# Precept 7: Population Projections

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## 1 Closed Populations

### 1.1 Steps

1. Calculate survivorship
2. Calculate survivorship for the last age group
3. Project current population forward (for survivorship)
    - Special consideration for open ended age interval
4. Calculate the number of births
5. Calculate the number of female births
6. Survive the female births to the end of the interval

### 1.2 Equations

#### 1.2.1 Survivorship

Calculate survivorship:

$$ {}_5N_x^F(t+5) = {}_5N_{x-5}^F(t) \cdot \frac{{}_5L_x}{{}_5L_{x-5}} $$

Calculate survivorship for the last age group:

$$ {}_\infty N_x^F(t+5) = \left[ {}_5N_{x-5}^F(t) + {}_\infty N_x^F(t) \right] \frac{T_x}{T_{x-5}} $$

Note: This is for when you have the same number of age groups as projection groups

#### 1.2.2 Reproduction

Calculate the number of births :

$$B\left[t,t+5\right]=\sum_{x=\alpha}^{\beta-5}{}_5F_x\cdot 5\cdot\left[\frac{{}_5N_x^F(t)+{}_5N_x^F(t+5)}{2}\right]$$

Calculate the number of female births:

$$B\left[t,t+5\right]=\frac{1}{1+SRB}\cdot B\left[t,t+5\right]$$

Survive the female births to the end of the interval:

$${}_5N_0^F(t+5)=B^F\left[t,t+5\right]\cdot\frac{{}_5L_0}{5\cdot l_0}$$

# 2 Open Populations

## 2.1 Steps

1. Calculate survivorship
2. Calculate survivorship for the last age group
3. Project current population forward (for survivorship)
   - This is where you bring in immigration, half at the beginning of the interval, half at the end
   - Special consideration for open ended age interval
4. Calculate the number of births (for citizens only)
5. Add in number of births to immigrants who come at the beginning of the cycle
6. Calculate the number of female births
7. Survive the female births to the end of the interval

## 2.2 Equations

#### 2.2.1 Survivorship

Calculate survivorship:

$${}_5N_x^F(t+5)=\left[{}_5N_{x-5}^F(t)+\frac{{}_5I_{x-5}^F\left[t,t+5\right]}{2}\right]\cdot\frac{{}_5L_x}{{}_5L_{x-5}}+\frac{{}_5I_x^F\left[t,t+5\right]}{2}$$

Calculate survivorship for the last age group:

$${}_\infty N_x^F(t{+}5)=\left[{}_5N_{x-5}^F(t)+{}_\infty N_x^F(t)+\frac{{}_5I_{x-5}^F\left[t,t+5\right]+{}_\infty I_x^F\left[t,t+5\right]}{2}\right]\cdot\frac{T_x}{T_{x-5}}+\frac{{}_\infty I_x^F}{2}$$

Note: This is for when you have the same number of age groups as projection groups

### 2.2.2 Reproduction

$$\Delta B\left[t,t+5\right]=\sum_{x=\alpha}^{\beta-5}\frac{5}{4}\cdot{}_5F_x\cdot\left({}_5I_x^F\left[t,t+5\right]+{}_5I_{x-5}^F\left[t,t+5\right]\cdot\frac{{}_5L_x}{{}_5L_{x-5}}\right)$$

$${}_5N_0^F\left(t+5\right)=B^F\left[t,t+5\right]\cdot\frac{{}_5L_0}{5\cdot l_0}+\frac{{}_5I_0^F\left[t,t+5\right]}{2}$$