

Problem Set 2: Solutions

Demography Camp

Summer 2013

Question 1

On December 31 2012, Belleville, Illinois had a population of 32,324 people. Over the next 12 months, 3213 babies were born, 4573 people died, 2015 people moved into Belleville, and 3073 people left.

Part 1

Write you the population balance equation for Belleville, IL for 2013.

$$\begin{aligned} P(\text{Belleville 2013}) &= P(\text{Belleville 2012}) + B(\text{Belleville 2013}) - D(\text{Belleville 2013}) \\ &\quad + I(\text{Belleville 2013}) - E(\text{Belleville 2013}) \end{aligned}$$

$$P(\text{Belleville 2013}) = 32,324 + 3213 - 4573 + 2015 - 3073$$

Part 2

What is the population of the town at the end of the year?

$$P(\text{Belleville 2013}) = 29,906$$

Question 2

Nigeria's population in 2010 was 158,423,182 with a growth rate of 2.5% a year.

The United States population was 309,349,689 in 2010 with a growth rate of 0.7% a year.

Part 1

If these rates remain constant into the future, in what year will Nigeria and the United States have the same size population?

$$P(\text{Nigeria}) \cdot e^{r(\text{Nigeria}) \cdot t} = P(\text{United States}) \cdot e^{r(\text{United States}) \cdot t}$$

$$\frac{P(N)}{P(U)} = \frac{e^{r(U) \cdot t}}{e^{r(N) \cdot t}}$$

$$\frac{P(N)}{P(U)} = e^{r(U) \cdot t - r(N) \cdot t}$$

$$\frac{P(N)}{P(U)} = e^{t \cdot (r(U) - r(N))}$$

$$\log\left(\frac{P(N)}{P(U)}\right) = t \cdot (r(U) - r(N))$$

$$\frac{\log\left(\frac{P(N)}{P(U)}\right)}{r(U) - r(N)} = t$$

$$\frac{\log\left(\frac{158,423,182}{309,349,689}\right)}{0.007 - 0.025} = t$$

$$t = 37.17792 \text{ years}$$

The populations will be of equal size in 2047.

Part 2

How big will these twin populations be?

Choose either country (you should get the same answer!):

$$P(2047) = P(2010) \cdot e^{r \cdot t}$$

Nigeria:

$$401,303,393 = 158,423,182 \cdot e^{(0.025 \cdot 37.17792)}$$

USA:

$$401,303,393 = 309,349,689 \cdot e^{(0.007 \cdot 37.17792)}$$

Question 3

Who is the "at-risk" population if I am calculating a:

Part A

Divorce Rate?

Married people

Part B

Maternal Mortality Rate?

Pregnant and recently postpartum women

Question 4

Imagine a population that had 100 people at time t , 130 people at time $t+1$, and 150 people at time $t+2$. What would be the estimated number of person years lived between time t and $t+2$ if:

Part A

You used the midpoint to estimate person years?

$$\text{Midpoint Population} \cdot \text{Length of Interval}$$

$$130 \cdot 2 = 260 \text{ Person Years}$$

Part B

You used the average of the beginning and end to estimate person years?

$$\text{Average Population} \cdot \text{Length of Interval}$$

$$\frac{100 + 150}{2} \cdot 2 = 250 \text{ Person Years}$$