Final Exam
Answers

Demography Camp

Summer 2013

Question 1

Age	n	lx	ndx	nLx	Tx	ex
20	5	80223.8806	5223.880597	375000	5375000	67
25	5	75000	5000	350000	5000000	66.7
30	5	70000				

Part A

${}_5L_{20} = 375000$

Part B

$l_{30} = 70000$

Part C

$e_{25} = 66.7$

Question 2

US life expectancy at birth in 2000 was 77 years. In no more than two sentences, provide an explanation of what this means in language suitable for non-demographers.

If someone was exposed for their whole life to the mortality schedule observed in 2000, they would live on average 77 years.

Question 3

Suppose there are two populations (A and B) which are currently of equal size and have been growing at a constant rate of .02 and .01 respectively. If these rates persist into the future, what will be the ratio of the population size of A relative to B in 50 years?

$$e^{50\cdot.02} = X \cdot e^{50\cdot.01}$$

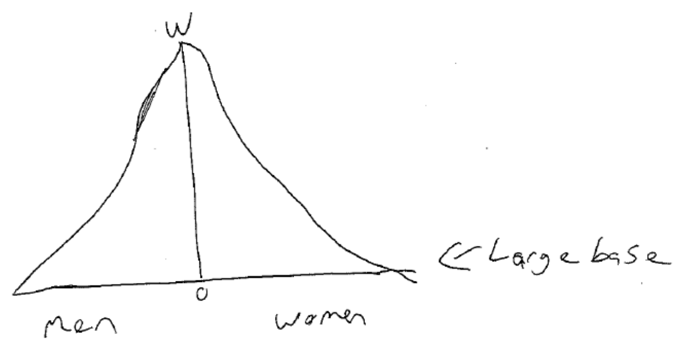
$$2.7182818 = X \cdot 1.6487213$$

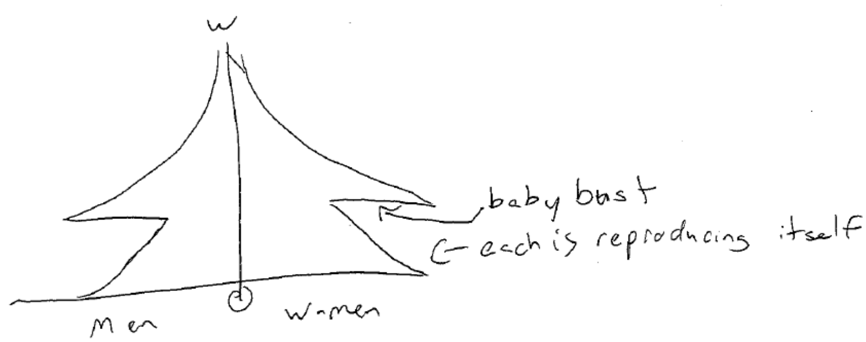
$$X = 1.6487212$$

Population A will be 1.65 times larger than popualtion B

Question 4

Draw a population pyramid for a country with high fertility. Make sure to include labels! Now imagine that tomorrow the TFR dropped to replacement level (and the mortality rates do not change). Draw the population pyramid for this population 20 years after the drop in fertility.





Question 5

According to the UN the population of Mexico (in thousands) was 27,737 in 1950 and 36,964 in 1960

Part A

Compute the annual rate of growth between 1950 and 1960:

$$P2 = P1e^{rt}$$

$$36,964 = 27,737 \cdot e^{r \cdot 10}$$

$$r = 0.029$$

Part B

Project the population to 2000 using that rate of growth

$$P2000 = 36,964 \cdot e^{40 \cdot 0.0287177204}$$

$$P2000 = 116589$$

Part C

The population in 2000 was 98,872 (in thousands). What does that say about the annual rate of growth in the last half of the 20th century?

The annual growth rate between 1960 and 2000 was less than it was between 1950 and 1960.

Question 6

Life expectancy is usually a better summary of mortality than crude death rates. Can you think of a situation when it makes no difference which one you use?

If you had a stationary population. The relationship is $CDR = \frac{1}{e_0}$

Question 7

Once upon a time, the total fertility rate in a certain country was 6.4 children per woman, which was 2.5 times the level needed for replacement. Approximately what proportion of newborn girls survived to the mean age of childbearing?

Assume sex ratio at birth is 105 boys per 100 girls

$$Replacement TFR = 6.4/2.5 = 2.56$$

$$ReplacementTFR \cdot \frac{B_f}{B_f + B_m} \cdot p(\bar{m}) = 1$$
$$p(\bar{m}) = \frac{1}{TFR} \cdot \frac{B_f + B_m}{B_f}$$
$$p(\bar{m}) = \frac{1}{2.56} \cdot \frac{100 + 105}{100} = 0.8$$

80% of newborn girls survived to the mean age of childbearing

Question 8

Here's part of an abridged life table for males in rural India in 1957-1958. Note the wide age interval after 5 years.

Age	${}_nq_x$	l_x	${}_nL_x$	T_x
0	0.15358	100000	89724	4391556
1	0.15764	84642	313163	4301832
5	0.05373	80094.18534	696511	3988669
15	0.1267	67468	1913868	3292158
45	0.36845	58920	1010622	1378290
65	0.53604	37210.926	274453	367668
75+	1	17264	93215	93215

Part A

Whats the probability of dying before age one? 0.15358

Part B

Calculate the mean age at death of children dying before age one (no assumptions needed).

$${}_nL_x = n \cdot l_{x+n} +_n a_x \cdot_n d_x$$
$${}_nd_x = l_x - l_{x+n}$$
$${}_1L_0 = 1 \cdot l_1 +_n a_x \cdot (l_0 - l_1)$$
$$89724 = 1 \cdot 84642 +_n a_x \cdot (100000 - 84642)$$
$${}_na_x = 0.33090246$$

A third of a year

Part C

Estimate the expectation of life at birth.

$$\frac{T_0}{l_0} = \frac{4391556}{100000} = 43.91556$$

Part D

What is the probability of surviving to age 65?

$$\frac{l_{65}}{l_0} = \frac{37210.926}{100000} = 37.2\%$$

Part E

How much longer would you expect a 65 year old to live?

$$\frac{T_{65}}{l_{65}} = \frac{367668}{37210.926} = 9.88\ years$$

Part F

Calculate the probability that a newborn will die between ages 15 and 65.

$$\frac{{}_{50}d_{15}}{l_0} = \frac{l_{15} - l_{65}}{l_0} = \frac{67468 - 37210.926}{100000} = .30257074 = 30\%$$

Part G

What is the mortality rate for age group 75+?

$${}_{\infty}m_{75} = \frac{{}_{\infty}d_{75}}{{}_{\infty}L_{75}} = \frac{17264}{93215} = .18520624 = 185\ deaths\ per\ 1000\ PYs$$

Question 9

Back in 1950, the crude death rates (CDR) for Sweden and Japan were very similar, 10.0 and 10.9, respectively.

Part A

The direct standardization rate for Japan using the Swedish age distribution as the standard was 15.7. Explain what this estimate means.

Japan would have 15.7 deaths per 1000 population if it had Sweden's age structure.

Part B

Summarize in a few sentences what these results say about the age distributions and the mortality levels in the two countries

Japan has generally higher age specific mortality than Sweden, but a younger age structure

Question 10

In what way(s) is the General Fertility Rate a better measure of fertility levels than the Crude Birth Rate? What are the limitations to using GFR (name at least 2)?

The GFR provides a better estimate than the CBR in that it restricts the population in the denominator to women of childbearing age, thus attempting an adjustment for the age structure of the population. Two limitations are the added data needed (you need to have a break down of you population estimates by age and sex) and this still does not take into account the age structure in the childbearing ages.

Question 11

In a time when marriage and childbearing are being delayed, what are the limitations on using the period TFR to gage trends in cohort TFR?

When childbearing is being delayed, the period TFR will underestimate the cohort TFR, since births that will eventually happen are not occurring in the observation period.

Question 12

What value must the period NRR have if period TFR is at replacement? Why?

When NRR is equal to 1, each generation of new born girls will produce on average 1 daughter each

Question 13

Suppose the following age-specific fertility rates are detected for a given calendar year:

Ages	Rate	Interval	Rate · Interval
14-18	0.005	5	0.025
19-20	0.015	2	0.03
21-29	0.085	9	0.765
30-34	0.04	5	0.2
35-49	0.002	15	0.03
			1.05

What is the period TFR for these data? 1.05

Question 14

How many females may we expect in the age group 20-24 at time t=5 who are survivors of women in the population at time t=0? Just show the formula(s) you would use.

$$ {}_5W_{20}^{(5)} = {}_5W_{15}^{(0)} \times \frac{{}_5L_{20}}{{}_5L_{15}} $$

Question 15

You are given a female population (in 5 year age groups), a female life table, and age specific fertility rates for a population in 2010. List the steps you would need to take and the assumptions you would need to make to predict the number of females under age 5 in 2015.

First, you need to make an assumption about the fraction female at birth. Then, you want to calculate the average number of women exposed to the ASFR in the interval

$$ {}_5\bar{W}_x = \frac{{}_5W_x^{(10)} + {}_5W_x^{(15)}}{2} $$

for x = 15, 20, ...45
From there, you can multiply the number of women by the ASFRs and fraction female at birth to compute the number of female births

(Female) births in a 5 year period = B = $5 \cdot \sum_{x=15}^{45} {}_5\bar{W}_x \cdot {}_5m_x$

Finally you survive your births to the end of the period.

$$ {}_5W_0^{(15)} = B \cdot \frac{{}_5L_0}{5 \cdot l_0} $$