LifetimeReport2

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August 30, 2018

a)

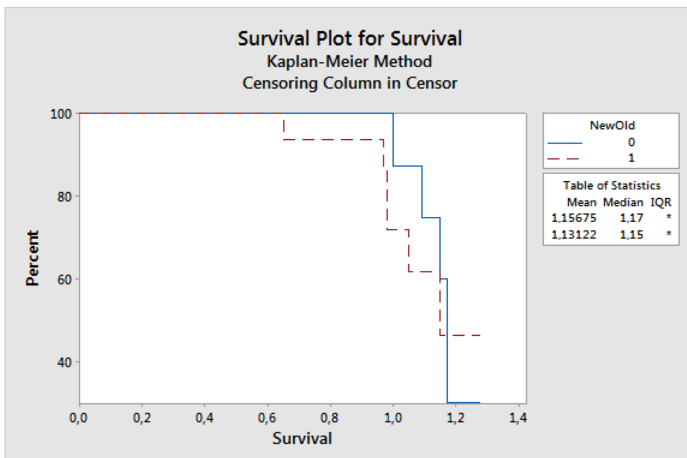

Figure 1: Kaplan-Meier plot for the survival functions of **NewOld**.

Let E_i denote the expected life time and M_i denote the median life the two data sets. Then we can see in Figure 1 that

$$E_0 = 1.15675 \qquad M_0 = 1.17$$
$$E_1 = 1.13122 \qquad M_1 = 1.15$$

From the log-rank test we conclude with p-value less than 0.001 that the two samples does not come from the same distribution. Further we see that given the age of a tire is below 1, the expected value and the median for the survival functions is 1.15675 and 1.17 respectively. In the other case, given that the age is above 1, we see that the mean and median for the survival functions will be 1.13122 and 1.15 respectively. These numbers are not significantly different, so it is reason to believe that Tire Age has no significant impact on the reliability for the tires.

b)

We perform a Weibull regression in MINITAB as described and the result is shown in Table 1. We see from 1 that the Shape parameter is calculated to be

$$\alpha = 16.7698$$

This value is greater than 1, which means that the underlying distribution has an increasing failure rate. The number is quite big and means that the graph of the probability density will have a clear spike after

Predictor	β_{Weib}	St.e.	p-value	Article β_{Cox}	Art St.e.
Tire age	-0.056	0.0717	0.435	2.109	1.393
Wedge gauge	0.618	0.2331	0.008	-9.686	4.638
Interbelt gauge	0.678	0.3058	0.027	-10.677	4.617
EB2B	0.735	0.5293	0.165	-13.675	8.112
Peel force	1.976	0.7806	0.011	-34.293	13.651
Carbon black	2.798	2.0750	0.178	-48.359	33.448
W x P	-1.239	0.4759	0.009	20.839	8.860
Shape	16.7698	4.32613			

Table 1: The parameters from the Weibull regression with the corresponding standard errors and the p-values. Covariates with significant effect are marked in bold. The last two columns are the values from Table 2 in the article.

some time. This means there will be a relatively small difference in lifetimes between tires with the same properties.

The estimated model can be shown from Table 1 to be

$$\begin{aligned}\ln(T) = &- 5.187 - 0.056x_{\text{Tire age}} + 0.618x_{\text{Wedge gauge}} + 0.678x_{\text{Interbelt gauge}} \\ &+ 0.735x_{\text{EB2B}} + 1.976x_{\text{Peel force}} + 2.798x_{\text{Carbon black}} - 1.239x_{\text{W x P}} + 0.0596W,\end{aligned}$$

where $W \sim \textit{Gumbell}(0,1)$ and the parameter for W equal to $1/\alpha$ [slide 9, p. 11]. We see that our values β_{Weib} does not correspond well to the respective values β_{Cox} from the article. This is not unexpected as they come from two different models, and we will calculate our corresponding β_{Cox}-values in (f). Further, the value for the log likelihood by MINITAB is calculated to be

$$l_{\text{full}} = 10.056.$$

c)

We now perform the regression with only the significant covariates shown in (b). Table 2 shows the new covariates. As the p-values are low, we have a good indication that the underlying model follows a Weibull model. As in (b), our values for β_{Weib} is not similar to the values β_{Cox} in Table 1. This will be elaborated in (f) and they are not expected to be so. From Table 2 our reduced model becomes:

$$\ln(T) \sim -1.360 + 0.571x_{\text{Wedge gauge}} + 0.468x_{\text{Interbelt gauge}} + 1.627x_{\text{Peel force}} - 1.094x_{\text{W x P}} + 0.0597W,$$

where $W \sim \textit{Gumbell}(0,1)$ according to the model obtained by this regression. The calculated value by MINITAB for the Log-Likelihood is

$$l_{\text{reduced}} = 6.621$$

Predictor	β_{Weib}	St.e.	p-value
Intercept	-1.360	0.316	>0.001
Wedge gauge	0.571	0.184	0.002
Interbelt gauge	0.468	0.145	0.001
Peel force	1.627	0.452	>0.001
W x P	-1.094	0.292	>0.001
Shape	16.749	4.124	

Table 2: Weibull regression with only the significant covariates with corresponding standard errors and p-values. The last two columns are the values from Table 2 in the article.

d)

We list the criterions for a good and a poor tire in Table 3. Further, we have calculated the lower quartiles, medians and the upper quartiles for the survival functions of a poor and a good tire in Table 4. Table 5 shows the estimated probabilities for a good and a bad tire to survive beyond the times 1.0, 1.2 and 1.5.

Definitions	Wedge gauge	Interbelt gauge	Peel force	W x P
Good Tire	1.2	1.0	2.0	2.4
Poor Tire	0.5	0.5	1	0.5

Table 3: Definition of a "Good" and "Poor" tire.

	Lower quartile	St.e.	Median	St.e.	Upper quartile	St.e.
Good Tire	1.417	0.318	1.493	0.338	1.556	0.356
Poor Tire	1.180	0.131	1.244	0.138	1.296	0.145

Table 4: Survival times for "Good" and "Poor" tires calculated from the model obtained in c).

Time	1	1.2	1.5
Good Tire	0.999	0.982	0.473
Poor Tire	0.982	0.684	$>$0.001

Table 5: Survival Probabilities beyond different times as calculated from the parameters from c) for tires defined as "Good" and "Poor" by their specifications.

e)

We have the following three Weibull regression models:

$$\begin{aligned} A_0 : \ln(T) = &- 5.187 - 0.056x_{\text{Tire age}} + 0.618x_{\text{Wedge gauge}} + 0.678x_{\text{Interbelt gauge}} \\ &+ 0.735x_{\text{EB2B}} + 1.976x_{\text{Peel force}} + 2.798x_{\text{Carbon black}} - 1.239x_{\text{W x P}} + 0.0596W, \end{aligned}$$

$$A_1 : \ln(T) = -1.360 + 0.571x_{\text{Wedge gauge}} + 0.468x_{\text{Interbelt gauge}} + 1.627x_{\text{Peel force}} - 1.094x_{\text{W x P}} + 0.0579W,$$

$$A_2 : \ln(T) = 1.159 + 0.115W,$$

where $W \sim \text{Gumbel}(0, 1)$ and A_0, A_1 and A_2 corresponds to the full model, the reduced model and the model with no covariates respectively. We compare two Weibull regression models with m and $n < m$ covariates by knowing that

$$2(l_m - l_n) \sim \chi^2_{m-n}, \tag{1}$$

where l_m and l_n are the respective Log-Likelihood values for the respective models. We must therefore perform three hypothesis tests for comparing all the models with each other. From Table 6, we can compute the respective critical values.

Model	# of covariates	l_i
A_2	7	10.056
A_1	4	6.621
A_0	0	-4.958

Table 6: Corresponding Log-Likelihood values for the regression models.

We start by calculating the critical value and setting the null- and the alternative hypothesis for A_0 and A_1:

$$2(l_7 - l_4) = 6.87 \sim \chi^2_3$$

$$H_0 : \ln(T) \sim A_2, \qquad H_1 : \ln(T) \sim A_1$$

We know that $\chi^2_{0.05,3} = 7.815$. As $2(l_7 - l_4)$ is less than $\chi^2_{0.05,3}$, we reject the null hypothesis. Hence, we prefer the reduced model over the full model.

We now do the same with A_2 and A_0:

$$2(l_7 - l_0) = 30.028 \sim \chi^2_7$$

$$H_0 : \ln(T) \sim A_2, \qquad H_2 : \ln(T) \sim A_0$$

From the statistics table we have that $\chi^2_{0.05,7} = 14.067$, which is less than $2(l_7 - l_0)$. Hence, we do not reject the null hypothesis and we therefore prefer the full model over the model with no covariates.

At last we compare A_1 with A_0

$$2(l_4 - l_0) = 23.158 \sim \chi^2_4$$

$$H_0 : \ln(T) \sim A_1, \qquad H_2 : \ln(T) \sim A_0$$

The statistics table says that $\chi^2_{0.05,4} = 9.488$. Since $2(l_4 - l_0)$ is bigger, we do not reject the null hypothesis and we will prefer A_1 over A_0.

From the three hypothesis tests, we conclude with preferring A_1 (the reduced model) over the other models.

f)

From [Slide 12, p. 18] and Table 7, we have that the Cox model can be written as

$$z(t, x) = z_0(t)e^{9.563x_{\text{Wedge gauge}}+0.468x_{\text{Interbelt gauge}}+1.627x_{\text{Peel force}}-1.094x_{\text{W x P}}}, \tag{2}$$

where $z_0(t)$ is any function of t. The main difference between the Weibull model and the Cox model is that $z_0(t)$ in the Cox model is arbitrary. Slide 12, p. 18 also gives the following relation between the covariates in the models:

$$\beta_{\text{Cox}} = -\alpha\beta_{\text{Weib}} \tag{3}$$

From this we can find our corresponding values for β_{Cox} and compare it with the values from the article in Table 7. We see that there is a strong correlation between our obtained values for β_{Cox} and the values found in the article.

Predictor	β_{Weib}	β_{Cox}	Article β_{Cox}
Intercept	-1.360		
Wedge gauge	0.571	-9.563	-9.313
Interbelt gauge	0.468	-7.838	-7.069
Peel force	1.627	-27.250	-27.411
W x P	-1.094	18.323	18.105

Table 7: Calculation of the estimated β_{Cox}-values in Column 3. Column 4 shows the respective values from Table 3 in the article.

Further we know from [Slide 12, p.3] that the hazard rate functions for the Weibull model can be written as

$$z(t) = \alpha t^{\alpha-1}\theta^{-\alpha}, \tag{4}$$

where

$$\ln(\theta) = -1.360 + 0.571x_{\text{Wedge gauge}} + 0.468x_{\text{Interbelt gauge}} + 1.627x_{\text{Peel force}} - 1.094x_{\text{W x P}} + 0.0579W.$$

The hazard rate functions for the Weibull- and the Cox model in Equation 2 and 4 may seemingly look identical. The difference lies in that in the Cox-model $z_0(t)$ can be arbitary. For the Weibull model, $z_0(t)$ will end up being uniquely determined.

g)

We have that a multiple survival regression model can be written as

$$\ln(T) = \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + \sigma U,$$

where $\sigma = 1/\alpha$. Then we have that

$$U_i = \frac{\ln(T) - \beta_0 - \beta_1 x_{ki} - \cdots - \beta_k x_{ki}}{\sigma}$$

Then the standardized residuals is defined to be

$$\hat{U}_i = \hat{\alpha}(\ln(Y_i) - \hat{\beta}_0 - \hat{\beta}_1 x_{1i} - \cdots - \hat{\beta}_k x_{ki})$$

Further, the probability plot for the standardized residuals for the full and the reduced Weibull model is plotted Figure 2. The full model looks to deviate most from our model with more extreme values, but also in terms of general deviation of the points. This verifies that the reduced model is more preferable. In Figure 4 and Figure 3 the same residual plots for the Lognormal- and the Loglogistics model are also considered. Now we plot the probability plots for the standardized residuals for the Loglogistics- and the

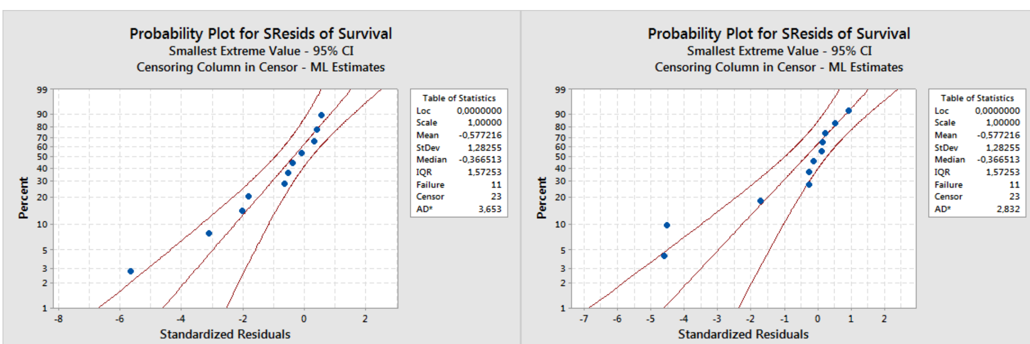

Figure 2: The probability plot for the standardized residuals for the full Weibull model (right) and the reduced Weibull model (left).

Lognormal models in Figure 4 and Figure 3 respectively. For the full model every model has at least one point that is either off or close to being off the confidence interval. The full Lognormal model looks to fit slightly better than the other full models, but the point does however has some deviation from the expected model. For the reduced model the residuals looks to fit the Loglogistics- and the Lognormal somewhat better than the Weibull model, but it is hard to tell which one is the best.

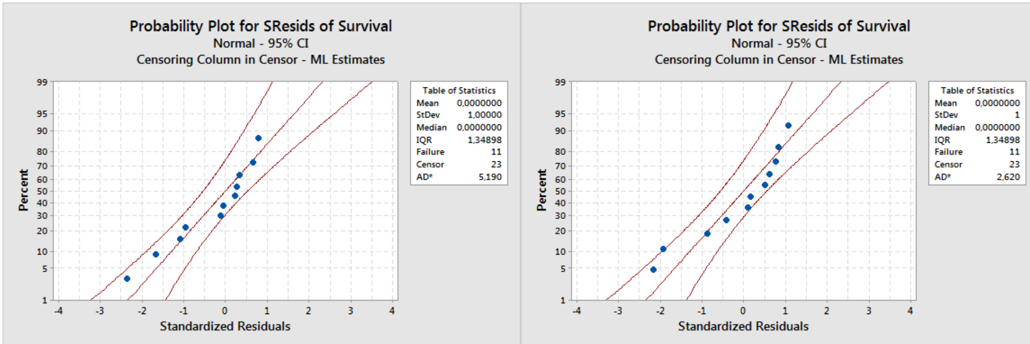

Figure 3: The probability plot for the standardized residuals for the full Lognormal model (right) and the reduced Lognormal model (left).

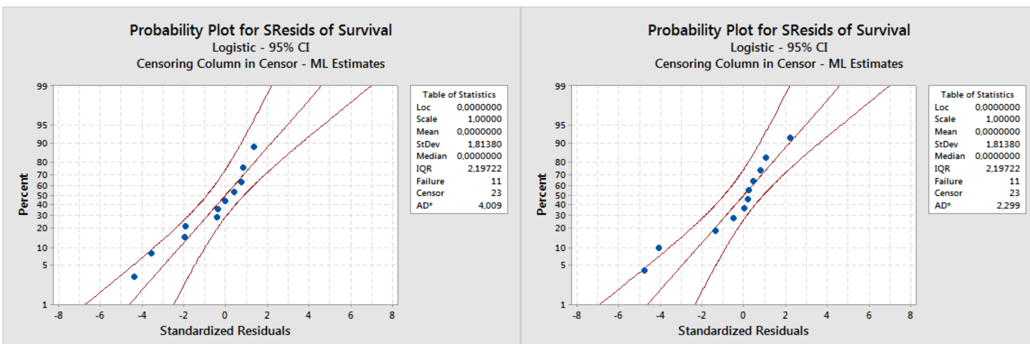

Figure 4: The probability plot for the standardized residuals for the full Loglogistics model (right) and the reduced Loglogistics model (left).

h)

Following the given instructions, we end up with Figure 5. The probability plot for the Weibull is know to come from the following equation

$$\ln(-R(t)) = \alpha \ln(t) - \alpha \ln(\theta),$$

where θ is the Scale-parameter, α is the slope-parameter and $R(t)$ is the survival function. Since α is identical for both lines, they will be parallell and $\alpha \ln(\theta)$ corresponds to the shift in the lines. The 'good' tires show very little deviation in the probability plot, while the bad tires show some deviation with one extreme point which is due to a tire that failed very early. MINITAB gives the covariate **PoorGood** a p-value of 0.010, which means that we can not reject the model as an Weibull model under 5% significance.

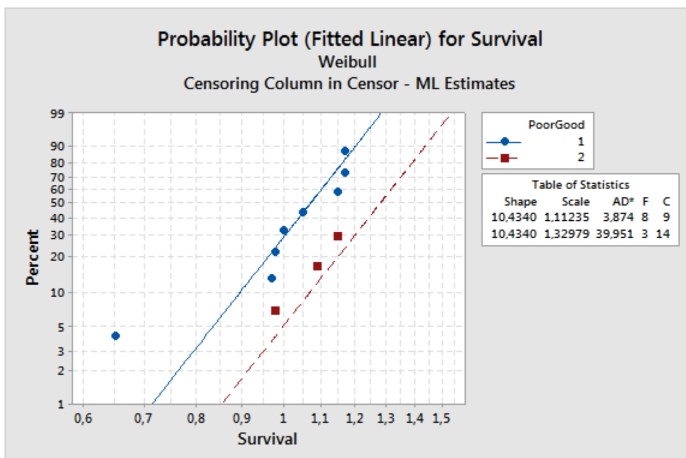

Figure 5: The probability plot for the survival functions with the variable **PoorGood** as response.