0.1 Independent component and dependent component

Let's say that we have two components A and B and B depends on A, but A is independent of B. We can integrate this system for a single time step in a compositional way and we can extract the linear part of the flow at the end of the timestep, let's call that linear part as M.

This linear part can be represented by the it's coefficients

$$M = \begin{bmatrix} M_{A,A} & M_{A,B} \\ M_{B,A} & M_{B,B} \end{bmatrix}$$

The rows in $M_{A,A}, M_{A,B}$ corresponds to variables existing in component $M_{A,A}$ and the rows in $M_{B,A}, M_{B,B}$ correspond to variables existing in B. All the coefficients of component A parameters are in the columns $M_{A,A}, M_{B,A}$ and the coefficients of B are in the columns $M_{A,B}, M_{B,B}$.

Since A is independent of B, we know that the linear part of the variables present in A won't have have any parameters of B, meaning the submatrix $M_{A,B}$ is only zeros:

In QR preconditioning, the QR factorization is done on matrix this matrix M, giving us the matrices Q and R, we are interested in Q, let's say that it has the form:

$$Q = \begin{bmatrix} Q_{A,A} & Q_{A,B} \\ Q_{B,A} & Q_{B,B} \end{bmatrix}$$

What I initially wanted to prove is that $Q_{A,B}$ can only have zeros. I don't think that is the case now. For example,

$$\begin{bmatrix} 1 & 0 \\ 3 & 4 \end{bmatrix}$$

has all nonzero elements in it's Q matrix.

Which also means that you would be able to compute the flowpipes at the further timesteps for variables in A separately, but they would depend on the same number of parameters as what was in the full system (not as the number that were in component A). Meaning that there wouldn't be those big performance gains.

An idea I have about this is that component A is still fully solvable, so maybe it is possible to have an solution for it and then make that solution suitable for solving B by making the same coordinate transformations as are done in QR for each step. I don't really have a good idea how to do it though right now.

When doing the QR decomposition on the whole component, the initial set becomes

$$T\vec{M} = (\vec{c} + Q\vec{a}) \circ TM_r$$

and you want to integrate only on the part $TM_l = \vec{c} + Q\vec{a}$.

This proposes some problems in the compositional case.

Let's say that you have 2 variables x_1 and x_2 and x_1 is independent of x_2:

$$\begin{aligned} \tfrac{dx_1}{dt} &= f_1(x_1) \\ \tfrac{dx_2}{dt} &= f_2(x_1, x_2) \end{aligned}$$

So you can solve x_1 separately. It makes sense to assume that the initial conditions for x_1 won't include the parameters of x_2, giving the possibility of taking advantage of that and having smaller data structures coming from that.

This is property won't likely hold after doing the QR composition, the model $\vec{c}+Q\vec{a}$ won't guarantee zero elements in Q. The result of that is that you need to use more parameters in order to solve independent components (the variables in independent component still solely depend on the initial conditions of those variables, but these initial conditions are include more parameters). There is still some performance gain coming from this, but it might be trivial (you may calculate a subset of remainders instead of all of them).

0.2 2 Independent components

A and B are independent.

Linear part is:

$$M = \begin{bmatrix} M_{A,A} & M_{A,B} \\ M_{B,A} & M_{B,B} \end{bmatrix}$$

in this case, both $M_{A,B}$ and $M_{B,A}$ are filled with zeros.

After QR, the Q is:

$$Q = \begin{bmatrix} Q_{A,A} & Q_{A,B} \\ Q_{B,A} & Q_{B,B} \end{bmatrix}$$

and both $Q_{A,B}$ and $Q_{B,A}$ seem to be only filled with zeros.

Guaranteeing this (given that $M_{A,B}$ and $M_{B,A}$ are filled with zeros) is similar to algebra question I had before.

I think this is very restrictive though, so I'm not sure how interesting this is because of that.