0.1 Multiplication

Multiplication by integers: inital definition

When adding equal numbers, we can use the multiplication symbol $\cdot$ to write our calculations more compact:

Example

$$4 + 4 + 4 = 4 \cdot 3$$

$$8 + 8 = 8 \cdot 2$$

$$1 + 1 + 1 + 1 + 1 = 1 \cdot 5$$

The language box

A calculation involving multiplication includes severeal *factors* and one *product*. In the calculation

$$4 \cdot 3 = 12$$

both 4 and 3 are factors, while 12 is the product.

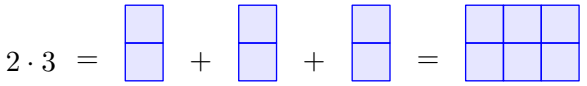
Common ways of saying $4 \cdot 3$ include

- "4 times 3"
- "4 multiplied by 3"
- "4 and 3 multiplied together"

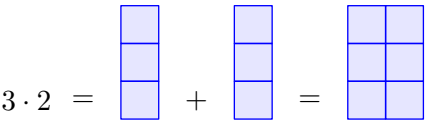
A lot of texts use $\times$ instead of $\cdot$. In computer programming, $*$ is the most common symbol for multiplication.

Multiplication involving amounts

Let us illustrate $2 \cdot 3$:



Now notice the product of $3 \cdot 2$:



0.1 Multiplication is commutative

The order of the factors has no impact on the product.

Example

$$3 \cdot 4 = 12 = 4 \cdot 3$$

$$6 \cdot 7 = 42 = 7 \cdot 6$$

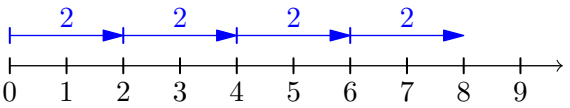
$$8 \cdot 9 = 72 = 9 \cdot 8$$

Multiplication on the number line

We can also use the number line to calculate multiplications. In the case of $2 \cdot 4$ we can think like this:

"$2 \cdot 4$ means moving 2 places to the right, 4 times."

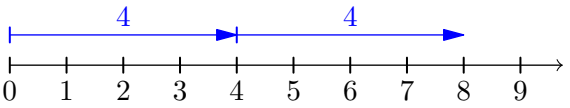
$$2 \cdot 4 = 8$$



We can also use the number line to prove to ourselves that multiplication is commutative:

"$4 \cdot 2$ means moving 4 places to the right, 2 times."

$$4 \cdot 2 = 8$$



Final definition of multiplication by positive integers

It may be the most intuitive to interpret "2 times 3" as "3, 2 times". Then it follows:

$$\text{"2 times 3"} = 3 + 3$$

In this section we introduced $2 \cdot 3$, that is "2 times 3", as $2 + 2 + 2$. With this interpretation, $3 + 3$ corresponds to $3 \cdot 2$, but the fact that multiplication is a commutative operation (*Rule 0.1*) ensures that the one interpretation does not exclude the other; $2 \cdot 3 = 2 + 2 + 2$ and $2 \cdot 3 = 3 + 3$ are two expressions of same value.

0.2 Multiplication as repeated addition

Multiplication involving a positive integer can be expressed as repeated addition.

Example 1

$$4 + 4 + 4 = 4 \cdot 3 = 3 + 3 + 3 + 3$$

$$8 + 8 = 8 \cdot 2 = 2 + 2 + 2 + 2 + 2 + 2 + 2$$

$$1 + 1 + 1 + 1 + 1 = 1 \cdot 5 = 5$$

Notice

The fact that multiplication with positive integers can be expressed as repeated addition does not exclude other expressions. There's nothing wrong with writing $2 \cdot 3 = 1 + 5$.