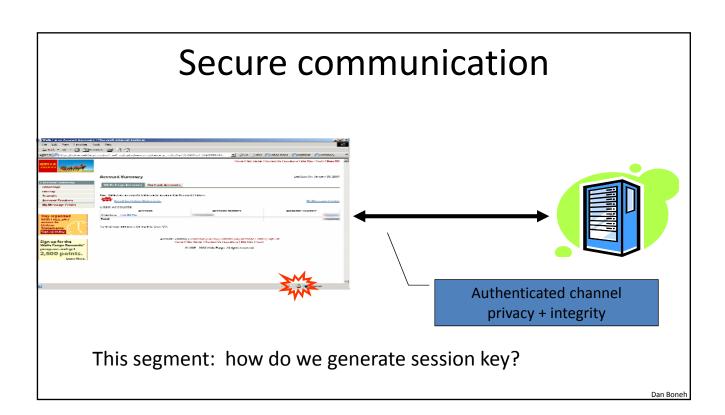


## Software Security Foundations: Crypto concepts II

Dan Boneh, Stanford University

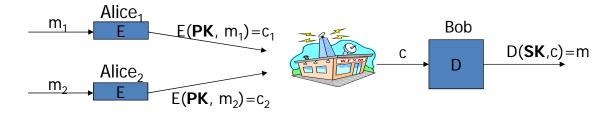




# Public key encryption

# Public-key encryption

Tool for managing or generating symmetric keys



- E Encryption alg. PK <u>Public</u> encryption key
- D Decryption alg.
- SK Private decryption key

Algorithms E, D are publicly known.

### Building block: trapdoor permutations

- 1. Algorithm KeyGen: outputs PK and SK
- 2. Algorithm F(PK, ·): a one-way function
  - Computing y = F(PK, x) is easy
  - One-way: given random y finding x is difficult
- 3. Algorithm  $F^{-1}(SK, \cdot)$ : Invert  $F(PK, \cdot)$  using trapdoor SK

$$F^{-1}(SK, y) = x$$

Dan Bonel

# Example: RSA

1. KeyGen: generate two equal length primes p, q

set 
$$N \leftarrow p \cdot q$$
 (3072 bits  $\approx$  925 digits)

set 
$$e \leftarrow 2^{16} + 1 = 65537$$
;  $d \leftarrow e^{-1} \pmod{\varphi(N)}$ 

$$PK = (N, e)$$
;  $SK = (N, d)$ 

2. RSA(PK, x):  $x \rightarrow (x^e \mod N)$ 

Inverting this function is believed to be as hard as factoring N

3. 
$$RSA^{-1}(SK, y) : y \rightarrow (y^d \mod N)$$

# Public Key Encryption with a TDF

KeyGen: generate PK and SK

 $c_0$ 

#### Encrypt(PK, M):

- choose random  $x \in domain(F)$  and set  $k \leftarrow H(x)$
- $\quad \quad c_0 \leftarrow \text{ F(PK, x)} \quad , \quad c_1 \leftarrow \text{ E(k, M)} \qquad \quad \text{(E: symmetric cipher)}$
- send  $c = (c_0, c_1)$

Decrypt(SK, c=(c<sub>0</sub>,c<sub>1</sub>)): 
$$x \leftarrow F^{-1}(SK, c_0)$$
,  $k \leftarrow H(x)$ ,  $M \leftarrow D(k, c_1)$ 

security analysis in crypto course

)an Ronah

# Digital Signatures

# Digital signatures

Goal: bind document to author

• Problem: attacker can copy Alice's sig from one doc to another

Main idea: make signature depend on document

**Example**: signatures from trapdoor functions (e.g. RSA)

sign(SK, m) := 
$$F^{-1}$$
(SK, H(m))  
verify(PK, m, sig) := accept if  $F$ (PK, sig) = H(m)

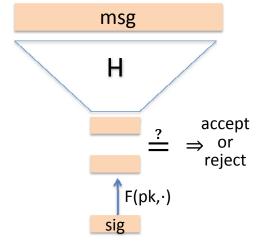
Dan Boneh

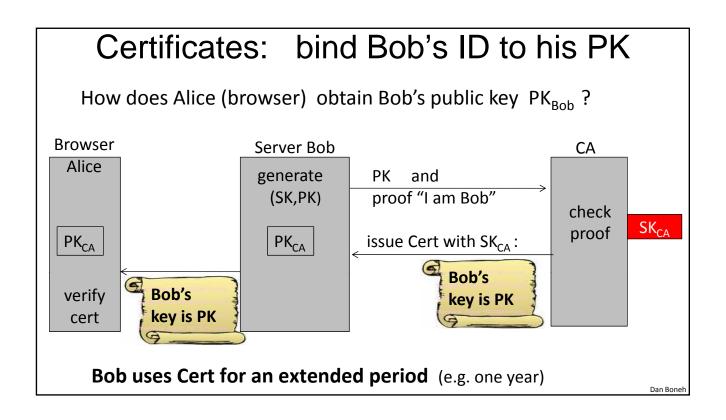
# Digital Sigs. from Trapdoor Functions

#### sign(sk, msg):

# msg H ↓ F<sup>-1</sup>(sk,·) sig

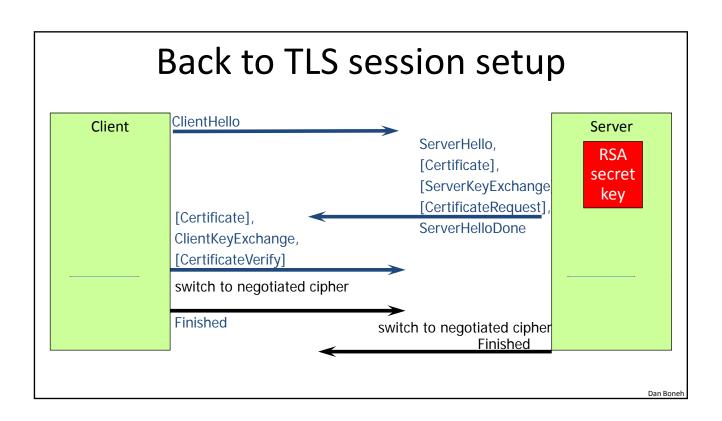
#### verify(pk, msg, sig):

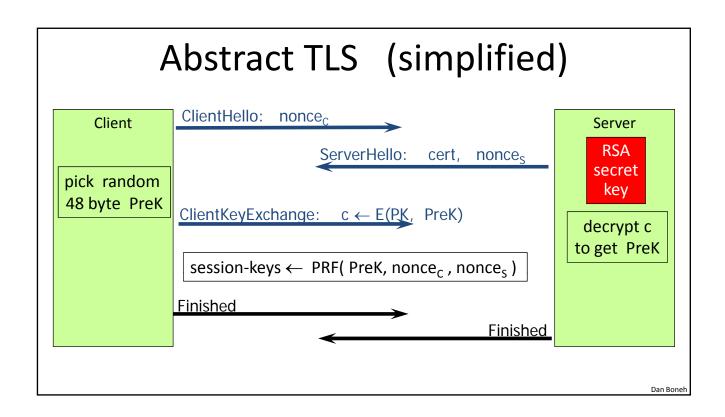






# Schematic SSL session setup





# **Properties**

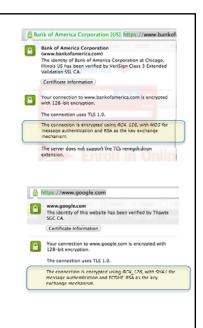
**Nonces**: prevent replay of an old session

#### No forward secrecy:

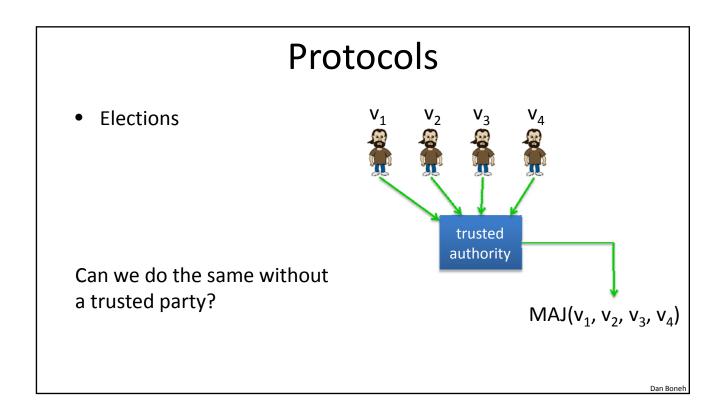
- Compromise of server secret key exposes old sessions
- TLS has support for forward secrecy

#### One sided identification:

- Browser identifies server using server-cert
- TLS has support for mutual identification
  - Rarely used: requires a client PK/SK and client-cert

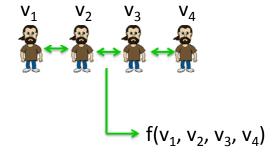


# A Brief Overview of Modern Crypto Tools





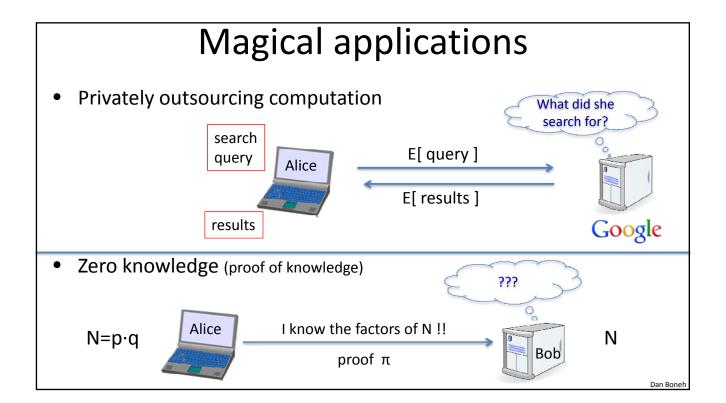
- Elections
- Private auctions



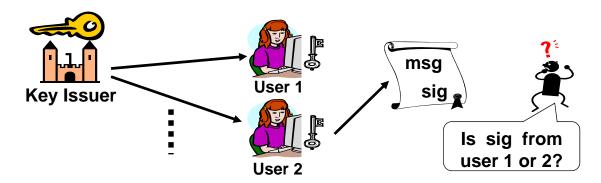
Goal: compute  $f(v_1, v_2, v_3, v_4)$ 

"Thm:" anything the can done with trusted auth. can also be done without

• Secure multi-party computation



# Privacy: Group Signatures

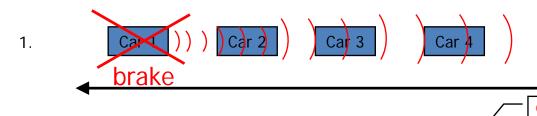


Simple solution: give all users same private key, but also need to:

- revoke signers when needed, and
- trace: trapdoor for undoing sig privacy.

Dan Boneh





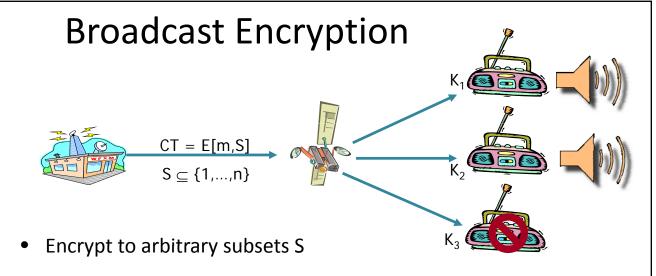
2.

Require authenticated (signed) messages from cars.

Prevent impersonation and DoS on traffic system.

<u>Privacy problem</u>: cars broadcasting <u>signed</u> (x,y, V).

Clean solution: group sigs. Group = set of all cars.



- Short ciphertexts
- Collusion resistance: secure even if all users in S<sup>c</sup> collude

Dan Boneh

# Summary

#### RSA Trapdoor permutation:

- Enables public-key encryption and digital signatures
- Used in TLS session setup

#### Certificates:

- Bind public key to an identity
- Used in TLS to identify server (and possibly client)

Modern crypto: goes far beyond basic encryption and signatures