

# Properties of Electromagnetic Waves

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## Abstract

Through a set of different experiments sharing a common apparatus, polarisation, transmission, reflection, refraction, diffraction, interference and general propagation of electromagnetic waves is examined, as is the law of reflection, Snell's law of refraction, and Bragg's law. As the experiments are conducted using micro waves, all relevant dimensions are held on a macro level. This is found to increase some of the uncertainties involved, but seems otherwise to cause no problems.

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## 1 Introduction

The objective of this report is to investigate different properties of electromagnetic waves. As electromagnetic waves propagates through vacuum at a higher rate than through matter, they are not waves in the classical sense, that is, they are not "energy propagating through the movement of matter". Electromagnetic waves are held to be quantified into particles, and indeed, current scientific theory holds that *all* particles have a wave nature [1], although the wave nature of most particles is difficult to observe.

In this report we observe the wave-properties of electromagnetic waves through a series of experiments sharing a common apparatus. The theory of how electromagnetic waves work, comes at the start of this report and is grouped according to the following experiments. After the theory section the common apparatus is described. The method, results discussion and apparatus specific for each experiment are then presented together. Lastly, in the end, there is a conclusion on the observed properties of electromagnetic waves, as derived from the different experiments.

## 2 Theory

### 2.1 General Theory

The frequency  $f$  of an electromagnetic wave propagating in vacuum is given by the simple formula

$$f = \frac{c}{\lambda}, \quad (1)$$

where  $\lambda$  is the wavelength, and  $c$  is the speed of light in vacuum. Due to Einstein's work with special relativity,  $c$  is now recognised as an universal constant, and is thus *defined* to be [2]

$$c = 299\,792\,458 \text{ m s}^{-1}. \quad (2)$$

Generally speaking, the speed  $v$  of an electromagnetic wave through a medium is different from  $c$ , and given by

$$v = (\epsilon\mu)^{-1/2}, \quad (3)$$

where  $\epsilon$  is the medium's permittivity, given by

$$\epsilon = \epsilon_r \epsilon_0, \quad (4)$$

and  $\mu$  is its permeability, given by

$$\mu = \mu_r \mu_0. \quad (5)$$

Here,  $\epsilon_r$  and  $\mu_r$  is the relative permittivity and permeability of the medium in question, respectively, and  $\epsilon_0$  and  $\mu_0$  is the permittivity and permeability of vacuum, respectively.

For air, however, one has that [3]

$$\epsilon_{\text{r,air}} = 1.000\,589\,86 \pm 0.000\,000\,50, \quad (6)$$

where  $\epsilon_{\text{r,air}}$  is measured at standard conditions for temperature and pressure using radio frequencies of 0.9 MHz; and [4]

$$\mu_{\text{r,air}} = 1.000\,000\,37, \quad (7)$$

making

$$v_{\text{air}} \approx 299\,704\,024 \text{ m s}^{-1} \approx c; \quad (8)$$

where formula (3) has been used.

Thus, in the remaining of this report  $c$  will be used as an approximation of  $v_{\text{air}}$ ,  $\epsilon_{\text{r,air}}$  and  $\mu_{\text{r,air}}$  will be considered equal to 1, and the effects of the experiments' surrounding air will be neglected.

Unless stated otherwise, the formulas used are taken from the information papers for the third laboratory exercise in the course TFY4160/FY1002 Bølgefysikk at Norges teknisk-naturvitenskapelige universitet (NTNU) [5]. Vector quantities are marked with **bold-font**, i. e.  $\mathbf{E}$  is understood to be a vector.

### 2.2 Polarisation

Electromagnetic waves are described by the time variation in the magnitude and direction of the electric and magnetic fields. As both  $\mathbf{E}$  and  $\mathbf{B}$  are perpendicular to the direction of propagation the waves are transverse. A

linearly polarised wave where the electric field vector is in the  $y$ -direction is described by the electric field

$$\mathbf{E}_y = \hat{\mathbf{e}}_y E_{0y} \cos(kz - \omega t), \quad (9)$$

where  $k$  is the wavenumber,  $\omega$  is the angular frequency. The magnitude of  $\mathbf{E}$  in a direction at an angle  $\theta$  to the  $y$ -direction is

$$E_\theta = E_{0y} \cos(kz - \omega t) \cos \theta. \quad (10)$$

The intensity

$$I = c\epsilon_0 \langle E^2 \rangle \quad (11)$$

of the electric field, where  $c$  is -, and  $\epsilon_0$  is -, in the direction given by  $\theta$  is

$$I = I_0 \cos^2 \theta, \quad (12)$$

where

$$I_0 = 2c^2 \epsilon_0^2 E_{0y}^2. \quad (13)$$

A monochromatic wave propagating in the  $z$ -direction in a homogeneous medium might be described by the electric field as the superposition

$$\mathbf{E} = \mathbf{E}_x + \mathbf{E}_y \quad (14)$$

between the two orthogonal linearly polarised waves (9) and

$$\mathbf{E}_x = \hat{\mathbf{e}}_x E_{0x} \cos(kz - \omega t) \quad (15)$$

where  $E_{0x}$  is the maximum magnitude in the  $\hat{\mathbf{e}}_x$  direction, and  $\epsilon$  is the phase difference between the waves.

Passing through a circuit the portion of the electric field that is parallel to the circuit will induce a current. In turn the current induces an electric field that cancel out the parallel portion of the electric field.

### 2.3 Intensity as a Function of Distance and Direction

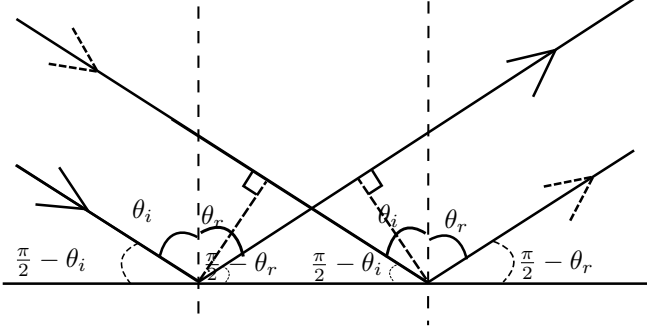
When the energy  $E$  in a wavefront is maintained as the wave propagates, the power  $P$  of the wavefront stays the same while it propagates. The waves from a point source that emits monochromatic electromagnetic waves equally in all directions will have wave fronts shaped like spheres. The intensity is

$$I = \frac{P}{A}, \quad (16)$$

where  $A$  is the total area of the wavefront at the distance  $r$ , because the energy density in the wave front is equal on the wavefront surface as the waves are emitted equally in all directions from the point source. As the area may be multiplied by a constant independent of the radius of the circle to retain a portion of a sphere

$$I \propto \frac{1}{r^2}, \quad (17)$$

for a wavefront shaped as a portion of a sphere with the distance  $r$  from the source. As Huygens' law states that every point on a wave front on an electromagnetic wave will behave as a point source [5], equation (17) should hold also when the electromagnetic wave is being emitted from a horn, specifically, it should hold in the situation described herein.



**Figure 1:** An electromagnetic wave reflected by a smooth surface. The black lines with arrowheads shows the boundary of the wavefront, and the arrow indicates the direction of the wave. The incident angle is  $\theta_i$ , and the angle of reflection is  $\theta_r$ .

## 2.4 Transmission

The penetration depth  $\delta$  of a electromagnetic wave in a material is given by

$$\delta = (\pi\sigma\mu f)^{-1/2}, \quad (18)$$

where  $f$  is the frequency of the electromagnetic wave, and  $\sigma$  and  $\mu$  is the conductivity and permeability, respectively, of the given material.

The size of the factor  $\sigma\mu$  varies greatly according to the material; for metals and electric conductors it can be in the range of about ten to a positive power, whereas for other materials it can be as small as ten to a negative power of sizes more than twenty<sup>1</sup>. Its unit is of course seconds per square metre, which can be seen from the formula.

## 2.5 Reflection

An electromagnetic wave hits a smooth surface at an angle  $\theta_i$  to the normal of the surface and is reflected at an angle  $\theta_r$  as in figure 1. Because the velocity of the incoming wavefront equals the velocity of the outgoing wavefront

$$\theta_i = \theta_r \quad (19)$$

as shown geometrically in figure 1.

## 2.6 Standing Waves and its Relation to the Wavelength $\lambda$

The electric field from an incoming electromagnetic wave that is partially reflected by a plane perpendicular to the direction of propagation is the superposition

$$\mathbf{E}_i = \mathbf{E}_a + \mathbf{E}_b, \quad (20)$$

of the electric field  $\mathbf{E}_a$  from a wave that is reflected without loss and  $\mathbf{E}_b$  from a wave that is not reflected. When the plane of reflection is at a distance

$$L = n \cdot \frac{\lambda}{2}, \quad n = 1, 2, 3 \dots \quad (21)$$

<sup>1</sup>This can for example be seen by looking at the relevant factors for air [4, 6, 7] and copper [7–9], which gives  $\sigma\mu \approx 10^{-21} \text{ s m}^{-2}$  and  $\sigma\mu \approx 70 \text{ s m}^{-2}$ , respectively.

from the wavesource, the wave described by  $\mathbf{E}_a$  and the reflected wave described by  $\mathbf{E}_r$ , will result in a standing wave, as they are in phase with each other [5].

While there is a standing wave the electric field

$$\mathbf{E} = \mathbf{E}_a + \mathbf{E}_r + \mathbf{E}_b, \quad (22)$$

at the point of reflection will equal  $\mathbf{E}_b$ , as  $\mathbf{E}_a$  and  $\mathbf{E}_r$  cancel each other out. The intensity (11) of the electric field at the endpoint will be minimal when there is a standing wave, as any other field (22) would give a greater intensity.

## 2.7 Refraction

Snell's law of refraction states that

$$n_a \sin \theta_a = n_b \sin \theta_b, \quad (23)$$

when an electromagnetic wave in material a meets the boundary between material a and material b, where  $\theta_a$  is the angle between the normal of the boundary surface and the incoming wave, and  $\theta_b$  is the angle between the normal of the boundary surface and the refracted ray, where  $n_a$  and  $n_b$  are material constants. Generally  $n_i$  is determined by

$$n_i = \sqrt{\epsilon_{r_i} \mu_{r_i}} \quad (24)$$

where  $\epsilon_{r_i}$  is the relative permittivity of medium  $i$ , and  $\mu_{r_i}$  is its relative permeability.

Taking material b to be the material outside the prism in figure 2, and material a to be the material inside the prism, the wave indicated in the figure will not alter direction when passing into the prism. When passing out of the prism  $\theta_a = \theta_1$ , so that

$$\theta_3 = \arcsin\left(\frac{n_a}{n_b} \sin \theta_1\right) \quad (25)$$

## 2.8 Interference

A linearly polarised electromagnetic wave that meets a wall with two identical slits perpendicular to the direction of the electric field describing the wave as in figure 3, will result in two equal electromagnetic waves

$$E_n = E_0 \sin(kr_n - \omega t) \quad (26)$$

at the distance  $r_n$  to slit number  $n$  on the other side of the wall. The electromagnetic field will be a superposition of the electric fields

$$E = E_1 + E_2, \quad (27)$$

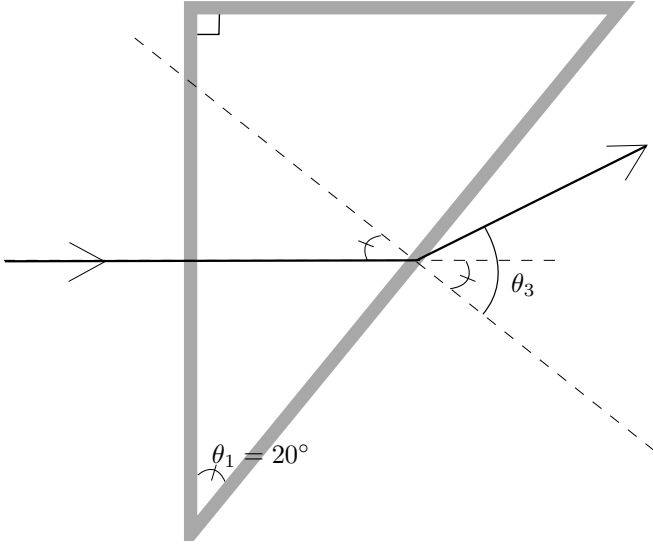
where  $E_1$  and  $E_2$  respectively are the waves from slit 1 and 2. This makes the intensity given from equation (11) at a point on the far side of the wall to be

$$I = 4I_0 \cos^2\left(\frac{\pi a \sin \gamma}{\lambda}\right), \quad (28)$$

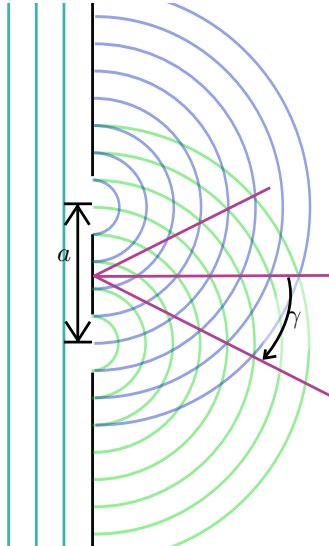
with the approximation<sup>2</sup>

$$r_1 - r_2 = a \sin \gamma, \quad (29)$$

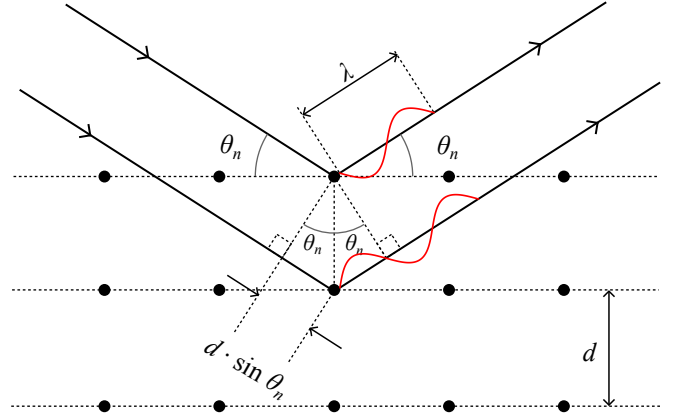
<sup>2</sup>Derived from the geometry when  $\mathbf{r}_1$  and  $\mathbf{r}_2$  are approximated to be parallel, and holds when the distances  $r_1$  and  $r_2$  are much greater than the distance  $d$  between the slits.



**Figure 2:** A solid prism of material a, contained by material b everywhere outside the prism. The black line indicates at which direction electromagnetic waves travel in the calculation in the text. When the wave leaves the prism  $\theta_1$  is the angle of incidence, and  $\theta_3$  is the angle of refraction.



**Figure 3:** Two slit experiment. A wave (light blue) hits a wall (black) with two slits so that there is an interference pattern on the other side of the wall. Distances  $r_1$  and  $r_2$  are marked in green and blue, and the purple line marks the lines of which angles are calculated by equation (30).



**Figure 4:** Schematic illustration of Bragg diffraction in a crystal. The black dots are atoms,  $\lambda$  is the wavelength of the reflected electromagnetic wave (partially shown, in red),  $\theta_n$  is the angle of the  $n$ -th order constructive interference maximum, measured as illustrated, and  $d$  is the grid constant, i.e. the distance between the atoms. Here a first order constructive maximum is illustrated.

where  $\gamma$  is the angle between the normal of the plane with the slits and line from the point in between the slits to the point where the intensity is considered, and  $a$  is the distance between the centre of the slits. It follows from equation (28) that the intensity is at local maximum when

$$a \sin \gamma = n\lambda \quad (30)$$

## 2.9 Diffraction

An idealised and simplified illustration of Bragg diffraction in a crystal is pictured in figure 4. In the illustration, the distance between the crystal and the source of the electromagnetic wave is assumed to be much greater than the distance  $d$  between the atoms, as is the distance between the crystal and the observer (i.e. the sensor; here a Schottky diode placed in a receiver). Because of this, the incoming waves can be assumed to be parallel within a satisfactory approximation, as can the reflected waves.

Given these assumptions, Bragg's law can be expressed as

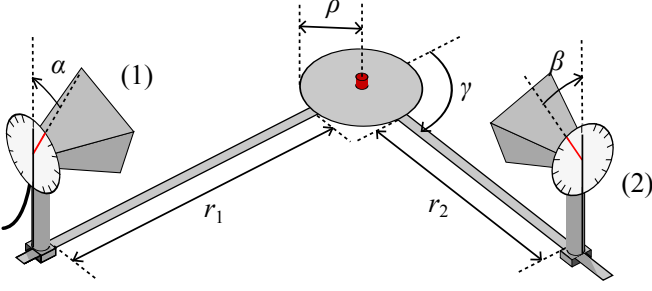
$$2d \cdot \sin \theta_n = n\lambda, \quad (31)$$

where  $d$  is the grid constant,  $\theta_n$  is the angle of the  $n$ -th reflection intensity maximum,  $n$  is an integer and  $\lambda$  is the wavelength. Simple algebra gives

$$d = \frac{n\lambda}{2 \sin \theta_n}. \quad (32)$$

## 3 Apparatus

The apparatus is a set of an emitter of monochromatic electromagnetic waves and an apparatus that register these waves, and various objects of appropriate dimensions that might be fastened or held between the wave source and the element that measure the waves. The basic setup of the apparatus is shown in figure 5. The Gunn diode and the Schottky diode are both placed so



**Figure 5:** The basic apparatus used in the experiments.  $\alpha$  and  $\beta$  indicates the angles at which the diodes are polarised. The wave source (1) emits vertically polarised waves when  $\alpha = 0$ , and the receiver (2) measures vertically polarised waves when  $\beta = 0$ . The distances  $r_1$  and  $r_2$  are measured from the effective points of emission and reception at 5 cm [10] into the transmitter and receiver. We define  $r = r_1 + \rho + r_2$ . A rotatable mirror, isoprism or simulated crystal might be installed on the pulley.

that the effective points of emission and reception are at 5 cm [10] into the horns. The wave source emits linearly polarised microwaves generated by a Gunn Diode in a 10.525 GHz resonance cavity [10]. The waves are led from the resonance cavity by a square horn.

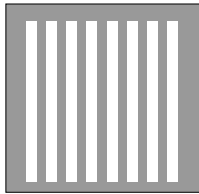
The wave receiver leads the electromagnetic waves through a square horn into a resonance cavity of 10.525 GHz where a Schottky diode rectifies the component parallel to the diode (at the angle  $\beta$  in figure 5) and shows the current  $D$  through the diode [10]. Even though  $D$  is non-linear, and will not be proportional to the electric field, nor the irradiance, it will be used as an approximation for the intensity  $I$  in this report. The meter for the current  $D$  will be constantly (re-)calibrated, and a measured  $D$  should therefore only be compared with other measured  $D$ -s within the same calibration.

## 4 Experiments

### 4.1 Polarisation

These experiments investigate properties of the polarisation of electromagnetic waves.

#### 4.1.1 Method



**Figure 6:** Polarisation filter in steel of size approximately 600 cm<sup>2</sup> with equal parallel holes shaped as columns at equal distances.

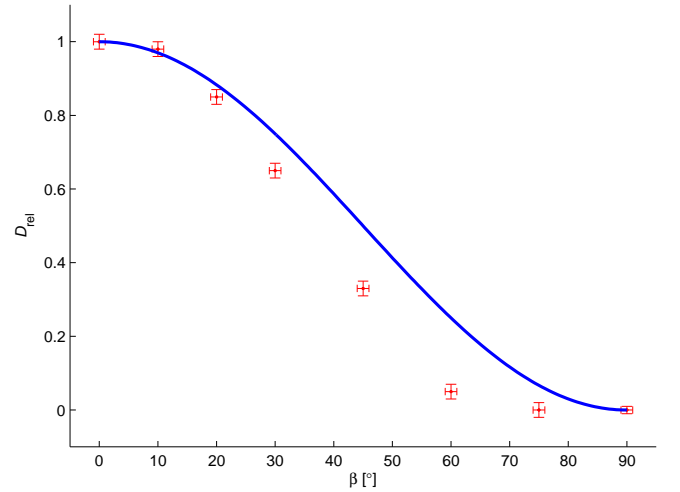
The distance  $r$  is held constant through the experiments on polarisation. First the angles  $\alpha$ ,  $\beta$  and  $\gamma$  are kept at zero degrees and the measured  $D$ s are compared when the polarisation filter in figure 6 is placed between

the emitter and the receiver with its columns vertically and horizontally. Second the angles  $\alpha$  and  $\gamma$  are kept at zero degrees and the measured  $D$ s are compared when  $\beta$  ranges from zero to ninety degrees through steps of ten and fifteen degrees.

#### 4.1.2 Results and Discussion

The current  $D$  is zero when the columns of the polarisation filter are vertical and parallel to the direction of the electric field from the linearly polarised waves.  $D$  when the columns are horizontal is equal to  $D$  measured without a filter. This is just as anticipated in the theory section.

The second series of measurements is plotted against the theoretical alteration of intensity in figure 7. As the relationship between  $D$  and  $I$  is not linear the plot of  $D_{\text{rel}} = D/D_0$  is not expected to have the same values at the same angles. Figure 7 shows that this is not the case. As the difference between the values for  $D$  and the theoretical value for  $I$  is greater here than in the other experiments, something seems to be amiss.



**Figure 7:** Measured  $D$ s relative to  $D_0$  when  $\beta = 0$  in the polarisation experiment plotted with error bars in both directions, and the theoretical value for  $I$  calculated from equation (12).

Potential sources of error are the waves not being completely linearly polarised and background noise from waves reflected in different ways. If the Gunn diode had emitted microwaves polarised in multiple directions  $D_{\text{rel}}$  should not have reached zero as it does. If the background noise from waves polarised in such a manner that they are measured by the receiver does decrease relatively to the polarised waves, or form an interference pattern at the measuring point  $D$  would drop faster than theoretical  $I$  as it does. As we observed  $D_{\text{rel}}$  to alter at a maximum of 0.07 when altering the environment of the apparatus, this might not wholly explain the difference.

As the great disadvantage of not knowing the relationship between  $D_{\text{rel}}$  and  $I_{\text{rel}}$  approximated by  $D_{\text{rel}}$  hinders us from being able to require a precise correlation between the items graphed in figure 7, we shall have to be contented with concluding that  $D_{\text{rel}}$  shows the ten-

dency expected of  $I$  in equation (12) when it varies from maximum to a minimum of zero through a path that is gradual near the endpoints and steep in the middle as the angle  $\beta$  is altered from 0 to 90°.

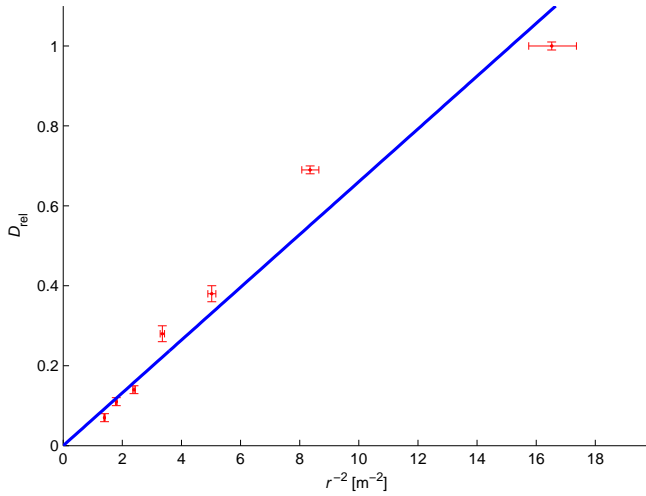
## 4.2 Intensity as a Function of Distance

This experiment investigates how the Intensity of a wave varies with the distance from the source of the wave.

### 4.2.1 Method

The angles  $\alpha$ ,  $\beta$  and  $\gamma$  from figure 5 are kept at zero degrees, and  $D$  is calibrated to  $D_0 = 1$  at a distance  $r = 40$  cm. Then  $D$  is measured at distances  $r$  from 40 cm through 100 cm at intervals of 10 cm.

### 4.2.2 Results and Discussion



**Figure 8:** Data for intensity varying with distance between source and receiver. Dimensionless  $D_{\text{rel}} = D/D_0$  plotted against  $r^{-2}$  and the quadratic regression (blue) of the data (red). For the data, horizontal and vertical error bars are also plotted.

As the distance between the Gunn diode and where electromagnetic waves are measured increases, the shape of the effective point of emission will be of less consequence, and the emitter is modelled as a point source in this experiment. Equation 17 and predicts how the intensity should vary and is plotted along with  $D_{\text{rel}}$  in figure 8. The uncertainty in  $D_{\text{rel}}$  is given by the uncertainty in the reading of the meter on the apparatus, and the uncertainty in the distance to the effective point of emission and reception.

As the relation between measured  $D$  and the  $I$  is not linear  $D_{\text{rel}}$ , they are not supposed to vary in the same way, which they do not. They show the same tendency, and we observe that the intensity of the electromagnetic waves decrease with distance from a wave source. As  $D_{\text{rel}}$  differ more from the expected tendency of  $I_{\text{rel}}$  at shorter distances  $D$  the effective point of emission not being shaped appropriately for approximation as a point source on these distances might be suspected but for the

uncertainties of the experiment hindering us from concluding as the result might be due to arbitrary errors from background noise.

## 4.3 Intensity as a Function of Direction

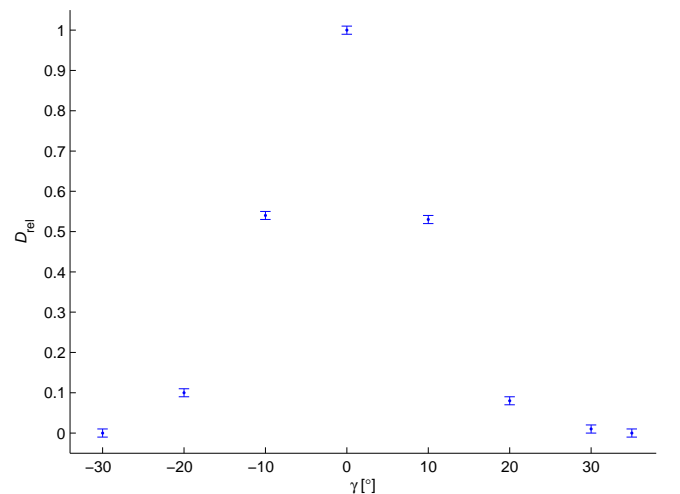
This experiment tests how the intensity of the electromagnetic waves vary at the same distance but at different directions from the wave source.

### 4.3.1 Method

The angles  $\alpha$  and  $\beta$  are kept at zero degrees,  $r_2$  is kept constant and  $r_2$  is kept at minimum while  $\gamma$  is varied from  $-30^\circ$  through  $40^\circ$  by intervals of  $10^\circ$  while  $D$  is measured. The current  $D$  is calibrated to 1 at  $0^\circ$  prior to the measurements.

### 4.3.2 Results and Discussion

The emitter emits microwaves focused in the forward direction as stated in the apparatus section. This means that the intensity should be strongest at  $\gamma = 0$  and fall as the size of  $\gamma$  grows. This is exactly the case, as figure 9 shows. Even though a general lack of knowledge of the apparatus leading to not knowing what the pattern in figure 9 should be renders us unable to know if the pattern is off from what it ought to be, we might assume that it should be symmetrical, as the instrument is assumed to be symmetrical. This is the case, but for the values where the size of  $\gamma$  is maximal. This is explainable by the environment being different on the two sides of the direction of propagation leading to different patterns of reflection, which has relative influence to a certain degree where the size of  $\gamma$  is maximal.



**Figure 9:** Data for intensity varying with the direction relative to the direction of propagation that the receiver is placed at. Dimensionless  $D_{\text{rel}} = D/D_0$  with uncertainty plotted against  $\gamma$ . Vertical error bars are included.

### 4.3.3 Discussion

## 4.4 Transmission

In this experiment various materials are tested for their ability to let through electromagnetic waves.

### 4.4.1 Method

The angles  $\alpha$ ,  $\beta$  and  $\gamma$  are kept at zero, and the  $r$  remains constant through the experiment. This experiment uses three plates, described in table 1, all with an area of  $A \approx 600 \text{ cm}^2$ .

**Table 1:** Schematic representation of data concerning the plates used in the transmission experiment.

Material	Approximate thickness (cm)
Plastic	2
Steel	0.1
Aluminium	0.001

The plates are put in between the wave source and the receiver and  $D$  is calibrated to see if the receiver measures any waves, and in what  $D$  changes when the plates are put between the wave source and the receiver.

### 4.4.2 Results and Discussion

The penetration depth of microwaves at frequencies of approximately 10 GHz will be 1 mm when

$$\sigma_1 \mu_1 = 3 \cdot 10^{-4} \text{ s m}^{-2}, \quad (33)$$

according to equation(18). The theoretical result is that the signal should be stopped by the steel and metal plates, and undisturbed by the plastic, as the values of  $\sigma\mu$  are far above (33) for aluminium and steel, and far below for plastic.

Both the plate made from steel and the plate made from aluminium blocks all signals to the receiver, while the receiver measures a higher value for  $D$  when the plate of plastic is between the wave source and the receiver, than when it is not.

Equation (18) states that the electromagnetic wave is transmitted a distance  $\delta$  into the material. The signal being stronger when transmitted through the plastic might be due to a focusing of the electromagnetic waves through this plastic, but this is not tested further in this experiment. The waves penetrate exactly the plates the theory predicts that it should.

## 4.5 Reflection

This experiment tests the law of reflection (19).

### 4.5.1 Method

The angles  $\alpha$  and  $\beta$  are kept at zero, and the distances  $r_1$  and  $r_2$  are kept constant at 34.0 cm and 93.8 cm. On the pulley in figure 5 a mirror is placed with its normal at the

angle  $\phi_1$  with the initial wave. For any  $\phi_1$  the angle  $\phi_2$  between the normal of the mirror and the arm carrying the wave receiver where  $D$  is maximum is found. This is done for angles  $\phi_1$  from  $10^\circ$  through  $70^\circ$  at intervals of  $20^\circ$ .

### 4.5.2 Results and Discussion

The resulting reflection angles  $\phi_2$  given the incident angles  $\phi_1$ , are listed in table 2.

**Table 2:** Resulting angles  $\phi_2$  for the incident angles  $\phi_1$ . The precision of  $\phi_2$  is determined by the fluctuation and accuracy of  $D$  at the incident angle  $\phi_1$ .

$\phi_1$ ( $^\circ$ )	$\phi_2$ ( $^\circ$ )
10	$22 \pm 10$
30	$32 \pm 2$
50	$50 \pm 1$
70	$64 \pm 2$

Comparing the results in table 2 with the predictions made by (19), its seen that the measured results agree with the predictions to a high degree for  $\phi_1 = 30^\circ$  and  $\phi_1 = 50^\circ$ , relatively well also for  $\phi_1 = 70^\circ$ , and not so well for  $\phi_1 = 10^\circ$ . The rather huge uncertainty in measured  $\phi_2$  for  $\phi_1 = 10^\circ$  is also noted.

To understand these data, it is important to realise some of the limitations in the used apparatus. When trying to measure  $\phi_2$  for  $\phi_1 = 10^\circ$ , the arms carrying the receiver and the transmitter partly crashed, bending and twisting as a result. In addition to these technical shortcomings, it should be remembered that at  $\phi_1 = 10^\circ$ , the reflected waves are likely to be travelling in an almost opposite direction of the incident waves, and thus interesting interference patterns may occur, distorting our measurements as a result.

The slight disagreement between predictions and measured data at  $\phi_1 = 70^\circ$  is a bit harder to understand, but as  $\phi_1$  gets close to  $90^\circ$ , the results in section 4.3.2, plotted in figure 9, suggest that the incident waves should disturb the measurements of the reflected waves in an increasing manner.

## 4.6 Determination of the Wavelength $\lambda$ from Measurements of Intensity Fluctuations Due to Standing Waves

### 4.6.1 Method

The angles  $\alpha$ ,  $\beta$  and  $\gamma$  are kept at zero degrees, and the distance  $r$  between the wave source and the measurer of the electric field is initially set at approximately 40 cm. The Schottky diode is moved apart from the wave source so that  $D$  varies, and  $r$  is recorded for the minimum values of  $D$ .

### 4.6.2 Results

The measured half-wave wavelengths,  $\lambda_{\frac{1}{2}}$ , and their uncertainty,  $\Delta\lambda_{\frac{1}{2}}$ , are listed in table 3. Using these data, the wavelength  $\lambda$  is found to be

$$\lambda = (2.86 \pm 0.45) \text{ cm}, \quad (34)$$

where the uncertainty is the standard deviation of the measured half-wave wavelengths listed in table 3 multiplied by two.

**Table 3:** Measured half-wave wavelengths  $\lambda_{\frac{1}{2}}$  with uncertainty  $\Delta\lambda_{\frac{1}{2}}$ .

$\lambda_{\frac{1}{2}}$ (cm)	$\Delta\lambda_{\frac{1}{2}}$ (cm)
1.3	$\pm 0.1$
1.7	$\pm 0.1$
1.4	$\pm 0.1$
1.5	$\pm 0.1$
1.3	$\pm 0.1$
1.5	$\pm 0.1$
1.5	$\pm 0.1$
1.0	$\pm 0.1$
1.8	$\pm 0.1$
1.3	$\pm 0.1$

The frequency is then, with the help of (1), found to be

$$f = (10.5 \pm 1.6) \text{ GHz}, \quad (35)$$

where the uncertainty is calculated using Gauss' law of uncertainty propagation.

### 4.6.3 Discussion

The frequency of the electric waves radiating from the microwave emitter is given as [10]

$$f = 10.525 \text{ GHz}. \quad (36)$$

Comparing this to (35), one realises that the calculated wavelength (34) must fit well with the given frequency, albeit the uncertainty is quite high.

The relatively high uncertainty is of no surprise, however, since the experiment was executed in a *non-non*-reflecting environment.

## 4.7 Refraction

### 4.7.1 Method

The distance  $r$  is constant through this experiment. A container in EPS shaped as a prism with angles at  $20^\circ$ ,  $70^\circ$  and  $90^\circ$  and walls of even thickness is placed on a platform on the pulley in figure 5 with the middle of the outer wall of the longest cathetus directly above the centre of the pulley so that the wave might follow the path indicated in figure 2. First the container is empty and the difference in  $D$  between the container being present and absent is considered. Then the container is filled

with pellets of styrene of approximately 2 mm in diameter, and the angle  $\gamma$  in which  $D$  is maximum is observed. This is done because the wave has its maximum intensity where it follows the path indicated in figure 2.

### 4.7.2 Results and Discussion

The measuring of  $\gamma$  and the geometry of figure 2 makes the measured value

$$\theta_{3m} = (34 \pm 1)^\circ \quad (37)$$

for the angle  $\theta_3$ .

Theoretical value  $\theta_{3t}$  for  $\theta_3$  is given by equation (25) and (24), where  $\epsilon_r$  is given as [11]

$$\epsilon_{r_{\text{polystyrene}}} = 2.6 \quad (38)$$

for a crystal of polystyrene, whereas the effects of the air surrounding the experiment—as already stated—is neglected.

The theoretical value is written as

$$\theta_{3t} = 33.5^\circ, \quad (39)$$

where  $\theta_{3t}$  is shortened and its uncertainty is neglected, as it is much smaller than the uncertainty of the measured value.

The theoretical angle of refraction is within the uncertainty of the measured value of refraction, which suggests that it was fair to neglect the effect of the EPS container, and to view the contained styrene pellets as equivalent to a solid block of styrene.

## 4.8 Young's Two Slit Experiment

This experiment looks at the interference pattern of two monochromatic electromagnetic waves.

### 4.8.1 Method

Three metal plates, where one is narrow and the two other ones are of equal size, are affixed to the pulley in figure 5 so that the centre of the narrow plate is at the centre of the pulley, the plates are parallel, and have an equal gap of 1.5 cm between them. The angles  $\alpha$  and  $\beta$  are held constant to  $\pi/2$ , so that the waves are polarised horizontally. The set up is adjusted to be as much like figure 3 as possible. Let  $\gamma = 0$  and adjust  $r$  so that  $D$  shows a local maximum. Keep  $r$  constant and rotate the arm of the receiver to observe  $\gamma$  where  $D$  shows a relative maximum.

### 4.8.2 Results

The gaps between the plates are

$$G = (15.5 \pm 0.5) \text{ mm}, \quad (40)$$

and the theoretical and practical values  $\gamma_t$  and  $\gamma_n$  for the maximum given by equation (30) is presented in table 4.

For the calculation of  $\gamma_t$ , equation (30), the wavelength measured in the standing waves experiment (34) and

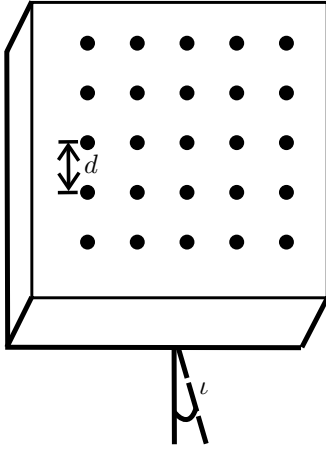
$$a = (7.5 \pm 0.5) \text{ cm} \quad (41)$$

are used.



**Table 4:** Results from Young’s two slit experiment.  $\gamma_t$  is theoretical values for  $\gamma$  at maximum, and  $\gamma_m$  is the corresponding measured values. The uncertainties in  $\gamma_t$  is found by use of Gauss’ law of uncertainty propagation applied on (30), using (34) as  $\lambda$  and (41) as  $a$ .

$\gamma_t$ (°)	$\gamma_m$ (°)
0	$0 \pm 1$
$22 \pm 4$	$21 \pm 1$
$-22 \pm 4$	$-21 \pm 1$
$50 \pm 9$	$47 \pm 1$



**Figure 10:** The artificial “crystal” used in the Bragg diffraction experiment. Distance  $d$  between the metallic pebbles placed in a container of EPS.

#### 4.8.3 Discussion

The data from the two slit experiment presented in table 4 shows great coherence between the theoretical and measured values of the angle  $\gamma$  where there are maximums. The uncertainty in the theoretical value is great because it is derived from the measured value of the wavelength. The average theoretical value and the average measured value are systematically off by one degree so that the size of the theoretical value is higher. About the maximum point, the intensity varies only slightly as the derivative in the maximum point is zero. Since the difference between the measured and theoretical values for  $\gamma$  is systematically off, a plausible explanation is that the measuring of the angle was systematically off by about one degree in the direction of greater size of  $\gamma$ . This is an error that was initially accounted for with the uncertainty in the measured angle. It might also be a coincidence that the average of the values are systematically off, and with the great uncertainty in the theoretical value the whole widths of the practical values lie within the theoretical value.

## 4.9 Determination of the Grid Constant $d$ of a Cubic “Crystal” from $\theta_n$ Using Bragg’s Law

### 4.9.1 Method

The distances  $r_1$  and  $r_2$  are kept at constant values, and the angles  $\alpha$  and  $\beta$  are kept constant and equal to zero degrees. The artificial “crystal” from figure 10 is placed with its centre directly above the centre of the pulley and two sides parallel to the initial direction of the propagation of the wave. When the “crystal” is rotated an angle  $\iota$  the arm that carries the receiver should be rotated an angle  $\gamma = 2\iota$ . When  $D$  is at its local maximum  $\iota$  is recorded. The distance  $d_{\text{actual}}$  between the metallic spheres in the artificial “crystal” is measured.

### 4.9.2 Results

Trough simple measurements with a metre-stick,  $d$  was found to be

$$d_{\text{actual}} = (3.83 \pm 0.02) \text{ cm}, \quad (42)$$

where the uncertainty is calculated using Gauss’ law of uncertainty propagation.

The angle of the first order constructive interference maximum,  $\theta_1$ , was found to be somewhere between  $(25 \pm 1)^\circ$  and  $(26 \pm 1)^\circ$ , and thus

$$\theta_1 = (25.5 \pm 1.0)^\circ. \quad (43)$$

The angle of the second order constructive maximum,  $\theta_2$ , was found to be

$$\theta_1 = (47.0 \pm 1.0)^\circ. \quad (44)$$

Using (32), our measurements of  $\lambda$  (34) and of  $\theta_1$  (43), one finds

$$d_{\theta_1} = (3.32 \pm 0.54) \text{ cm}, \quad (45)$$

where the uncertainty is found by use of Gauss’ law of uncertainty propagation.

Using (44), one gets

$$d_{\theta_2} = (3.91 \pm 0.62) \text{ cm}, \quad (46)$$

where the uncertainty is calculated in a similar manner as in (45).

The mean of these two values gives

$$d_\theta = (3.61 \pm 0.41) \text{ cm}, \quad (47)$$

and, once again, Gauss’ law of uncertainty propagation has been used to determine the uncertainty.

### 4.9.3 Discussion

Comparing (47) with (42), one sees that the grid constant found by using Bragg’s law and the angles of the  $n$ -th order constructive interference maximum,  $d_\theta$ , is slightly lower than the directly measured grid constant  $d_{\text{actual}}$ , but not by so much that the actual value (that is, (42)) is not within its uncertainty range.

Thus, even though Bragg's law is derived for microscopic wavelengths and real crystals, where one can safely assume the incoming electromagnetic waves hitting two neighbour atoms to be parallel—as illustrated in figure 4—its still not *entirely* unusable when one can safely assume incoming electromagnetic waves to *not* be parallel.

## 5 Conclusion

Although the great defect the lack of knowledge of the exact connection between the measured current  $D$  and the intensity  $I$  represents, rendering the uncertainty of the observation of  $I$  considerably, the collection of experiments illustrates at an acceptable level of accurateness the foreseen qualities of the electric fields alteration due to the propagation of electromagnetic microwaves focused in a forward direction. The experimental results concur with the theory that electromagnetic waves propagate with the properties of reflection, polarisation, interference, refraction and diffraction as is expected of a wave.

## References

- [1] Walter Greiner. *Quantum Mechanics: An Introduction*. Springer. 2001
- [2] The Committee on Data for Science and Technology. *Fundamental Physical Constants from NIST*. <http://physics.nist.gov/cgi-bin/cuu/Constants>, visited October 23, 2013.
- [3] L. G. Hector and H. L. Schultz. *The Dielectric Constant of Air at Radiofrequencies*. Physics, 7:133–136, 1936.
- [4] B. D. Cullity and C. D. Graham. *Introduction to Magnetic Materials*. 2nd edition. p. 16. 2008.
- [5] Unknown author(s). NTNU. *Bølgeoptikk*. Retrieved from [home.phys.ntnu.no/brukdef/undervisning/tfy4160\\_lab/kompendie/BolgeoptikkTFY.pdf](http://home.phys.ntnu.no/brukdef/undervisning/tfy4160_lab/kompendie/BolgeoptikkTFY.pdf) October 15, 2013.
- [6] S. D. Pawar, P. Murugavel and D. M. Lal. *Effect of relative humidity and sea level preassure on electrical conductivity of air over Indian Ocean*. Journal of Geophysical Research, 114:D02205. 2009.
- [7] E. Lillestøl, O. Hunderi and J. R. Lien. *Generell fysikk for universiteter og høyskoler. Bind 2 Varmelære og elektromagnetisme 2*. opplag. Universitetsforlaget. p. 332. 2006.
- [8] D. Giancoli. *Physics for Scientists and Engineers with Modern Physics*. 4th edition. Upper Saddle River, New Jersey: Prentice Hall. p. 658. 2008.
- [9] Ibid. [7], p. 378.
- [10] PASCO. *Complete Microwave Optics System*. [http://www.pasco.com/file\\_downloads/product\\_manuals/Exp-Guide-Microwave-Optics-WA-9314C.pdf](http://www.pasco.com/file_downloads/product_manuals/Exp-Guide-Microwave-Optics-WA-9314C.pdf), downloaded October 22, 2013.
- [11] The University of Sydney. *Physics Resources Database*. [http://www.physics.usyd.edu.au/teach\\_res/db/d0006c.htm](http://www.physics.usyd.edu.au/teach_res/db/d0006c.htm), visited October 23, 2013.

## A Scattering of light

Bjarne Å. Bergtun and Kristin B. Bakka attended “lekelab” the 24th of October. We took a fascination to the experiment on scattering of light. When the white light from the sun hits the particles in the atmosphere light with high frequencies collide more with the particles and are scattered more when there are more particles. At the sunset the light that reaches us pass from the sun and through more of earth's atmosphere so that the light with the high frequencies is scattered and the red light with the low frequencies are usually seen. In the middle of the day when the sun is almost in the vertical direction, the light does not have to go through so much atmosphere, and more of the blue light reaches us. As the blue light has more energy than the red light, the red light is “drowned” and we see the blue light.

In the experiment we also played with glass prisms and looked at how the different colored light have different wavelength, resulting in different angles of reflection and refraction, making the light split into a rainbow as it enters the prism, and the rainbow to widen for each time the light is reflected or refracted. :)