0.1 Indexes

0.1.1 Introduction

In economy, an **index** is the value of a ratio indicating the change of a quantity. For example, the Norwegian ice cream "kroneis" cost 0.75 kr when it was released in 1953, while in 2021 it cost 27 kr. Then the ratio of the price in 2021 to the price in 1953 is

$$\frac{\text{price in 2021}}{\text{price in 1953}} = \frac{27}{0.75} = 36$$

In this conjuncture 36 is the index of the price change of "kroneis" between 1953 and 2021.

0.1.2 Consumer price index and base year

The **Consumer price index** (CPI) is an index indicating a comparative price level of urban merchandise and services, such as

- Food and non-alcoholic beverages

- Alcoholic beverages and tobacco

- Clothing and footwear

- Housing, water, electricity, gas and other fuels

- Furnishings, household equipment and routine maintenance

- Health

- Transport

- Communications

- Recreation and culture

- Education

- Restaurants and hotels

To compare something a reference is needed, and the consumer price index use the price level of the aforementioned wares and servise in 2015 as a reference. Hence, 2015 is called the **base year**[1], with it's index set to 100.

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[1]The year set as the base year changes with time. Before 2015 became the base year, 1998 was.

0.1 Base year

The index of a base year equals 100. 2015 is the base year of the consumer price index.

The below table presents the CPI for the years 2011 - 2020

År	KPI
2020	112,2
2019	110,8
2018	112,2
2017	105,5
2016	103,6
2015	100
2014	97,9
2013	95,9
2012	93,9
2011	93,3

Table 1: Consumer price index for the years 2011-2021. Numbers from SSB.

According to the table, we can, for example, read the following:

- Since CPI in 2017 was 105.5, the prices have increased by 5.5% since 2015.

- Since CPI in 2011 was 93.3, the prices were 7,7% lower in 2011 than in 2015.

0.2 Percentage change relative to the base year

$$\text{index} - 100 = \text{percentage change relative to the base year}$$

Example 1

In july 2021 CPI for food was 109,4. How much have the price for food changed compared to the base year?

Answer

$109.4 - 100 = 9.4$. The price of food has increased by 9,4% compared to the base year.

> **Example 2**
>
> In july 2021 CPI for footwear was 98.0. How much have the price for footwear changed compared to the base year?
>
> **Answer**
>
> $98{,}0 - 100 = -2$. Hence, the price of footwear is reduced by 2% compared to the base year.

0.1.3 Kroneverdi

As mentioned, a "kroneis" cost 0.75 kr in 1953, and 27 kr in 2021. When at two distinct points of time we have to pay a *different* price for the *same* merchandise it will often mean that the[1] **krone value** has changed; *1 kr in 1957 was more worth than 1 kr i 2021.*

The krone value for a given year is defined as 100 divided by the CPI of that year:

> **0.3 Kroneverdi**
> $$\text{kroneverdi} = \frac{100}{\text{KPI}}$$
> *Note*: Kroneverdien for basisåret (2015) er 1.

> **Example 1**
>
> CPI in 2012 was 93.9. Calculate the krone value in 2012.
>
> **Answer**
> $$\text{krone value in 2012} = \frac{100}{93.9}$$
> $$\approx 1.06$$
>
> This means that 1 kr in 2012 was equivalent to 1,06 kr in the base year.

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[1]Here, 'krone' can be substituted by any currency.

0.4 Real value

The real value of an amount of money yields the value of these money in the base year.

$$\text{real value} = \text{original value} \cdot \text{currency value}$$

Example

In 1928, the CPI was 3.2, and in 2020, the CPI was 112.2. Which had the greater real value, 10,000 NOK in 1928 or 350,000 NOK in 2020?

Answer

We have that

$$\text{Krone value in 1928} = \frac{100}{3.2}$$

Thus,

$$\text{The value of 10,000 NOK from 1928 in base year} = 10,000 \text{ NOK} \cdot \frac{100}{3.2}$$
$$= 312,500 \text{ NOK}$$

Furthermore,

$$\text{Krone value in 2020} = \frac{100}{112.2}$$

Thus,

$$\text{The value of 350,000 NOK from 1928 in base year} = 350,000 \text{ NOK} \cdot \frac{100}{112.2}$$
$$\approx 311,943 \text{ NOK}$$

Therefore, 10,000 NOK was worth more in 1928 than what 350,000 NOK was worth in 2020.

0.1.4 Real Wages and Nominal Wages

How well off we are depends on how much we earn and what the price level is. Suppose you had an annual salary of 500,000 NOK in both 2020 and 2019. *Table 1* then tells us that you had better purchasing power in 2019 because the price level (CPI) was lower than in 2020.

An increase in the price level is the same as a decrease in the value of the currency. This means that if your salary was the same in 2019 and 2020, the *real value* of your salary is higher in 2019 than in 2020. The original salary and the real value of the salary are so commonly used in statistics that they have their own names:

0.5 Real Wages and Nominal Wages

Nominal wages are the wages received in a given year.

Real wages are the real value of the nominal wages.

Example

In 2016, Per earned 450,000 NOK, while in 2012, he earned 420,000 NOK. In 2016, the CPI was 103.6, while in 2012, it was 93.9. In which of these years did Per have better purchasing power?

Answer

To find out in which year Per had better purchasing power, we check which year had the highest real wage[1] (see Rule 0.4):

$$\text{Real wage in 2016} = 450{,}000 \cdot \frac{100}{103.6} \text{ NOK}$$
$$\approx 434{,}363 \text{ NOK}$$
$$\text{Real wage in 2012} = 420{,}000 \cdot \frac{100}{93.9} \text{ NOK}$$
$$\approx 447{,}284 \text{ NOK}$$

Per's real wage was highest in 2012; therefore, he had better purchasing power in that year.

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[1]We obtain the CPI values from *Table 1* for this calculation.

0.6 Value Following an Index

A value is said to have *followed an index* if the value and index at two points in time are equal.

$$\frac{\text{value at time 1}}{\text{index at time 1}} = \frac{\text{value at time 2}}{\text{index at time 2}}$$

Example 1

The table below shows an overview of prices recorded in a store for two items at two different points in time.

	2010	2020
chocolate	11.00 NOK	13.40 NOK
soda	12.50 NOK	19.00 NOK

In 2010, the CPI was 92.1, and in 2020, the CPI was 112.1. Has the price of any of the items followed the index?

Answer

We have that

$$\frac{\text{price of chocolate in 2010}}{\text{CPI in 2010}} \approx \frac{11.00}{92.1} \approx 0.119$$

$$\frac{\text{price of chocolate in 2020}}{\text{CPI in 2020}} \approx \frac{13.40}{112.1} \approx 0.119$$

Furthermore,

$$\frac{\text{price of soda in 2010}}{\text{CPI in 2010}} \approx \frac{12.50}{92.1} \approx 0.136$$

$$\frac{text price of soda in 2020}{\text{CPI in 2020}} \approx \frac{19.00}{112.1} \approx 0.169$$

So, it's reasonable to say that the price of chocolate has followed the index, while the price of soda has not.

0.2 Loans and Savings

0.2.1 Loans

Sometimes we don't have enough money to buy what we want and, therefore, need to take out a loan from a bank. The bank then lends us a certain *loan amount* in exchange for us paying it back, along with *interest*, over a specified period. Typically, we pay the bank what is called the *installment amount* along the way, which consists of *principal* and interest. The amount we owe the bank at any given time is called *debt.*

Suppose a bank lends us 100,000 NOK, which is the loan amount. The loan is to be repaid over 5 years, with one installment amount each year, and the interest rate is 10

- **The sum of all installments should equal the loan amount**.

  To simplify our example, we decide to repay the loan with equal installments each year. Since 100,000 NOK should be distributed equally over 5 years, the annual installment should be $\frac{100,000}{5}$ NOK = 20,000 NOK.

- **What is paid in installments should be deducted from the debt**.

  The initial debt is 100,000 NOK, but in the first year, we pay 20,000 NOK in installments, so the debt becomes 100,000 NOK− 20,000 NOK = 80,000 NOK. In the second year, we pay another 20,000 NOK, making the debt 80,000 NOK − 20,000 NOK = 60,000 NOK, and so on for the next three years.

- **Interest should be calculated on the debt**.

  Since the debt in the first year is 100,000 NOK, we must pay 100,000 NOK × 0.1 = 10,000 NOK in interest. In the second year, with a debt of 80,000 NOK, we must pay 80,000 NOK × 0.1 = 8,000 NOK in interest, and so on for the next three years.

- **The installment amount is the sum of the principal and interest**.

  From the first and third points, we deduce that:

	1st year	2nd year
Installment amount	20,000 NOK + 10,000 NOK = 30,000 NOK	20,000 NOK + 8,000 N = 28,000 NOK

And so on for the next three years.

- **The loan is complete when the debt is zero, and all interest is paid**.

  If we have paid installments of 20,000 NOK for 5 years, the debt is reduced to zero. If we have also paid all the interest, the loan is complete.

  Note: You always have the right to pay larger installments than originally agreed upon. If you pay off the entire debt, provided that any interest has also been paid, the loan will be terminated.

Amortizing Loans and Annuity Loans

Two common types of loans are *amortizing loans* and *annuity loans.* The loan from the example we just saw is an amortizing loan because the installments are of equal size. If the installment amounts had been equal, it would have been an annuity loan instead. If the loan amount, interest rate, and repayment period are the same, an amortizing loan will always result in lower overall expenses. However, for individuals, annuity loans are very popular because it is easier to plan finances when paying the same amount each time.

Credit Cards

Credit cards are a payment card that works in such a way that if you, for example, use the card to pay 10,000 NOK, you are borrowing the money from the bank. After a period agreed upon with the bank, it will charge interest on your debt. When you pay this debt is partly up to you, but generally, credit cards have very high-interest rates, so it is wise to pay it off before the interest starts accruing!

0.7 Loans

loan amount	The amount we borrow from the bank.
debt	What we owe the bank at any given time.
interest	Percentage of debt to be paid.
principal	What we pay down on the debt.
	The sum of the principal equals the loan amount.
	New debt = old debt − principal
interest	debt · interest
installment	principal + interest
amortizing loan	A loan where the installments are of equal size.
annuity loan	A loan where the installment amounts are equal.
credit card	A payment card that creates a loan from the bank.

Example 1

From a bank, you borrow 300,000 NOK with a 3**a)** What is the annual principal payment? **b)** What is your debt after paying the third installment amount? **c)** How much do you have to pay in interest for the fourth installment amount? **d)** What is the amount of the fourth installment?

Solution:

a) Since 300,000 NOK is to be paid over 5 years, the annual principal payment is

$$\frac{300{,}000 \text{ NOK}}{5} = 60{,}000 \text{ NOK}$$

b) When the third installment is paid, you have paid three installments. That means your debt is

$$300{,}000 - 60{,}000 \cdot 3 = 300{,}000 - 180{,}000 = 120{,}000 \text{ NOK}$$

So, 120,000 NOK.

c) From the answer to part b), we know that the debt is 180,000 NOK when the fourth installment is to be paid. 3

$$180{,}000 \cdot 0.03 = 5{,}400 \text{ NOK}$$

So, 5,400 NOK.

d) The installment amount is equal to the principal plus interest. Based on the answers to parts a) and c), we know that the fourth installment amount is

$$60{,}000 \text{ NOK} + 5{,}400 \text{ NOK} = 65{,}400 \text{ NOK}$$

So, the fourth installment amount is 65,400 NOK.

Example 2

From a bank, you borrow 100,000 NOK with a 6.4% annual interest rate. The loan is to be repaid as an annuity loan over 5 years, and the bank has calculated that the installment amount will be 24,000 NOK.

Calculate the principal and interest for the first installment amount.

Answer

In the first year, the debt is 100,000 NOK, and you must pay 6.4

$$100{,}000 \cdot 0.064 = 6{,}400 \text{ NOK}$$

So, you must pay 6,400 NOK in interest in the first year.

We have that

$$\text{installment amount} = \text{principal} + \text{interest}$$

So, the principal is

$$\text{principal} = \text{installment amount} - \text{interest} = 24{,}000 \text{ NOK} - 6{,}400 \text{ NOK} = 17{,}600$$

So, you must pay 17,600 NOK in principal in the first year.

0.2.2 Savings; Deposit Interest and Expected Return

Deposit Interest

We have seen that we must pay interest when we borrow money from a bank, but if we instead put money (make a deposit) in a bank, we *earn* interest:

0.8 Deposit Interest

Deposit interest is a percentage increase in the money you have in the bank, repeated over fixed time intervals (monthly, annually, etc.).

Example 1

You deposit 20,000 NOK in a bank that offers a 2% annual savings interest rate. How much money do you have in the bank after 8 years?

Answer

To calculate deposit interest, we can use Rule **??**. Since the interest rate is 2%, the growth factor is 1.02. The original value is 20,000, and the number of changes (time) is 8:

$$20{,}000 \cdot 1.02^8 \approx 23{,}433$$

So, you have approximately 23,433 NOK in the bank after 8 years of saving.

Expected Return

Another way to save money is to invest in a mutual fund. In this case, we talk about *expected return*:

0.9 Expected Return

Expected return specifies an *expected* percentage increase of an investment, repeated over fixed time intervals.

Example 1

You invest 15,000 NOK in a mutual fund that expects a 5% annual return. How much is the investment worth after 8 years

with such a return?

Answer

For expected return, we can also use Rule **??**. The growth factor is 1.05, the original value is 15,000, and the number of changes (time) is 8:

$$15{,}000 \cdot 1.05^8 \approx 22{,}162$$

After 8 years, it is expected that the investment is worth 22,162 NOK.

Saving with Deposit Interest or Mutual Funds?

Usually, the expected return on a mutual fund is higher than the deposit interest you get in a bank, but the downside is that expected return does not provide any guarantees. Expected return only indicates the increase experts anticipate. If you're lucky, the increase will be higher; if you're unlucky, it will be lower and may even result in a *reduction* of your investment. In the worst case, although extremely rare, your entire investment may end up being worth 0 NOK.

Deposit interest rates can also change somewhat over time, but they can never lead to a reduction in your investment.

0.3 Taxation

If you have an income, you usually have to pay a portion of that money to the state. This money is called *tax* (and sometimes *duty*). The purpose of tax is to provide the state with the means to offer services to its citizens, such as education, healthcare, and more. Today, taxes are largely calculated by computer systems, but it is your responsibility to ensure that the calculations are correct - and that's why it's important to understand how the tax system works.

> **Note!**
>
> In exam questions and in real life, you will quickly realize that tax systems are presented somewhat differently than in this book. This is because tax rules can change from year to year, and in this book, we have based our explanations on the tax rules of Norway in 2018. The most important thing is not to memorize these specific rules but to learn what is meant by the terms *gross income, deductions, taxable income, social security contribution*, and *net income*.

0.3.1 Gross Income, Deductions, and Taxable Income

Most people have to pay 23% of what is called *taxable income*, which is *gross income* minus *deductions*. Gross income is the salary you receive from your employer, while deductions can be various things. *Personal deductions* and *minimum deductions* are something all taxpayers receive. Additionally, you can

Taxable income is sometimes called *taxable basis*.

receive deductions if you pay *union dues* or have donated money to charitable causes.

Union dues are what you pay to be a member of a trade union.

> **0.10 Gross Income, Deductions, and Taxable Income**
>
> $$\begin{array}{rl} & \text{gross income} \\ - & \text{deductions} \\ \hline = & \text{taxable income} \end{array}$$

Example

Magnus's gross income is 500,000 NOK. He receives a 56,000 NOK personal deduction, a 97,600 NOK minimum deduction, and he also pays 1,000 NOK for annual membership in the union *Tekna*.

How much does Magnus have to pay if he is taxed at 23

Answer

We start by calculating the taxable income, which is the gross income minus the deductions:

	500,000	gross income
−	56,000	personal deduction
−	97,600	minimum deduction
−	1,000	union dues
=	345,400	taxable income

Example

Jonas and his grandmother, Line, both have a salary of 150,000 NOK. Jonas is 18 years old, and Line is 71 years old.

a) How much does Jonas have to pay in social security tax?

b) How much does Line have to pay in social security tax?

Answer

a) Since Jonas is between 17 and 69 years old, he has to pay 8.2

$$150{,}000 \cdot 0.082 = 12{,}300$$

So, Jonas has to pay 12,300 NOK in social security tax.
b) Since Line is over 69 years old, she has to pay 5.1

$$150{,}000 \cdot 0.051 = 7{,}650$$

So, Line has to pay 7,650 NOK in social security tax.

Example

If you earn 550,000 NOK, the calculation of progressive tax is as follows:

Trinn 1	As the entire salary is over 237,900 NOK, you must pay tax on $(237{,}900 - 169{,}000)\,NOK = 68{,}900\,NOK$. The tax for trinn 1 is then $68{,}900\,NOK \cdot 0.014 \approx 965\,NOK$.
Trinn 2	Since 550,000 NOK is over 237,900 NOK but below 598,050 NOK, you must pay tax on $(550{,}000 - 237{,}900)\,NOK = 312{,}100\,NOK$. The tax for trinn 2 is then $312{,}100\,NOK \cdot 0.033 \approx 10{,}299\,NOK$.
Total	In total, you must pay $965\,NOK + 10{,}299\,NOK = 11{,}264\,NOK$ in progressive tax.

0.3.2   Accounting

In a budget, you list *expected* incomes and expenses, while in an *accounting* record, you list *actual* incomes and expenses. The difference between the budget and accounting is called the *deviation.* For the deviation, it is common to calculate 'accounting − budget' for incomes and results, while for expenses, you calculate 'budget − accounting'. This is because we want positive numbers if the incomes are higher than expected, and negative numbers if the expenses are higher than expected.

Example

In the example from the previous subsection (**??**), we set up a monthly budget for Lisa. In March, it turned out that these were her actual incomes and expenses:

- She didn't work as much as she had planned. Net pay was 3,500 NOK.

- She spent 4,200 NOK on food.

- She received 4,360 NOK in away-from-home grant.

- In birthday gifts, she received a total of 2,000 NOK.

- She spent about 3,600 NOK on clothing, leisure activities, etc.

Set up an accounting record for Lisa's month of March.

Answer

Incomes	Budget	Accounting	Deviation
Salary	4,000	3,500	−500
Grant	4,360	4,360	0
Birthday Gifts	0	2,000	2,000
Total	8,360	9,860	2,000

Expenses	Budget	Accounting	Deviation
Food	4,500	4,200	300
Clothing, Leisure, etc.	1,200	3,600	−2,400
Total	5,700	7,800	1,900

Result	2,660	2,060	−600

So, Lisa ended up with a surplus of 2,060 NOK, but 600 NOK less than expected based on the budget.