

PDA to CFG

Let $M = (\{q_0, q_1\}, \{0, 1\}, \{X, Z_0\}, q_0, Z_0, \delta, \emptyset)$ where δ is defined as:

$$\begin{aligned}\delta(q_0, 0, Z_0) &= \{(q_0, XZ_0)\} & \delta(q_0, 0, X) &= \{(q_0, XX)\} \\ \delta(q_0, 1, X) &= \{(q_1, \Lambda)\} & \delta(q_1, 1, X) &= \{(q_1, \Lambda)\} \\ \delta(q_1, \Lambda, X) &= \{(q_1, \Lambda)\} & \delta(q_1, \Lambda, Z_0) &= \{(q_1, \Lambda)\}\end{aligned}$$

$$\begin{aligned}S &\rightarrow [q_0, Z_0, q_0]q_0, Z_0, q_1 & [q_0, X, q_1] &\rightarrow 1 \\ [q_1, X, q_1] &\rightarrow 1|\Lambda & [q_1, Z_0, q_1] &\rightarrow \Lambda\end{aligned}$$

$$\begin{aligned}\frac{[q_0, Z_0, p] \rightarrow 0[q_0, X, q][q, Z_0, p]}{[q_0, Z_0, q_0] \rightarrow 0[q_0, X, q_0][q_0, Z_0, q_0]} \\ [q_0, Z_0, q_1] \rightarrow 0[q_0, X, q_1][q_1, Z_0, q_1] \\ [q_0, Z_0, q_1] \rightarrow 0[q_0, X, q_0][q_0, Z_0, q_1] \\ [q_0, Z_0, q_0] \rightarrow 0[q_0, X, q_1][q_0, Z_1, q_0]\end{aligned}$$

$$\begin{aligned}\frac{[q_0, X, p] \rightarrow 0[q, X, q_0][q, X, p]}{[q_0, X, q_0] \rightarrow 0[q_0, X, q_0][q_0, X, q_0]} \\ [q_0, X, q_1] \rightarrow 0[q_1, X, q_0][q_1, X, q_1] \\ [q_0, X, q_1] \rightarrow 0[q_0, X, q_0][q_0, X, q_1] \\ [q_0, X, q_0] \rightarrow 0[q_0, X, q_1][q_1, X, q_0]\end{aligned}$$

After eliminating the useless stuff (by observation) we get:

$$\begin{aligned}S &\rightarrow [q_0, Z_0, q_1] & [q_1, Z_0, q_1] &\rightarrow \Lambda & [q_1, X, q_1] &\rightarrow \Lambda|1 \\ [q_0, Z_0, q_1] &\rightarrow 0[q_0, X, q_1][q_1, Z_0, q_1] & [q_0, X, q_1] &\rightarrow 1|0[q_0, X, q_1][q_1, X, q_1]\end{aligned}$$

Another example is $L = \{01^n0 | n \geq 0\}$

Let $M = (\{q_0, q_1, q_2\}, \{0, 1\}, \{X, Z_0\}, q_0, Z_0, \delta, \emptyset)$ where δ is defined as:

$$\begin{aligned}\delta(q_0, 0, Z_0) &= \{(q_1, XZ_0)\} & \delta(q_1, 1, X) &= \{(q_1, X)\} \\ \delta(q_1, 0, X) &= \{(q_2, \Lambda)\} & \delta(q_2, \Lambda, Z_0) &= \{(q_2, \Lambda)\}\end{aligned}$$

$$\begin{aligned}S &\rightarrow [q_0, Z_0, q_0][q_0, Z_0, q_1][q_0, Z_0, q_2] \\ [q_1, X, q_2] &\rightarrow 0 & [q_2, Z_0, q_2] &\rightarrow \Lambda\end{aligned}$$

$$\begin{aligned}\frac{[q_1, X, q] \rightarrow 1[q_1, X, q]}{[q_1, X, q_0] \rightarrow 1[q_1, X, q_0]} \\ [q_1, X, q_1] \rightarrow 1[q_1, X, q_1] \\ [q_1, X, q_2] \rightarrow 1[q_1, X, q_2]\end{aligned}$$

$$\begin{aligned}\frac{[q_0, Z_0, p] \rightarrow 0[q_1, X, q][q, Z_0, p]}{[q_0, Z_0, q_0] \rightarrow 0[q_1, X, q_0][q_0, Z_0, q_0]} \\ [q_0, Z_0, q_1] \rightarrow 0[q_1, X, q_1][q_1, Z_0, q_1] \\ [q_0, Z_0, q_2] \rightarrow 0[q_1, X, q_2][q_2, Z_0, q_2] \\ [q_0, Z_0, q_1] \rightarrow 0[q_1, X, q_0][q_0, Z_0, q_1] \\ [q_0, Z_0, q_2] \rightarrow 0[q_1, X, q_0][q_0, Z_0, q_2] \\ [q_0, Z_0, q_0] \rightarrow 0[q_1, X, q_1][q_1, Z_0, q_0] \\ [q_0, Z_0, q_2] \rightarrow 0[q_1, X, q_1][q_1, Z_0, q_2] \\ [q_0, Z_0, q_0] \rightarrow 0[q_1, X, q_2][q_2, Z_0, q_0] \\ [q_0, Z_0, q_1] \rightarrow 0[q_1, X, q_2][q_2, Z_0, q_1]\end{aligned}$$

After eliminating the useless stuff (by observation) we get:

$$\begin{aligned}S &\rightarrow [q_0, Z_0, q_2] & [q_1, X, q_2] &\rightarrow 0 & [q_2, Z_0, q_2] &\rightarrow \Lambda \\ [q_1, X, q_2] &\rightarrow 1[q_1, X, q_2] \\ [q_0, Z_0, q_2] &\rightarrow 0[q_1, X, q_2][q_2, Z_0, q_2]\end{aligned}$$