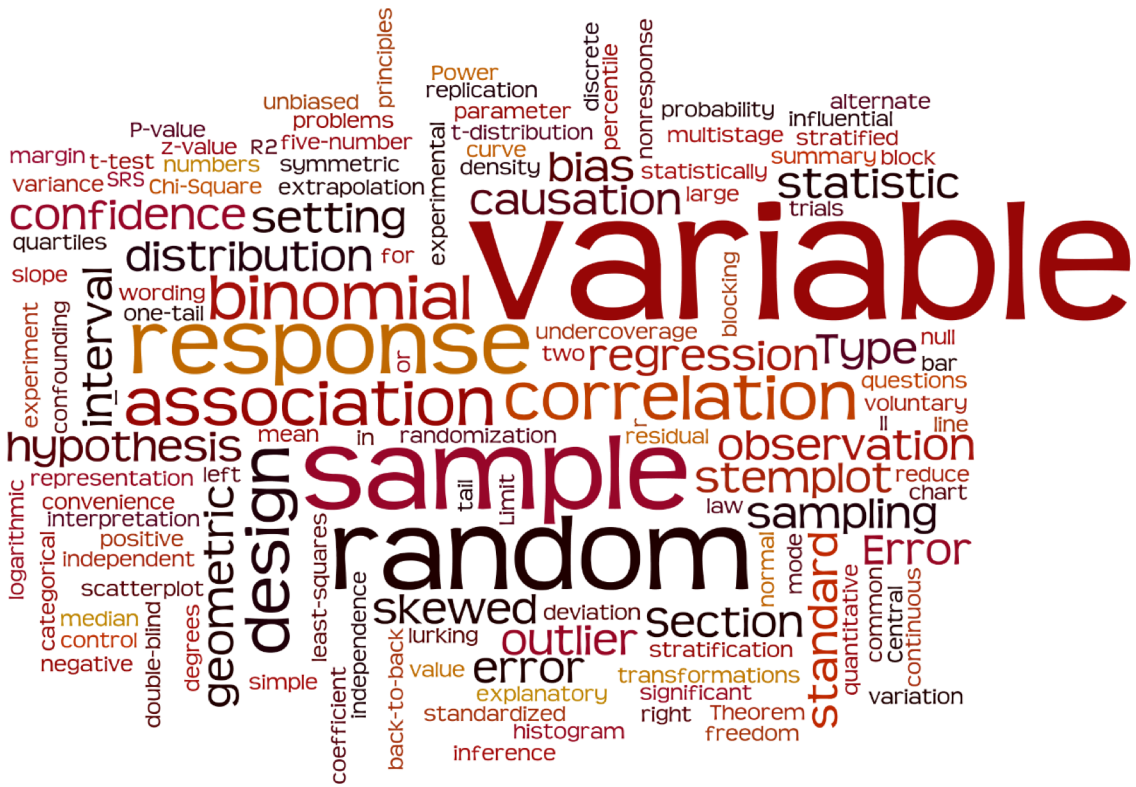
Introduction to Quantitative Methods



Kristin Eccles

Outline

Review: Normal Distribution

Philosophy

Inferential statistics – tests

One sample difference of means

Decisions and testing errors

One- vs. two-tailed tests

Assumptions

t-distribution and *t*-test

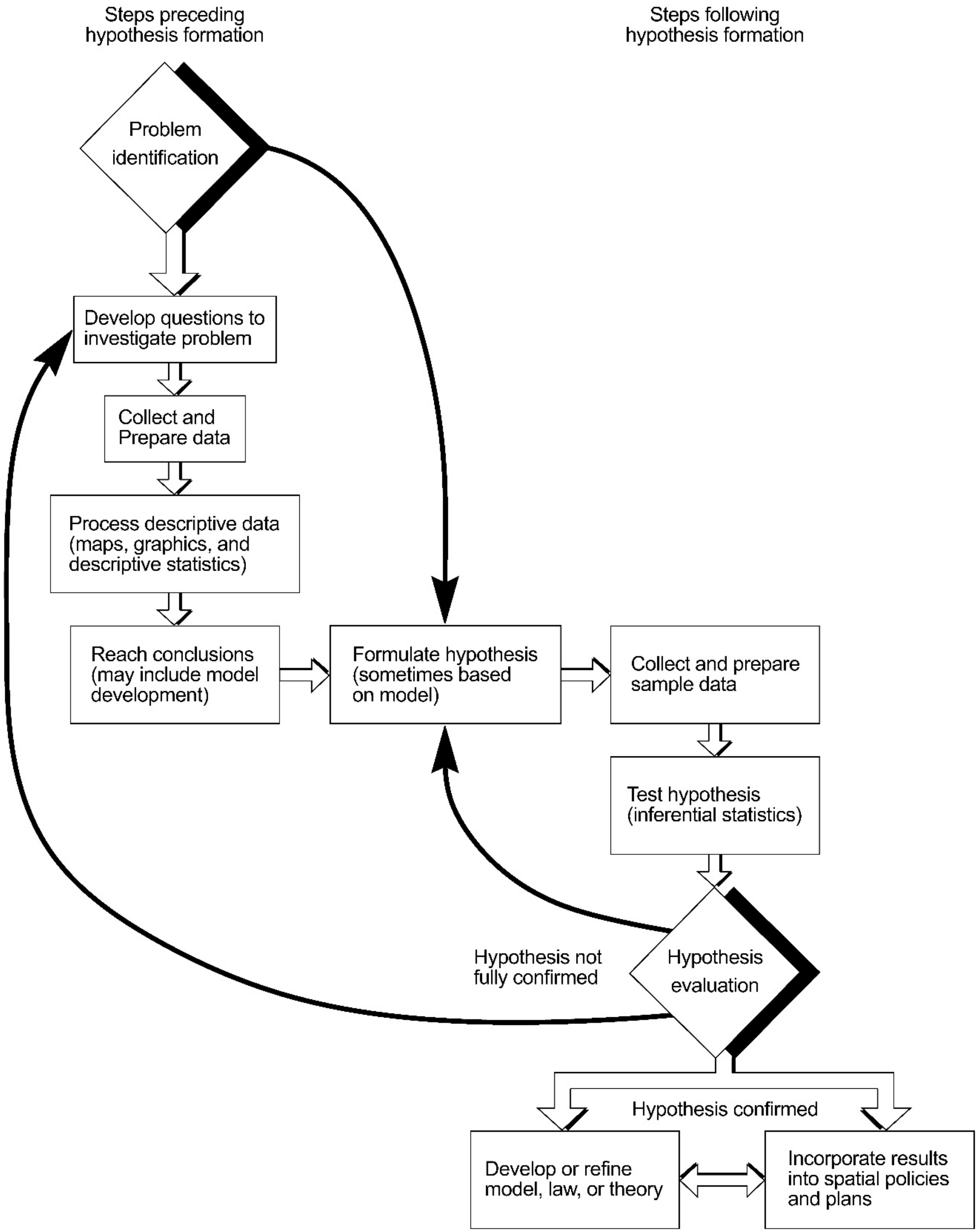


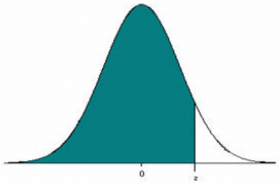
FIGURE 1.1
The Role of Statistics in the Geographic Research Process

Z-distribution (Normal)

Table of Standard Normal Probabilities for Negative Z-scores

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table of Standard Normal Probabilities for Positive Z-scores



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Note that the probabilities given in this table represent the area to the LEFT of the z-score.
The area to the RIGHT of a z-score = 1 – the area to the LEFT of the z-score

Samples vs. Populations

- Inferential statistics seeks to understand the characteristics of a population, but does so via a sample
- Characteristics of a sample are called statistics: $\bar{x}, s, s^2, n$
  - These will change with each sampling
- Characteristics of a population are called parameters: μ, σ, σ^2, N
  - These do not vary at all
- In theory, all population parameters are exact, whereas a degree of uncertainty and random error exists with a sample (see error and uncertainty topics)
- In reality, populations are too vast to determine their parameters
  - you can't really measure them without error anyway

Hg in Perch, Dickie Lake, ON

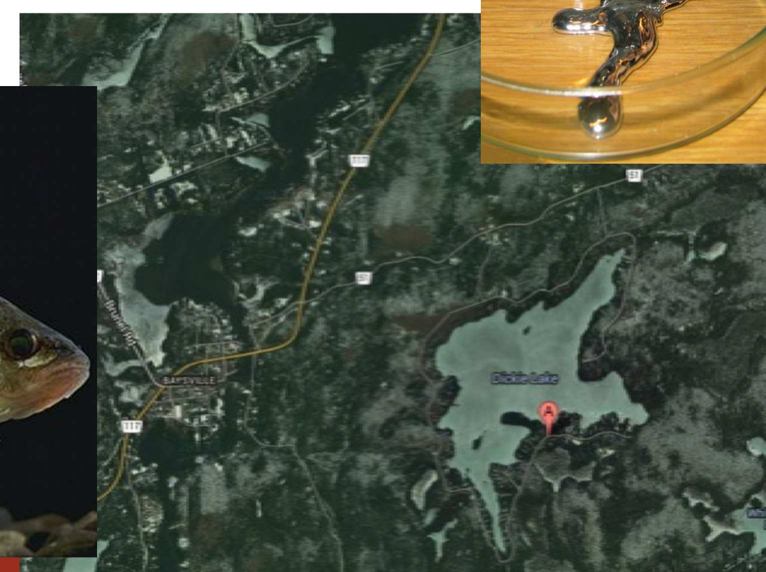
Mercury (Hg) is a contaminant in many lakes, including Dickie Lake in Muskoka, ON

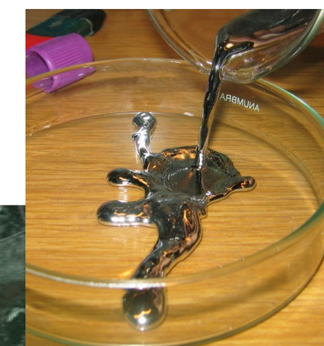
Hg residue sampled in 80 yellow perch

Mean: 150 ppb, Standard deviation: 31 ppb (parts per billion)

Is the mercury level in Dickie Lake perch safe for a pregnant women to eat a serving weekly....?

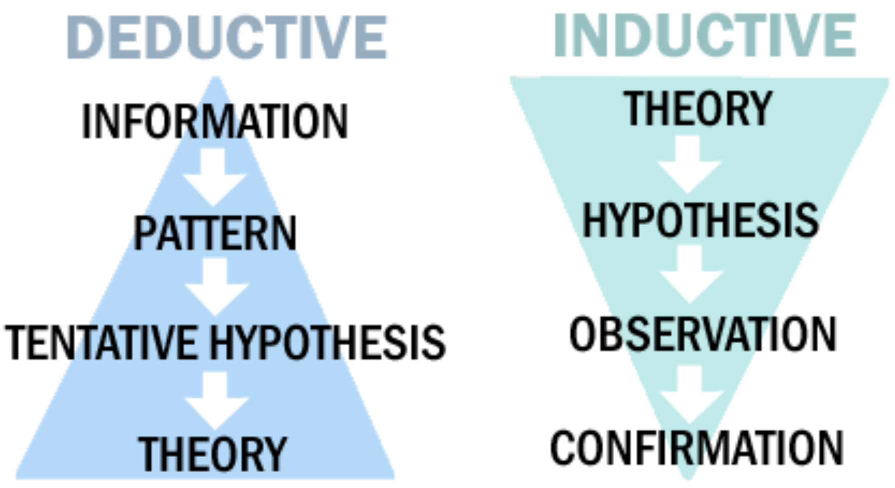
Not in the sample, but rather the ***population***





Philosophy of science

- **Deduction**
  - Draw specific conclusions from general principles
  - Proposing a new idea (hypothesis) based on examination of existing theory
- **Induction**
  - Draw general conclusions from specific observations
  - Looks for trend or pattern then generalizing/ extrapolating
  - Prone to error
- How can we create new scientific knowledge?
  - Testing falsifiable claims with observations



Inductive vs. Deductive

- Deduction
  - All men are mortal. Socretes is a man. Therefore socretes is mortal.
  - Monocot flower parts are in multiples of three. Apple flowers have five petals. Therefore, apple trees are not monocots.
- Induction
  - Every swan I have ever seen is white. Therefore, all swans are white.
  - All basketball players in your school are tall, so all basketball players must be tall.

Hypothetico-deductive method

- Observe and take data
- Make a research hypothesis about those data using inductive reasoning
- Make a prediction that follows using deductive reasoning
- Devise an experiment/test of that prediction
- Corroboration – do the results support the hypothesis?
  - YES = More predictions, more tests
  - NO = Falsification – generate new hypothesis
- N.B. Research hypotheses are never proven, but they can be falsified

Where do statistics fit in?

- Experimental/measurement error
  - There is always uncertainty
  - Is your method precise enough?

- Good enough? Close enough?
  - Must rely on probability to determine this
  - Cutoff is given by the statistical significance

- Statistical hypothesis – relates to the test (not exactly the same as a research hypothesis, but similar)

Steps in Classical/ Traditional Hypothesis Testing

Steps in Classical/ Traditional Hypothesis Testing	
Step 1	State the null and alternative hypothesis
Step 2	Select the appropriate statistical test
Step 3	Select level of significance
Step 4	Delineate regions of rejection and non-rejection of the null (α)
Step 5	Calculate test statistic
Step 6	Make decision regarding null and alternative hypothesis

Step 1: State the null and alternative hypothesis

- Evaluates the validity of our prediction using a statistical test
  - This will say if the result is statistically significant or not
- Two hypotheses:
  - H_0 – the null hypothesis
    - Status quo, no change, nothing special going on, conservative
  - H_A – the alternate hypothesis (aka H_1)
    - Your hypothesis – there is a difference/change.
- They are mutually exclusive and exhaustive!
- You will test the null hypothesis- Why?

Null and Alternative Options

H_0: $\mu = \mu_h$

H_A : $\mu \neq \mu_h$ (two-tailed)

H_0: $\mu \leq \mu_h$

H_A : $\mu > \mu_h$ (upper-tailed)

H_0: $\mu \geq \mu_h$

H_A : $\mu < \mu_h$ (lower-tailed)

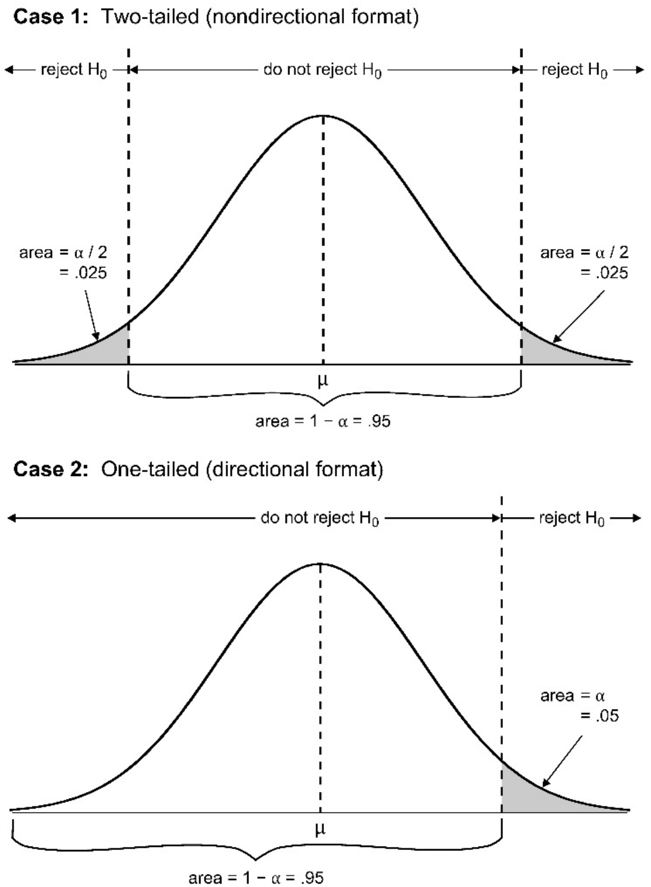


FIGURE 9.1
General Regions of Rejection and Non-rejection of Null Hypothesis: Significance Level (α) = .05

Step 2: Choose a statistical test

- It is recommended that pregnant women don't eat more than one serving of fish containing Hg levels of 144 ppb
- The Dickie Lake sample (n=80) had a mean Hg concentration of 150 ppb and a standard deviation of 31 ppb
- Is the sample mean (μ) significantly higher than the recommended amount?

- **One sample difference of means**
- The null hypothesis assumes that no difference exists
- The alternative hypothesis assumes that some difference exists between the sample mean (μ) and the comparison value (μ_h)

Step 2: Choose a statistical test

- H_0: $\mu \leq 144$
- H_A: $\mu > 144$

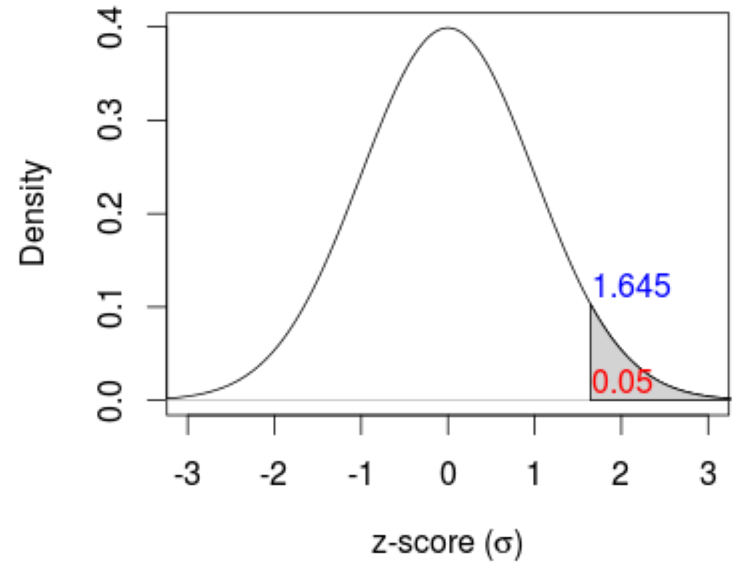
NOTE: This is the traditional way of writing a hypothesis

Step 3: Select Level of Significance

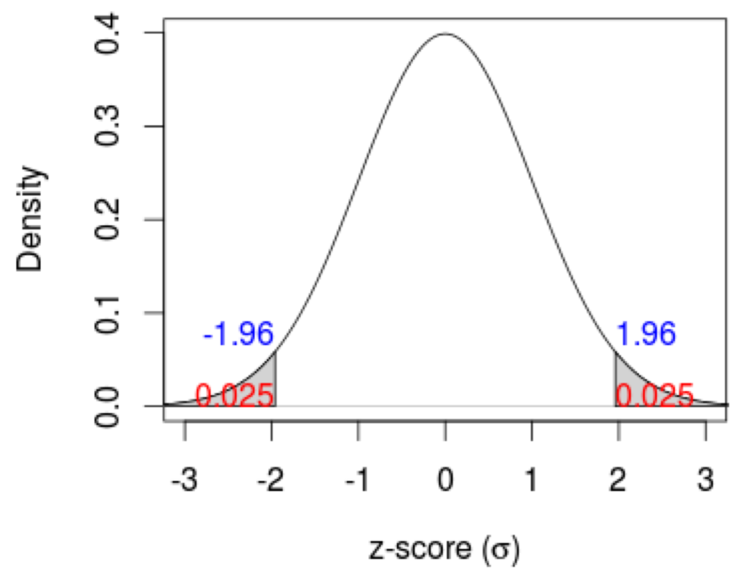
- What **significance level** should we adopt?
  - How sure of this result do we want to be?
  - This should be decided ahead of time (*a priori)*
  - 95%? → 0.95 → $\alpha = 0.05$ (standard/typical)
  - Any *probability* of obtaining a more extreme test statistic due to random chance

Step 4: Delineate regions of rejection

- H_A contains > or <
- The extreme can *only be* in **one** tail
- Critical *p*-value is α



- H_A contains ≠
- The extreme can be in *either* tail
- Critical *p*-value is α/2 in *either* tail
- BUT note... when you do a 2-sided test, with Excel or R, the *p*-value is listed for *both sides together*. So, in that case just compare *p* to α



Critical Thresholds

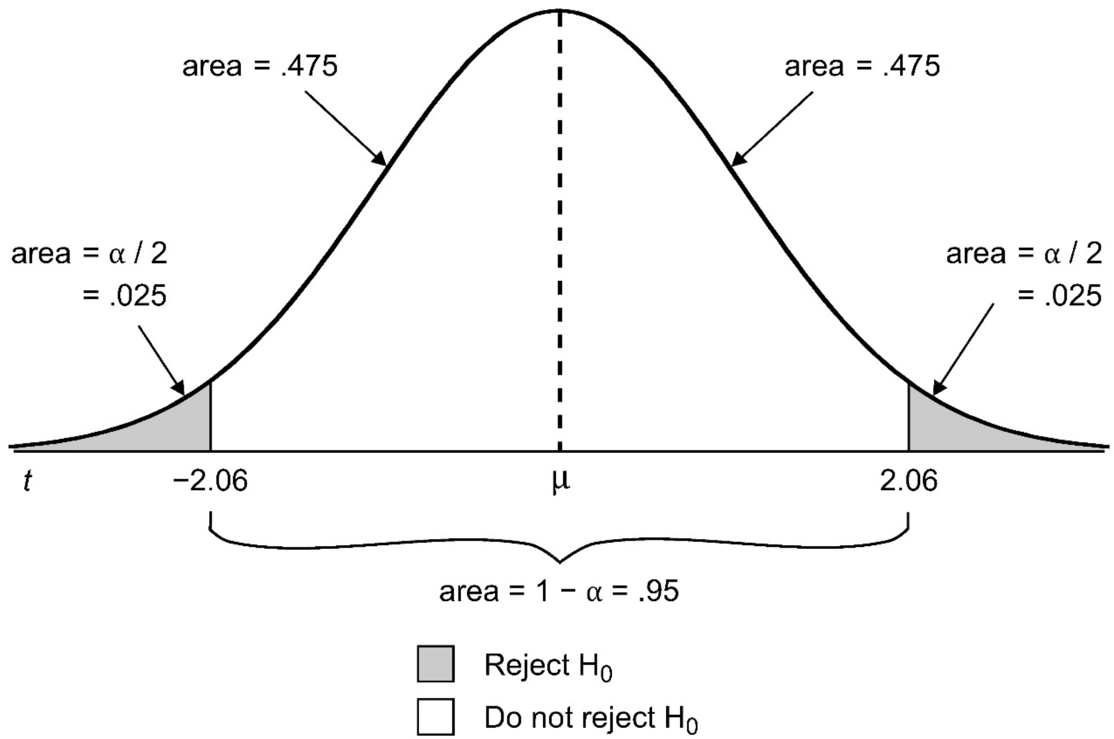


FIGURE 9.2
Normal Distribution Values Associated with a Significance Level (α) = .05: Two-Tailed Case

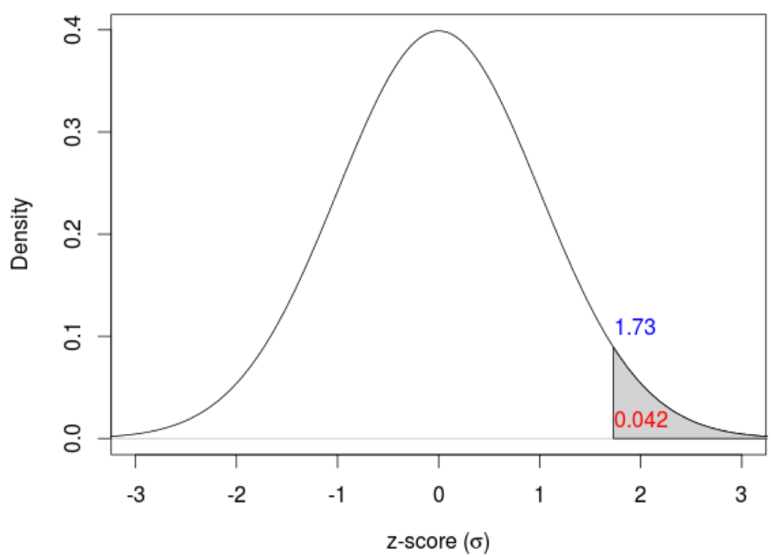
Step 5: Calculate the test statistic

- Test statistic is Z →

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{(n)}} \qquad Z = \frac{150 - 144}{31/\sqrt{80}}$$

Standard error of the mean (**SEM**)= $s/\sqrt{(n)}$

- Z = 6/3.47 = 1.73
- What is the probability of getting a larger Z?
- P(Z > 1.73) = 0.042
  - Look up in a table/computer
  - Known as the ***p*-value**
- Ask: Is the test statistic (Z value) extreme?
- If so, what is the chance of getting more than this value due to random chance?



Z is given in blue; P(>Z) is in red

What is the p-value?

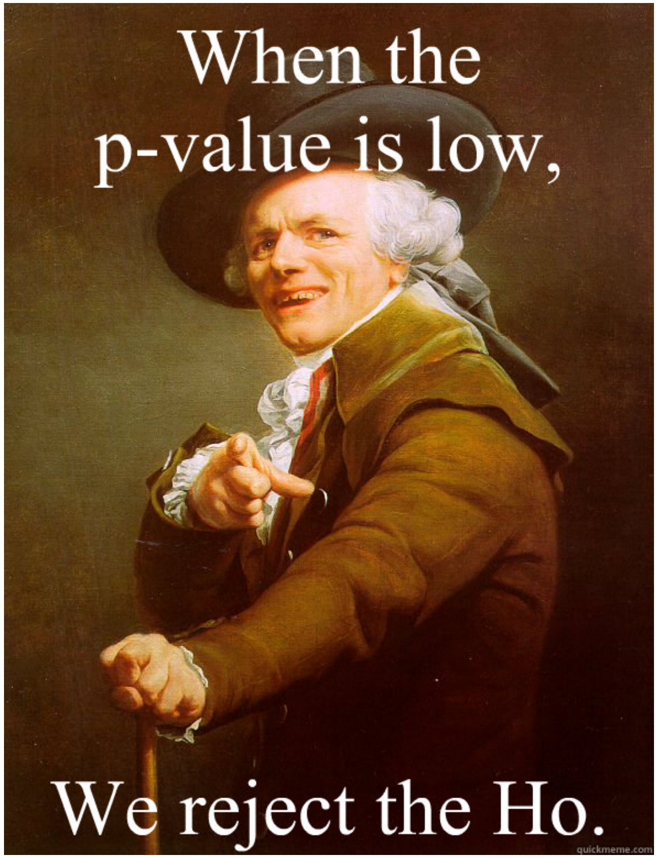
- **The P value** a.k.a. calculated probability
- The probability of finding a more extreme, results when the null hypothesis (H_0) of no difference is true
- The probability of making a **type I error**

Step 6: Make decision regarding null and alternative

- Now decide if this significance is high enough to:
  - Reject the null hypothesis – which supports the alternate hypothesis (technically accept H_A or H_0)

    OR

  - Fail to reject the null hypothesis – which implies the alternate hypothesis is not supported (don't accept H_A or H_0)
  - In our case: $p = 0.042$ vs. $\alpha = 0.05$; *this **is** significant*
  - **Decision: Reject H_0 (and accept H_A)**

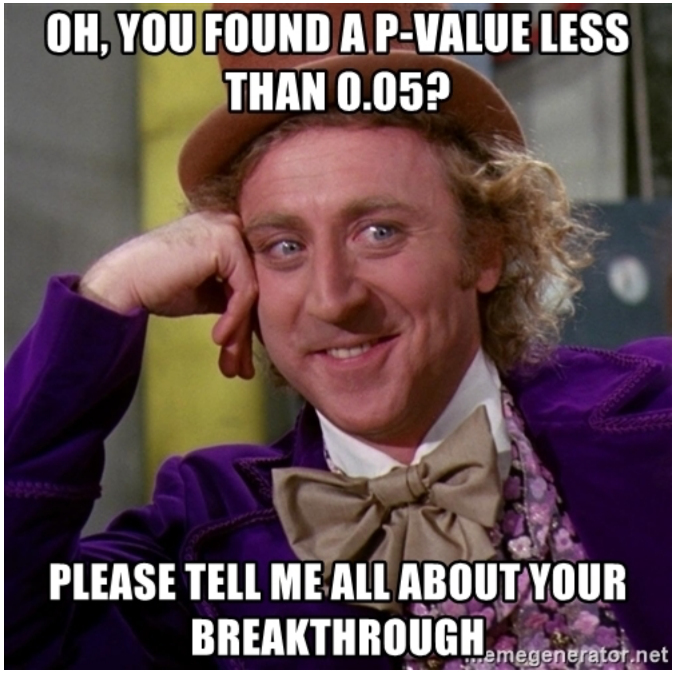


Terminology

- We always talk about the null (remember)
- Why?
  - We are always trying to falsify our results
- Terminology
  - We accept the null (there is no difference/relationship)
  - We fail to reject the null (there is evidence of a difference/relationship)
    - This links back to our philosophy of science
    - We didn’t prove anything is true- just that its not, not true
    - Yes confusing- I know!

Burden of proof

- If we have enough evidence to reject H_0, then we prove H_0 false (based on p-value)
  - BUT it is possible that H_0 is actually true
- If we don't have enough evidence to reject H_0, then we accept H_0 by default
  - BUT it is possible that H_0 is actually false



Decision errors

		The unknowable truth	
		H_0 is true / H_1 is false	H_0 is false / H_1 is true
Your test decision	Do not reject H_0	OK	Type II Error
	Reject H_0	Type I Error	OK

- Type I error = significance = alpha (α)
  - You set the significance level and therefore the Type I error you are willing to accept
- Type II error = 1-power = beta (β)
  - Depends on the experimental design (sample size)
  - Hard to detect and they are worse

Decision errors

		Truth	
		H_0 is true / H_A is false **Actually no fire**	H_0 is false / H_1 is true **There is a fire**
Fire alarm response	H_0= No fire **Alarm doesn't go off**	OK	Type II Error (bad ☹)
	H_A= Fire **Alarm goes off**	Type I Error (False alarm)	OK

H_0= No fire
H_1= Fire

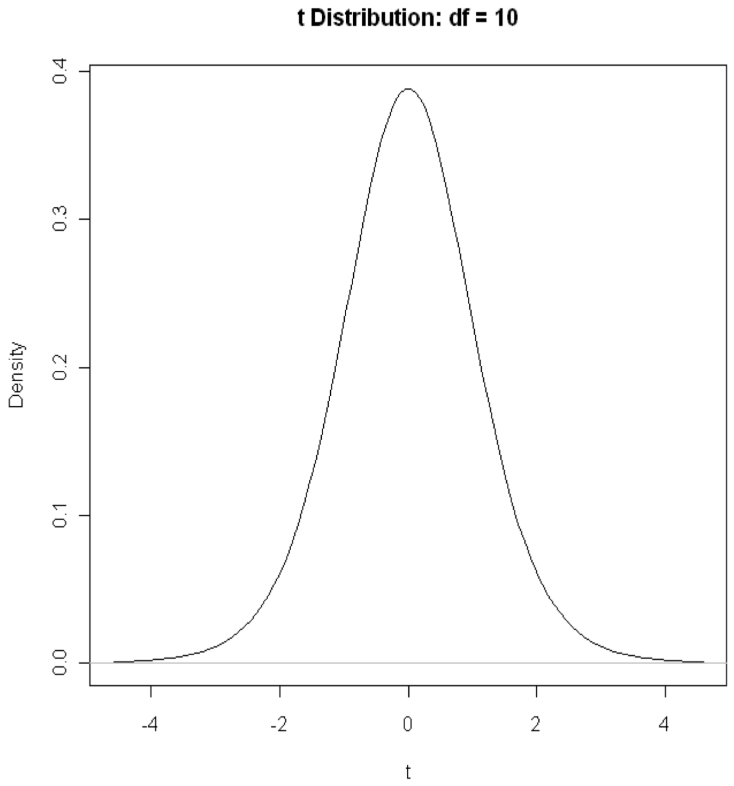
Test assumptions

- An important consideration!!
- Z-test assumes:
  - Random sample
  - Population that was sampled has a normal distribution
  - Variable is measured at the interval/ratio scale
  - Sample size is > 30
- If these requirements are not met, then *chose another test*!

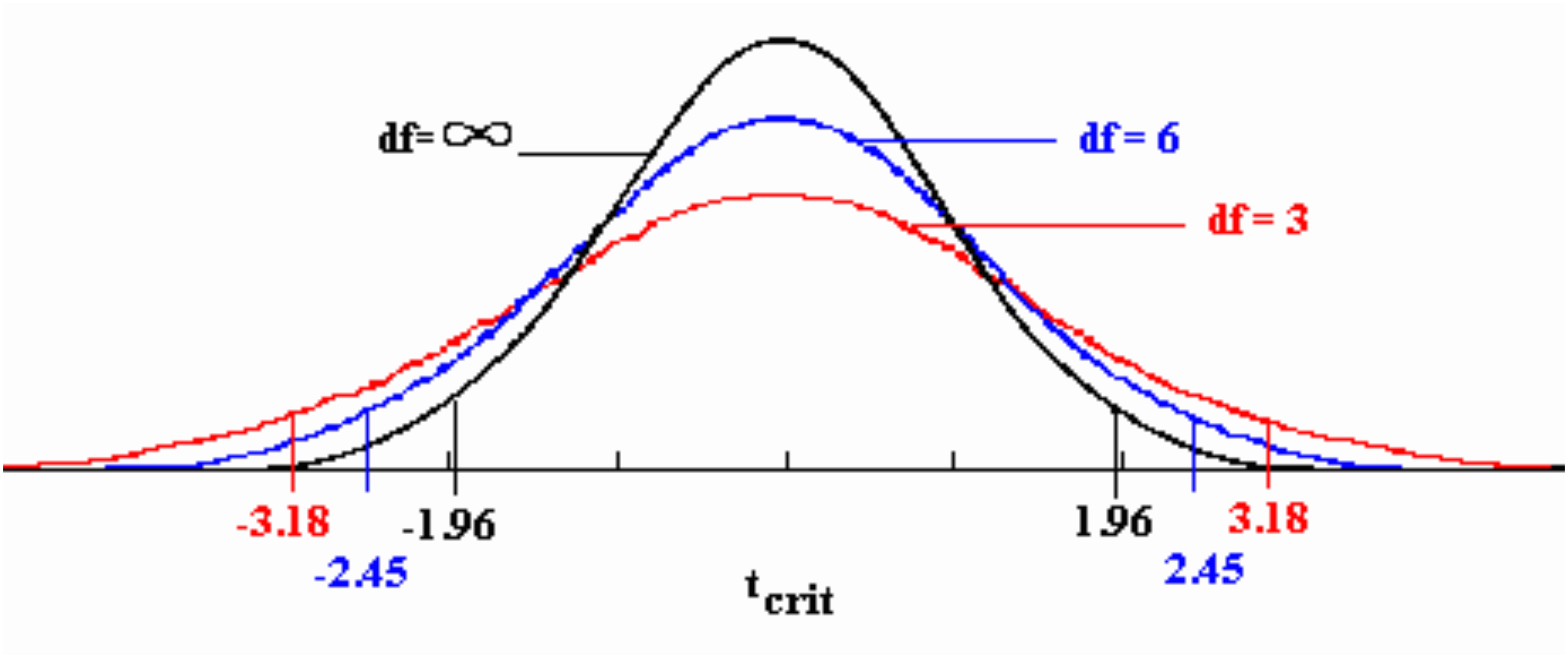
- **Note: <u>All</u> statistical tests have assumptions.**

Introducing the *t* distribution- Small Sample Sizes

- A probability density function, like the normal distribution, but for small sample sizes
- A bit broader than the normal distribution
  - Needs 1 parameter:
    - degrees of freedom
    - df = n-1
- Degrees of freedom:
- how many values in a
- calculation are free to vary?
- df for mean, std?



T vs. Z distribution



T-Distribution

- Approximates a normal distribution for small sample sizes (<30)
  - T-distribution is flatter and more spread out than the normal distribution
  - ”Fatter tails”
- Why?
  - Remember the central limit theorem, that a sample distribution will follow a normal distribution **as long as the sample size is sufficiently large**
  - But sample sizes are sometimes small
  - And we do not always know the standard deviation of the population
  - When either of these problems occur we use the distribution of the **t statistic** (also known as the **t score**)

Student's t-test

- t-test assumptions:
  - Random sample
  - Population that was sampled has a normal distribution
  - Variable is measured at the interval/ratio scale
- t -statistic – pretty similar to the Z statistic
  - **NB: some use n-1 to calculate the SEM**
    - **Why?**

$$t = \frac{\bar{x} - \mu}{s/\sqrt{(n)}}$$

- look up critical value using degrees of freedom
  - (df = n-1)
  - For perch example: t = 1.731, df = 79
  - p-value = 0.0436 (still significant)
- Can you use t-test instead of Z-test for large sample sizes?
  - Yes, it will work very well, so t-tests are more common than z-tests

T-look up table

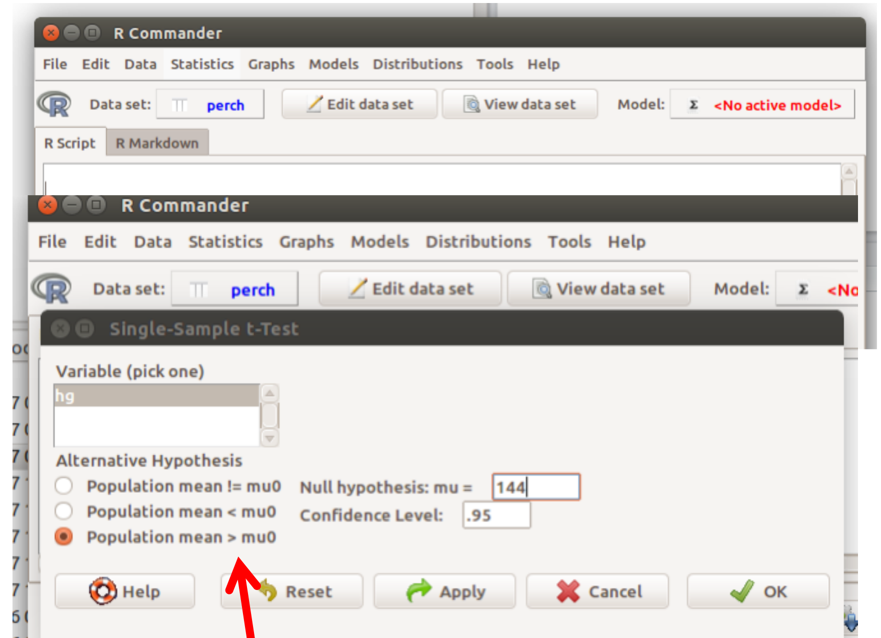
cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

t-test with R/Rcmdr

Go to MENU/Statistics/Single Sample t test

Complete the form:

- Population mean > mu0
- Null hypothesis mu = 144



Rcmdr> t.test(perch$hg, alternative='greater', mu=144, conf.level=.95)

test hypothesis (H_1)

One Sample t-test

data: perch$hg

t = 1.7311, df = 79, p-value = 0.04366

alternative hypothesis: true mean is greater than 144

95 percent confidence interval:

144.2314 Inf

sample estimates:

mean of x

150

test statistic (*t*) & *p*-value: the probability of getting a more extreme *t* due to random chance

confidence interval

sample mean

Confidence intervals of the mean (CI)

- The probability that the interval surrounding the sample mean ($\bar{\mathbf{x}}$) actually contains the mean of the population (μ)
- Often expressed as a percent:
  - 95% and 99% confidence intervals are standard
- Can calculate any confidence interval:

$$P\left(\bar{x} - \left|z_{\alpha/2}\right| \times SEM < \mu < \bar{x} + \left|z_{\alpha/2}\right| \times SEM\right) = 1 - \alpha$$

- Where α (the **alpha value**) is the complement of the confidence interval probability
  - $\alpha = 1 - (CI/100)$
- Assumes a large sample size ($n > 30$)

Confidence Interval continued

- In practice, the confidence interval can be understood as the mean $\pm$ the Margin of Error (ME)
  - the region between the **lower bound** (mean – ME) and the **upper bound** (mean + ME)
- It is where you expect to find the *population mean* (μ), but remember: you cannot be certain of this, so you need to assign a probability
- =CONFIDENCE.NORM(α, stdev, n)

$$ME = \left|z_{\alpha/2}\right| \times SEM$$

One sample test of proportions

- Where $\hat{p}$ is the proportion from your sample and p_0 is the hypothesized proportion

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$

- Assumptions:
  - Independent samples
  - Random/probabilistic sampling
  - Large sample size
- Note that R uses a different formula to test proportions

Summary

Steps in hypothesis testing

Define H0

Define HA

Determine α

Choose an appropriate test
One or two-tailed?

Assumptions met?

Calculate the test statistic or p-value

Compare the p-value or test statistic to the α or critical value

Make a decision

Report your findings

Resources

Hypothesis Testing using Z and T scores:

https://www.khanacademy.org/math/statistics-probability/significance-tests-one-sample/tests-about-population-mean/v/hypothesis-testing-and-p-values