Mortality and its Measurement:
Life Table Construction

Demography Camp

Summer 2013

1 Cohort Experience

Traces out the life-time experience of a **hypothetical** group of individuals all born at the same time and all subject throughout their lifetimes to the risks of dying at each age, either observed in a particular population or recorded in a particular mortality schedule

- Most life tables are **period** life tables (i.e., the level of mortality is assumed constant)
- The group of individuals is called a **cohort** a group of persons who begin experience at the same time or during the same interval, e.g., birth cohort of 1985, or of 1980-1985; also marriage cohort of 1985.

1.1 Life table formulas

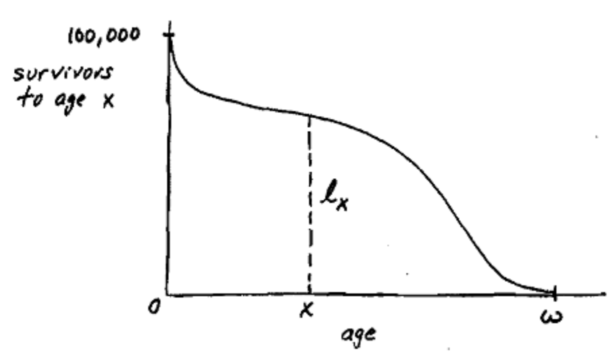

Figure 1: The l_x curve

This is a hypothetical population. No population ever subject to these mortality rates throughout their lifetimes because observed rates change over time.

- l_x: number still alive at exact age x
- l_0: **radix** of life table (usually 1 or 100,000)
- l_{55}: # who would survive to age 55
- $\frac{l_x}{l_0}$: proportion surviving (out of an initial birth cohort) to exact age x; also denoted by p(a) or p(x)
- ω: oldest age attained by anyone in the population.
- ${}_np_x$: proportion surviving from exact age x to x+n
  - $l_{x+n}l_x$
- ${}_nd_x$: number of deaths between ages x and x+n
- ${}_nq_x$: proportion of those who are living at exact age x who die before reaching age x+n
  - $1 -_n p_x$
  - ${}_nq_x \neq_n m_x$; age-specific mortality rates are based on people living **during** the interval, not on those living at the the **beginning** of the interval
- ${}_nL_x$: number of person years lived by the cohort between exact ages x and x+n
  - $\int_x^{x+n} ly\ dy$: area under l_x curve between x and x+n
  - If deaths between ages x and x+n are evenly distributed over the interval, then: ${}_nL_x = \frac{n}{2}(l_x + l_{x+n})$
    - Can't always make this assumption, especially under ages 2 or 3
- ${}_na_x$: the average number of person years contributed by those who die in the interval
- T_x: total number of person years remaining to be lived by the cohort beyond age x
  - $\sum_x^{\infty} {}_nL_x$
  - $T_{50} = {}_5L_{50} + {}_5L_{55} + {}_5L_{60} + \cdots + {}_5L_{\omega-5}$
  - $T_x = \int_x^{\omega} ly\ dy$: area under the l_x curve to the right of x

1.1.1 Life Expectancies

$$Life\ Expectancy = \frac{\text{Total person years to be lived}}{\text{Population alive to live them}}$$

- $\mathring{e}_x = \frac{T_x}{l_x}$
- $\mathring{e}_0 = \frac{T_0}{l_0}$ = average duration of life starting at birth
- $\mathring{e}_0 = \int_0^\omega p(a)\, da$, where p(a) is the survival function, $\left(\frac{l_x}{l_0}\right)$

1.2 Relationships

- ${}_nd_x = l_x \cdot {}_nq_x = l_x - l_{x+n}$
- ${}_nq_x = \frac{{}_nd_x}{l_x}$
- ${}_nm_x = \frac{{}_nd_x}{{}_nL_x}$

1.3 Central problem of life table construction

How to construct the ${}_nq_x$ values when all we observe are ${}_nM_x$ age-specific death rates in a population?
Two steps:

1. Assume ${}_nm_x = {}_nM_x$
   - Not a bad assumption provided age intervals are relatively small
2. Derive ${}_nq_x$ from ${}_nm_x$
   - First assume that deaths in an interval are evenly distributed. Not bad, except for the young ages
   - Then, ${}_nL_x = n \cdot (l_x - \frac{{}_nd_x}{2})$
   - $l_x = \frac{{}_nL_x}{n} + \frac{{}_nd_x}{2}$
   - $\therefore {}_nq_x = \frac{{}_nd_x}{l_x} = \frac{{}_nd_x}{\frac{{}_nL_x}{n} + \frac{{}_nd_x}{2}} \cdot \frac{\frac{n}{{}_nL_x}}{\frac{n}{{}_nL_x}}$
   - $\therefore {}_nq_x = \frac{n \cdot \left(\frac{{}_nd_x}{{}_nL_x}\right)}{1 + \frac{n}{2} \cdot \left(\frac{{}_nd_x}{{}_nL_x}\right)}$
   - Remember that ${}_nm_x = \frac{{}_nd_x}{{}_nL_x}$,
   - ${}_nq_x = \frac{n \cdot {}_nm_x}{1 + \frac{n}{2} \cdot {}_nm_x}$

One remaining actuarial problem how to calculate ${}_nL_x$ from l_x? Usually done by assuming deaths are evenly distributed in an age interval

- i.e. ${}_nL_x = \frac{n}{2}\,(l_x + l_{x+n})$
- Ok from ages 3 if n=1 or 5-65 if n=5

- ${}_1L_0 \approx 0.3l_0 + 0.7l_1$: better than using equal weight. Why?

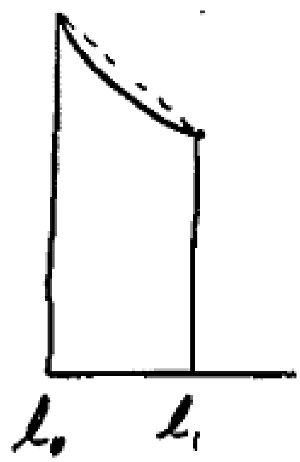

Figure 2: The l_x curve between 0 and 1. Strong curvature. Gives lower figure than assumption of straight line.

- ${}_1L_1 \approx 0.4l_1 + 0.6l_2$
- ${}_1L_x = a_x \cdot l_x + (1 - a_x)l_{x+1}$ is exactly correct if a_x is the mean number of years lived in the interval by those who die in the interval