TMA4285 Time Series Models, Kalman filter

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August 30, 2018

1 State space model

First equation is underlying mechanism, second equation is observation. First observation has its own distribution.

$$\begin{aligned} Y_{t+1} &= AY_t + Ga_{t+1}\ , & a_t &\sim N(0,\Sigma) \\ Z_t &= HY_t + b_t\ , & b_t &\sim N(0,\Omega) \\ Y_0 &\sim N(\mu_0, \Sigma_0) \end{aligned} \tag{1}$$

2 Filtering Equations

We want to know something about the reals states Y from observing the observable states Z. Define

$$\hat{Y}_{t+1|t} = E(Y_{t+1}|Z_0,\dots,Z_t)\ , \tag{2}$$
$$V_{t+1|t+1} = Var(Y_{t+1}|Z_0,\dots,Z_t)\ , \tag{3}$$

which are from 0 to t according to lecture notes page 48. [1] The filtering equations are

$$\begin{aligned} \hat{Y}_{t+1|t+1} &= \hat{Y}_{t+1|t} + K_{t+1}(Z_{t+1} - H\hat{Y}_{t+1|t}\ , \\ \hat{Y}_{t+1|t} &= A\hat{Y}_{t|t}\ , && (4) \\ V_{t+1|t+1} &= (I - K_{t+1}H)V_{t+1|t}\ , \\ V_{t+1|t} &= AV_{t|t}A^T + G^T\ , && (5) \\ K_{t+1} &= V_{t+1|t}H^T(HV_{t+1|t}H^T - \Omega)^{-1}\ . \end{aligned}$$

2.1 Proofs of Filtering equations

Derive (4) from (1) and (2) like this:

$$\begin{aligned} \hat{Y}_{t+1|t} &= E(Y_{t+1}|Z_0,\dots,Z_t)\ , \\ \hat{Y}_{t+1|t} &= E(AY_t + Ga_{t+1}|Z_0,\dots,Z_t)\ , \\ \hat{Y}_{t+1|t} &= A\hat{Y}_{t|t}\ . \end{aligned}$$

[1] Might it sometimes in derivations mean for $s \leq t$?

Derive (5) from (1) and (3) like this:

$$V_{t+1|t} = Var(Y_{t+1}|Z_0, \ldots, Z_t)\ ,$$
$$V_{t+1|t} = Var(AY_t + Ga_{t+1}|Z_0, \ldots, Z_t)\ ,$$
$$V_{t+1|t} = AV_{t|t}A^T + G\Sigma G^T.$$

Now when this is done we try our hand at the derivation from the LF in H12:

3 Done! :D

And we should check that everything turns out alright for $\Omega = 0, H = I$, like some other exam. :) It does