0.1 Addition

Column addition

This method builds on the base-10 positional notation, in turn adding the ones, the tens, the hundreds and so on.

Example 1

$$\begin{array}{cccc} & 2 & 3 & 4 \\ + & 6 & 1 & 2 \\ \hline = & 8 & 4 & 6 \end{array}$$

Example 2

$$\begin{array}{cccc} & \overset{1}{2} & 7 & 3 \\ + & & 8 & 6 \\ \hline = & 3 & 5 & 9 \end{array}$$

Example 3

$$\begin{array}{cccc} & {}^{1} & \overset{1}{8} & 5 \\ + & & 7 & 9 \\ \hline = & 1 & 6 & 4 \end{array}$$

Example 4

$$\begin{array}{ccccc} & \overset{1}{3} & \overset{1}{9} & \overset{1}{7}, & 2 \\ + & & 8 & 5, & 9 \\ \hline = & 4 & 8 & 2, & 1 \end{array}$$

Example 1 (explanation)

$$\begin{array}{cccc} & 2 & 3 & \color{blue}{4} \\ + & 6 & 1 & \color{blue}{2} \\ \hline & & & \color{green}{6} \end{array}$$

(a)

$$\begin{array}{cccc} & 2 & \color{blue}{3} & 4 \\ + & 6 & \color{blue}{1} & 2 \\ \hline & & \color{green}{4} & 6 \end{array}$$

(b)

$$\begin{array}{cccc} & \color{blue}{2} & 3 & 4 \\ + & \color{blue}{6} & 1 & 2 \\ \hline = & \color{green}{8} & 4 & 6 \end{array}$$

(c)

a) We add the ones: $4 + 2 = 6$

b) We add the tens: $3 + 1 = 4$

c) We add the hundreds: $2 + 6 = 8$

Example 2 (explanation)

$$\begin{array}{cccc} & 2 & 7 & 3 \\ + & & 8 & 6 \\ \hline & & & 9 \end{array}$$

(a)

$$\begin{array}{cccc} & {}^{1} & & \\ & 2 & 7 & 3 \\ + & & 8 & 6 \\ \hline & & 5 & 9 \end{array}$$

(b)

$$\begin{array}{cccc} & {}^{1} & & \\ & 2 & 7 & 3 \\ + & & 8 & 6 \\ \hline = & 3 & 5 & 9 \end{array}$$

(c)

a) We add the ones: $3 + 6 = 9$

b) We add the tens: $7 + 8 = 15$. Since 10 tens equals 100, we add 1 to the hundreds position, and write the remaining 5 tens at the tens position.

c) We add the hundreds: $1 + 2 = 3$.

The language box

Writing 1 on a place value to the left is calle ”carrying 1 over”.

0.2 Subtraction

Column subtraction

This method is founded on the base-10 positional notation, in turn subtracting the ones, the tens, the hundreds and so on. It is also based on the perspective of numbers as amounts, so it does not allow negative differences. (see the explanation of *Example 2*).

Example 1

$$\begin{array}{cccc} & 7 & 8 & 9 \\ - & 3 & 2 & 4 \\ \hline = & 4 & 6 & 5 \end{array}$$

Example 2

$$\begin{array}{ccc} & & {\scriptstyle 10} \\ & \cancel{9} & 3 \\ - & 5 & 7 \\ \hline = & 3 & 6 \end{array}$$

Example 3

$$\begin{array}{cccc} & & {\scriptstyle 10} & {\scriptstyle 10} \\ & \cancel{5} & \cancel{6} & 4 \\ - & 4 & 7 & 8 \\ \hline = & & 8 & 6 \end{array}$$

Example 4

$$\begin{array}{cccccc} & & \cancel{\scriptstyle 10} & & & {\scriptstyle 10} \\ & \cancel{2} & 0 & \cancel{6} & , & 1 \\ - & & 3 & 1 & , & 7 \\ \hline = & 1 & 7 & 4 & , & 4 \end{array}$$

Example 1 (explanation)

$$\begin{array}{cccc} & 7 & 8 & 9 \\ - & 3 & 2 & 4 \\ \hline & & & 5 \end{array}$$

(a)

$$\begin{array}{cccc} & 7 & 8 & 9 \\ - & 3 & 2 & 4 \\ \hline & & 6 & 5 \end{array}$$

(b)

$$\begin{array}{cccc} & 7 & 8 & 9 \\ - & 3 & 2 & 4 \\ \hline = & 4 & 6 & 5 \end{array}$$

(c)

(a) We find the difference between the ones: $9 - 4 = 5$

(b) We find the difference between the tens: $8 - 2 = 6$.

(c) We find the difference between the hundreds: $7 - 3 = 4$.

Example 2 (explanation)

$$\begin{array}{r|c|c}
 & & {\scriptstyle 10} \\
 & \cancel{9} & 3 \\
- & 5 & 7 \\
\hline
 & & 6
\end{array}
\qquad\qquad
\begin{array}{r|c|c}
 & & {\scriptstyle 10} \\
 & \cancel{9} & 3 \\
- & 5 & 7 \\
\hline
= & 3 & 6
\end{array}$$

(a) *(b)*

(a) We notice that 7 is larger than 3, and thus we take 1 ten from the 9 at the tens position. This is marked by drawing a line across 9. Then we find the difference between the ones: $13 - 7 = 6$

(b) Since we took 1 from the 9 tens, it is now only 8 tens. We find the difference between the tens: $8 - 5 = 3$.

The table method

The table method takes advantage of subtraction being the inverse operation of addition. For example, the answer to the question "What is $789 - 324$?" is the same as the answer to the question "How much must i add to 324 in order to get 789?". With the table method you can freely chose which numbers to add as long as you end up with the targeted number.

Example 1

$789 - 324 = 465$

$$\begin{array}{r|l}
 & 324 \\
\hline
6 & 330 \\
70 & 400 \\
389 & 789 \\
\hline
465 &
\end{array}$$

Example 2

$83 - 67 = 16$

$$\begin{array}{r|l}
 & 67 \\
\hline
3 & 70 \\
13 & 83 \\
\hline
16 &
\end{array}$$

Example 3

$564 - 478 = 86$

	478
2	480
20	500
64	564
86	

Example 4

$206{,}1 - 31{,}7 = 174{,}4$

	31,7
0,3	32
70	102
104,1	206,1
174,4	

Example 1 (explanation)

$789 - 324 = 465$

	324

(a)

	324
6	330

(b)

	324
6	330
70	400

(c)

	324
6	330
70	400
389	789

(d)

	324
6	330
70	400
389	789
465	

(e)

(a) We start at 324.

(b) We add 6, and get $324 + 6 = 330$

(c) We add 70, and get $70 + 330 = 400$

(d) We add 389, and get $389 + 400 = 789$. Now we have reached 789.

(e) We find the sum of the numbers we added: $6 + 70 + 389 = 465$

0.3 Multiplication

Multiplying by 10, 100, 1 000 etc.

0.1 Å gange heltall med 10, 100 osv.

- When multiplying an integer by 10, the product can be found by adding the digit 0 behind the integer.
- When multiplying an integer by 100, the product can be found by adding the digits 00 behind the integer.
- The same pattern applies for the numbers 1 000, 10 000 etc.

Example 1

$$6 \cdot 10 = 60$$

$$79 \cdot 10 = 790$$

$$802 \cdot 10 = 8020$$

Example 2

$$6 \cdot 100 = 600$$

$$79 \cdot 100 = 7\,900$$

$$802 \cdot 100 = 80\,200$$

Example 3

$$6 \cdot 1\,000 = 6\,000$$

$$79 \cdot 10\,000 = 790\,000$$

$$802 \cdot 100\,000 = 80\,200\,000$$

0.2 Multiplying decimal numbers by 10, 100, etc.

- When multiplying an integer by 10, the product is found by moving the dot one position to the right.
- When multiplying an integer by 100, the product is found by moving the dot one position to the right.
- The same pattern applies for the numbers 1 000, 10 000 etc.

Example 1

$$7.9 \cdot 10 = 79. = 79$$

$$38.02 \cdot 10 = 380.2$$

$$0.57 \cdot 10 = 05.7 = 5.7$$

$$0.194 \cdot 10 = 01.94 = 1.94$$

Example 2

$$7.9 \cdot 100 = 790. = 790$$

$$38.02 \cdot 100 = 3802. = 3\,802$$

$$0.57 \cdot 100 = 057. = 57$$

$$0.194 \cdot 100 = 019.4 = 19.4$$

Example 3

$$7.9 \cdot 1\,000 = 7900. = 7\,900$$

$$38.02 \cdot 10\,000 = 380020. = 380\,200$$

$$0.57 \cdot 100\,000 = 57000. = 57\,000$$

Note

Rule 0.1 is just a special case of Rule 0.2. For example, applying Rule 0.1 when calculating $7 \cdot 10$ yields the same answer as when applying Rule 0.2 when calculating $7{,}0 \cdot 10$.

Multiplying by 10, 100 etc. (explanation)

The Base-10 positional notation is founded on groups of tens, hundreds, thousands etc., and tenths, hundredths, thousandths etc. (see Rule **??**). When multiplying a number by 10, all the ones in the number will form a group of tens, all the tens will form a group of hundreds an so on. Hence, every digit is moved one position to the left. Similarly, every digit is moved one position to the left when multiplying by 100, three places when multiplying by 1 000 etc.

Expanded form

Multiplication with expanded form can be applied on multi digit numbers. The method is based on the distributive law (Rule **??**).

Example 1

2	4	$\cdot$	3	=	7	2
2	0	$\cdot$	3	=	6	0
	4	$\cdot$	3	=	1	2
					7	2

Example 2

$$279 \cdot 34 = 9486$$

$$\begin{aligned} 200 \cdot 30 &= 6000 \\ 70 \cdot 30 &= 2100 \\ 9 \cdot 30 &= \underline{\ \ 270} \\ & \quad 8370 \end{aligned} \qquad \begin{aligned} 200 \cdot 4 &= 800 \\ 70 \cdot 4 &= 280 \\ 9 \cdot 4 &= \underline{\ \ 36} \\ & \quad 1116 \end{aligned} \qquad \begin{aligned} &8370 \\ &\underline{1116} \\ &9486 \end{aligned}$$

Example 1 (explanation)

24 can be written as $20 + 4$, so

$$24 \cdot 3 = (20 + 4) \cdot 3$$

Moreover, by Rule **??**,

$$\begin{aligned} (20 + 4) \cdot 3 &= 20 \cdot 3 + 4 \cdot 3 \\ &= 60 + 12 \\ &= 72 \end{aligned}$$

Example 2 (explanation)

We have

$$\begin{aligned} 279 &= 200 + 70 + 9 \\ 34 &= 30 + 4 \end{aligned}$$

Thus

$$279 \cdot 34 = (200 + 70 + 9) \cdot (30 + 4)$$

Moreover,

$$\begin{aligned} (200 + 70 + 9) \cdot (30 + 4) &= 200 \cdot 30 + 70 \cdot 30 + 9 \cdot 30 + 200 \cdot 4 + 70 \cdot 4 + 9 \cdot 4 \\ &= 9486 \end{aligned}$$

The compact method

The compact method is based on the same principles as the expanded form method, only with a shorter way of writing.

Example 1

$$279 \cdot 34 = 9486$$

$$\begin{array}{r}
{\scriptstyle 2\,3} \\
\color{blue}{886} \\
{\scriptstyle 2\,2} \\
\color{green}{617} \\
\hline
9486
\end{array}$$

Example 1 (explanation)

First we multiply the digits of 279 by 4:

- $9 \cdot 6 = 36$, so we write 6 at the ones position and carry over 3.
- $7 \cdot 4 = 28$, so we write 8 at the tens position and carry over 2.
- $2 \cdot 4 = 8$, so we write 8 at the hundreds position.

Then we multiply the digits of 279 by 30. This in turn can be simplified to multiplying by 3, as long as we shift the digits one position to the left, relative to when we multiplied by 4:

- $9 \cdot 3 = 27$, so we write 7 at the tens position and carry over 2.
- $7 \cdot 3 = 21$, so we write 1 at the hundreds position and carry over 2.
- $2 \cdot 3 = 6$, so we write 6 at the thousands position.

0.4 Division

Division by 10, 100, 1 000 etc.

0.3 Deling med 10, 100, 1 000 osv.

When dividing a decimal number by 10, the quotient is found by moving the dot one position to the left.

When dividing a decimal number by 100, the quotient is found by moving the dot two positions to the left.

The same pattern applies for the numbers 1 000, 10 000 etc.

Example 1

$$\begin{aligned} 200 : 10 &= 200.0 : 10 \\ &= 20.00 \\ &= 20 \end{aligned}$$

$$\begin{aligned} 45 : 10 &= 45.0 : 10 \\ &= 4.50 \\ &= 4.5 \end{aligned}$$

Example 2

$$\begin{aligned} 200 : 100 &= 200.0 : 100 \\ &= 2.000 \\ &= 2 \end{aligned}$$

$$\begin{aligned} 45 : 100 &= 45.0 : 100 \\ &= 0.450 \\ &= 0.45 \end{aligned}$$

Example 3

$$143.7 : 10 = 14.37$$

$$143.7 : 100 = 1.437$$

$$143.7 : 1\,000 = 0.1437$$

Example 4

$$93.6 : 10 = 9.36$$

$$93.6 : 100 = 0.936$$

$$93.6 : 1\,000 = 0.0936$$

Division by 10, 100, 1 000 osv. (explanation)

The Base-10 positional notation is founded on groups of tens, hundreds, thousands etc., and tenths, hundredths, thousandths etc. (see Rule **??**). When dividing a number by 10, all the ones in the number will form a group of tens, all the tens will form a group of ones an so on. Hence, every digit is moved one position to the right. Similarly, every digit is moved two positions to the right when multiplying by 100, three places when multiplying by 1 000 etc.

Long division

Long division is based on the perspective of numbers as amount (see page **??**).

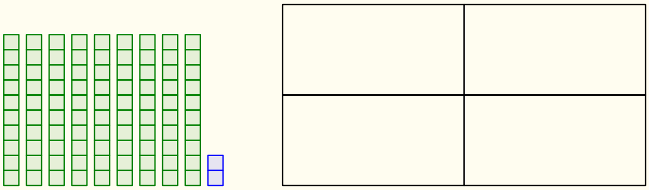
Example 1

9	2	:	4	=	2	3
8						
1	2					
1	2					
	0					

Example 1

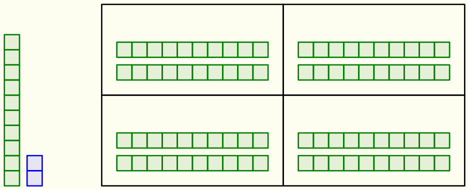
8	9	4	:	3	=	2	9	8
6								
2	9							
2	7							
	2	4						
	2	4						
		0						

Example 1 (explanation)

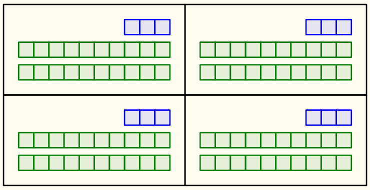


The above figure illustrates the amount 92, which we shall divide into 4 equal groups.

- We start by distributing as many tens as possible. Of the 9 tens, each group can get 2. In total we have distributed $2 \cdot 4 = 8$ tens.



- Now we are left with 1 ten and 2 ones, which equals 12 ones. Of the 12 ones, each group can get 3. In total we have distributed $3 \cdot 4 = 12$ ones.



- The amount we started with, 92, is now equally distributed, and our calculation is done. In each group we got 23.

The table method

The table method takes advantage of division being the inverse operation of multiplication. For example, the answer to the question "What is 76 : 4?" is the same as the answer to the question "Which number yields 76 when multiplied by 4?". In the same way as with the table method for subtraction, the numbers can be chosen more ore less freely.

Example 1

$92 : 4 = 23$

$\cdot 4$		
10	40	40
10	40	80
3	12	92
23		

Example 2

$894 : 3 = 298$

$\cdot 3$		
200	600	600
30	90	690
30	90	780
30	90	870
8	24	894
298		

Example 3

$894 : 3 = 298$

$\cdot 3$		
300	900	900
-2	-6	894
298		

Note: Same task as in *Example 2* but a different calculation.

Example 1 (explanation)

Since we are to divide 92 by 4, we multiply numbers by 4 until we reach 92.

$\cdot\, 4$		
10	40	40

(a)

$\cdot\, 4$		
10	40	40
10	40	80

(b)

$\cdot\, 4$		
10	40	40
10	40	80
3	12	92

(c)

$\cdot\, 4$		
10	40	40
10	40	80
3	12	92
23		

(d)

(a) We multiply 10 by 4, which equals 40. So far we have reached 40.

(b) We multiply 10 by 4, which equals 40. So far we have reached $40 + 40 = 80$.

(c) We multiply 3 by 4, which equals 12. So far we have reached $80 + 12 = 92$, which was our target.

(d) We add the numbers we multiplied by 4: $10 + 10 + 3 = 23$.

Tip

It is wise to look back at calculations done with the table method and search for improvements. In *Example 1* on page 14, we could have started off multiplying by 20. This is almost as easy as multiplying by 10, and it takes us closer to the target.

Divisjon med rest

The value of a division calculation is far from always an integer. One way of expressing the answer is by using the term **remainder**. What it means is best explained through an example:

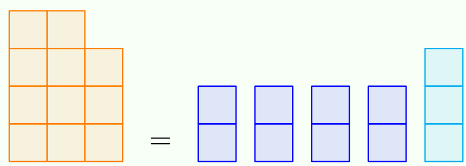
Example 1

Calculate $11 : 4$ using remainder.

Answer

The largest integer we can multiply by 4 without having a product exceeding 11 is 2. $2 \cdot 4 = 8$, so we have $11 - 8 = 3$ left to reach 11.

$$11 = 2 \cdot 4 + 3$$



This means that

$$11 : 4 = 2 \text{ and } 3 \text{ in remainder}$$

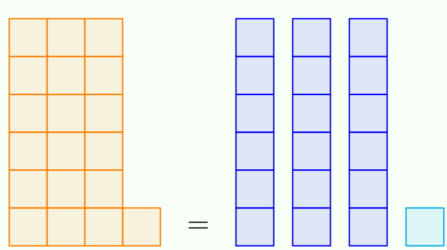
Example 2

Calculate $19 : 3$ using remainder.

Answer

The largest integer we can multiply by 3 without exceeding 19 is 6. $6 \cdot 3 = 18$, so we have $19 - 8 = 1$ left to reach 19.

$$19 = 6 \cdot 3 + 1$$



This means that

$$19 : 3 = 6 \text{ and } 1 \text{ in remainder}$$

Example 3

Calculate $94 : 4$ using remainder.

Answer

Using long division

$$94 : 4 = 23 \text{ og } 2 \text{ i rest}$$

$$\begin{array}{|c|c|c|c|c|c|c|}
9 & 4 & : & 4 & \rightarrow & 2 & 3 \\
8 & & & & & & \\
\cline{1-1}
1 & 4 & & & & & \\
1 & 2 & & & & & \\
\cline{1-2}
 & 2 & & & & &
\end{array}$$

Note: Since it's wrong to use $=$ in the above calculation, we have chosen to use $\rightarrow$.

Using the table method

$$94 : 4 = 23 \text{ og } 2 \text{ i rest}$$

$$\begin{array}{r|c|c}
\cdot 4 & & \\
\hline
20 & 80 & 80 \\
3 & 12 & 92 \\
\hline
23 & &
\end{array}
\qquad\qquad 94 - 92 = 2$$

The language box

Performing a **modulo-operation**, we find the remainder of a division calculation. This is often abbreviated as mod. For example is

$$11 \bmod 4 = 3 \qquad , \qquad 19 \bmod 3 = 1$$

In addition to `mod`, `%` and `//` are common symbols for this operation in programming languages.

Divisjon using mixed numbers

Example 1

Calculate $11 : 4$ using mixed numbers.

Answer

$$11 : 4 = 2 \text{ and } 3 \text{ as remainder} = 2 + \frac{3}{4}$$

Example 2

Calculate $19 : 3$ using mixed numbers.

Answer

$$19 : 3 = 6 \text{ and } 1 \text{ in remainder} = 6 + \frac{1}{3}$$

Example 1 (explanation)

We start by noticing that $4 = \frac{4}{1}$. Hence

$$11 : 4 = 11 : \frac{4}{1}$$

From Rule **??** it follows that

$$11 : \frac{4}{1} = 11 \cdot \frac{1}{4}$$

Moreover, $11 = 2 \cdot 4 + 3$, and thus

$$11 \cdot \frac{1}{4} = (2 \cdot 4 + 3) \cdot \frac{1}{4}$$

Now, by Rule **??**,

$$\begin{aligned}(2 \cdot 4 + 3) \cdot \frac{1}{4} &= 2 \cdot 4 \cdot \frac{1}{4} + 3 \cdot \frac{1}{4} \\ &= 2 + \frac{3}{4}\end{aligned}$$

0.4.1 Division using decimal numbers

Example 1

Calculate 11 : 4 using a decimal number.

Answer

With long divsion

$$11 : 4 = 2{,}75$$

1	1	:	4	=	2	,7	5
	8						
	3	0					
	2	8					
		2	0				
		2	0				
			0				

With the table method

$$11 : 4 = 2{,}75$$

$\cdot 4$		
2	8	8
0,5	2	10
0,25	1	11
2,75		

Example 1; long division (explanation)

Since we divide 4, we seperate 11 in 4 equal groups.

- We can equally distribute 8 of the 11 ones in 4 groups. Then there are 3 ones left. This equals 30 tenths.
- 28 of the 30 tenths can be equally distributed in 4 groups. Then there are 2 tenths left. This equals 20 hundredths.
- 20 of the 20 hundredths can be equally distributed in 4 groups.
- Now the whole amount of 11 is equally distributed, so our calculation is finished.

0.5 Calculation with time

Seconds, minutes and hours are organized in groups of 60:

$$1 \text{ minute} = 60 \text{ second}$$
$$1 \text{ hour} = 60 \text{ minute}$$

This means that *crossovers* arise in the calculations when we reach 60.

Example 1

$2\,\text{h}\ 25\,\text{min}\ + 10\,\text{h}\ 45\,\text{min} = 13\,\text{h}\ 10\,\text{min}$

Method 1

		10 h 45 min
15 min	15 min	11 h 00 min
10 min	25 min	11 h 10 min
2 h	2 h 25 min	13 h 10 min

Method 2

		10:45
00:15	00:15	11:00
00:10	00:25	11:10
02:00	02:25	13:10

Example 2

$14\,\text{h}\ 18\,\text{min}\ - 9\,\text{h}\ 34\,\text{min} = 4\,\text{h}\ 44\,\text{min}$

Method 1

	9 h 34 min
26 min	10 h 00 min
18 min	10 h 18 min
4 h	14 h 00 min
4 h 44 min	

Method 2

	09:34
00:26	10:00
00:18	10:18
04:00	14:18
04:44	

0.6 Avrunding og overslagsregning

Avrunding

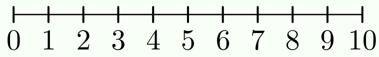
When **rounding**, we decrease the amounts of digits other than 0 in a number. Also, we can round off to the *closest one*, *the closest ten* and such.

Example 1

When rounding off to the *closest ten* we round off

- 1, 2, 3 and 4 *down* to 0, because they are closer to 0 than to 10.
- 6, 7, 8 and 9 *up* to 10 because they are closer to 10 than to 0.

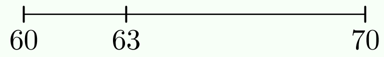
5 avrundes også opp til 10.



Example 2

- **63 rounded off the the closest 10 = 60**

  Because 63 is closer to 60 than 70.



- **78 rounded off to the closest ten = 80**

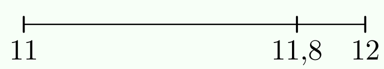
  Because 78 is closer to 80 than 70.

- **359 rounded off to the closest hundred = 400**

  Because 359 is closer to 400 than 300.

- **11,8 rounded off to the closest one = 12**

  Because 11,8 is closer to 12 than 11.



0.6.1 Estimate

Rather than knowing the exact answer to a calculation, at times there is more important to quickly determine the outcome *approximately*, preferably by mental arithmetic. When finding an approximate answer, the result is called an **estimate**. An estimate involves rounding off[1] numbers so that the calculation is easier to perform.

> **The language box**
>
> That something is "about the same as" is often written as "cirka" ("ca."). The symbol for "cirka" is $\approx$.

Estimates when adding or multiplying

Let us estimate the calculation

$$98.2 + 24.6$$

We have 98,2 $\approx$ 100. If we write 100 instead of 98.2 in our calculation, the result will slightly exceed the exact answer. Therefore, if we are to round off 24.6 we should round it off downwards. 24.6 is pretty close to 20, we can write

$$98.2 + 24.6 \approx 100 + 20 = 120$$

As this example showcases, when making an estimate involving addition, we should try to round off one of the numbers (upwards) and the other number (downwards).

The same applies for estimates involving multiplication. Let us estimate

$$1\,689 \cdot 12$$

We round off 12 to 10. In that case, the estimate will result in an answer lower than the exact anser, so to account for this we round off 1 689 uo to 1 700. Now

$$1\,689 \cdot 12 \approx 1\,700 \cdot 10 = 17\,000$$

---

[1] Note: Rounding off when performing estimates does not necessarily involve rounding off to the closes one, ten or such.

Estimation when subtracting and dividing

Let us make an estimate of

$$186{,}4 - 28{,}9$$

If we round off 186,4 up to 190, the result will be slightly larger than the exact answer. To account for this, we should subtract a bit more than in the original calculation. That can be done by rounding off 28,9 up (to 30):

$$\begin{aligned} 186.4 - 28{,}9 &\approx 190 - 30 \\ &= 160 \end{aligned}$$

The same principle applies when an estimate involves division. Let us estimate

$$145 : 17$$

We round off 17 up to 20. This will make our result a bit smaller than the exact answer. Hence, we should also round off 145 upwards (to 150):

$$145 : 17 \approx 150 : 20 = 75$$

Overslagsregning oppsummert

0.4 Overslagsregning

- Ved addisjon eller multiplikasjon mellom to tall, avrund gjerne et tall opp og et tall ned.
- Ved subtraksjon eller deling mellom to tall, avrund gjerne begge tall ned eller begge tall opp.

Example

Estimate the calculations.

a) $23{,}1 + 174{,}7$ b) $11{,}8 \cdot 107{,}2$

c) $37{,}4 - 18{,}9$ d) $1054 : 209$

Answer

a) $32{,}1 + 174{,}7 \approx 30 + 170 = 200$

b) $11.8 \cdot 107.2 \approx 10 \cdot 110 = 1\,100$

c) $37.4 - 18.9 \approx 40 - 20 = 20$

d) $1\,054 : 209 \approx 1\,000 : 200 = 5$

Comment

There are no specific rules for what you *can* or *can not* do when making an estimate. Thereby, Rule 0.4 is strictly speaking not a rule but rather a useful tip.

It is also natural to raise the question how far away from the exact answer an estimate is allowed to be. Neither in this case are there any rules to follow. However, an estimate and the exact answer should have the same **order of magnitude**. Simply put, if the exact answer has something to do with thousands, then so should also the estimate. To be more concise, the exact answer and the estimate shall have the same power of 10 when written in standard form[1].

[1]See Section 0.7

0.7 Standard form

Note: In this section it is taken for granted that the reader is familiar with powers, which we study in Section **??**.

We can apply Rule 0.2 and Rule 0.3, and what we know about powers, to write numbers in **standard form**.

Let us look at the number 6 700. By Rule 0.2,

$$6\,700 = 6.7 \cdot 1\,000$$

Since $1000 = 10^3$, we have

$$6\,700 = 6{,}7 \cdot 1\,000 = 6.7 \cdot 10^3$$

$6.7 \cdot 10^3$ er 6 700 written in standard form because

- 6.7 is larger than or equal to 1, and smaller than 10.
- 10^3 is a power with base 10 and exponent 3, which is an integer.
- The product of 6,7 and 10^3 equals 6 700.

Let us look at the number 0.093. By Rule 0.3,

$$0.093 = 9.3 : 100$$

Dividing by 100 is the same as multiplying by 10^{-2}, so

$$0.093 = 9.3 : 100 = 9.3 \cdot 10^{-2}$$

$9{,}3 \cdot 10^{-2}$ is 0,093 written in standard form because

- 9.3 is larger than or equal to 1, and smaller than 10.
- 10^{-2} is a power with base 10 and exponent -2, which is an integer.
- The product of 9,3 and 10^{-2} equals 0.093.

0.5 Standardform

A number written on the form

$$a \cdot 10^n$$

where $1 \leq |a| < 10$ and n is an integer, is a number written in **standard form**.

Example 1

Write 980 in standard form.

Answer

$$980 = 9.8 \cdot 10^2$$

Example 2

Write 0.00671 in standardform.

Answer

$$0.00671 = 6.71 \cdot 10^{-3}$$

Tip

To write numbers in standard form you can do the following:

1. Move the decimal separator so that you get a number between 1 and 10.
2. Multiply this number by a power of ten with exponent equal to the amount of places you moved the decimal separator. If you moved the decimal seperator to the left-/right, the exponent is positive/negative.

Example 3

Write 9 761 432 in standard form.

Answer

1. We move the decimal separator 6 places to the left, and get 9.761432

2. We multiply this number by 10^6, and get

$$9\,761\,432 = 9.761432 \cdot 10^6$$

Example 4

Write 0.00039 in standard form

Answer

1. We move the decimal separator 4 places to the right, and get 3,9.

2. We multiply this number by 10^{-4}, and get

$$0{,}00039 = 3{,}9 \cdot 10^{-4}$$