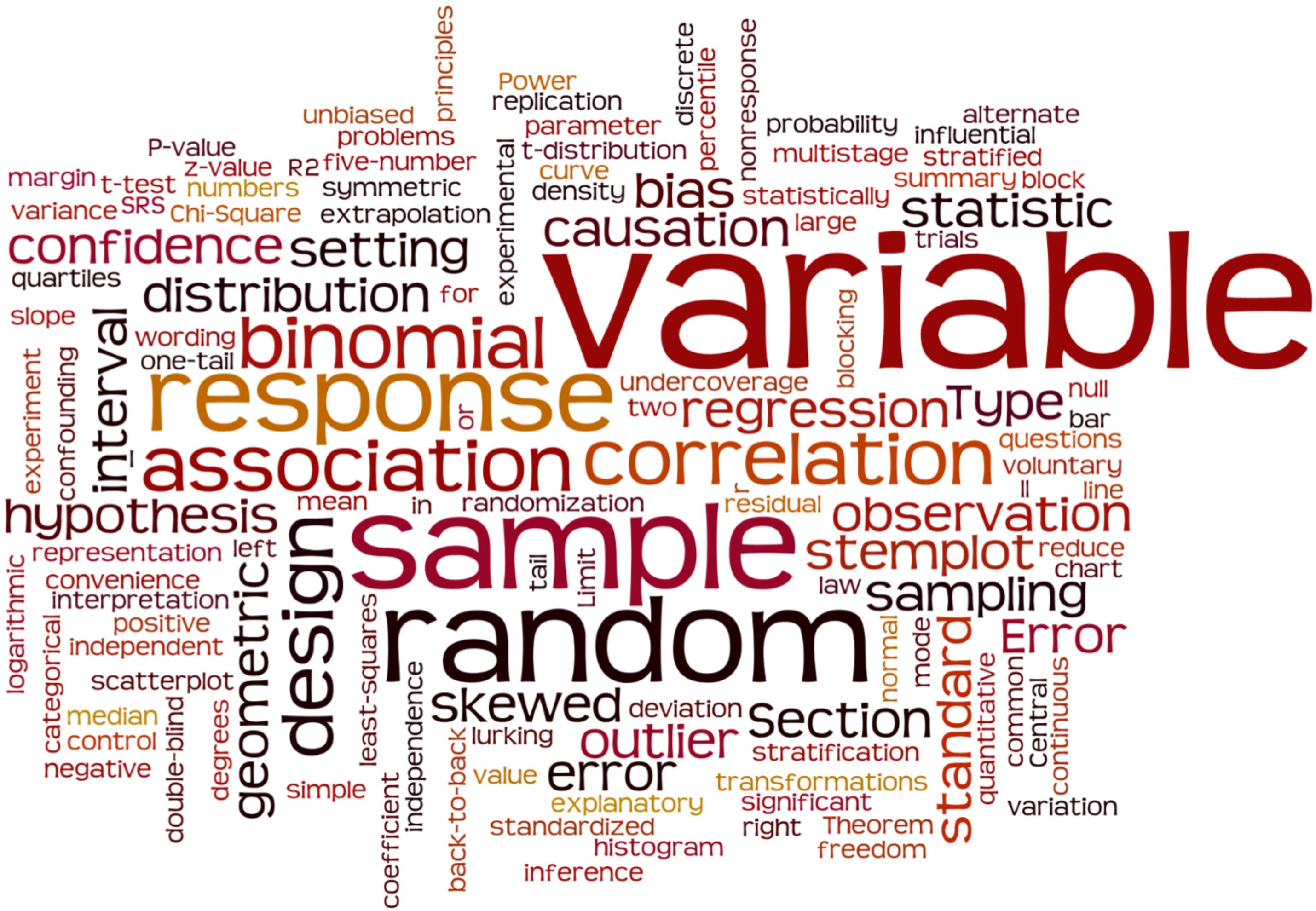
Introduction to Quantitative Methods



Nuclear disasters

Chernobyl (April 1986) – Level 7 event

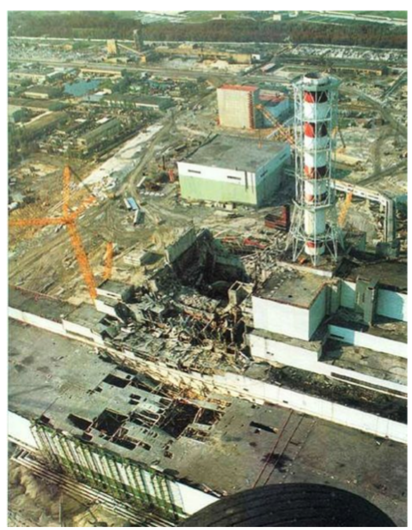
Power surge, explosion followed emergency shutdown, meltdown

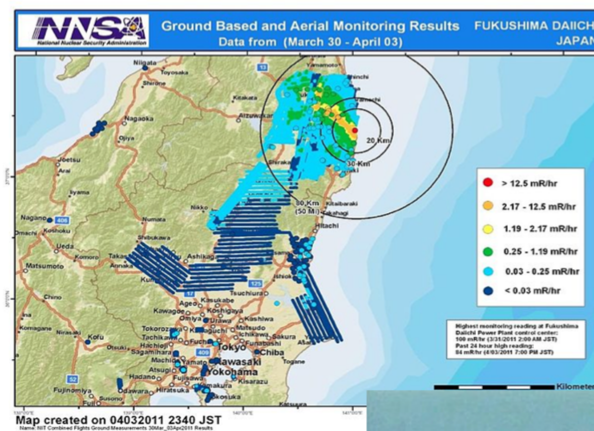
Contamination – mostly air

Fukushima (March 2011) – Level 7 event

Tsunami flooded plant, 3 reactors melted down

Contamination – mostly water





Images: Wikimedia; www.whoi.edu

Radiation doses

A boxplot can be helpful to compare two samples

Mean:
74.0 vs. 68.6
mSv/h

Is this close enough to be considered the same?

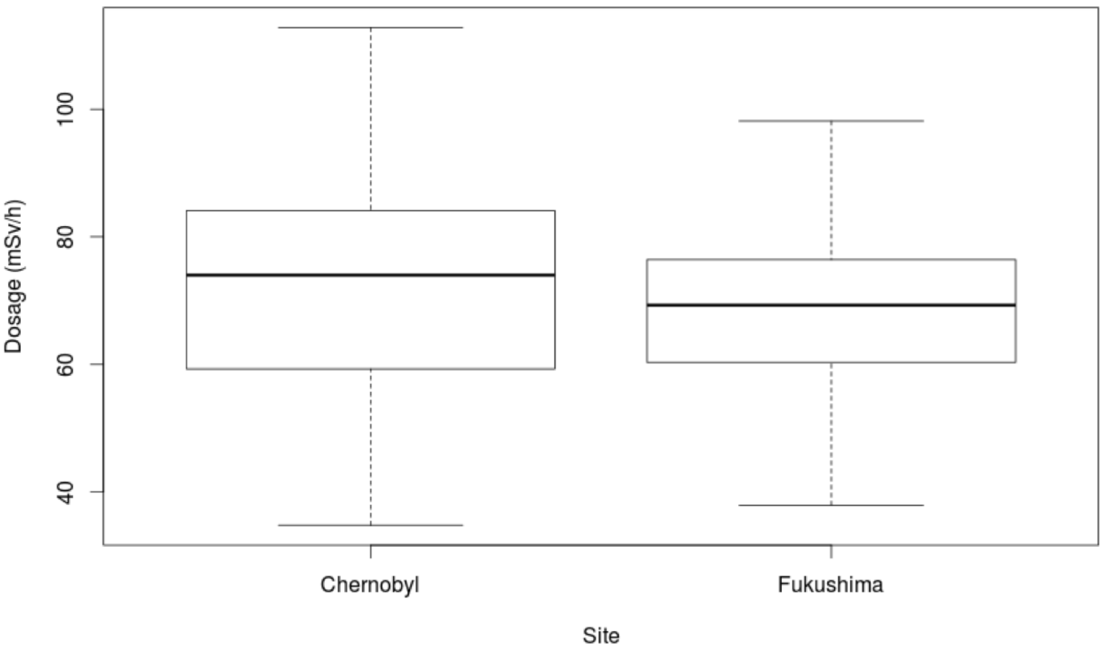


Fig. 1 Boxplot of the dosage near the Chernobyl and Fukushima nuclear disaster sites following core meltdown. Data are fabricated (actually, Chernobyl radiation levels were much higher)

Two sample difference of means (Unpaired)

- 2 samples (categories/groups) with continuous values
- Answers: Do the two populations have equal means?

- H0: $\mu_A = \mu_B$
- H1: $\mu_A \neq \mu_B$

Note, you may also see:
- H_0: $\mu_A - \mu_B = 0$
- H_1: $\mu_A - \mu_B \neq 0$

- Could also be one-tailed (aka: one-sided)
  - H1: $\mu_A > \mu_B$ or H1: $\mu_A < \mu_B$
- Example questions:
  - Was the radiation dose the same for Chernobyl vs Fukushima?
  - Are living costs higher in Ottawa or Montreal?

Two sample difference of means

- Difference of means
  - Close to zero, then not significant
  - If not close to zero, then significant
    - The two samples means are more different than could have happened due to random chance
- We use the t-distribution
  - Don't know the standard deviation of the **population**
  - We must estimate based on the **sample**

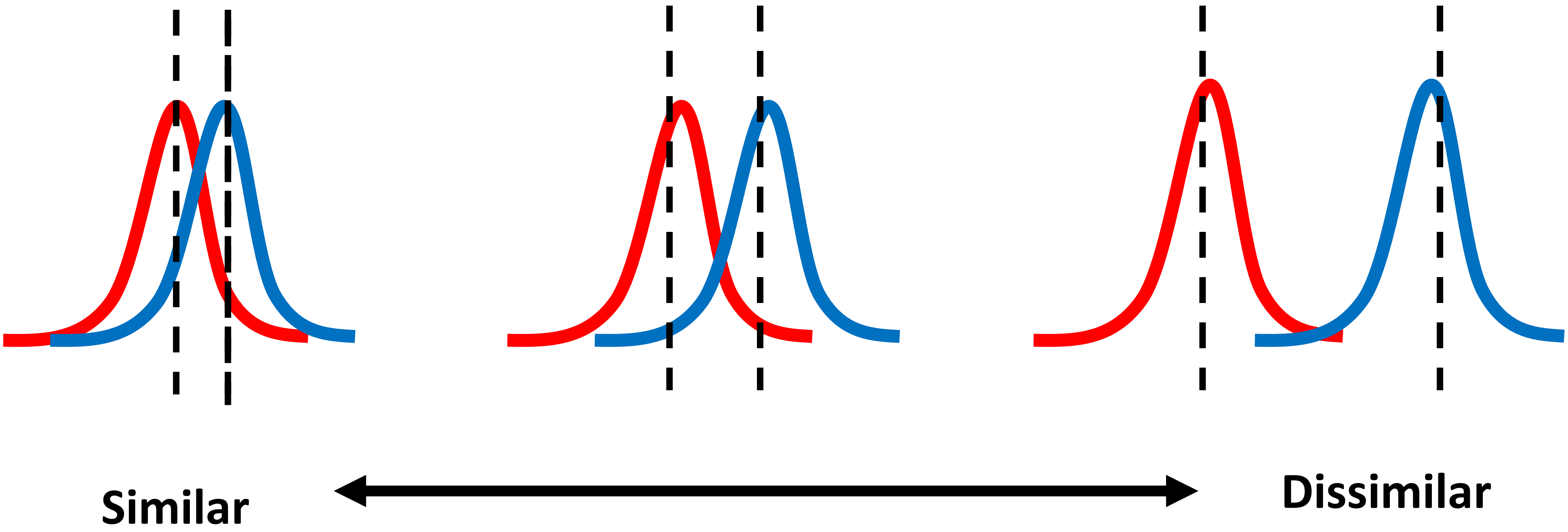
$$t = \frac{\bar{x} - \mu}{s/\sqrt{(n)}}$$

Student's t-test

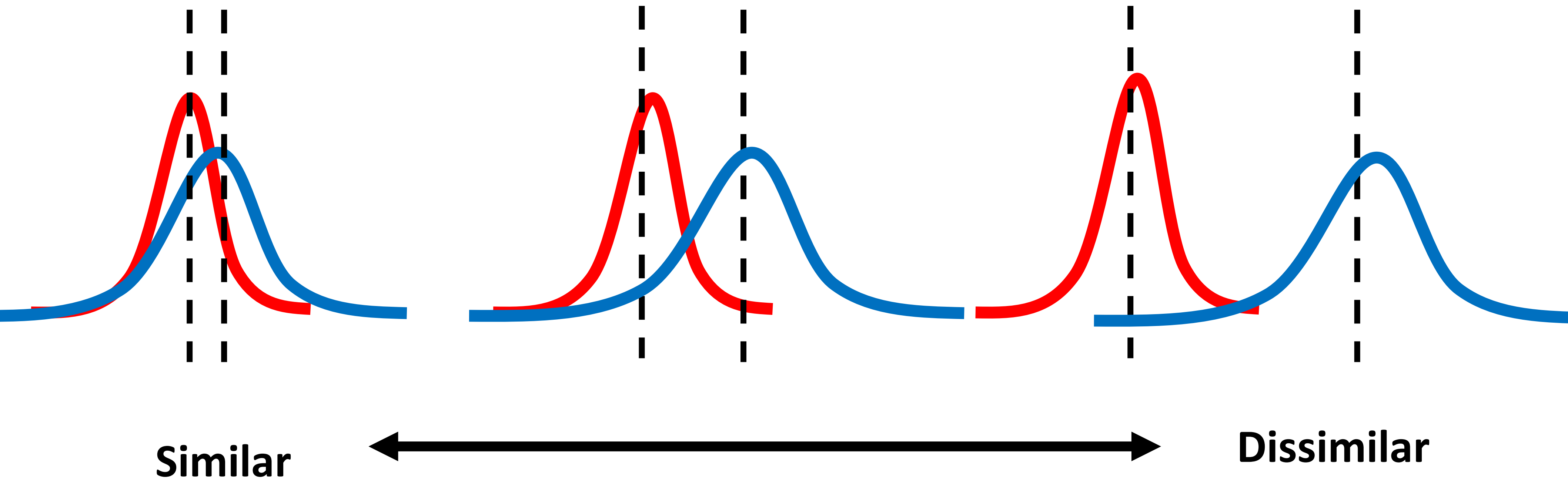
$$t = \frac{\bar{x_A} - \bar{x_B}}{s/\sqrt{(n)}}$$

Unpaired t-test

Two sample difference of means- Equal Variance



Two sample difference of means- Unequal Variance



Two sample difference of means- Unequal variance

- Calculating the denominator (SEM)
  - Tells you how much differences you would expect to see between $\bar{x_A} - \bar{x_B}$ due to random sampling error

$$t = \frac{\bar{x_A} - \bar{x_B}}{s/\sqrt{(n)}}$$

  - Now you have two s and two n
  - How to combine these:
    - Known as the Standard Error of the Difference (SED)
    - Unpooled S
    - degrees of freedom are corrected ***
    - This will affect the critical t-value

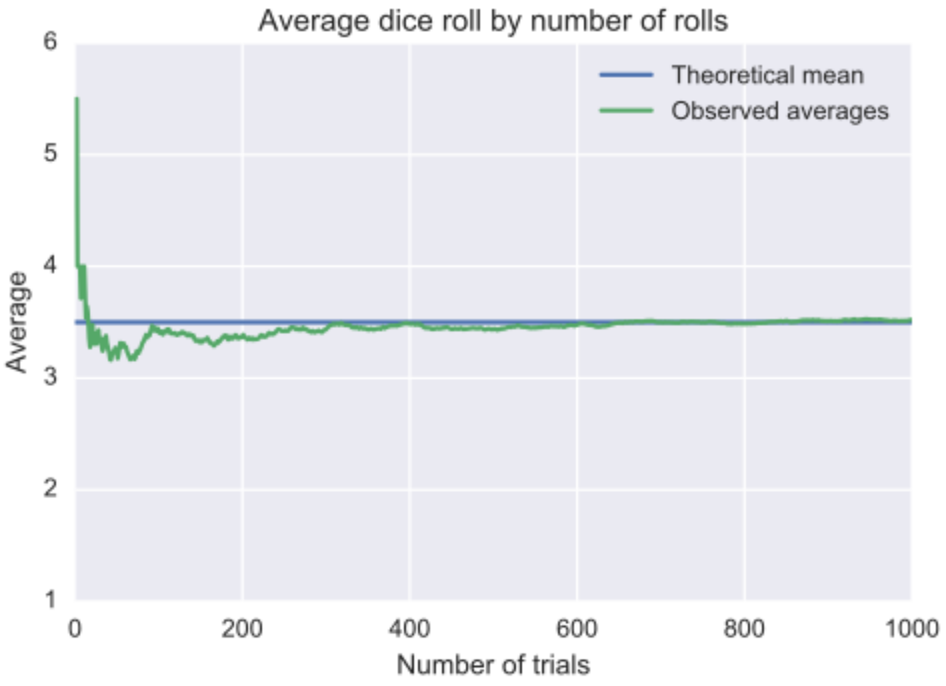
$$t = \frac{\bar{x_A} - \bar{x_B}}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

Welch's T-test

$$df = \frac{[(s_A^2/n_A) + (s_B^2/n_B)]^2}{\frac{(s_A^2/n_A)^2}{n_A - 1} + \frac{(s_B^2/n_B)^2}{n_B - 1}}$$

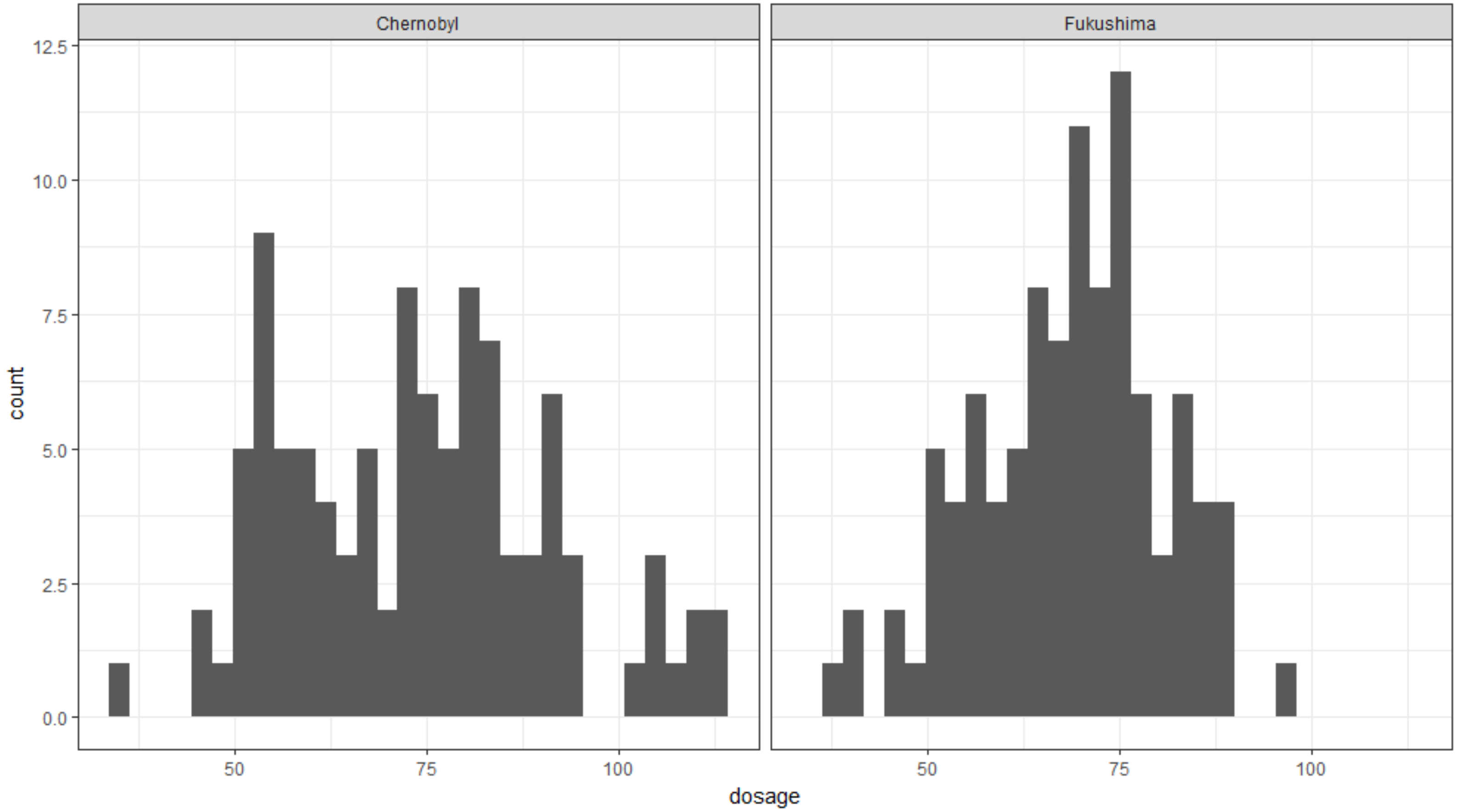
Law of Large Numbers

- Why do we care about df?
- Back to probability
- The law of large numbers is a principle of probability
- The frequencies of events with the same likelihood of occurrence even out, given enough trials or instances.
- As the number of experiments increases, the actual ratio of outcomes will converge on the theoretical, or expected, ratio of outcomes.



Unpaired t-test assumptions

- Two independent random samples
- Each population is normally distributed
- Variable is measured on an interval or ratio scale



Two sample difference of means- Equal Variance

This works well if the variances (or standard deviation) of both samples are *different*

If they are the *same* (or nearly so), you will get better results (higher statistical power) if you use the **pooled standard deviation** to calculate the SED:

The t-statistic will then look like:

For your reference only

$$t = \frac{\bar{x}_A - \bar{x}_B}{s_p\sqrt{\frac{1}{n_A} + \frac{1}{n_B}}}$$

$$s_p = \sqrt{\frac{\sum_{i=1}^{n_A}(x_i - \bar{x}_A)^2 + \sum_{i=1}^{n_B}(x_i - \bar{x}_B)^2}{(n_A - 1) + (n_B - 1)}}$$

degrees of freedom = $n_A + n_B - 2$

- But really all you need to know is:

1)There are two ways of *combining* 2 standard deviations together in a *t*-test

2)Which version to use

Steps in Classical/ Traditional Hypothesis Testing

Steps in Classical/ Traditional Hypothesis Testing	
Step 1	State the null and alternative hypothesis
Step 2	Select the appropriate statistical test
Step 3	Select level of significance
Step 4	Delineate regions of rejection and non-rejection of the null (α)
Step 5	Calculate test statistic
Step 6	Make decision regarding null and alternative hypothesis

1. State the null and alternative hypothesis

H0: $\mu_A = \mu_B$

H1: $\mu_A \neq \mu_B$

2. Select the appropriate statistical test

Difference of means, unpaired t-test

3. Select level of significance

$\alpha = 0.05$

4. Delineate regions of rejection and non-rejection of the null (α)

+/- 1.98

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	$\boxed{1.984}$	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

5. Calculate the test statistic

library(psych)
describe(dataset)

	vars	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurtosis	se
Chernobyl	1	100	74.04	17.24	74.00	73.12	18.69	34.79	112.81	78.02	0.28	-0.56	1.72
Fukushima	2	100	68.55	12.32	69.29	68.85	11.24	37.92	98.15	60.22	-0.24	-0.34	1.23

```
        Welch Two Sample t-test

data:  Chernobyl and Fukushima
t = 2.5913, df = 179.15, p-value = 0.01035
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 1.309376 9.671735
sample estimates:
mean of x mean of y 
 74.04123  68.55067 
```

6. Make decision regarding null and alternative hypothesis

We reject H_0

Real world interpretation: 74.04-68.55= 5.49 +/- 4.19 mSv/h difference between two sites.

Chernobyl 1986 vs 2006

- Test radiation dose after the disaster
- Return to the same place and retest to see if there is a difference
- We expect the mean radiation to (H1)
  - Be different?
  - Be less?
  - Be more?

Site	1986	2006
1	73.8	75.2
2	34.8	25.2
3	105.5	140.3
4	79.0	93.4
5	59.0	15.9
6	50.7	40.4
7	92.6	44.1
8	63.9	32.1

http://au.ibtimes.com/articles_slideshows/132654/20110411/25-years-after-chernobyl-then-and-now-photos__5.htm

Two sample difference of means (Paired)

- Paired t-test
- The difference between two samples that are dependent through pairing
- Find: $d_i = x_{1i} - x_{2i}$
  - H_0: $\mu_d = 0$
  - H_1: $\mu_d \neq 0$

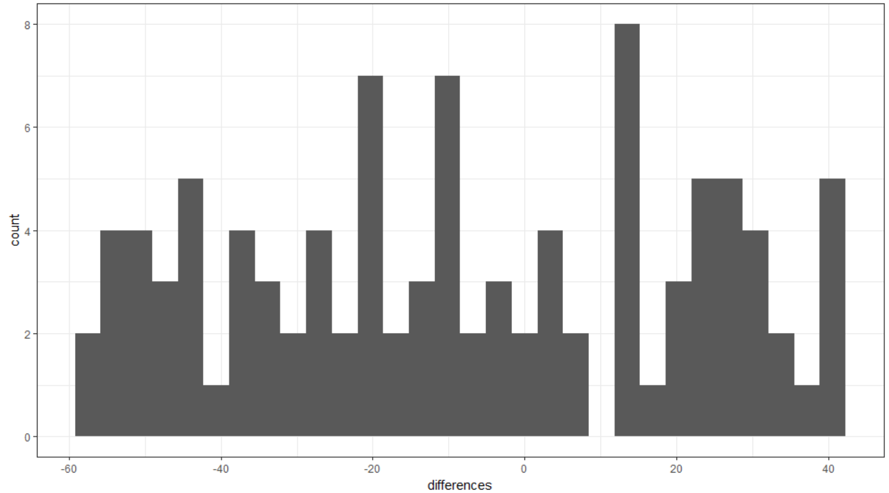
Note: you can do one-tailed tests too!

- n = the number of pairs
- s_d = the standard deviation of d
- df = n-1

$$t = \frac{\bar{d}}{s_d/\sqrt{(n)}}$$

Paired *t*-test

- You are doing the same thing as a one sample test of means on the variable differences (d)
- Has the following assumptions:
  - The sample scheme is independent and probabilistic
    - But for the second set of observations the sample is dependent/matched/paired
    - The **differences** are Normally distributed
    - Variables measured at the interval/ratio scale



1. State the null and alternative hypothesis

H_0: $\mu_A \leq \mu_B$

H_1: $\mu_A > \mu_B$

2. Select the appropriate statistical test

Difference of means, paired t-test

3. Select level of significance

$\alpha = 0.05$

4. Delineate regions of rejection and non-rejection of the null (α)

+ 1.660

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
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24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
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26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	$\boxed{1.660}$	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

Calculate test statistic

5. Calculate test statistic

```
> difference=paired$Chernobyl1986-paired$Chernobyl2006
> describe(difference)
   vars   n mean    sd median trimmed mad   min   max range  skew kurtosis   se
X1    1 100 8.02 29.32   9.91    8.07  37 -41.3 56.78 98.08 -0.01    -1.26 2.93
```

```
        Paired t-test

data:  paired$Chernobyl1986 and paired$Chernobyl2006
t = 2.7339, df = 99, p-value = 0.007415
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
  2.198241 13.834268
sample estimates:
mean of the differences
               8.016254
```

6. Make decision regarding null and alternative hypothesis

We reject H_0

Real world interpretation: Measured concentration in 1986 are 8.01 mSv/h higher than the measured concentrations in 2006

Independent vs. Dependent Samples

- Two samples are **independent** if the sample values selected from one population are not related or somehow paired or matched

- Two samples are dependent (or consist of matched pairs) if the members of one sample can be used to determine the members of the other sample
  - dependent, repeated, before and after, matched pairs, paired

Analysis of Variance (ANOVA)

- What if you want to compare more than 2 groups?

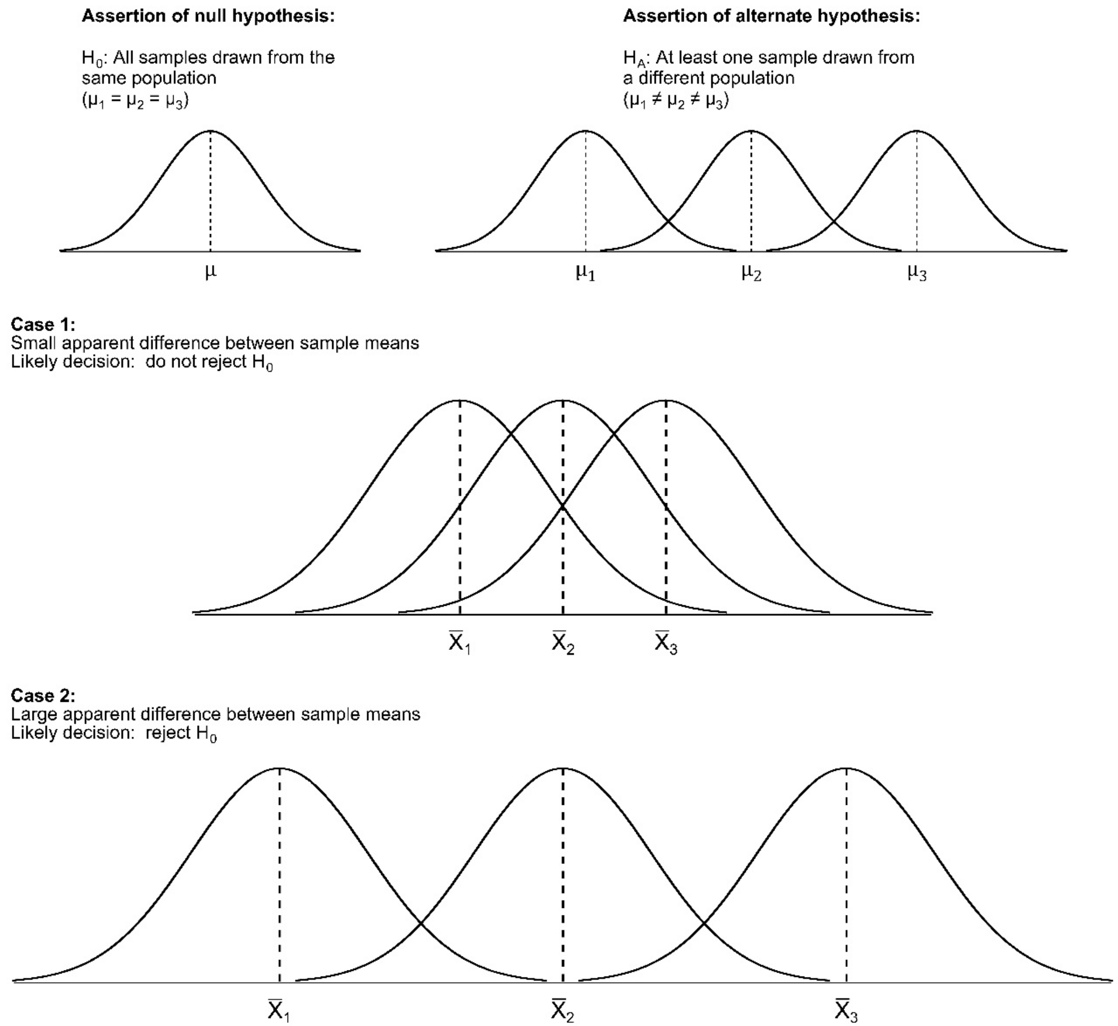


FIGURE 11.1
Null and Alternate Hypotheses in Analysis of Variance (ANOVA)

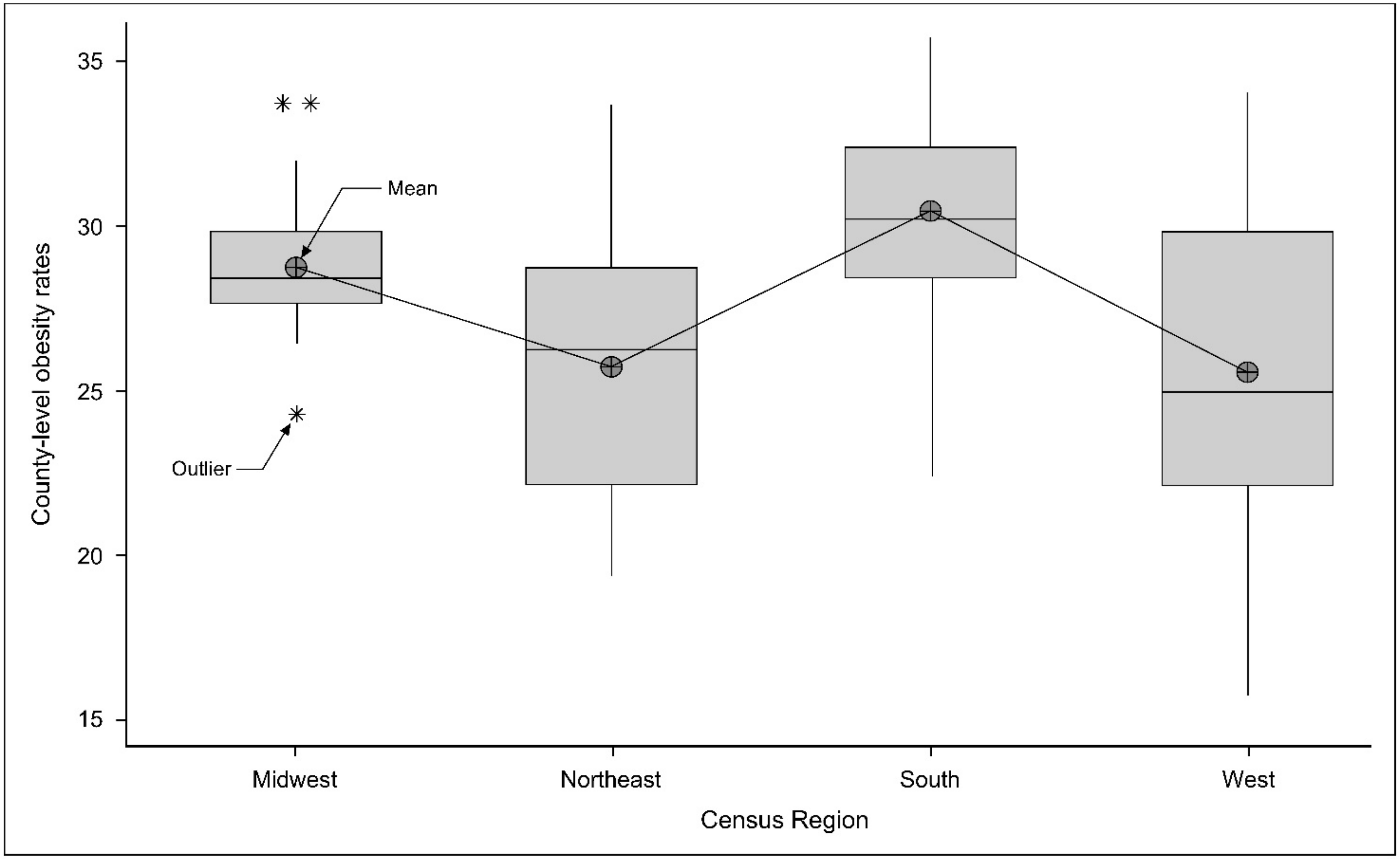
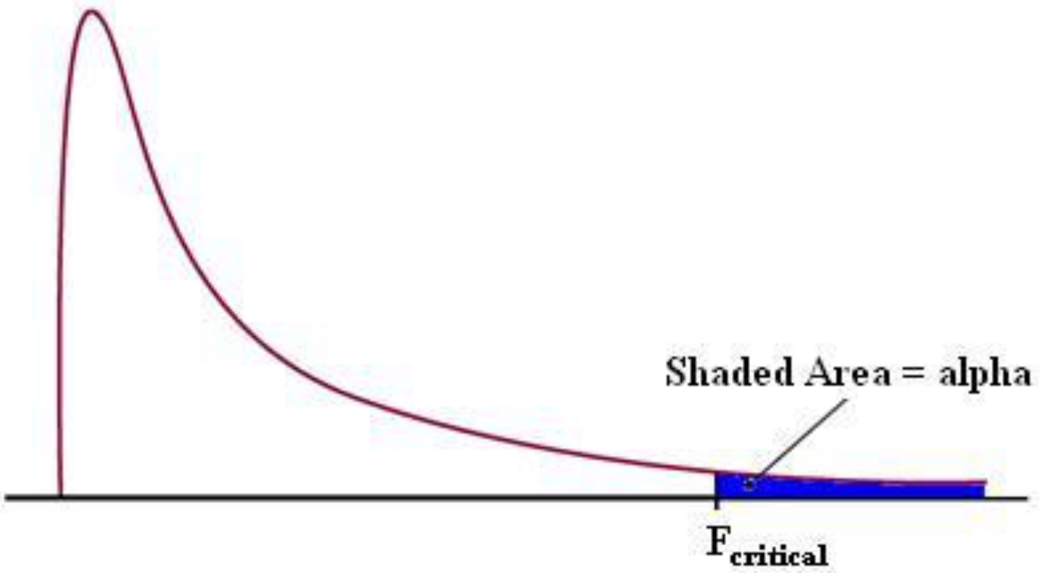


FIGURE 11.2
Boxplots of County-Level Obesity Rates by Census Region, 2008

F-distribution

- ANOVAs use a f-distribution
- From this you can only tell is there a difference between the means
- Does not tell you where the difference is:
- H_0: $\mu_A = \mu_B = \mu_C$
- H_1: $\mu_A \neq \mu_B \neq \mu_C$
- If fair to reject H_0 (you accept H_1) then you need to do follow up t-tests to tell you where the difference is
  - This is called a post-hoc test



Violation of assumptions

- What to do if the assumptions are violated?
- We will see after reading week

Example

2. On a salmon farm, 10 salmon are monitored to test the effects of a growth hormone. In a given year, we would expect that these salmon would gain 0.75 kg. Did the hormone treatment increase the average weight of the salmon above the natural weight growth? The data are listed below.

Source: http://labs.geog.uvic.ca/geog226/frLab7.html

	Sample 1	Sample 2
Salmon ID	Weight before hormone (kg)	Weight after hormone (kg)
1	4.5	5.6
2	5.0	5.8
3	4.8	5.8
4	5.2	5.7
5	4.8	7.2
6	5.8	7.3
7	4.6	6.0
8	4.9	6.9
9	4.7	6.6
10	5.1	6.9

Summary – checklist

- Is the data normally distributed?
  - Yes – go to *t*-test
  - No – consider non-parametric test (next week)
- Are samples dependent?
  - No- Use Unpaired t-test
  - Yes- Use a paired t-test
- 2 samples – check if variances are equal
  - Yes – use pooled variances
  - No – use Welch's *t*-test
- 1 sample – see last week's lecture
- 3 samples (ANOVA)