Precept 8: Stable Population

Kristin E. Bietsch

Spring 2012

1 Estimating Lotka's r

We will be estimating the intrinsic growth rate of Egypt in 1997. To do this, we will need to make an initial estimate of r, then refine this estimate using a loop.

To start, remember how Lotka related the NRR and r:

$$r = \frac{ln(NRR)}{T} \tag{1}$$

Where:

- The Net Reproduction Ratio is the average number of daughters that female members of a birth cohort would bear if they were subjected to observed age-specific maternity rates (${}_nF_x^F$) and mortality rates (${}_nL_x^f$):

$$NRR[0,T] = \sum_{x=\alpha}^{\beta-n} {}_nF_x^F[0,T] * \frac{{}_nL_x^F}{l_0} \tag{2}$$

- T is the mean length of a generation
  - Here we will assume that $T = 27$

Now we want to put this value into Lotka's equation, and see how close we are to 1.

$$y(r_n) = \sum_{a=15,5}^{45} e^{-r_n(a+2.5)} * {}_5L_a * {}_5m_a \tag{3}$$

To estimate a new r, we plug the $y(r_n)$ into:

$$r_{n+1} = r_n + \frac{y(r_n) - 1}{27} \tag{4}$$

Iterate until you (and Georges) are happy!

2 Construction of a Stable-Equivalent Population

First we want to calculate r, just as we did in the last do file. To do this, use Equation (1), Equation (3), and Equation (4).

From here, calculate the crude birth rate for the stable population with the fertility and mortality schedules of US women in 1991 and the intrinsic growth rate.

$$b = \frac{1}{\sum_{a=0,5}^{80} e^{-r(a+2.5)} \frac{{}_5L_a}{l_0}} \tag{5}$$

Now we can calculate the age distribution of the stable equivalent population (**Note:** the actual age distribution for the population is given to you).

$${}_5c_a^{stable} = be^{-r(a+2.5)} \frac{{}_5L_a}{l_0} \tag{6}$$

3 Population Momentum

Here I will walk you through how to calculate the population momentum for women.

3.1 Data requirements:

- The number of women in each age group
- Age-specific fertility rates
- Age-specific mortality rates

3.2 What you need to calculate:

- Net Reproduction Rate

$$NRR[0, T] = \sum_{a=15}^{45} {}_5m_a * {}_5L_a^F \tag{7}$$

- Maternity rates in the stable population

$${}_5m_a^* = \frac{{}_5m_a}{NRR} \tag{8}$$

- Mean age of childbearing (in the stationary population)

$$A^* = \sum_{a=15}^{45} (mid - age) * {}_5m_a{}^* * {}_5L_a^F \tag{9}$$

- Weights- The number of lifetime births above age x, divided by the mean age at childbearing

$$ {}_5w_a = \frac{(\frac{{}_5L_a^F}{2} * {}_5m_a^* + \sum_{y=a+5}^{45} {}_5L_y^F * {}_5m_a{}^*)}{A^*} \tag{10} $$

- Age structure before the fertility drop

$$ {}_nc_x^{pre-drop} = \frac{{}_nN_x}{N} \tag{11} $$

- Age structure after the fertility drop

$$ {}_nc_x^{post-drop} = \frac{{}_nL_x}{e_0} \tag{12} $$

3.3 A Simple Approximation

Presented below is a simple approximation of momentum developed by James Frauenthal. This can be used with $NRRs$ above and below 1, but I will explain things as if $NRR^{pre-drop} > 1$

Note: I am switching to Wachter notation for this section.

$$ K(U) \approx K(0-)\frac{b(0-)e_0}{\sqrt{NRR}} \tag{13} $$

Where:

- $K(U)$ is the ultimate size a population will achieve if NRR drops to 1 immediately
- $K(0-)$ is the population size just before the sudden drop in fertility rates
- $b(0-)$ is the crude birth rate just before the sudden drop in fertility rates
- e_0 is the life expectancy at birth (we assume this does not change after the drop in fertility)
- NRR is the net reproduction rate just before the drop in fertility

How does the approximation compare to the original?