Assignment 1, TMA4300 Computer intenstive statistical methods
Student numbers: 750338,

Problem A: Stochastic simulation by the probability integral transform

1) Exponential distribution

One may use the the probability integral transform (PIT) to generate random samples from a exponential distribution. Given the cumulative distribution function $F(x) = 1 - exp(-\lambda x)$ with rate λ, the inverse function is easily computed as $F^{-1}(F(x)) = -\frac{1}{\lambda}\log(1 - f(x))$. Using the PIT method an algorithm producing n samples is then given by:

In order to check if this is a proper generator:

1) Another probability distribution

Given the probability distribution:

$$f(x) = \frac{ce^{\alpha x}}{(1 + e^{\alpha x})^2}, \quad -\infty < x < \infty, \quad \alpha > 0, \tag{1}$$

where c is a normalizing constant. Integrating over the whole x-space, c must satisfy:

$$c\int_{-\infty}^{\infty} \frac{e^{\alpha x}}{(1 + e^{\alpha x})^2}dx = \frac{c}{\alpha}\left[\frac{e^{\alpha x}}{1 + e^{\alpha x}}\right]_{-\infty}^{\infty} = \frac{c}{\alpha} = 1, \tag{2}$$

which implies that $c = \alpha$.

Cumulative distribution and inverse function

The cumulative distribution can now be found by definition

$$F(x) = \alpha\int_{-\infty}^{x} \frac{e^{\alpha x'}}{(1 + e^{\alpha x'})^2}dx' = \left[\frac{e^{\alpha x'}}{1 + e^{\alpha x'}}\right]_{-\infty}^{x} = \frac{e^{\alpha x}}{1 + e^{\alpha x}}. \tag{3}$$

The inverse function, $g(y)$, can easily be found by direct computation:

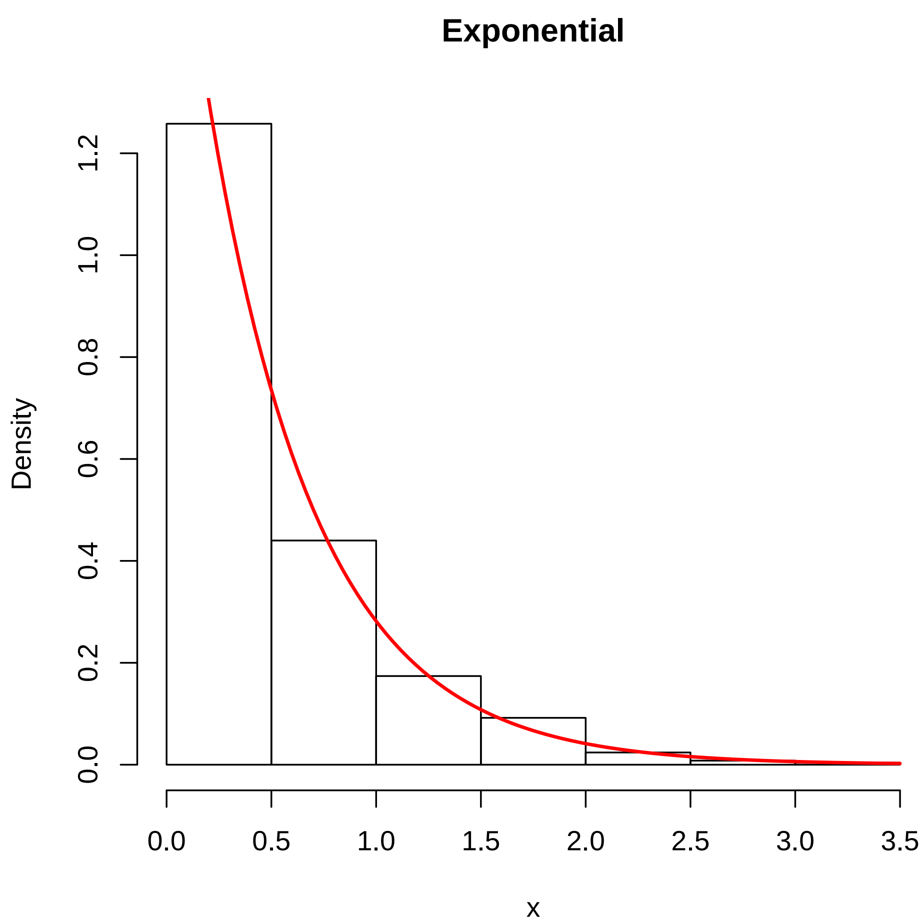
$$g(y) = \frac{1}{\alpha}\log\frac{y}{1 - y}. \tag{4}$$

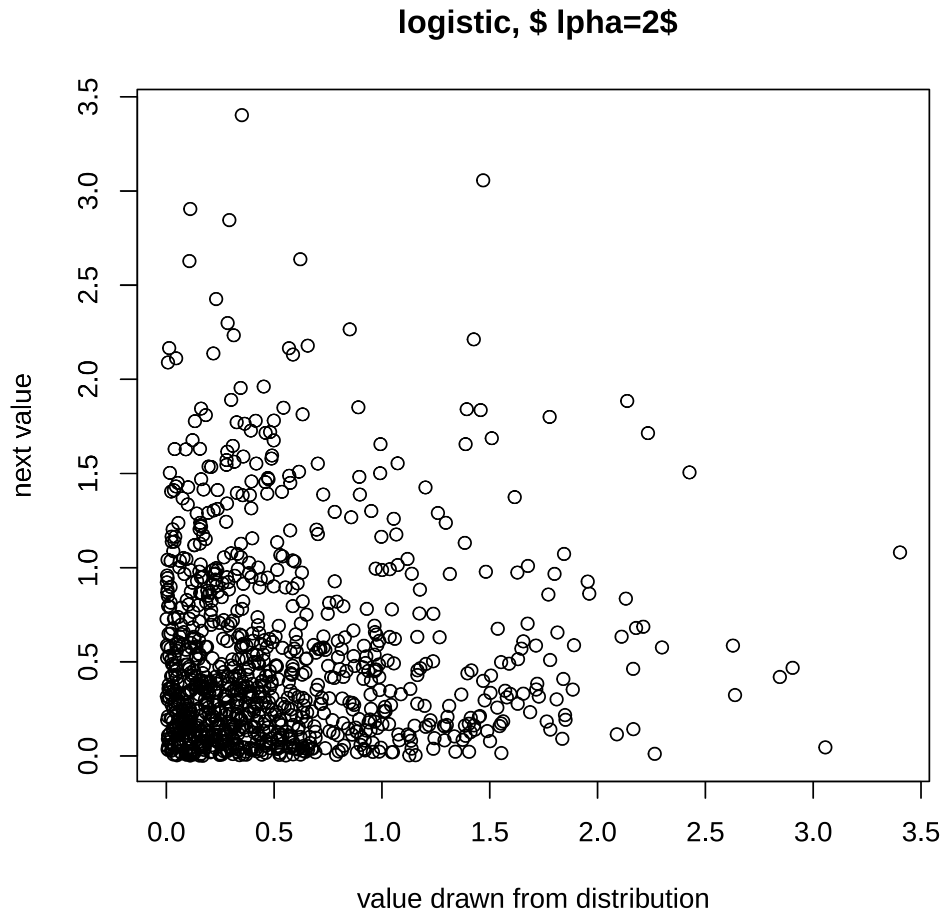
In fact, this distribution is a logistic distribution with mean 0 and scale parameter, $\frac{1}{\alpha}$. In order to generate samples from this distribution, the same method is used and the algorithm is given by:

As before we chech how proper this generator is:

```
> expgenerator = function (lambda, n){
+
+   u = runif(n);
+
+   x = (-1/lambda)*log(1-u)
+
+ return(x)
+
+ }
>
```

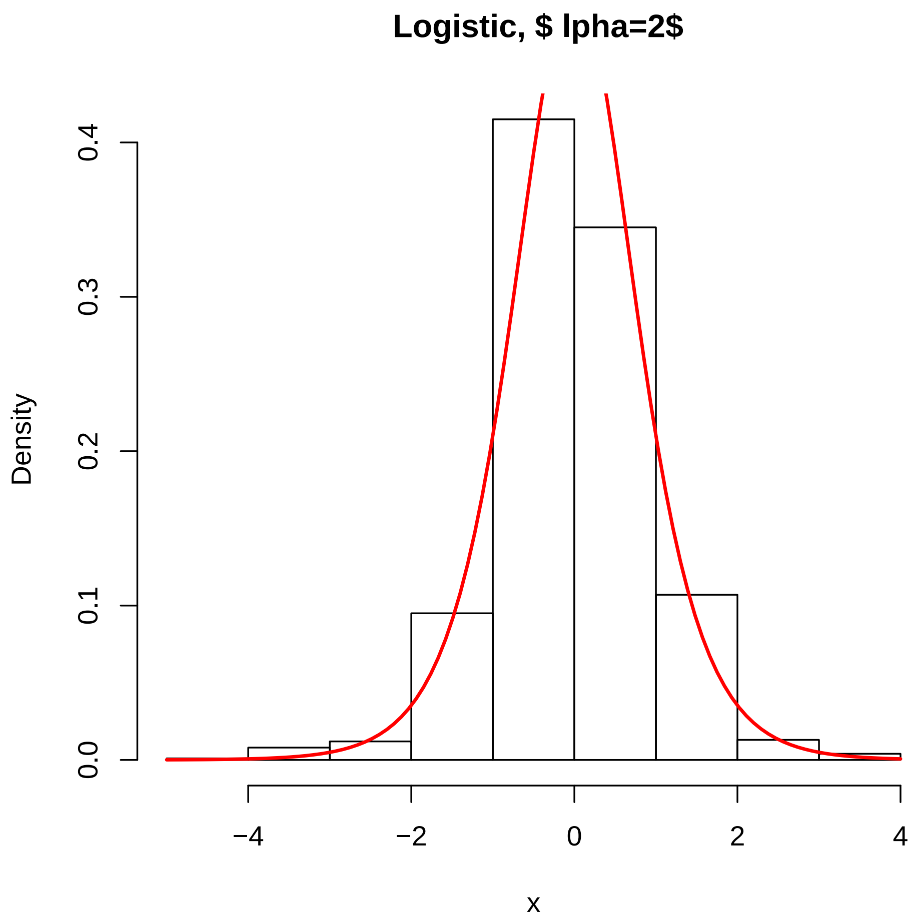
```
    rate
1.917844
```



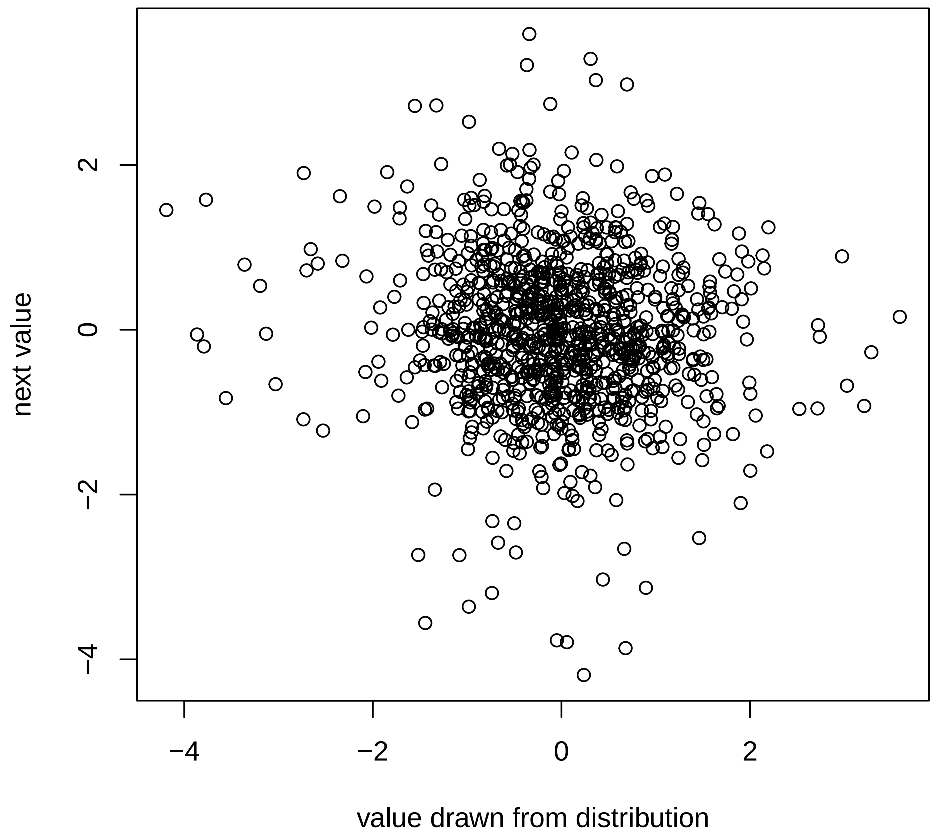



```
> logisticgenerator = function (alpha, n=1000){
+
+   u = runif(n);
+
+   x = (1/alpha)*log(u/(1-u))
+
+   return(x)
+
+ }
>
```

```
   location       scale
-0.02581635  0.51653561
```



logistic, $ lpha=2$



A3 - **(a)** With probability density function

$$g(x) = \begin{cases} 0 & x \leq 0 \\ cx^{\alpha-1} & 0 < x < 1 \\ ce^{-x} & 1 \leq x \end{cases} \tag{5}$$

the cumulative distribution becomes

$$G(x) = \begin{cases} 0 & x \leq 0 \\ \frac{c}{\alpha}x^{\alpha} & 0 < x < 1 \\ \frac{c}{\alpha} + \frac{c}{e} - ce^{-x} & 1 \leq x \end{cases} \tag{6}$$

and

$$1 = \lim_{x \to \infty} (\frac{c}{\alpha} + \frac{c}{e} - ce^{-x}) = c(\alpha^{-1} + e^{-1}) \tag{7}$$

such that $c = \frac{\alpha e}{e+\alpha}$. Since $G(1) = (1 + e^{-1})^{-1} = 0.731$ the inverse function will be defined differently beyond this point. The inverse function becomes

$$x = \begin{cases} (\frac{c}{\alpha}G)^{\frac{1}{\alpha}} & 0 < G < (1 + e^{-1})^{-1} \\ -log(1-G) + log(c) & (1 + e^{-1})^{-1} \leq G+;. \end{cases} \tag{8}$$

A3 - **(b)** recognizing the distribution and plotting it against the real thing.

C1 - **(a)** Box-Muller algorithm for iid standard normal. The goal is to generate n independent samples from the standard normal distribution. The density of a standard normal distribution is

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{x^2}{2}} \tag{9}$$

plus some explanaition of why thhe algorithm work (find it in my notes which are at home).

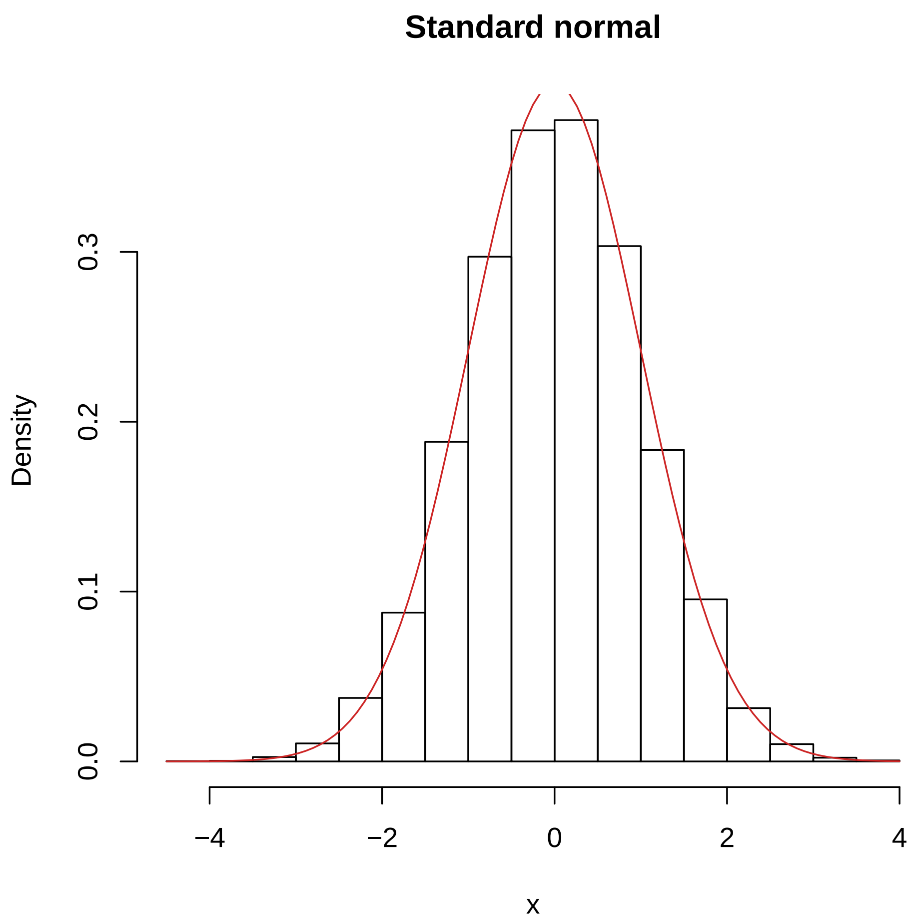
and now we draw from it to check wether it behaves as expected

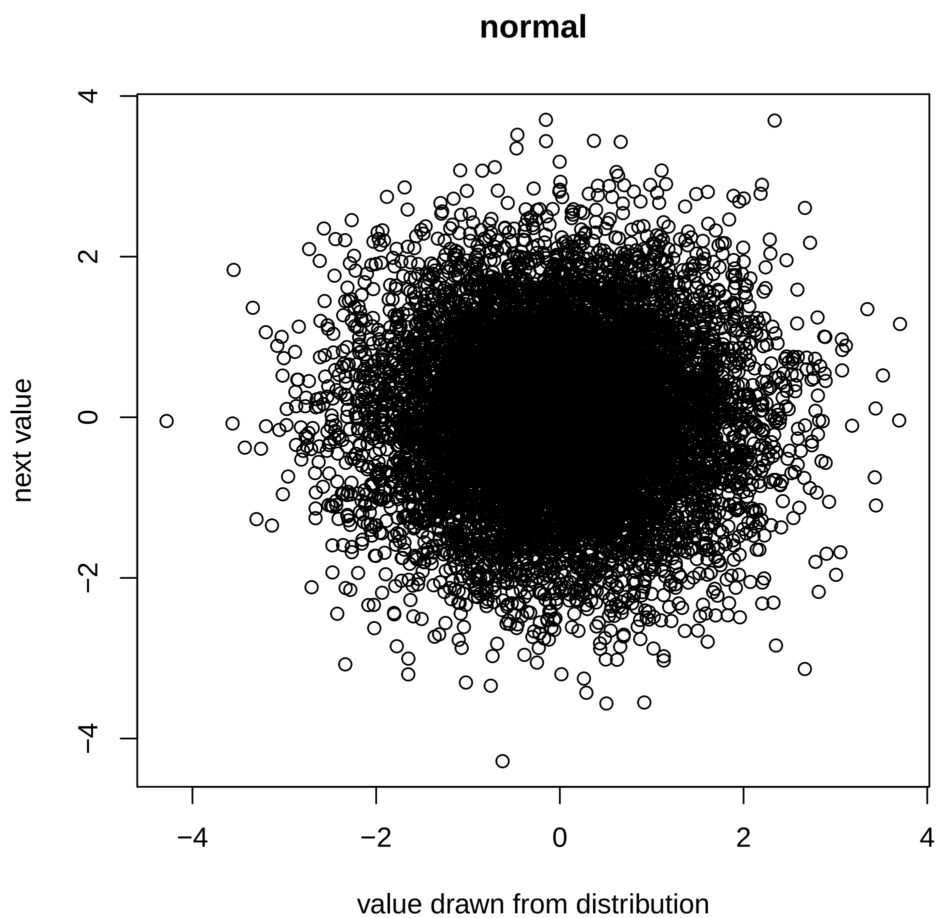
References

[1] http://www.investopedia.com/articles/pf/10/credit-score-factors.asp

```
> stdnormalgenerator = function (n=1000){
+
+   u = 2*pi*runif(n/2)
+   x = expgenerator(1/2,n/2)
+
+   return(c(sqrt(x)*sin(u), sqrt(x)*cos(u)))
+
+ }
```

```
         mean            sd
-0.0009305537  1.0113502094
```




normal
next value
value drawn from distribution
4
2
0
−2
−4
−4
−2
0
2
4