Number and Calculation

Getting the best results

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Number and calculation at the heart of mathematics

> ***Understanding about number... grows into skills valued by industry and higher education, and are the best starting points for equipping children for their future lives.***
>
> *Good Practice in Primary Mathematics* (2011), page 4

CONTENTS

The purpose of this report

This booklet provides an overview of Ofsted's reports *Good Practice in Primary Mathematics: Evidence from 20 Successful Schools* (2011) *and Mathematics: Made to Measure* (2012). It is written to help you reflect on your school's approach to teaching number and calculation. In particular, the 'Action Points' and 'Key Questions' will help you assess the teaching of calculation and the development of children's capacity to solve problems in your school.

This report identifies characteristics of effective practice in building pupils' secure knowledge, skills and understanding of number so that they demonstrate fluency in calculating, solving problems and reasoning.

In particular, pages 8 to 15 of this report look at effective calculation methods and policies. You can use these pages to review your school's practice in detail over time.

Ofsted and good practice in primary mathematics

Good Practice in Primary Mathematics (2011) reports on a survey conducted following a ministerial request for Ofsted to provide evidence on effective practice in the teaching of early arithmetic.

The 20 successful schools visited for Ofsted's report included ten schools from each of the maintained and independent sectors. The schools spanned a wide range of contextual characteristics, such as size and location, as well as varying attainment on entry to the school. Common features included:

- strong track records of high achievement in mathematics
- success in ensuring that the majority of their 11 year-olds become fluent in calculating, solving problems and reasoning about number
- pupils' progress has been significantly above the national average, and often outstanding, for at least the last four consecutive years (from results of Key Stage 2 national mathematics tests for maintained schools).

Mathematics: Made to Measure (2012) is based on evidence from inspections between January 2008 and July 2012 in maintained schools in England. It is also informed by the findings from *Good Practice in Primary Mathematics* (2011). The report highlights examples of best practice and includes recommendations for schools.

20 successful schools – policy and practice

A secure understanding of number is a pre-requisite before moving onto written methods.

Common features of the practice of the 20 schools in *Good Practice in Primary Mathematics* (2011) included careful attention to progression in the development of number skills and the associated language and notation. The report highlights practical, hands-on activities in the early years as essential for success with all four operations, with high importance given to the development and use of mathematical language and mental mathematics. The report emphasises that:

> *"At each stage in developing skills in addition, subtraction, multiplication and division, the schools follow a similar pattern in:*
>
> - *establishing pre-requisite knowledge of the number system such as place value, families of number facts and partitioning*
> - *calculating in practical contexts and using hands-on resources such as base-10 materials*
> - *developing mental methods supported by jottings and visual images such as number lines*
> - *establishing written forms of recording, moving towards more efficient methods over time."*
>
> ***Good Practice in Primary Mathematics: Evidence from 20 successful schools*** **(2011), page 11**

The 20 successful schools did not follow a single approach to when or whether traditional written algorithms for the four operations were introduced to children. However, all schools did consider the following to be essential precursors for learning traditional written methods:

- understanding place value
- fluency in mental methods
- a good recall of number facts such as multiplication tables and number bonds.

Problem solving

Successful schools also placed **a strong emphasis on problem solving**, including it as an integral part of each topic taught. Children encountered a wide range of problems with meaningful cross-curricular links exploited successfully.

Calculation policies

The report also highlighted that staff in all schools worked together to ensure a consistent approach to teaching and learning number and calculation as children moved from one year group to the next. Whether it be a written one or not, the report advocates **the creation of a policy to aid consistency.** *Mathematics: Made to Measure* (2012) supports the evidence gathered form the 20 successful schools by recommending that schools "tackle inconsistency of teaching", "increase the emphasis on problem solving" and choose "teaching approaches that foster pupils' deeper understanding through the use of practical resources, visual images and information and communication technology" *(Mathematics: Made to Measure, page 10).*

ABOUT THE AUTHOR

Lynn Churchman

Lynn Churchman has been a teacher, teacher trainer and Mathematics Adviser in two Local Authorities before moving into national roles.

On a national level, Lynn has been Specialist Subject HMI for mathematics with Ofsted and Manager of the mathematics team at QCA, responsible for the mathematics curriculum and test development. She is passionate about mathematics and the capacity of primary pupils to learn it successfully.

Lynn is now Director of the National Mathematics Partnership, a company dedicated to working with schools on raising achievement in mathematics and primary school improvement through mathematics.

Lynn is also a trustee of National Numeracy, a charity which supports children and adults with low levels of numeracy.

> ***A crucial element is the involvement of all staff in professional development on aspects of the policy.***
>
> *Good Practice in Primary Mathematics* (2011), page 30-31

Written methods – the big debate

The debate about what pupils should be able to do by the end of primary school often gets polarised into arguments about whether we should teach certain written algorithms and to which pupils.

The importance of written and mental methods

Since the introduction of a National Curriculum for primary schools in England in 1989, and all subsequent revisions, an important place has been accorded to the teaching of both mental and written (pencil-and-paper) methods of calculation, with calculator methods being introduced later on in Key Stage 2 (ages 7-11). Mental tests, non-calculator and 'calculator available' written tests have been part of the national assessment system at Year 6 (age 11) to reflect the requirements of this range of calculation methods. The intention was that learners were taught the skills to make informed and intelligent decisions about an appropriate strategy when faced with a problem.

The National Curriculum for mathematics, however, does not specify when or how standard written algorithms should be taught – this has been left to schools to decide in relation to their own pupils and teaching programmes. The Primary National Strategy framework adopted in schools from 1999 onwards, was more explicit. It encouraged increasing efficiency of methods of calculation for all four operations as pupils' progress through the primary years and provided examples and detailed guidance about progression in calculation development.

Written methods – an efficient and reliable set of tools

My experience of working with schools has led me to the conclusion that we must aim for the majority of children to be able to use a reliable and efficient written method for each operation with confidence and understanding by the age 11. This will require the use of what are commonly known as 'standard' written methods; methods that are efficient and work for any calculations, including those that involve whole numbers or decimals.

These written methods give children an efficient set of tools they can use when they are unable to carry out the calculation in their heads or do not have access to a calculator. Our aim should be for children to have a reliable, written method to which they can turn when the need arises. Anything less means that pupils may well struggle with secondary mathematics and thus be disadvantaged by the time they reach the end of compulsory schooling and are making their choices about progression routes beyond 16.

> *"We must aim for the majority of children to use a reliable and efficient written method for each operation with confidence and understanding by the age 11."*

REFLECTING ON YOUR PRACTICE KEY QUESTIONS

1. Do you have a clear and consistent approach to the teaching of number and calculation skills to enable all pupils to become fluent in calculating, solving problems and reasoning about number by age 11?
2. Are pupils always encouraged to use mental methods as a first resort when presented with a calculation to complete, reverting to a written method when they are not able to use a mental method readily?
3. Is development of mental methods supported by jottings and visual images? Are number lines well used? Does your approach work towards development of effective written methods?
4. Is your approach evaluated and reviewed on a regular basis?
5. Does the review include reflecting on children's performance year on year?
6. Are all teachers and teaching assistants involved in professional development activity on aspects of the policy and approach to ensure that it is consistently applied?

> *"A calculations policy will provide consistency in a school's work and enable staff to work together to ensure progression in skill development."*

Calculation policies

Evidence from *Good Practice in Primary Mathematics* (2011) highlighted the importance of a calculation policy in delivering consistency and progression in the teaching of number and calculation.

Consistency of approach

Policy and practice in the teaching of calculation in primary schools has developed significantly since the introduction of the National Numeracy Strategy in the late 90s and its continuation within the Primary National Strategy (w) until April 2011.

Most schools have developed a calculations policy to provide consistency in the school's work and to enable staff to work together to ensure progression in skill development.

Progression in methods

A key role for such policies is to enable all teachers and teaching assistants to see how the methods in any year build on what went before and feed into what is learned later. It can also be used to provide information to parents, engaging them and avoiding children getting contradictory advice.

ACTION POINTS

1. Read the 'key findings' section of *Good Practice in Primary Mathematics* (2011) and compare this with an overview of your own school's approach.
2. Use the information on the pages that follow to help you reflect in detail on each aspect of your own school's approach to the teaching and learning of arithmetic.

Progression in learning: the essential components

There are three inter-related aspects of learning that need to be developed alongside each other, throughout primary school, for children to develop competence, confidence and reliability when calculating.

Follow the significant steps below for successful development in the these three key areas.

Significant steps from Early Years Foundation Stage (age 3) to Year 6 (age 11)

The following steps show the progression that I recommend children follow for successful development in the key aspects of learning shown opposite.

1. Young children, as they start school, are introduced to numbers, what they mean and how they are connected through concrete practical experiences and the development of the associated labels and language.
2. They use concrete apparatus, such as Numicon Shapes, that embody the structure of number and how numbers combine to underpin secure conceptual development of number relationships.
3. They begin to understand that numbers can be combined and are introduced to the processes of calculation through practical, oral and mental activities.
4. They use practical structural equipment, e.g. base-10 apparatus and number rods, and learn how to use number lines and other models to support and explain their methods and reasoning.
5. They develop ways of recording to support their thinking and complete their calculations.
6. They increasingly use and learn the meaning of the associated signs and symbols.
7. They learn a range of number facts, such as times tables that enable them to develop speed and accuracy in their mental calculations.
8. As mental methods are strengthened and extended, so too are children's informal written methods.
9. These informal methods are gradually refined, leading to efficient written methods that can be used more generally.

Key aspects of learning

1. Secure understanding of numbers and the number system:

- *counting and ordering numbers*
- *developing a good sense of size and where numbers fit into the number system.*

2. Methods of calculation (mental, pencil-and-paper written and calculator methods)

- *the relationships between numbers and operations such as knowing that addition and subtraction are inverses*
- *key number facts, such as number bonds to 20, multiplication and division facts to 10 x 10*
- *arithmetic procedures such as methods of subtraction.*

3. Capacity and inclination to reason

- *about numbers and quantities*
- *to solve related problems.*

Outcomes Following these steps will mean that by the end of primary school, children are equipped with a **'toolkit' of mental, written and calculator methods** that they understand and can use correctly. When faced with a calculation, in a problem or an unfamiliar context, children are able to decide which method is most appropriate and apply this accurately. They have strategies and the inclination to check its accuracy and interpret the solution in the context of the problem.

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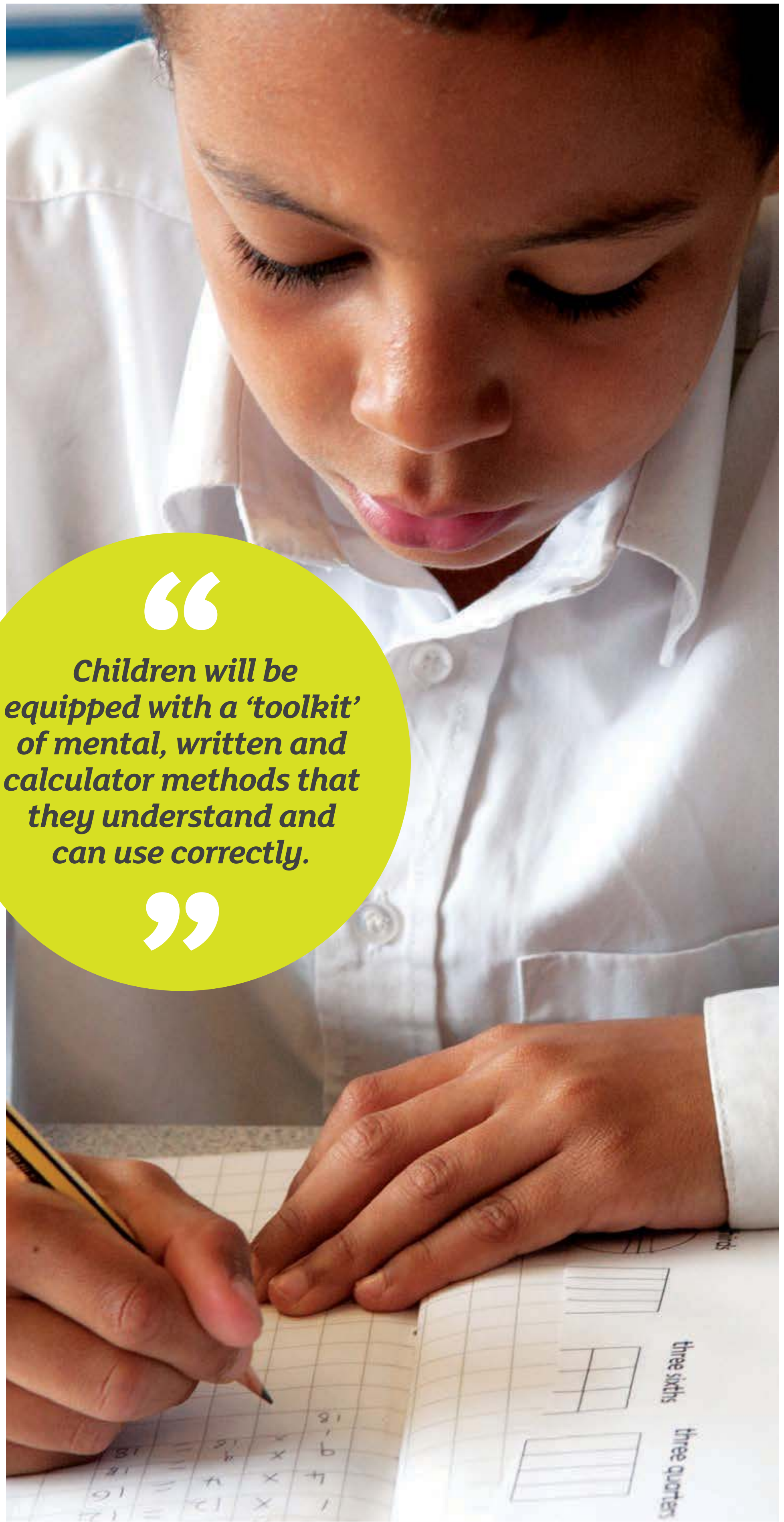

> ***"Children will be equipped with a 'toolkit' of mental, written and calculator methods that they understand and can use correctly."***

REFLECTING ON YOUR PRACTICE KEY QUESTIONS

1. Does your approach help provide secure beginnings for your youngest learners?
   - Are they able to recite number names and count reliably?
   - Do you show them how numbers relate?
   - Do they have access to practical, hands-on equipment?
2. In years 3 and 4 (age 7 - 9), do children develop their understanding of the wider number system?
   - Does your approach help them to build the crucial building blocks, such as knowing number pairs to 100?
   - Do children have access to practical structural equipment?
3. Do your children achieve mastery and confidence in years 5 and 6 (age 9-11)?
   - Are they encouraged to generalise, predict and explain?
   - Do you help them to refine their informal calculation methods into efficient written ones?
   - Do children continue to have access to practical structural equipment?
4. Is there a variety of concrete apparatus available in every classroom?

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Methods of calculation

> *When using or applying calculation strategies, children will need to consider what will be the most efficient and reliable way of doing the calculation.*

Since the introduction of a National Curriculum for primary schools in 1989, an important place has been accorded to the teaching of both mental and written (pencil-and-paper) methods of calculation. However, when and how formal written methods should be introduced has been the cause of much debate.

Good Practice in Primary Mathematics **(2011) did not silence the debate on when and how to introduced formal written methods, as there was not a common approach for all operations used across the 20 schools.**

However, a central message remains: when using or applying calculation strategies, children will need to consider what will be the most efficient and reliable way of doing the calculation.

In my experience, for pupils to develop fluency and confidence in choosing and using appropriate calculation strategies by the time they leave primary school, they should develop a 'toolkit' of methods. This fact is often understood and demonstrated by high-performing schools.

I recommend this 'toolkit' consists of:

1. **efficient mental methods** – likely to be used with numbers up to 100 or with easy larger numbers; for example, Year 3/4 pupils (age 7/8) should be able to rapidly and reliably mentally add three numbers such as 150 + 400 + 250 without reverting to a pencil-and-paper algorithm or entering the numbers into a calculator as both these latter methods will take longer and are unnecessary (Figure 1).
2. **efficient written methods** – for all four rules of number to be applied to whole numbers and to decimals.
3. **effective use of calculators** – including interpreting the display according to the context and nature of the computation and required solution and knowing how to enter and interpret money calculations and simple fractions.

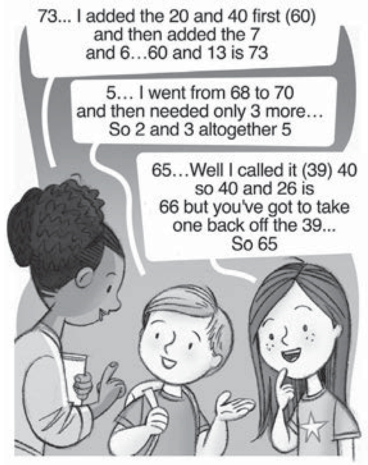

Figure 1: Pupils in Year 2 (age 6-7) use mental recall of addition and subtraction facts to 20 in solving problems involving larger numbers. They add and subtract numbers with two digits mentally

PUPIL REFLECTION

'Can I do this in my head?'

'Can I do this in my head using drawings or jottings?'

'Do I need a pencil and paper procedure?'

'Do I need a calculator because of the size or types of number?'

Addition and subtraction

It is important to build a strong sense of number before introducing formal written methods

In the Early Years Foundation Stage (EYFS) and Key Stage 1 (ages 4-7) schools tend to provide good experience of numbers and number bonds in a range of practical contexts including songs, rhymes, use of structural equipment (e.g. Numicon Shapes), water and sand play and through mathematical games of diverse assortment. *Good Practice in Primary Mathematics* (2011) supports this as the schools reported the need for practical, hands-on activities in the EYFS and in Key Stage 1 (ages 4 -6), with attention paid to developing children's use and understanding of mathematical language and mental mathematics.

Children should then quickly move on to learning to count and order numbers, understanding the number bonds to 10 and 20 and carrying out mental calculations using number bonds when solving practical problems involving addition and subtraction with numbers up to 100. Building the sense and experience of numbers is necessary to lay a strong foundation for the later learning of efficient written methods for addition and subtraction.

Column addition and subtraction

When it comes to teaching formal written methods for addition and subtraction, in practice, there is a reasonable consensus. Most schools teach column addition and subtraction methods to pupils at much the same age, as *Good Practice in Primary Mathematics* (2011) confirms:

> *"Most of the schools in the survey teach column addition and subtraction in lower Key Stage 2, with most of the maintained schools favouring Year 4 and the independent schools Year 3, although around half of the schools introduce them earlier for higher attaining pupils or later for lower attaining pupils."*
>
> ***Good Practice in Primary Mathematics: Evidence from 20 successful schools*** **(2011), page 11**

Develop essential knowledge and skills before introducing formal written methods

The successful schools in *Good Practice in Primary Mathematics (2011)* are generally agreed on the necessary prior knowledge and skills if pupils are to be successful with the compact written algorithms and select and use them appropriately to solve problems.

From my experience of raising achievement in schools, we need to ensure that children have developed the following abilities by the end of Key Stage 1 (age 7) if they are to secure good addition and subtraction knowledge and skills in Year 3 and 4 (ages 7-9):

- recall key number facts instantly – for example, all addition and subtraction facts for each number to 10 then to 20, sums and differences of multiples of 10
- appreciate that the arithmetic operations of addition and subtraction are the inverse of each other
- recognise that addition can be done in any order and use this to add mentally a one-digit number or a multiple of 10 to a one-digit or two-digit number
- partition two-digit numbers in different ways including into multiples of ten and one and add the tens and ones separately and then recombine.

When these abilities are secure, children can be taught the efficient column method (see figure 2) for addition and subtraction and will be able to carry out vertical calculations efficiently and accurately.

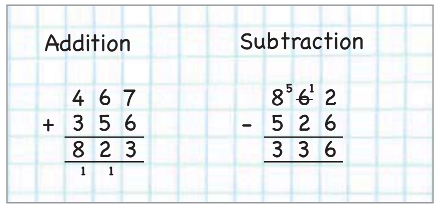

Figure 2: Example addition and subtraction calculations using the efficient column method

ACTION POINTS

1. Read pages 9–13 of *Good Practice in Primary Mathematics* (2011) describing effective practice in the teaching of column addition and subtraction and compare your provision with the good practice in the 20 successful schools.
2. View the Oxford School Improvement video 'Building a strong sense of number by age 11' and compare the general progress of your learners with that described in the video (www.oxfordprimary.co.uk).
3. Make any alterations to your school's programme for teaching mathematics in EYFS and Key Stage 1 (age 4–7) to ensure that it lays the necessary foundation for all learners to progress onto learning efficient column addition and subtraction methods in Year 3 (age 7–8) and 4 (age 8–9).
4. You may want to choose a sample of children in EYFS and Key Stage 1 (age 4–7) to review the progress your learners make in mental methods for addition and subtraction.
5. Audit the approach to teaching mental and written methods for addition and subtraction from Year 2 (age 6–7) into Year 3 and 4 (age 7–9). Do you need to make changes to your approach to teaching column addition and subtraction methods as the children progress into Year 3 (age 7–8) and on through Year 4 (age 8–9) so that they have efficient and reliable methods for calculations such as 145 + 89 + 374 and 524 - 376?

Multiplication

Good Practice in Primary Mathematics (2011) found that in successful schools the foundations for multiplication were laid in EYFS and Key Stage 1 (ages 4 – 6) and that "as with addition and subtraction, the schools emphasised practical, hands-on activities, with a high profile given to developing mathematical language and mental mathematics, coupled with visual images and physical representations of sharing, grouping, arrays and patterns."
(Good Practice in Primary Mathematics (2011) *page 14.)*

The report also found that the teaching of the formal arithmetic methods for long multiplication generally takes place in upper Key Stage 2 (ages 9–11), usually in Year 5 (age 9–10) for most pupils.

Develop essential knowledge and skills before introducing formal written methods

For pupils to learn the long multiplication algorithm successfully, I believe they should have previously developed the ability to:

- recall all multiplication facts to 10 × 10
- partition numbers into multiples of one hundred, ten and one
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value
- add two or more single-digit numbers mentally
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value
- add combinations of whole numbers using the column method.

For pupils to develop these necessary pre-requisites to long multiplication, I would recommend schools provide learners with opportunities to work on tasks such as:

- calculating the value of an unknown in a number sentence
- working out how many 2s make eight or how many 5s make 35
- mentally calculating doubles or remainders
- solving problems such as "There are 27 cubes. Make three towers the same height. How tall is each tower?

The grid method

The schools surveyed for *Good Practice in Primary Mathematics* (2011) usually introduced long multiplication through earlier work using the 'grid method' (see figure 3), to allow pupils to see how the two forms of recording align before moving to the more efficient method.

The report found that the advantages of the grid method were its use for work with multiplying decimals and for secondary mathematics topics such as multiplication of algebraic expressions such as $(2x + 3)(x – 6)$ and numerical expressions involving square roots, for example $(\sqrt{3} – 1)(2\sqrt{3} + 1)$.

246 x 37
The grid method

wwx	200	40	6
30	6000	1200	180
7	1400	280	42

6000 + 1200 + 180 = 7380 →
1400 + 280 + 42 = 1722 →
Answer 246 x 37 = 9102

The formal algorithm

$$\begin{array}{r} 246 \\ \times\quad 37 \\ \hline 7380 \\ \hline 1722 \\ \hline 9102 \end{array}$$

Figure 3: Long multiplication using the grid method and the formal algorithm

> **"There is a need for the development of practical, hands-on activities with attention paid to developing mathematical language and mental mathematics."**

Division

The schools in *Good Practice in Primary Mathematics* (2011) were confident that the majority of their pupils master the formal algorithms for addition, subtraction and multiplication (including long multiplication algorithm) but this was not necessarily the case for division.

The report showed that the schools differ in their approach to the teaching of the compact long division method. Some schools teach it only to the highest-attainers whilst others usually teach it to the majority of their Year 6 (P7) pupils. Schools reported that some pupils are not very secure in its use, especially when it comes to applying the algorithm in a context. In some of the independent schools in the survey (with transition to the next phase at age 13) they wait until Year 7 to teach the long division algorithm.

Regardless of when you choose to introduce the long division algorithm, it is vital to lay secure foundations first. My experience is that pupils should learn strategies to divide successfully in their heads and be able to:

- understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways
- recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5
- understand and use multiplication and division as inverse operations.

To carry out later written methods of division successfully, it is my belief that children also should be able to:

- understand division as sharing or repeated subtraction
- estimate how many times one number divides into another when there is a remainder – for example, how many 6s there are in 47, how many 23s there are in 92 or how many 52s there are in 600
- multiply a two-digit number by a single-digit number mentally
- subtract numbers using the column method.

ACTION POINTS

1. Read pages 14–20 of *Good Practice in Primary Mathematics* (2011) describing effective practice in the teaching of written algorithms for multiplication and division and compare your provision with the good practice in the 20 successful schools.
2. Audit your school's approach to teaching mental and written methods for short multiplication and division in lower Key Stage 2 (ages 7–9) to ensure that it develops firm foundations for work in upper Key Stage 2 (ages 9–11) towards the standard algorithms for multiplication and division involving two and three digit numbers.
3. Compare the progression described on page 12 of this report to the lines of progression in your own teaching programmes.
4. You may want to choose a sample of children in Key Stage 2 (ages 7-11) to review the progress your learners make in these respects.
5. Reflect on whether you now need to make changes to your approach to the teaching of multiplication and division.
6. Make any alterations to the school's programme for teaching efficient written methods of multiplication and division which lead to the standard algorithms for those pupils with the pre-requisite understanding to use them accurately and reliably.

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Progression in multiplication and division

For children to understand and use the efficient written method for long multiplication and division, I recommend schools first teach them to:

- represent repeated addition and arrays as multiplication, and sharing and repeated subtraction (grouping) as division; calculate the value of an unknown in a number sentence (e.g. □ ÷ 2 = 6, 30 – □ = 24; lower Key Stage 2, ages 7–9)
- use practical and informal written methods and related vocabulary to support multiplication and division, including calculations with remainders; use practical and informal written methods to multiply and divide two-digit numbers (e.g. 13 × 3, 50 ÷ 4); round remainders up or down, depending on the context (lower Key Stage 2, ages 7–9)
- use the symbols ×, ÷ and = to record and interpret number sentences involving multiplication and division (lower Key Stage 2, ages 7–9)
- use understanding of place value to multiply and divide whole numbers and decimals by 10, 100 or 1000; refine and use efficient written methods to multiply and divide HTU × U, TU × TU, and HTU ÷ U (Year 5, age 9–10)
- use efficient written methods to:
  1. multiply and divide integers and decimals by a one-digit integer
  2. multiply three-digit and four-digit integers by a two-digit integer
  3. divide a three-digit integer by a two-digit integer (Year 6).

> *Reflect on whether you need to make changes to your approach to the teaching of multiplication and division.*

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Calculation policies

In recent years most schools have developed a **calculations policy** to provide consistency across the school and to enable staff to work together to ensure progression in skill development. A key role for these policies is to enable all teaching staff to see how the methods in any year build on what went before and feed into what is learned later. They can also be used to provide information to parents.

Research indicates that a school's calculations policy is most effective when it is kept under regular review.

Good Practice in Primary Mathematics (2011) states that:

> *"A feature of strong practice in the maintained schools is their clear, coherent calculation policies and guidance, which are tailored to the particular school's context. They ensure consistent approaches and use of visual images and models that secure progression in pupils' skills and knowledge lesson by lesson and year by year.*
>
> *Good Practice in Primary Mathematics: Evidence from 20 successful schools (2011), page 7*

Many such policies contain an outline of the key pencil and paper procedures to be taught within the school. In some of these policies the significant role of mental calculation is made clear, whilst in others there is little link between the development of children's mental calculation strategies and the teaching of written calculation methods. It is important to recognise that the mental facility with numbers and the ability to calculate mentally lies at the heart of successful calculation methods. Pupils' mental facilities with number should be honed as they move into and through Key Stage 2 (age 7–11) rather than allowing the focus to move exclusively to written methods of calculation. Mental calculation and written recording should be seen as complementary to each other. In every written method there is an element of mental processing so the two should be continually developed alongside each other.

> ***"Mental calculation and written recording should be seen as complementary to each other."***

ACTION POINTS

1. Does your school has a calculations policy? If not, plan to implement one.
2. Consider bringing forward the review of your calculations policy, with this report in mind.
3. Audit the consistency and effectiveness of your school policy and practice in the development of pupils' mental and written calculation skills.
4. Consider all the questions listed on page 15 carefully.
5. Complete the review of your policy thoroughly and honestly. This should help you to identify your school's strengths and weaknesses in teaching calculation.
6. Once the review is complete, reflect on whether you now need to make changes to your calculations policy. If so, make sure these changes are made and communicated to all staff.

When considering progression in learning for the four operations and the development of calculation methods, it may help you to ask yourself these questions that I commonly use with schools:

Addition

- Which expanded method for addition is taught? Are we consistent?
- What is our approach to partitioning numbers? Does this support children's secure understanding of place value? For example, when adding two numbers do children only think to partition the second number?
- Are pupils encouraged to move onto a standard efficient written method as soon as their understanding is secured?

Subtraction

- How do we teach subtraction in each class? Do we agree as a staff to teach complementary addition rather than teach decomposition?
- Is there enough emphasis on 'difference' (how many more/ how many less) in Key Stage 1 (ages 4–7) or do children only view subtraction as 'take away'?
- Are pupils encouraged to move onto a standard efficient written method as long as their understanding is secured?

Multiplication

- Do we currently teach the grid method of multiplication?
- Do we use arrays in Key Stage 1 (ages 4–7)?
- Does our approach support the secure development of children's understanding of place value?
- Are pupils encouraged to move onto a standard efficient written method as long as their understanding is secured?

Division

- Is our approach consistent to teaching division consistent? Do we use chunking or not?
- Have empty number lines been used to demonstrate the idea of counting on in multiples of the divisor?
- If we use chunking, are the foundations taught in Years 2 and 3 (ages 6–8) (e.g. is there enough emphasis placed on grouping or are children only taught how to share?)?
- Is partitioning used regularly to support division?
- Are pupils encouraged to move onto a standard efficient written method as long as their understanding is secured?

CALCULATIONS POLICY **KEY QUESTIONS**

1. **Is there whole-school agreement on the methods in the policy?**
   - If not, are there alternatives, which all staff agree on?
   - Is the policy clear on which mental calculation skills and strategies are to be developed in each year group?
2. **Is there enough emphasis on the use of number lines and other models and images to support the development of mental and written calculation skills in each year group?**
3. **Are the expected methods of recording clear? Are they adopted or should they be extended?**
4. **Do children have the necessary mental calculations skills and strategies needed for the written methods detailed in the policy for each year group ?**
5. **Does the policy encourage children to estimate the approximate size of a calculation and use this to check the 'reasonableness' of their answer?**
6. **Are children given regular opportunities to share their methods and strategies and to refine and improve them?**
   - Are children given regular opportunities to use and apply calculation methods efficiently to solve a range of problems?
7. **Is there a commitment to how often the policy is reviewed?**

Problem solving and reasoning

Problem solving, reasoning and explaining lie at the heart of mathematics and in pupils' capacity to use their arithmetic skills in a variety of contexts. However it is being a mathematical thinker and problem-solver that leads to real success in mathematics.

Good Practice in Primary Mathematics (2011) made clear that this aspect of learning was a high priority in the successful practice evident in the 20 schools:

> *"The emphasis almost all of the schools placed on pupils using and applying their arithmetic skills to solving a wide range of problems was striking. Diverse opportunities were provided within mathematics, including measures and data handling, and through thematic and cross-curricular work. Pupils' extensive experience of solving problems deepens their understanding and increases their fluency and sense of number."*
>
> *Good Practice in Primary Mathematics: Evidence from 20 successful schools* (2011), page 20

This is supported by *Mathematics: Made to Measure* (2012) supports this with the recommendation that schools "increase emphasis on problem solving across the mathematics curriculum" *(Mathematics: Made to Measure, page 10).*

Give children the time and opportunity to solve problems

Successful schools in *Good Practice in Primary Mathematics* (2011) capitalise on opportunities for problem solving within and beyond the daily mathematics lesson to develop learners' skills as problem solvers. Children need to solve problems to become problem solvers. This may seem obvious, but it means that pupils should be given the time and opportunity to tackle problems regularly if they are to be confident and competent problem solvers. Problem solving therefore should be integrated into mathematics teaching and learning, and become a regular part of the children's work through being embedded into everyday lessons.

As children acquire more number skills and understanding, the problems they are presented with should involve increasingly complex calculations set in wider-ranging contexts. As they progress into and through Key Stage 2 (ages 7-11), the problems presented should move from one-step to multi-step problems that are more complex and where less routine approaches are needed to solve them.

Is your curriculum structured to develop problem-solving abilities?

There are numerous approaches to solving maths problems but in general it requires the development of pupils' abilities to:

- look for a pattern or structure in the context or problem
- translate the problem into a diagram, picture or concrete/ visual model (model-draw)
- guess and check – trying a solution and improving
- make a systematic list to make sure all possible answers have been found
- use logical reasoning – exploring, predicting, testing and explaining
- work backwards.

Schools looking to improve their pupils' achievements in mathematics therefore should consider whether their curriculum is structured to develop these capacities in learners of all ages alongside developing the knowledge and understanding of the arithmetic skills and procedures in the curriculum (figure 4).

> ***"Children need to solve problems to become problem solvers."***

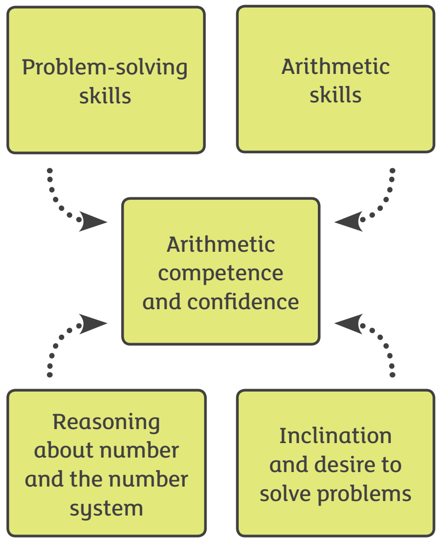

Figure 4: Four significant components for arithmetical competence and confidence.

ACTION POINTS

1. Read pages 20–23 of *Good Practice in Primary Mathematics* (2011) describing the schools' approaches to problem solving.
2. View the Oxford School Improvement video 'Developing Mathematical Habits of Mind' and compare your school's provision for children to learn these with what is described in the video (www.oxfordprmary.co.uk).
3. Audit the school's approach to developing pupils' problem-solving and reasoning skills. Choose a sample of children in each year group to consider the extent to these skills are developed in each year group.
4. Reflect on whether you now need to make changes to your approach to the development of problem-solving and reasoning skills as pupils progress through school.
5. Make a plan of needs (e.g., teaching programmes, resources, staff training) and draft an action plan to outline your intended developments.

A supportive learning environment

Just as a rich literacy environment supports children's development in reading and writing, a number-rich environment supports the development of pupils' number skills and their understanding of numbers and how they are related.

A visually-rich mathematics environment, including a range of number images, gives children opportunities to learn about numbers and how useful they are in daily life and to develop a practical feel for what are essentially very abstract concepts. *Good Practice in Primary Mathematics* (2011) discusses the importance of a mathematically rich environment and how every opportunity can be used to use mathematics in everyday life.

A classroom rich in images

A classroom and corridors rich in images for number supports children in their day-to-day mathematics work. For instance they should be able to glance at the imagery and number lines around them to confirm their thinking and – for example - check how a numeral is correctly written (see figure 5).

A classroom equipped with a range of resources

All classrooms should be equipped with a range of resources (structured and unstructured) to support children's learning of arithmetic including those shown opposite.

Good Practice in Primary Mathematics (2011) also featured use of such objects and images as evidence of good practice:

> *"Such use of practical equipment or visual images helps pupils to connect the recorded method with what is happening physically during the addition/subtraction, and was a key feature of most of the schools visited."*
>
> *Good Practice in Primary Mathematics: Evidence from 20 successful schools* (2011), page 27

> *"Almost all the schools visited consider fluent and versatile use of the number line as crucial in developing understanding from Key Stage 1 onwards of the concepts underpinning the four operations. Moreover, it provides an important image for mental calculations."*
>
> *Good Practice in Primary Mathematics: Evidence from 20 successful schools* (2011), page 21

Technology to support learning

The use of technology to support learning was highlighted in *Good Practice in Primary Mathematics* (2011) .

> *"Examples of good practice observed during the survey include the use of the interactive whiteboard to show visual images to support calculations, for example in exchanging one block of 10 for 10 units in column subtraction."*
>
> *Good Practice in Primary Mathematics: Evidence from 20 successful schools* (2011), page 29

Interactive whiteboards can be used to aid the comparison of different methods of calculation.

> **"A classroom and corridors rich in images for number supports children in their day-to-day mathematics work."**

> **"A visually-rich mathematics environment gives children opportunities to learn about numbers and how useful they are in daily life."**

A school rich in mathematical images and objects

The wider school environment – the hall, corridors and outdoor play areas – should be rich in mathematical images and objects. For example, having a number line from 0 – 1000 starting at an entrance and going along one or more corridors and around to the hall gives children a real sense of the size of 1000. It additionally provides children and teachers with the opportunity to talk about larger numbers and patterns within the 100's in a practical and natural way as children's understanding of numbers and the number system develops.

Figure 5: A classroom and corridors rich in images for number supports children in their day-to-day mathematics work.

Figure 6: Unstructured concrete materials – pebbles, fingers, shells, lengths of ribbon etc – for counting and comparing quantities

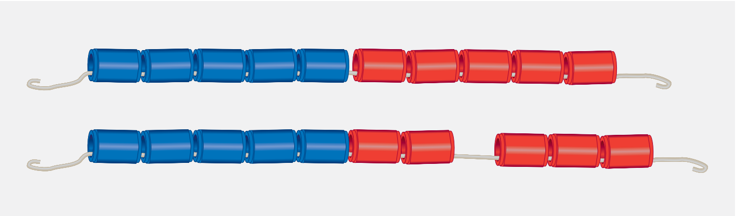

Figure 7: 10 and 20 bead number strings (structured) for addition and subtraction

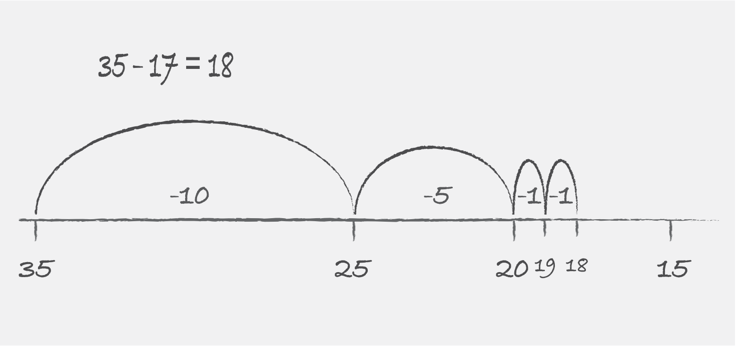

Figure 8: Empty number lines (unstructured) for addition and subtraction

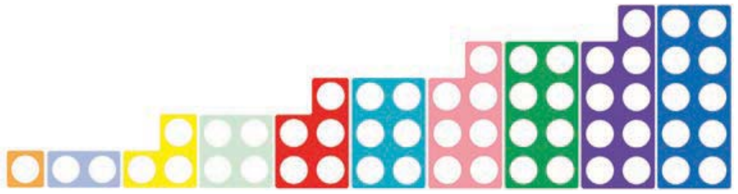

Figure 9: Structured concrete materials - such as Numicon Shapes and number rods – to support children's understanding of the pattern and structure in the number system

ACTION POINTS

1. View the Oxford School Improvement video 'A number-rich environment' and compare your school's provision with what is described in the video (www.oxfordprimary.co.uk).
2. Read pages 11–13 of *Good Practice in Primary Mathematics* (2011) describing how the successful schools make significant use of the 'Empty Number Line' as a model to underpin the development of calculation skills.
3. Audit the school's resources for mathematics to assess the extent to which the resources are fully available and used to support the learning of number and calculation for all children.
4. Audit the school's classrooms and wider school environment to assess the extent to which it is fully used to support children's learning of number and calculation.
5. Reflect on whether you now need to make changes and make a plan of action to strengthen this aspect of your school's provision.

Working with parents and carers

Number is essentially an abstract concept for young children and some children take longer than others to get a confident understanding of the way numbers are used and how the number system works.

The more enjoyable and fun experiences a child has at home, the sooner they become comfortable and confident with their early number work in school.

The majority of parents and carers want to do the very best for their children in mathematics as in other aspects. However, where mathematics is concerned, they are often fearful of giving their children contradictory advice and are not always confident that they understand the 'modern methods' of calculation. Many are therefore not clear what the school's approach is to developing their children's calculation skills is and how they can best help.

Many of the schools in *Good Practice in Primary Mathematics* (2011) communicated regularly with parents and carers, with the importance of mathematics highlighted through newsletters.

INVOLVING PARENTS KEY QUESTIONS

1. How are parents informed about your approach to the teaching of number and calculation?
   - Does your calculations policy address how you will provide information for parents?
2. Do parents understand that there are many ways to help their children at home with number and calculation skill development?
3. What opportunities are there for parents to see your approaches to calculation in action, such as demonstration lessons?
4. How effective is your school website in giving parents information about the teaching of calculation methods?
   - Does it suggest useful links to other sources of online information, for example so that parents can see how number and calculation skills develop?
5. How do you communicate with parents about their child's progress in mathematics and how they can help?
   - What strategies do you use to communicate with 'hard to reach' parents?
   - Does your feedback to parents on their child's progress with number and calculation highlight ways in which they could help?
6. Do you direct them to resources they can use? For example, are they aware of the free resources available on the Oxford Owl website: www.oxfordowl.co.uk
7. What is the role of governors, especially parent governors, in supporting mathematics?
   - How informed and confident are your governors about the approach to calculation in your school?

ACTION POINTS

1. Audit the school's approach to involving parents/carers using the questions opposite to start your thinking.
2. Revisit your school website, as if you were a parent, and appraise it critically for what it says about mathematics or ask a parent governor and your numeracy coordinator to take on this task.
3. View the Oxford Owl website at www.oxfordowl.co.uk/maths and decide if there are particular resources you would like to recommend to parents to support their children's learning at home.
4. View the parents page at http://www.nationalnumeracy.org.uk, a website concerned with numeracy development for all ages.
5. Consider running a session for parents on 'approaches to calculation' where you share with them the school's approach to the teaching of the arithmetic operations.
5. Reflect on whether you now need to make changes and make a plan of action to strengthen this aspect of your school's provision.

Helping you achieve the best results

Use these questions as a checklist to ensure you have put every measure in place to help children succeed.

QUESTIONS FOR MATHS SUBJECT LEADER	YOUR COMMENTS
Do I have a systematic approach to teaching number and calculation?	
Is our calculation policy up-to-date?	
Is my staff as effective as it can be?	
Am I catching every child the moment they fall behind?	
Do I have practical resources to ensure all my children have a hands-on experience of mathematics?	
Are children surrounded by a number-rich learning environment?	
Are my parents fully involved in helping support their children's arithmetic progress?	

ACTION POINTS

1. Read the 'key findings' of *Good Practice in Primary Mathematics* (2011) and the recommendations for schools in *Mathematics: Made to Measure* (2012). Compare with an overview of your own school's approach.
2. Watch the Lynn Churchman Oxford School Improvement videos on www.oxfordprimary.co.uk with your senior management team.
3. Consider the three inter-related aspects of learning on page 6 of this report and reflect on whether your calculation policy allows for development in all areas.
4. Review your calculation policy and compare with the progression outlined on pages 8 to 12 of this report.
5. Consider whether your mathematics curriculum allows for children to develop the skills necessary for problem-solving and reasoning whilst also developing their arithmetic skills.
6. Review your calculations policy, in consultation with your staff, in line with points 4, 5 and above.
7. Consider how you can improve the learning environment in your school to make it more number-rich.
8. Reflect on how you can involve parents further in their children's arithmetic.

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Getting the best results

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our customer care line on
01536 452610.

Read Ofsted's full reports, *Good Practice in Primary Mathematics: Evidence from 20 Successful Schools* (2011) and *Mathematics: Made to Measure* (2012) at:
http://www.ofsted.gov.uk/inspection-reports/our-expert-knowledge/mathematics

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