Sensors

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1 Introduction

We are given a set of points on a sphere of radius 1, which we can view as sensors on the surface of the Earth. The sensors are used to gather data and form a sensor network with parameters r and R, where each sensor gathers data from the surrounding area in the shape of a circle of radius R and each sensor can communicate with other sensors which are at most r away.

The goal of this project was to determine r and R, so that:

1. numbers r and R are as small as possible (that would decrease the cost of sensors),
2. the sensor network is connected (i.e. the Vietoris-Rips graph is connected),
3. the sensor network covers the whole sphere (the Čech complex should be homotopy equivalent to the sphere, i.e. the Euler characteristic of the Čech complex should be that of a sphere).

Once we established the parameters r in R, we have also returned a list of obsolete sensors, i.e. sensors, whose removal would not change the desired properties 2. and 3. of the sensor network.

We implemented our method in Python using the GUDHI library for Vietoris-Rips complex. To generate the Čech complex we have used the MiniBall algorithm on the Vietoris-Rips complex from the third homework.

2 Methodology

2.1 Finding R and r

We start with small value of r and increment the value by small amount ϵ until all the sensors are connected. We check if the sensors are connected by checking the number of components of the Vietoris-Rips graph. To generate the Vietoris-Rips graph and get the number of components we use the GUDHI library [1]. Once we find the appropriate r we set the initial value of R to $\frac{r}{2}$.

We start increasing the value of R by a small amount ϵ until the sensors can cover the whole sphere. In order to determine if the whole sphere is covered we calculate the Euler characteristic of the Čech complex and compare it to Euler characteristic of a sphere.

2.2 Finding obsolete sensors

Once we have obtained R and r we can find the sensors that are obsolete. We simply remove each sensor from the list of sensors and check if the whole sphere is still being covered using the Euler characteristic of the generated Čech complex. If a sphere is covered we can add the removed sensor to the list of obsolete sensors.

3 Results

In this project we perform 3 different experiments. In the first experiment we want to find appropriate R and r as well as obsolete sensors from a list of 54 sensors given to us. We find out that there are 2 obsolete sensors and minimum R and r equal to 0.360 and 0.500 respectively.

For the second experiment we randomly sample 50 points on a sphere as presented on figure 3. We calculate that the minimum R and r are equal to 0.595 and 0.650 respectively and that the 31 sensor are obsolete.

For the third experiment we use a Fibonacci spiral to generate 50 uniformly distributed points on a sphere as presented on figure 5. We calculate that the minimum R and r are equal to 0.415 and 0.510 respectively and that 3 sensor are obsolete.

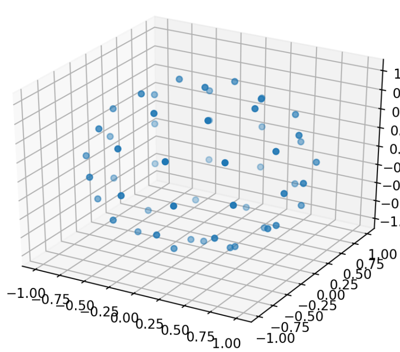

Figure 1: Given points distributed on a sphere.

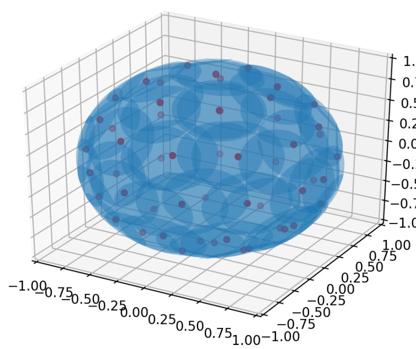

Figure 2: Sphere completely covered by sensors that were given.

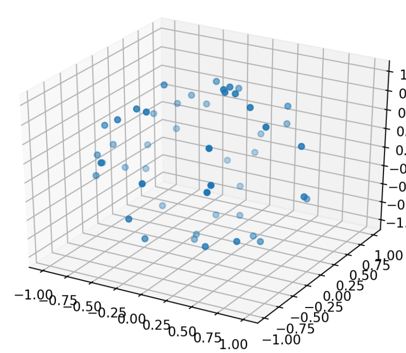

Figure 3: Randomly sampled points on a unit sphere.

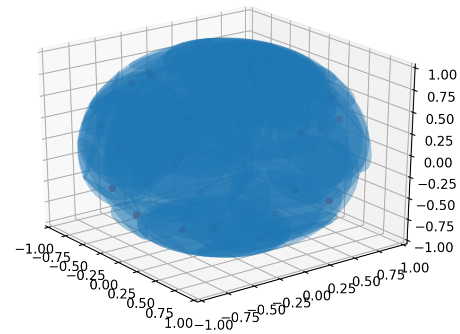

Figure 4: Sphere completely covered by sensors that were randomly sampled points on a unit sphere.

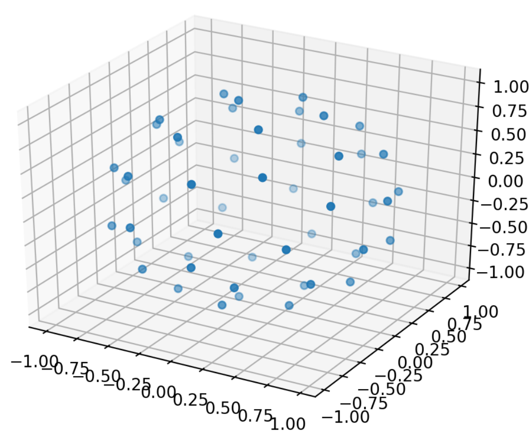

Figure 5: Fibonacci spiral generated equidistant points on a unit sphere.

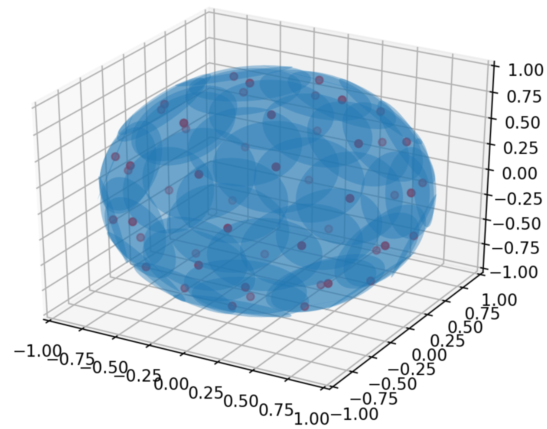

Figure 6: Sphere completely covered by a sensors uniformly distributed on the surface.

4 Division of work

Kristjan worked on implementing the minimization function and plotting. Jan worked on generating Čech complex and Vietoris-Rips complex. Both authors contributed equally to the report, presentation and code of the project.

5 Conclusions

We presented the problem of finding near optimal R and r for the sensors distributed on a unit sphere. We described the methodology and experiments done on different distributions of sensors as well as providing visualization for the experiments. In further work we could try to use different distance measure (orthodromic distance) and also experiment with lower number of sensors.

Literature

[1] Clément Maria, Jean-Daniel Boissonnat, Marc Glisse, and Mariette Yvinec. The gudhi library: Simplicial complexes and persistent homology. In *International Congress on Mathematical Software*, pages 167–174. Springer, 2014.