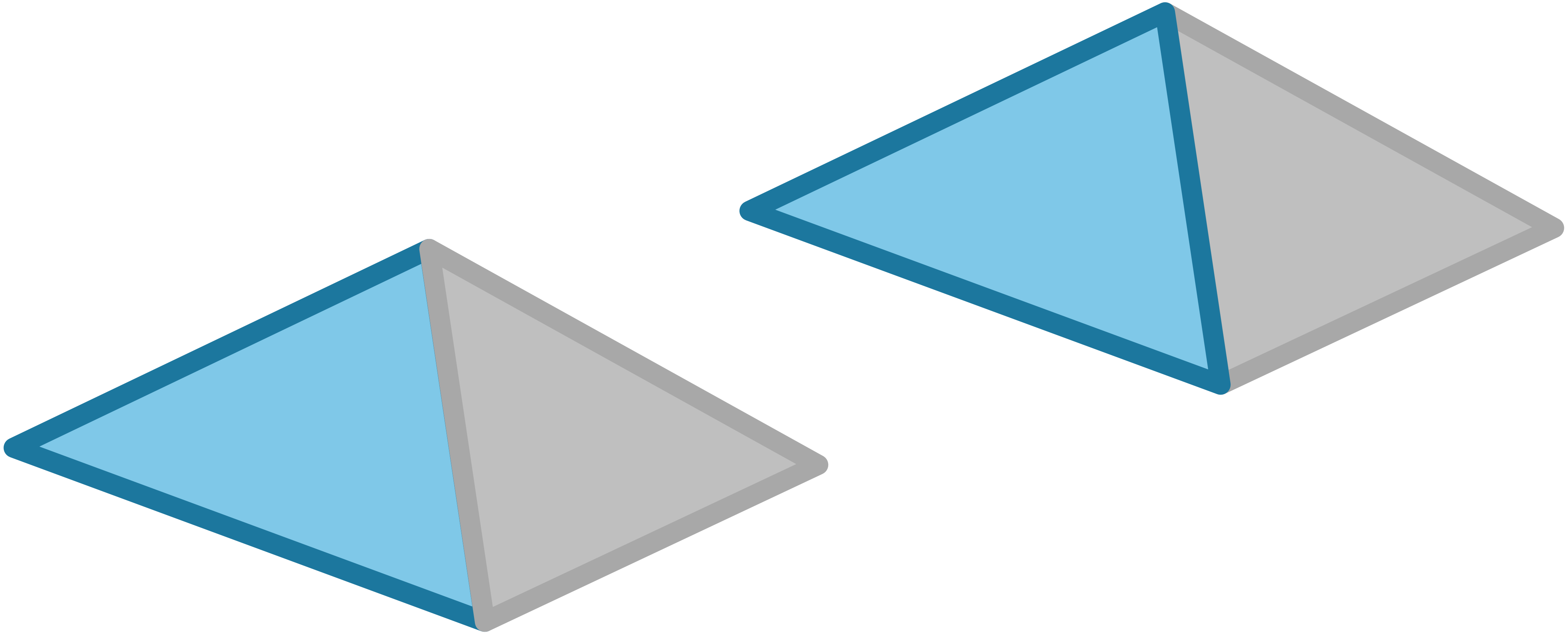
1.2 -1.4 1.2	1.0 -1.2 1.2	0.8 -1.0 1.2	0.6 -0.8 1.2	0.4 -0.6 1.2	0.2 -0.4 1.2	0.0 -0.2 1.2	-0.2 0.0 1.2
1.2 -1.2 1.0	1.0 -1.0 1.0	0.8 -0.8 1.0	0.6 -0.6 1.0	0.4 -0.4 1.0	0.2 -0.2 1.0	0.0 0.0 1.0	-0.2 0.2 1.0
1.2 -1.0 0.8	1.0 -0.8 0.8	0.8 -0.6 0.8	0.6 -0.4 0.8	0.4 -0.2 0.8	0.2 0.0 0.8	0.0 0.2 0.8	-0.2 0.4 0.8
1.2 -0.8 0.6	1.0 -0.6 0.6	0.8 -0.4 0.6	0.6 -0.2 0.6	0.4 0.0 0.6	0.2 0.2 0.6	0.0 0.4 0.6	-0.2 0.6 0.6
1.2 -0.6 0.4	1.0 -0.4 0.4	0.8 -0.2 0.4	0.6 0.0 0.4	0.4 0.2 0.4	0.2 0.4 0.4	0.0 0.6 0.4	-0.2 0.8 0.4
1.2 -0.4 0.2	1.0 -0.2 0.2	0.8 0.0 0.2	0.6 0.2 0.2	0.4 0.4 0.2	0.2 0.6 0.2	0.0 0.8 0.2	-0.2 1.0 0.2
1.2 -0.2 0.0	1.0 0.0 0.0	0.8 0.2 0.0	0.6 0.4 0.0	0.4 0.6 0.0	0.2 0.8 0.0	0.0 1.0 0.0	-0.2 1.2 0.0
1.2 0.0 -0.2	1.0 0.2 -0.2	0.8 0.4 -0.2	0.6 0.6 -0.2	0.4 0.8 -0.2	0.2 1.0 -0.2	0.0 1.2 -0.2	-0.2 1.4 -0.2

$P = \alpha P_0 + \beta P_1 + \gamma P_2$

triangle = $\{P \mid \alpha+\beta+\gamma=1,\ 0<\alpha<1,\ 0<\beta<1,\ 0<\gamma<1\}$

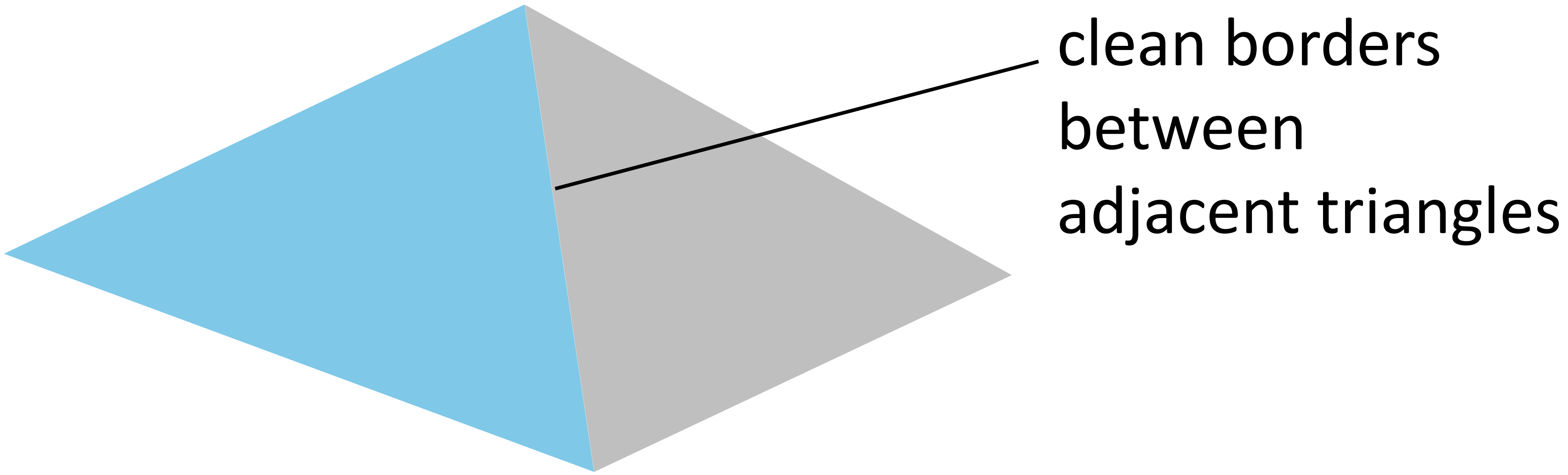
Avoiding to Draw Borders Twice

- don‘t draw the outline of the triangle!
  - result would depend on rendering order



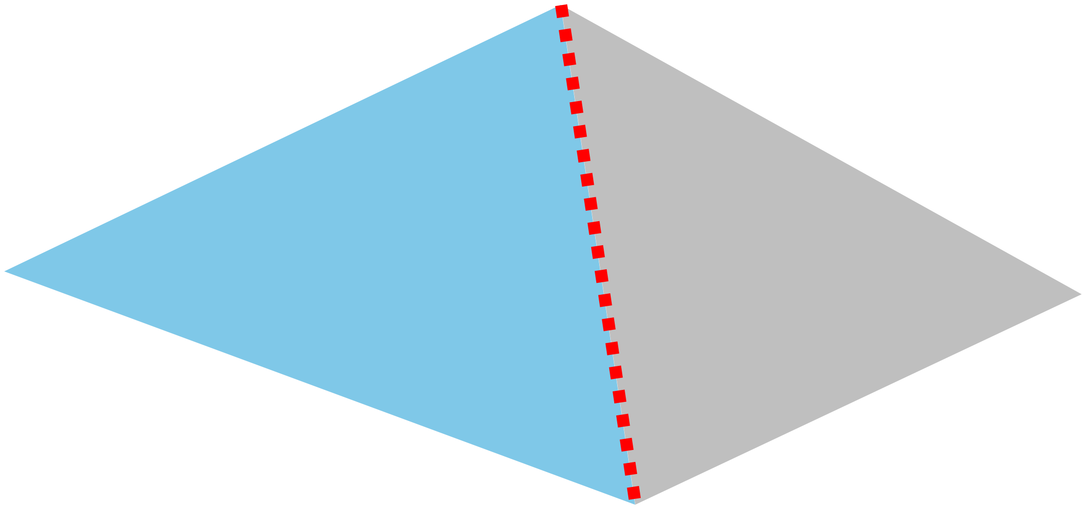
Avoiding to Draw Borders Twice

- don‘t draw the outline of the triangle!
  - result would depend on rendering order
- draw only pixels that are inside exact triangle
  - i.e. pixels with $\alpha = 0$ or $\beta = 0$ or $\gamma = 0$ are not drawn

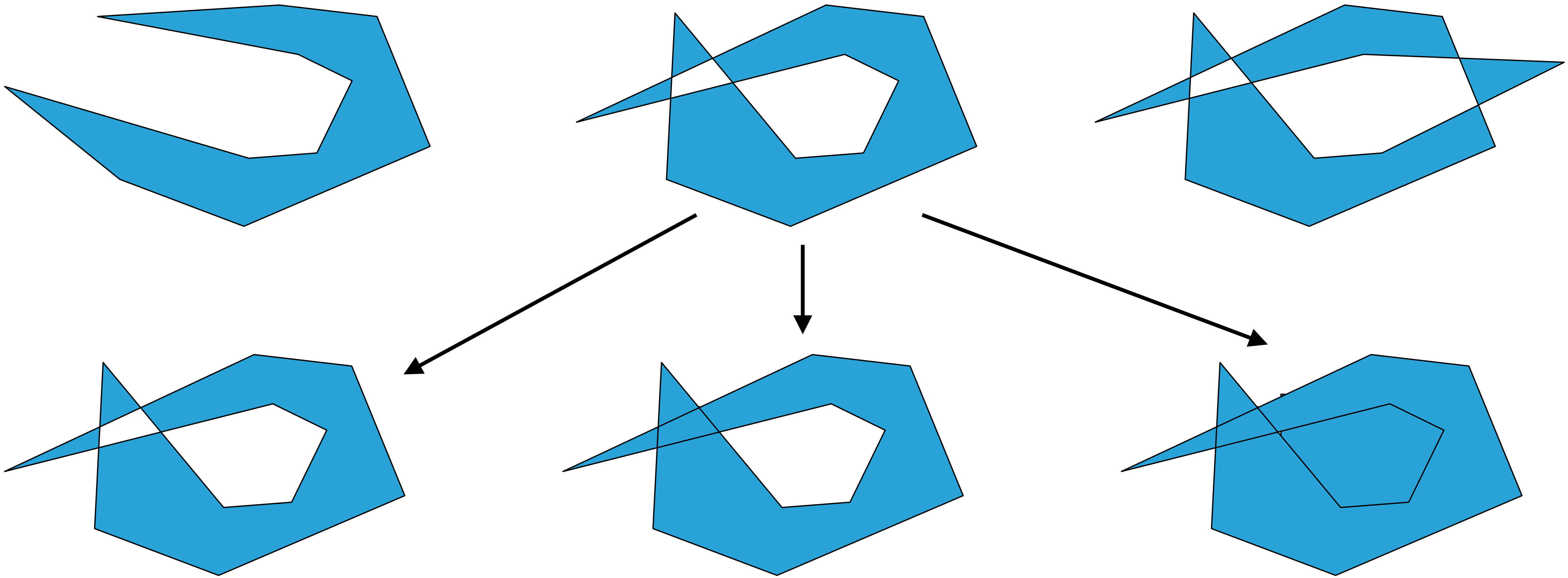


Pixels exactly on a Border ...!?

- holes if both triangles leave pixels away
- simplest solution: draw both pixels
- better: arbitrary choice based on some test
  - e.g. only right boundaries



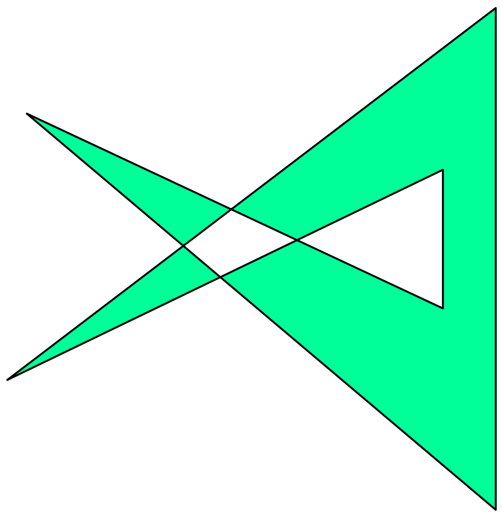
What is Inside a Polygon?

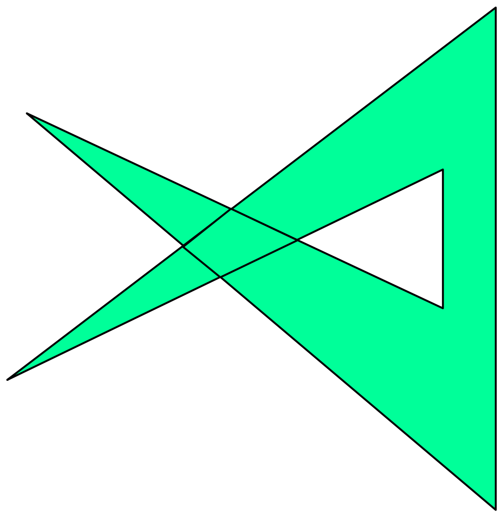


odd-even rule | nonzero-winding- number rule | ???

Inside-Outside Tests

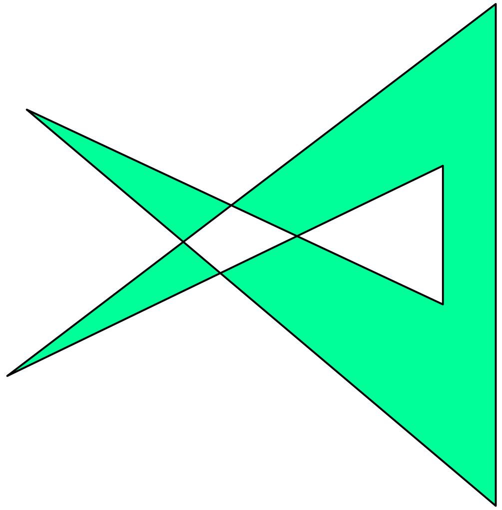
- area-filling algorithms
  - “interior”, “exterior” for self-intersecting polygons?
  - odd-even rule
  - nonzero-winding-number rule
  - same result for simple polygons

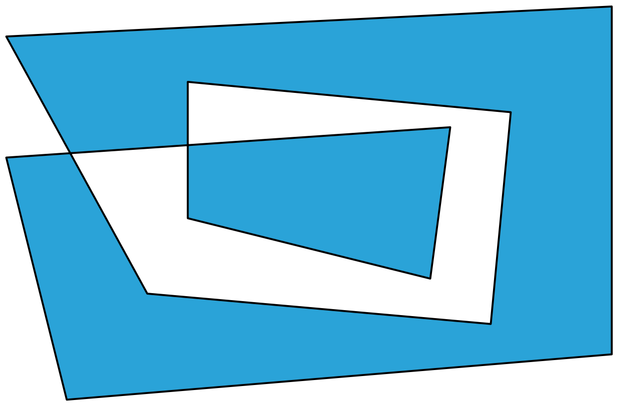


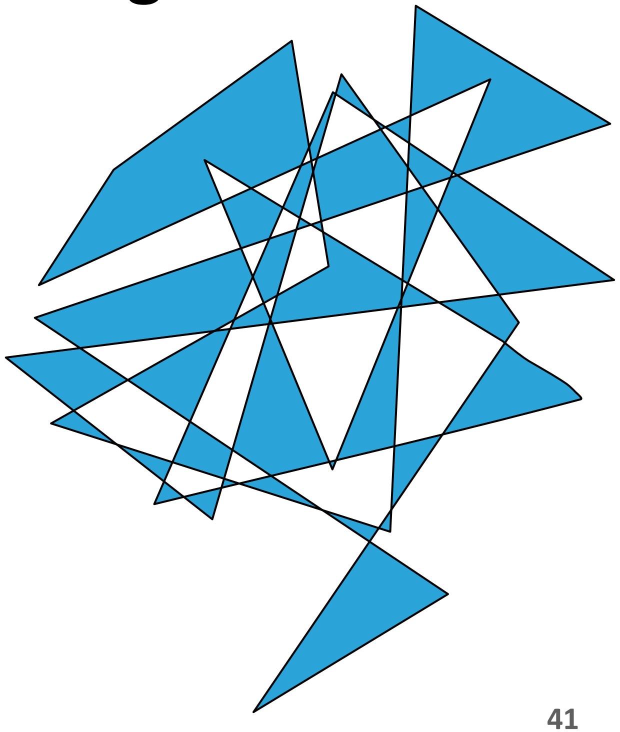


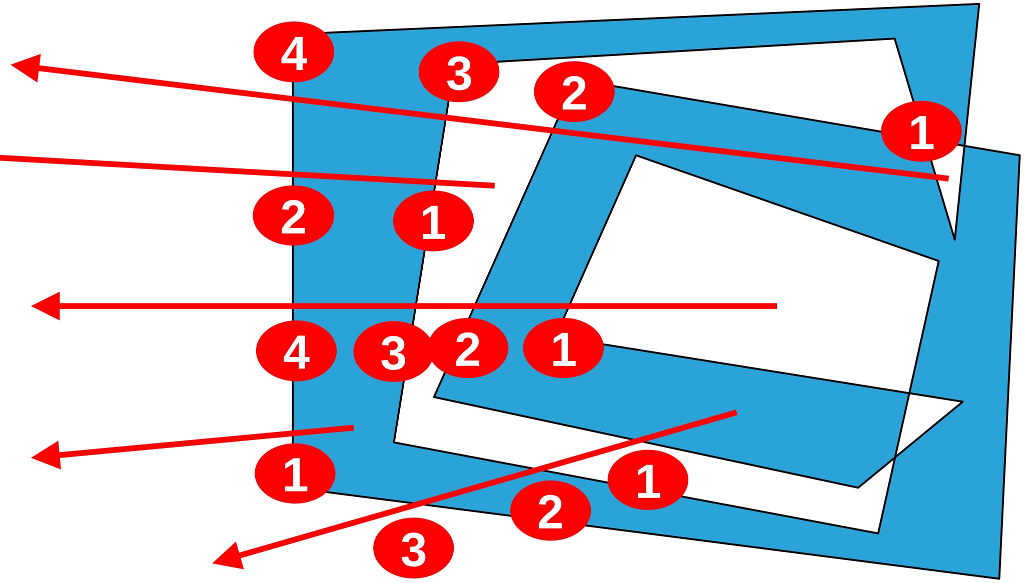
What is Inside?: Odd-Even Rule

- inside/outside switches at every edge
- straight line to the outside:
  - **even** number of edge intersections = **outside**
  - **odd** number of edge intersections = **inside**



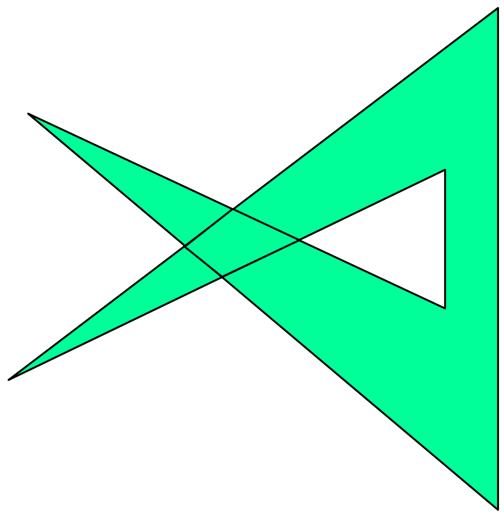


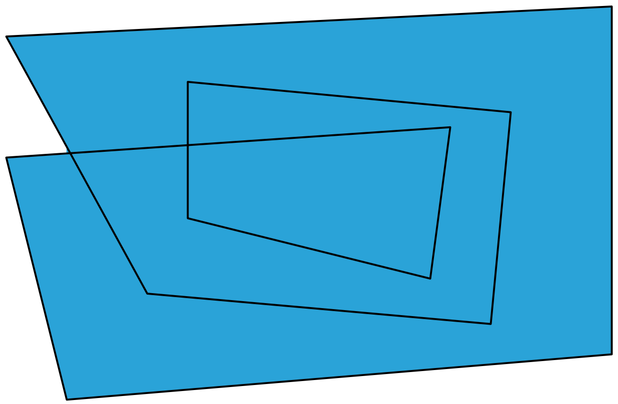


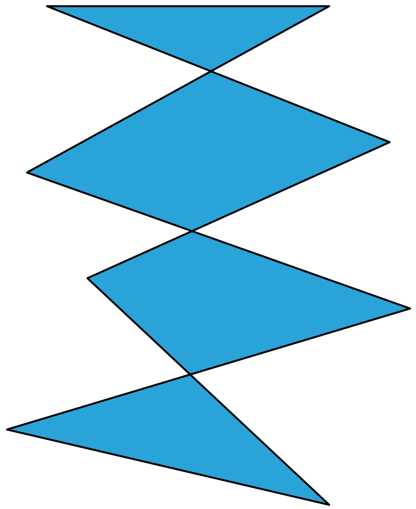


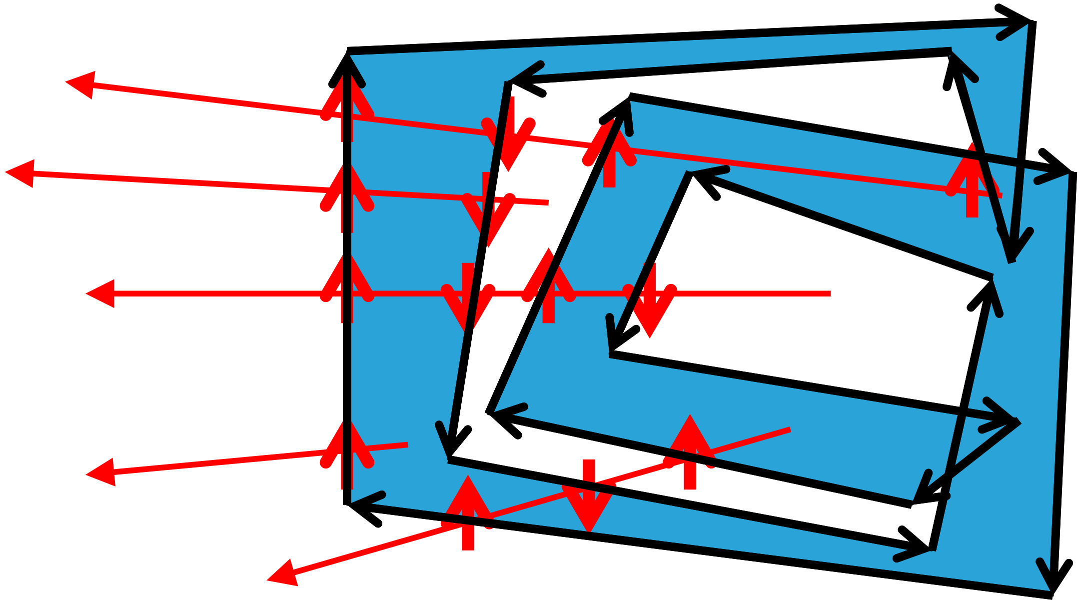
What is Inside?: Nonzero Winding Number

- point is inside if polygon surrounds it
- straight line to the outside:
  - **same** number of edges up and down = **outside**
  - **different** number of edges up and down = **inside**



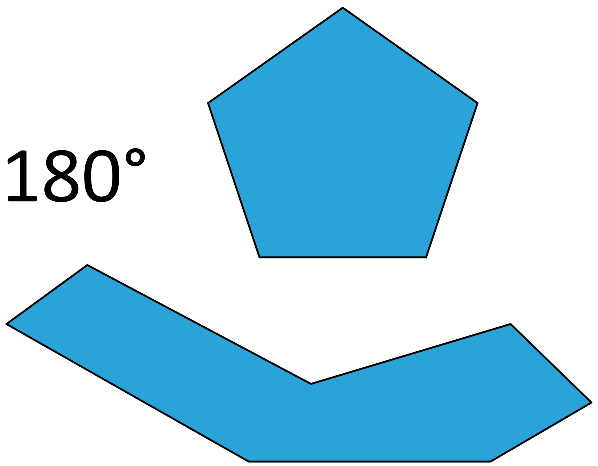






Polygon Fill Areas

- polygon classifications
  - ***convex:*** no interior angle > 180°
  - ***concave:*** not convex



- concavity test
  - *vector method*
    - all vector cross products have the same sign ⇒ convex
  - *rotational method*
    - rotate polygon-edges onto x-axis, always same direction ⇒ convex