Heat Pump

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Abstract

Heat transfer from a cold reservoir to a hot reservoir and the required work is examined by using R-134a as a cooling liquid in a heat pump that transfers heat between two buckets of 4.49 ± 0.2 L of water with initial temperatures 16.1 ± 0.1 °C and 16.5 ± 0.1 °C , and final temperatures 0.1 ± 0.2 °C and 37.7 ± 0.2 °C exploiting isenthropic and isenthalpic pressure changes of the cooling liquid and heat connected to isobaric phase transfers. After 10 minutes the efficiency of the corresponding Carnot cycle, the theoretical cycle and the machinery of the experiment were $\eta_{\mathrm{C}} = 15.7 \pm 0.16$, $\eta_{\mathrm{t}} = 11.0 \pm 1.3$ and $\eta_{\mathrm{exp}} = 2.76 \pm 0.08$, which points to an energy loss per time in the physical heat pump of
$\Delta P_{10} = (8.7 \pm 0.3) \cdot 10^2$ W, which equals 75% of the power input. The energy loss is expected to be mostly due to friction and other energy loss in the compressor and its wiring.

1 Introduction

Heat transfer from a hot place to a cold place is something we are used to from our everyday life. -10 °C is a common outside temperature in Norway in the winter while 20 °C is more popular as a temperature indoors. Even when a building is insulated, heat will leak from indoors to outdoors under such conditions. The heat transfer will make inside temperature drop and outdoor temperature increase. In this case it would be great to be able to transfer heat from a cold place to a hot place. This is what a heat pump does. Peter von Rittinger designed and built the first energy-conserving heat pump in 1857 [1]. Today the heat pump is used both in refrigerators to cool, and in houses to heat.

In this experiment a heat pump is used to transfer heat from a cold place to a hot place. To make this happen the heat pump must do work. If doing this should make any sense, the heat transferred to the hot place should be bigger than the work we apply, else it had been smarter to convert work into heat directly. We will investigate the amount of heat it is possible to transfer to the hot place per amount of work done by the heat pump at different temperatures of the hot and cold places.

2 Theory

The first law of thermodynamics [2] states that for any process where heat Q passes to a system which does work W, the increase in internal energy

$$\Delta U = Q - W, \tag{1}$$

because it equals the sum of added energy.

The second law of thermodynamics is by the Clausius statement [3] that it is not possible for the only result of a process to be heat transported from a body of lower temperature to a body of higher temperature, as illustrated in figure 1.

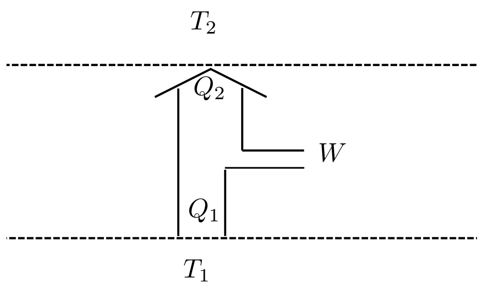

Figure 1: When $T_1 < T_2$ a work W is necessary to transfer a heat Q_1 from body one to body two. Total heat transfer to body two when there is no energy loss to the surroundings is $Q_2 = Q_1 + W$.

This motivates the definition of energy conversion efficiency

$$\eta = \frac{E_{\mathrm{output}}}{E_{\mathrm{input}}}, \tag{2}$$

where E_{output} is the amount of energy converted to the desired body in the desired form of energy, and E_{input} is the amount of energy used in the process.

In a Heat Pump heat is transported from a body with lower temperature to a body with higher temperature as illustrated in figure 1. The desired output energy of the process is heat transported to a body with higher temperature, so that

$$\eta_{\mathrm{HeatPump}} = \frac{Q_2}{W}. \tag{3}$$

Enthalpy H is defined [2] as

$$H = U + pV, \tag{4}$$

where V is volume and p is pressure. Due to this,

$$\Delta H = \Delta U + V\Delta p + p\Delta V. \tag{5}$$

By equation (1) and (4) and the volume work being

$$W = p\Delta V, \tag{6}$$

isobaric processes will have

$$\Delta H = Q. \tag{7}$$

For an isentropic process the small changes in entropy

$$dS = \frac{đQ_{\mathrm{rev}}}{T}, \tag{8}$$

equals zero throughout the process, so that the heat is also zero. For the isentropic process [2]

$$\Delta H = W, \tag{9}$$

as is graphed in the preasssure-enthalpy diagram in figure 2.

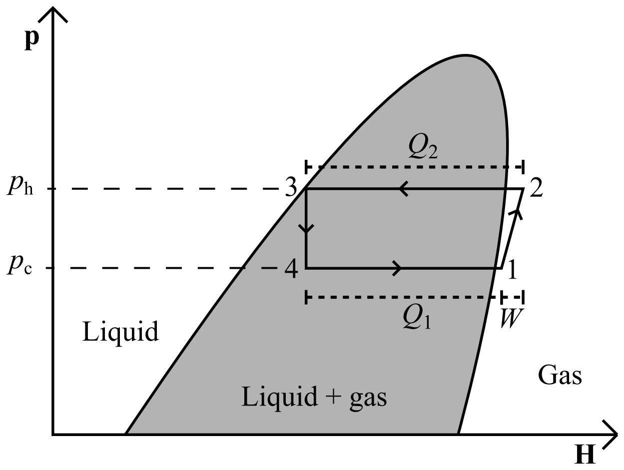

Figure 2: The figure shows a pressure-enthalpy diagram of a typical cooling liquid and the processes the cooling liquid in a heat pump goes through. The enumerations from one to four indicates different states of matter and pressures of the conducting material. (1, 2) gas sate, (3) liquid, (4) liquid + gas (mainly liquid), (2, 3) high pressure, (1, 4) low pressure. p_{c} is the pressure over the cooling liquid in the low temperature state and p_{h} is the pressure in the high temperature state. The energy transfers Q_1, Q_2 and W are the same as in figure 1. The figure is inspired by figure 1.4 in the laboratory guide [2] in the course TFY4165 at NTNU

A heat pump is typically two bodies thermally connected through a substance or cooling liquid that goes through a process cycle. The process cycle might consist of four processes that are isenthalpic, isobaric and isentropic as illustrated in the pressure-enthalpy diagram in figure 2. By looking at the values of the cooling liquids enthalpy in point 1, 2 and 3, the theoretical efficiency

$$\eta_{\mathrm{HP,t}} = \frac{H_2 - H_3}{H_2 - H_1}, \tag{10}$$

as $Q_2 = H_2 - H_3$ and $W = H_2 - H_1$. A heat pump uses that the temperature at wich a liquid evaporates depends on the pressure the liquid is under, and that the temperature stays constant while the internal energy increases when a liquid evaporates. At point 1 in figure 2 work is done to increase the pressure of the gas isentropically so that the temperature and evaporation point of the cooling liquid is higher than the temperature T_{h} of the hot body. The cooling liquid is now in state 2 in figure 2. In the process from point 2 to point 1 the cooling liquid will give off heat Q_2 to the hot body until they reach the same temperature. Since the evaporation point is higher than T_{h} the gas will condensate while emitting heat. The cooling liquid is now in point 3 in figure 2 and goes through an isenthalpic expansion from point 3 to 4. This expansion decreases the pressure, and thereby the evaporation point, as well as the temperature[1], until both the temperature and evaporation point is lower than T_{c}. Likewise as with the condensation, the cooling liquid will absorb heat, Q_1, from the cold reservoir when it evaporates while being heated by the cold body, the process between point 4 and 1 in figure 2. To maximize the efficiency of the heat pump the pressures p_{c} and p_{h} is chosen so that the evaporation points are as close to respectively T_{c} and T_{h} as possible. This way the difference in pressure is minimized as well as the work needed to raise the pressure from p_{c} to p_{h}, see figure 2.

Heat capacity C is defined [3] as

$$C \equiv \frac{Q}{\Delta T}, \tag{11}$$

where Q is the the heat absorbed/emitted and ΔT is the change in temperature. The specific heat capacity, c_m, of a substance is the heat capacity per unit of mass. The volumetric heat capacity of a substance is given by

$$c_{vol} = \frac{c_m}{\rho}, \tag{12}$$

where ρ is the mass density of the substance. Using equation (11) and (12) the necessary heat transfer to change the temperature of a substance with ΔT is

$$Q = c_{vol} \cdot V \cdot \Delta T, \tag{13}$$

where V is the volume of the substance.

The energy, W, used by an electrical machine is

$$W = P \cdot t, \tag{14}$$

where P is the electrical power and t is the time. By using equation (13) and (14), equation (3) can be rewritten to

$$\eta_{\mathrm{HP}} = \frac{c_{vol} \cdot V_{\mathrm{h}} \cdot \Delta T_{\mathrm{h}}}{P \cdot t}, \tag{15}$$

where V_{h} is the volume of the hot reservoir and ΔT_{h} is the change in the temperature in the hot reservoir.

A Carnot heat pump is a heat pump with maximal efficiency and no energy loss. The cycle process consists of isothermic and isentropic processes which makes the Carnot efficiency [3]

$$\eta_{\mathrm{HP,C}} = \frac{T_{\mathrm{h}}}{T_{\mathrm{h}} - T_{\mathrm{c}}} = \frac{T_{\mathrm{h}}}{\Delta T}, \tag{16}$$

where T_{c} and T_{h} is the respective temperatures in the cold and the hot heat reservoir.

[1]This is only the case of a Van der Wall's fluid (non ideal fluid) were the Joule-Thomson coefficient is greter than 0, which is the case for specific intervals of the temperature and pressure of the given fluid.

3 Method

The heat pump and the equipment used in the experiment is illustrated in figure 3, and the cooling liquid used in the heat pump is R-134a. When it is running the heat pump draws heat from the cold bucket (blue), where the pressure is low, and gives off heat to the warm bucket (red), were the pressure is high. There is a regulator linked up to the high pressure side of the heat pump to make sure the pressure never exceeds 15 bar. If this happens the regulator cuts of the compressors power. The expansion valve is also linked to the pipe by the outlet of the evaporator (grey line in figure 3). This is because the expansion valve is controlled by the cooling liquid's temperature on both sides of the evaporator to make sure the pressure fall from p_h to p_c is correct. Otherwise the heat pump works as described in the theory section.

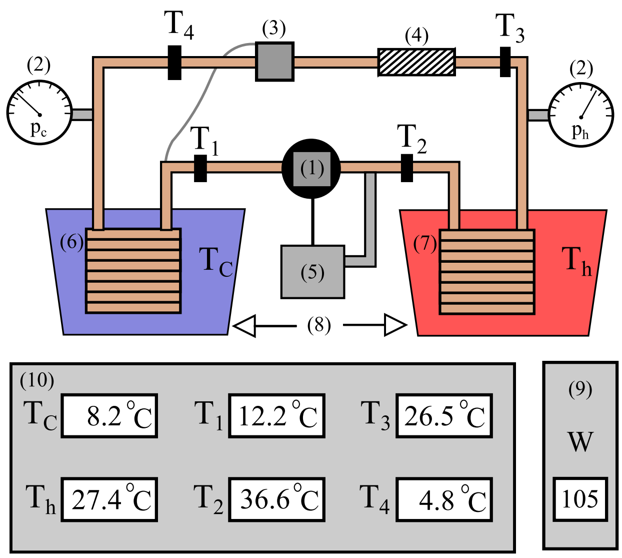

Figure 3: The figure shows a simplified version of the heat pump used in the experiment. (1) compressor, (2) pressure meters, (3) expansion valve, (4) filter, (5) regulator, (6) evaporator, (7) condenser, (8) buckets with water, (9) watt meter, (10) display panel for the thermometers (black squares marked with T and one inside each bucket).

Initially the buckets contain 4.5 L of water at approximately 15 °C. Turning on the compressor and noting of the temperatures, pressures and the compressor's power is delayed with 2 minutes after the buckets are placed in the position in figure 3, to allow for the thermometers in the buckets to stabilize. Then the mentioned variables are noted after 1 minute and for every 2 minutes over a time period of approximately 24 minutes, until the temperature in the cold bucket reaches 0 °C. During the time of the experiment the water in the buckets are constantly stirred to make the temperature in the buckets even.

4 Results and discussion

Figure 4 gives a graphic representation of the change in temperature of the water in the buckets during the experiment. All uncertainties in calculated values are derived using Gauss' law of uncertainty propagation.

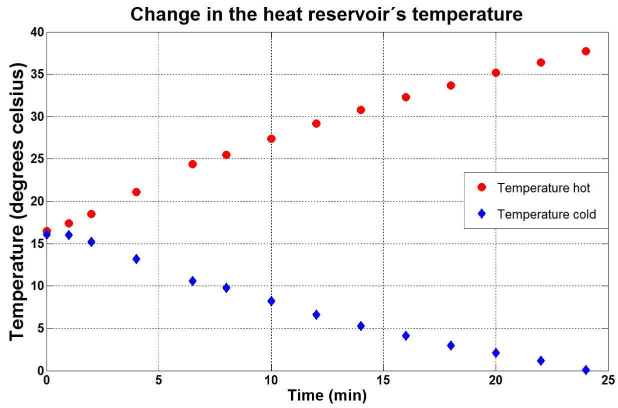

Figure 4: The figure shows a diagram of the change in temperature of the water in the two heat reservoirs. The red dots mark the warm bucket while the blue mark the cold bucket.

The specific heat capacity of water (H_2O) is $c_m = 4.2 \pm 0.02$ kJ/(K kg) [2][2] and the mass density of water is $\rho = 1.00$ kg/L [3], making the volumetric heat capacity of water $c_{vol} = 4.2 \pm 0.03$ kJ/(K L). The heat pump's experimental efficiency and theoretical maximal efficiency (Carnot) stated in table 1 is calculated from equation (15) and(16), with the differences in time t and temperature ΔT_h being the differences between the values of time and temperature in the sets of measurements preceding and succeeding the set of measurements from which the power P is taken.

Table 1: Table with the calculated theoretical (Carnot) and experimental efficiency of the heat pump.

Time (t) (min)	η_C (1)	η_exp (1)
1	207.4 ±29.6	2.92±0.09
2	88.3 ± 5.35	3.61±0.11
4	37.2 ± 0.94	3.63±0.11
6.5	21.6 ± 0.31	3.22±0.10
8	19.0 ± 0.24	2.60±0.08
10	15.7 ± 0.16	2.76±0.08
12	13.4 ± 0.12	2.58±0.08
14	11.9 ± 0.09	2.35±0.07
16	10.8 ± 0.08	2.22±0.07
18	9.99± 0.07	2.20±0.07
20	9.31± 0.06	2.05±0.06
22	8.79± 0.05	1.92±0.06

Table 1 shows that the theoretically maximal efficiency of a Carnot heat pump is more than three times the size of observed efficiency for every point in the cooling process. This can be explained by several factors. First

[2] c_m changes with temperature. In the given interval of the temperature in this experiment it changes with about 0.02 kJ/(K kg) [4].

[3] The uncertainty of ρ is negligible by comparison to the uncertainty of c_m.

the Carnot efficiency is above the maximum efficiency the heat pump may reach; As the heat pump contains an isentalpic process where the Carnot machine has an isentropic process, the heat pump does not work like a Carnot machine and may not reach the efficiency of the Carnot machine.

The theoretical efficiency of an ideal heat pump that use the processes in figure 2 is found by equation (10). The neccessary enthalpies are found from the pressure-enthalpy diagram of the cooling liquid R-134a in figure 5 by using the measurements of temperature 1 through 4 and p_c and p_H.

Figure 5 shows that the heat needed to evaporate the cooling liquid changes with the pressure at which the liquid evaporates. At 0 °C, $p \approx 3$ bar, the evaporation heat $Q_\mathrm{evap} \approx 202$ kJ/kg, while at 40 °C the evaporation heat $Q_\mathrm{evap} \approx 170$ kJ/kg, giving a difference of 32 kJ/kg.

The figure shows that if a heat pump is made with this R-134a as the cooling liquid at low pressures, the theoretical efficiency will be larger than at higher pressures, because the work required in the compression is approximately equal at equal differences in temperature between the hot and the cold reservoir, while the evaporation heat is bigger at lower temperatures. With the same gap in temperature between the hot reservoir and the cold reservoir the Carnot efficiency from equation (16) will drop with the temperature of the hot reservoir, while the theoretical efficiency of the R-134a heat pump will increase. As the Carnot efficiency according to the theory will be greater than the efficiency of the heat pump even at these temperatures, the difference between the efficiencies at temperatures about 290 K of the material is not expected to be minimal.

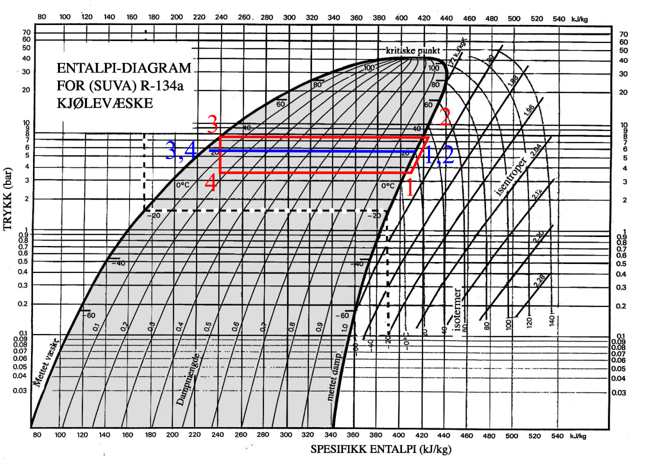

Figure 5: The figure shows the enthalpy diagram of the cooling liquid R-134a. The blue line represents the cooling liquid's cycle when the experiment started, while the red square represents the cooling liquid's cycle 10 minutes after the experiment started. The numbers indicates where in the cycle the cooling liquid is, see figure 2. The figure is a copy of figure 1.9 in the laboratory guide [2] in the course TFY4165 at NTNU

The processes are graphed in figure 5 for the process after 10 minutes, and for the process when $T_\mathrm{c} \approx T_\mathrm{h}$ in respectively red and blue. This gives the energy conversion efficiency after 10 minutes [4] to be

$$\eta_\mathrm{t} = 11 \pm 1.3,$$

which is 400% of the calculated experimental efficiency and 70% of the efficiency to a Carnot heat pump at this point. The reasons why experimental efficiency is so much lower than the theoretically calculated maximal efficiency will coincide with the following reasons why the Carnot efficiency is much higher than the observed efficiency as they come from the errors made in propositions such that the system does not lose heat to the surroundings.

Neither the copper pipes or the water buckets (heat reservoirs) are insulated. This way heat may leek in or out of the system and thereby disrupt the experimental values. The hot water will if hotter than the room temperature emit heat to the surrounding air, and vice versa. The cold water will always absorb heat from the surrounding air, and likewise the pipes may emit/absorb heat. Since it is the change in T_h that is used to calculate the efficiency it is the mentioned effect on the hot water that is of interest. Though there is a heat emitting effect for $T_\mathrm{h} > T_\mathrm{room}$, there is no way of telling the magnitude of it because there was not taken any measurements of how fast heat was emitted from the hot water. Either way it is possible to neglect this effect for the period of time when $T_\mathrm{h} \approx T_\mathrm{room}$, which was the case for the time between 4 and 10 minutes. Taken the pipes in consideration, the surface area of the pipes is so small compared to the water buckets' area, so the heat emitting/absorbing effect may be neglected for the pipes.

Possibly the biggest reason for why the experimental efficiency deviates from the theoretical value for the heat pump is the fact that most machines have some form of inner resistance. In this case the compressor may run with a lot of friction, causing a major energy loss to heat and sound. This also applies to the electrical wiring. This suspicion was confirmed by the fact that the compressor got warmer during the experiment, which is especially not unusual for old electrical equipment. As the total deviation ΔP of heat emitted per time to the hot reservoir that theoretically should have been produced versus what was emitted is

$$\Delta P = (\eta_\mathrm{t} - \eta_\mathrm{exp}) \cdot P_\mathrm{exp}, \tag{17}$$

making the power loss $\Delta P_{10} = (8.7 \pm 0.3) \cdot 10^2$ W of the heat pump after 10 minutes. Supposing the compressor's power loss is the only significant source, then the compressor's power loss equals 75% of the power input. Although it is confirmed that there was a power loss in the compressor, the magnitude is not possible to calculate. Therefor it can only be concluded that this power loss had some part in the low value of the heat pump's efficiency.

In this experiment both the hot and cold reservoirs were water. If a heat pump should be used to heat a house in Norway, the heat pump would most probably

[4]Calculated from the measurements $T_1 = 12.2$ °C, $T_2 = 36.6$ °C, $T_3 = 24.8$ °C, $T_4 = 4.8$ °C, $p_\mathrm{c} = 3.62$ bar and $p_\mathrm{h} = 7.8$ bar

transfer heat from air to air, outdoors to indoors. This is possible because of the same thermodynamic principles as for water to water. Both water and air have heat capacity, which means that they both have the ability to absorb or emit heat through thermal contact with the cooling liquid. The differences between the media is that air has lower heat capacity and thermal conductivity than water, and they may both work as heat reservoirs for a heat pump. Due to air's low thermal conductivity, it would have to be circulated to even out the temperature, as the water also was in this experiment, to keep the difference in temperature as little as possible i order to optimize the heat pump's efficiency.

In a practical case the temperatures could be -10 to -20 °C outdoors, and 25 °C indoors. In order for the heat pump to work without fear of damage due to overpressure in the pipelines, the cooling liquid must be chosen wisely. R-134a could have been used and would have worked with a pressure between 1 and 8 bar, although other cooling liquids do exist that are more suitable. Accordingly a heat pump can be used to transport heat from a cold reservoir to a hot one, as long as the reservoirs consist of matter with a nonzero heat capacity, but practically the reservoirs should have a large enough thermal conductivity so that the heat is quickly exchanged with the whole reservoirs.

5 Conclusion

From the results the efficiency of the heat pump was between 3.66 and 1.92 during the experiment, which is less than a third of the Carnot efficiency at any point. There are four reasons why the experimental efficiency is smaller. First the heat pump's processes are not equal to those of a Carnot heat pump and thereby the efficiency will be smaller. Secondly the cooling liquid's properties make the efficiency smaller at a higher versus a lower temperature when the difference in temperature between the reservoirs is constant. Meanwhile the Carnot efficiency becomes smaller by the same arguments. Thirdly there was a power loss in the heat pump's compressor making the work input more than need be which makes the efficiency smaller. Lastly the heat pump and the heat reservoirs were not insulated, making them able to absorb/emit heat to the surrounding air and disturb the measurements.

At 10 minutes into the experiment the experimental efficiency was $\eta_{\mathrm{exp}} = 2.76 \pm 0.08$, while the Carnot efficiency was $\eta_{\mathrm{C}} = 15.7 \pm 0.16$ and the theoretical maximal efficiency was $\eta_{\mathrm{t}} = 11.0 \pm 1.3$. Some of the difference between η_{C} and η_{t} can be explained with the first two arguments listed above, while the difference between η_{exp} and η_{t} can be explained with the second two. If the compressor's power loss should explain the whole difference, the power loss must have been 75%.

It is possible to use a heat pump between two reservoirs of air, but the cooling liquid used in the heat pump must be chosen based on the temperatures at which it shall operate to make the heat pump as efficient as possible.

References

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