Introduction

Demography Camp

Summer 2013

1 Mathematics for Demography

1.1 Summations

We often have a series of numbers for different sub-groups (such as number of births classified by age of the mother). The summation sign ($\sum$) simply tells us to add them all up! There will often be information around the summation sign ($\sum_a^b c$). The bottom tells us where to start, the top tells us where to stop, and the middle right tells us what we are adding up.

$$\sum_{start}^{stop} add\ me!$$

1.1.1 An Example of Something You Will See in Demography

$$\sum_{15-19}^{45-49} ASFR = TFR$$

1.2 Natural Logarithms

Logarithms are common in demography, often used when we talk about population growth. **Note:** When demographers refer to logarithms, we mean natural logarithms (log_e or ln). Here, and in future classes, natural logarithms will be denoted simply as log.

1.2.1 Definition of a Logarithm

From wikipedia: The logarithm of a number is the exponent to which another fixed value, the base, must be raised to produce that number. For example, the logarithm of 1000 to base 10 is 3, because 1000 is 10 to the power 3: $1000 = 10 \times 10 \times 10 = 10^3$. More generally, if $x = b^y$, then y is the logarithm of x to base b, and is written $y = log_b(x)$, so $log_{10}(1000) = 3$.

1.2.2 A Few Rules

Product:

$$log_b(xy) = log_b(x) + log_b(y)$$

$$ln(xy) = ln(x) + ln(y)$$

Quotient:

$$log_b\left(\frac{x}{y}\right) = lob_b(x) - log_b(y)$$

Power:

$$log_b(x^p) = p \cdot log_b(x)$$

Inverse Function:

$$log_b(b^x) = x \cdot log_b(b) = x$$
$$b^{log_b(y)} = y$$

$$e^{ln(x)} = x$$
$$ln(e^x) = x$$

An Example of Something You Will See in Demography

$$P(t) = P(1)e^{rt}$$

1.3 Derivatives and Integrals

In demography, things are constantly changing: babies are always being born and people are always dying. Therefore, while we sometimes make things simpler by dealing with discrete time, we often need to think in continuous time.

From wikipedia: In calculus, the derivative is a measure of how a function changes as its input changes, the derivative at a point equals the slope of the tangent line to the graph of the function at that point.

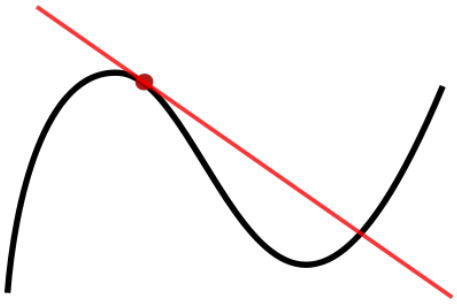

Figure 1: Derivative

A definite integral of a function can be represented as the signed area of the region bounded by its graph.

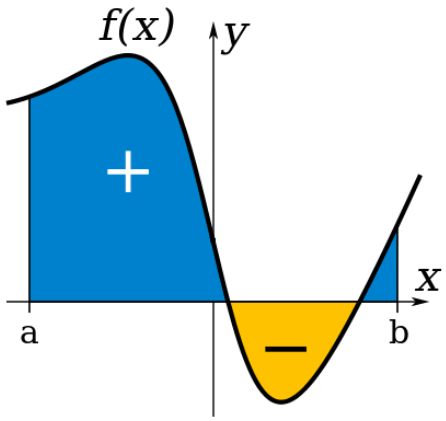

Figure 2: Integral

1.3.1 An Example of Something You Will See in Demography

To find the instantaneous death rate at age 52, take the derivative of the survival function. To find the number of years lived by the population between 19 and 28, integrate the survival function from 19 to 28.

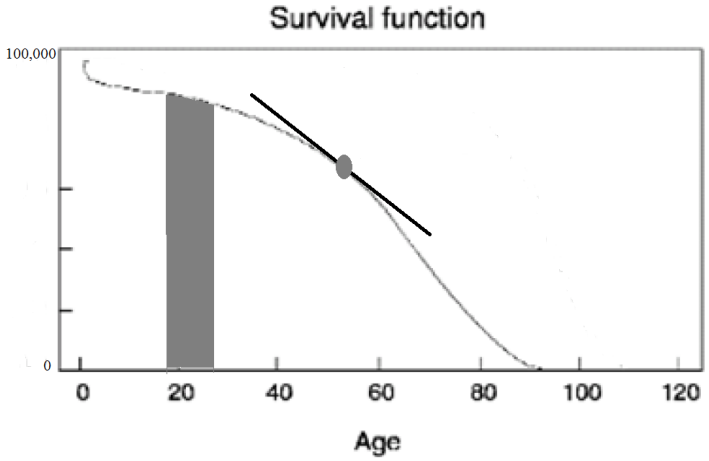

Figure 3: Survival Function

2 Demography Word Bank

- **Period:** happens in a specific period of time (0 to t, 1992, etc.)
- **Cohort:** group that experiences a demographic event during a specific time interval (ex. the marriage cohort of 1992, the birth cohort of 1960, the 2013 Princeton demography cohort)
- **Person Year:** the number of years lived in a period by a population (often thought of as the amount of time exposed to risk of an event)
- **Crude:** measurements are crude when they include people-years in the denominator who are not actually (or equally) exposed to the risk of an event (for example, men are not exposed to the risk of giving birth, and 10 year olds have a much lower risk of dying in the next year than 100 year olds)
- **Decremental:** this is used for measurements where people are removed from the population after the event occurs (for example, once a person dies they no longer contribute person years to the crude death rate denominator)
- **Non-decremental:** this is used for measurements where people are not removed from the population after the event occurs (for example, in one measurement of the risk of first marriage, people who marry are not removed from the population contributing to the person years at risk)
- **Monotonic:** a monotonic function is a function between ordered sets that preserves the given order (for example, a decreasing monotonic function is never increasing)

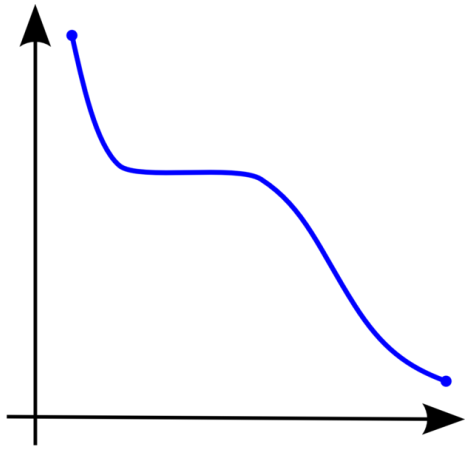

Figure 4: A monotonically decreasing functon