0.1 Introduction

In a **undersøkelse** we collect information. This information is often words or numbers, and is called **data**. A collection of data is called a **data set**.

For example, say you ask two people whether they like caviar. The one answers ”yes”, the other ”no”. Then ”yes” and ”no” are the data (answers) you have collected, and {”yes”, ”no”} is your data set.

Roughly speaking, statistics involves two things; *presenting* and *interpreting* data sets. For both purposes we have some terms which we will study in the following sections, helped by different examples of studies. These examples are found on page 2.

There are no universal laws telling you how to present or interpret data sets, however, you should follow these two guidelines:

- Let it always be clear exactly what you have studied, and what data you have collected.
- Always be aware the methods you use to interpret the data.

The language box

Persons participating in a survey where they are asked to answer questions are called **respondents**.

Survey 1

10 persones tested how many seconds they could hold their breath. These were the results (in seconds):

47 124 61 38 97 84 101 79 56 40

Survey 2

15 persons were asked how many apples they eat during a week. The answers were these:

7 4 5 4 1 0 6 5 4 8 1 6 8 0 14

Survey 3

300 persones where asked to name their favorite animal.

- 46 persons answered tiger
- 23 persons answered lion
- 17 persons answered crocodile
- 91 persons answered dog
- 72 persons answered cat
- 51 persons answered other animals

Survey 4

Mobile phones with smart-functions (app-based) came to the Norwegian market in 2009. The table[1] below shows the total sale of mobile phones during the time period 2009-2014, and the share with and without smart-functions. The numbers express the amount of 1 000 phones.

År	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**
totalt	2 365	2 500	2 250	2 200	2 400	2 100
wtho. sm.f.	1 665	1 250	790	300	240	147
wth. sm.f.	700	1 250	1 460	1 900	2 160	1 953

[1] Numbers imported from medienorge.uib.no.

0.2 Ways of presenting

When presenting data sets, it should be easy to see for others what we have found. This can be accomplished by using frequency tables, bar charts, sector graphs, or line graphs.

0.2.1 Frequency table

In a **frequency table** the data set are organized in a table showing the amount of times each unique answer appears. This amount is called the **frequency**.

Example - Survey 2

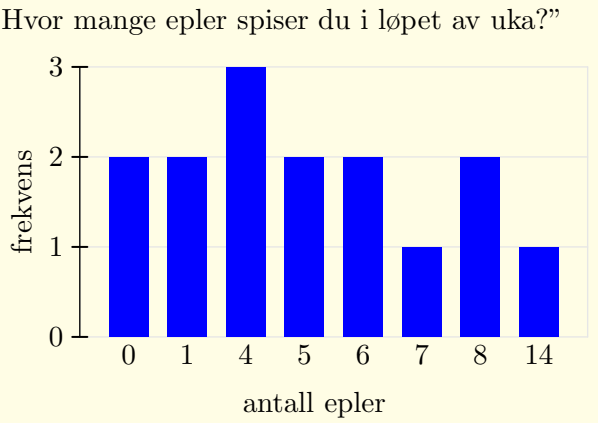
In this survey we have two 0's, two 1's, three 4's, two 5's, two 6's, one 7, two 8's and one 14. In a frequency table we then write

amount of apples	frequency
0	2
1	2
4	3
5	2
6	2
7	1
8	2
14	1

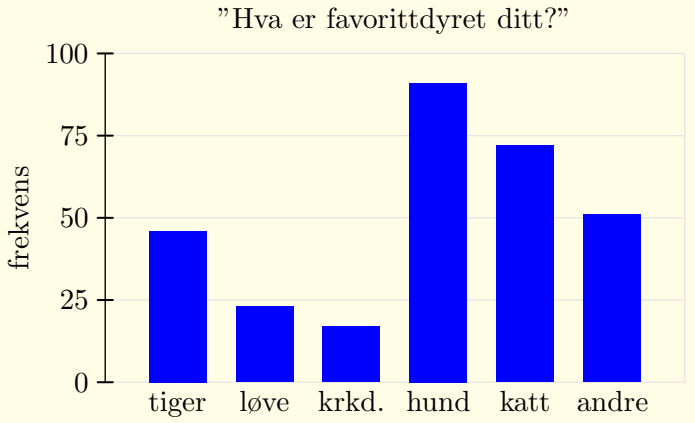
0.2.2 Søylediagram (stolpediagram)

In a bar chart the frequencies are represented by bars.

Survey 2



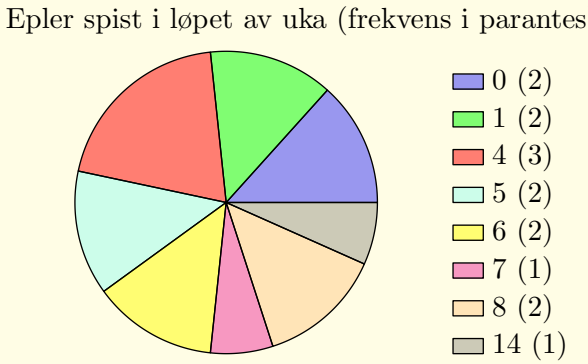
Survey 3



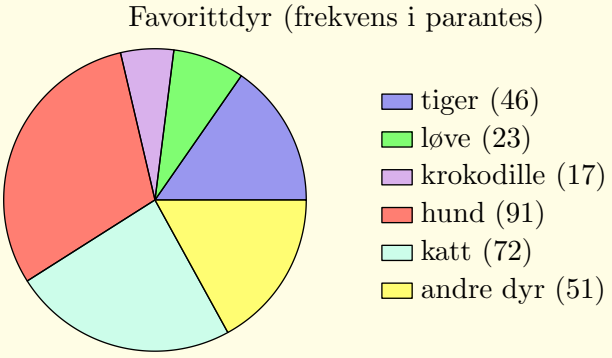
0.2.3 Sektordiagram (kakediagram)

In a sector graph the frequencies are represented by sectors in a circle.

Survey 2



Survey 3



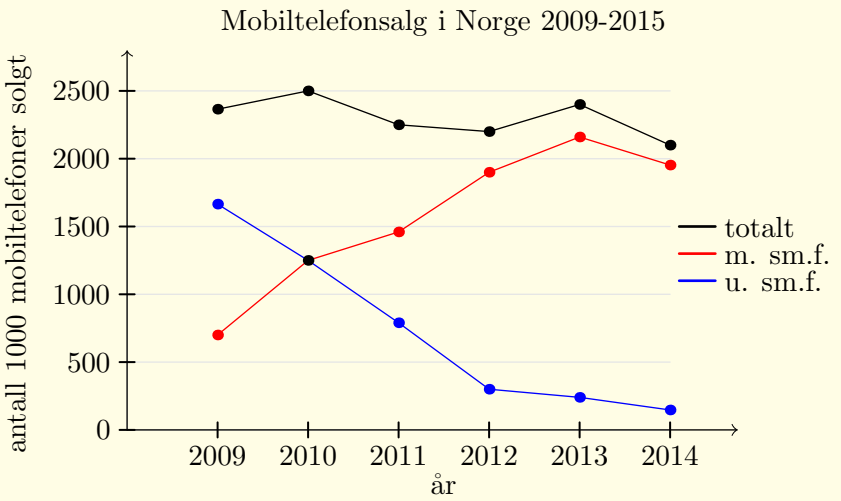
Making a sector graph from scratch

There are a lot of software available that generates sector graphs. However, if you were to make one from scratch, you will need basic knowledge of angles and fraction shares (see MB).

0.2.4 Linjediagram

In a line graph the data is represented as points in a coordinate system, with lines drawn between the points. Line graphs is typically used for describing evolving data.

Survey 4



0.3 Interpretation; central tendencies

In a data set there are often answers which are totally or approximately equal, and which reoccur. This means we can tell something about things that apply to the many; a **central tendeny**. The most common measures of central tendencies are the mode, the mean and the median.

0.3.1 Mode

0.1 Mode

The **mode** is the value occurring the most in the data set.

Example - Survey 2

In this dataset 4 has the highest occurrence (three), so 4 is the mode.

Multiple modes

If multiple values have the highest occurance in the data set, the data set has multiple modes.

0.3.2 Mean

When a data set includes numbers, we can find the sum of their values. When raising the question what the **mean** is, we ask this:

"If all the numbers had equal value, and the sum were still the same, what would the value be?"

The question is answered by the aid of division[1]:

0.2 Gjennomsnitt

$$\text{mean} = \frac{\text{sum of the values of the data set}}{\text{amount of values}}$$

The language box

The mean, as defined here, is also called the **average**. Also, there are multiple types of means. More specifically, the mean from Rule 0.2 is called the **arithmetic mean**.

Example - Survey 1

We sum the values from the data set, and divide by the amount of values:

$$\begin{aligned}\text{mean} &= \frac{47 + 124 + 61 + 38 + 97 + 84 + 101 + 79 + 56 + 40}{10} \\ &= \frac{727}{10} \\ &= 72.7\end{aligned}$$

Hence, the 10 participators held their breath for 72.7 seconds on average.

[1] See MB, side 23.

Example - Survey 2

Method 1

$$\begin{aligned}\text{mean} &= \frac{7+4+5+4+1+0+6+5+4+8+1+6+8+0+14}{15}\\ &= \frac{73}{15}\\ &\approx 4.87\end{aligned}$$

Method 2

We expand our frequency table 3 to find the sum of the values from the data set. (we have also included the sum of the frequencies):

Amount of apples	Frequency	amount · frequency
0	2	$0 \cdot 2 = 0$
1	2	$1 \cdot 2 = 2$
4	3	$4 \cdot 3 = 12$
5	2	$5 \cdot 2 = 10$
6	2	$6 \cdot 2 = 12$
7	1	$7 \cdot 1 = 14$
8	1	$8 \cdot 2 = 16$
14	1	$14 \cdot 1 = 14$
sum	15	73

Now

$$\begin{aligned}\text{mean} &= \frac{73}{15}\\ &\approx 4.87\end{aligned}$$

Hence, on average, the 15 respondents ate 4.87 apples a week.

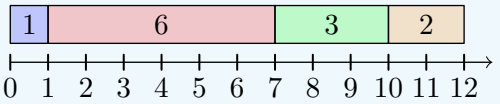
Example - Survey 4

(Calculations omitted. The values are rounded off to the nearest one.)

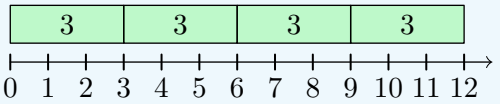
- Mean of total sale of mobiles: 2302
- Mean of sale of mobiles without smart-functions: 732
- Mean of sale of mobiles with smart-functions: 1570

Equal distribution

Note that the mean is about equal distribution. If we have 4 rectangles with respective lengths 1, 6, 3 and 2, their joint length is $1 + 6 + 3 + 2 = 12$.



Therefore, their mean length is $\frac{12}{4} = 3$. Thus, if we could reshape the rectangles so that their lengths were equal, with their joint length unchanged, they would each have length 3.



The average rate of change

Say you go for a jog, and measure your speed three times. Also, say that the data set you end up with is

$$10\,\mathrm{m/s} \qquad 10\,\mathrm{m/s} \qquad 10\,\mathrm{m/s}$$

Then your **average speed** was

$$\frac{10 + 10 + 10}{3}\,\mathrm{m/s} = 10\,\mathrm{m/s}$$

In other words; if your speed is the same alle the time[1], this speed is also you average speed. Consequently, the formula for the speed from Definition **??** is also the formula for the average speed. Alternatively stated, it is the formula for the average rate og change of length per time.

0.3 Average rate of change

If we *assume* or *hold* two quantities to be proportional, the proportionality constant from (**??**) is called the **averate rate of change**.

[1] In other words, your speed is *constant*.

Example - Survey 4

- For the years 2009 and 2010, the difference of smartphones sold to the difference of years is

$$\frac{1\,260 - 700}{2010 - 2009} = \frac{550}{1} = 550$$

  Therefore, between 2009 and 2010 the sale of smartphones have *increased* by 550 000 smartphones per year.

- For the years 2010 and 2014, the difference of smartphones sold to the difference of years is

$$\frac{1\,953 - 1\,250}{2014 - 2010} = \frac{703}{4} = 175{,}75$$

  Therefore, between 2010 and 2014 the sale of smartphones have *increased* by ca. 176 000 smartphones per year.

- For the years 2013 and 2014, the difference of smartphones sold to the difference of years is

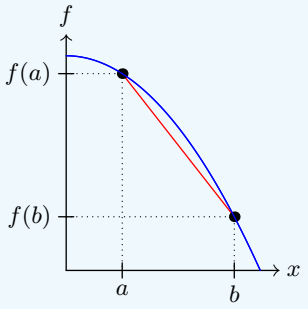
$$\frac{1\,953 - 2\,160}{2014 - 2013} = \frac{-207}{1} = -207$$

  Therefore, between 2013 and 2014 the sale of smartphones have *decreased* by ca. 207 000 smartphones per year.

The slope of the line through two points

Given a function $f(x)$. In MB we have seen that the slope of the line through the points $(a, f(a))$ og $(b, f(b))$ is

$$\frac{f(b) - f(a)}{b - a}$$



Comparing this expression with the calculations made on page 11, we realize that the expressions are, in general, identical. Hence, the slope of a line through two points yields the average rate of change between the two points.

0.3.3 Median

0.4 Median

Medianen er tallet som ender opp i midten av datasettet når det rangeres fra tallet med lavest til høgest verdi.

Hvis datasettet har partalls antall verdier, er medianen gjennomsnittet av de to verdiene i midten (etter rangering).

Survey 1

Vi rangerer datasettet fra lavest til høgest verdi:

38 40 47 56 61 79 84 97 101 124

De to tallene i midten er 61 og 79. Gjennomsnittet av disse er

$$\frac{61 + 79}{2} = 70$$

Altså er medianen 70.

Survey 2

Vi rangerer datasettet fra lavest til høgest verdi:

$$0 \quad 0 \quad 1 \quad 1 \quad 4 \quad 4 \quad 4 \quad 5 \quad 5 \quad 6 \quad 6 \quad 7 \quad 8 \quad 8 \quad 14$$

Tallet i midten er 5, altså er medianen 5.

Survey 4

(Utregning utelatt. Verdiene er rundet ned til nærmeste éner).

- Median for totalt salg av mobiler: 2307
- Median for salg av mobiler uten smartfunksjon: 545
- Median for salg av mobiler med smartfunksjon: 1570

0.4 Tolking av forskjeller; spredningsmål

Ofte vil det også være store forskjeller (stor spredning) mellom dataene som er samlet inn. De vanligste matematiske begrepene som forteller noe om dette er **variasjonsbredde**, **kvartilbredde**, **varians** og **standardavvik**.

0.4.1 Variasjonsbredde

0.5 Variasjonsbredde

Differansen mellom svarene med henholdsvis høgest og lavest verdi.

Survey 1

Svaret med henholdsvis høgest og lavest verdi er 124 og 38. Altså er

$$\text{variasjonsbredde} = 124 - 38 = 86$$

Survey 2

Svaret med henholdsvis høgest og lavest verdi er 14 og 0. Altså er

$$\text{variasjonsbredde} = 14 - 0 = 14$$

Survey 4

- Variasjonsbredde for mobiler totalt:

$$2\,500 - 2\,100 = 400$$

- Variasjonsbredde for mobiler uten smartfunksjoner:

$$1\,665 - 147 = 518$$

- Variasjonsbredde for mobiler med smartfunksjoner:

$$2\,160 - 700 = 1460$$

0.4.2 Kvartilbredde

0.6 Kvartilbredde og øvre og nedre kvartil

Kvartilbredden til et datasett kan finnes på følgende måte:

1. Ranger datasettet fra høgest til lavest verdi.
2. Skill det rangerte datasettet på midten, slik at to nye sett oppstår. (Viss det er oddetalls antall verdier i datasettet, utelates medianen).
3. Finn de respektive medianene i de to nye settene.
4. Finn differansen mellom medianene fra punkt 3.

Om medianene fra punkt 3: Den med høgest verdi kalles **øvre kvartil** og den med lavest verdi kalles **nedre kvartil**.

Survey 1

1. 38 40 47 56 61 79 84 97 101 124
2. 38 40 47 56 61 79 84 97 101 124
3. Medianen i det blå settet er 47 (nedre kvartil) og medianen i det røde settet er 97 (øvre kvartil).

   38 40 47 56 61 79 84 97 101 124
4. Kvartilbredde $= 97 - 47 = 50$

Survey 2

1. 0 0 1 1 4 4 4 5 5 6 6 7 8 8 14
2. 0 0 1 1 4 4 4 5 5 6 6 7 8 8 14
3. Medianen i det blå settet er 1 (nedre kvartil) og medianen i det røde settet er 7 (øvre kvartil).

   0 0 1 1 4 4 4 5 6 6 7 8 8 14
4. Kvartilbredde $= 7 - 1 = 6$

Survey 4

(Utregning utelatt)

- For mobiler totalt er kvartilbredden: 200
- For mobiler uten smartfunksjoner er kvartilbredden: 1010
- For mobiler med smartfunksjoner er kvartilbredden: 703

The language box

Nedre kvartil, medianen og øvre kvartil blir også kalt henholdsvis **1. kvartil**, **2. kvartil** og **3. kvartil**.

0.4.3 Avvik, varians og standardavvik

0.7 Varians

Differansen mellom en verdi og gjennomsnittet i et datasett kalles **avviket** til verdien.

Variansen til et datasett kan finnes på følgende måte:

1. Kvadrer avviket til hver verdi i datasettet, og summer disse.
2. Divider med antall verdier i datasettet.

Standardavviket er kvadratroten av variansen.

Example

Gitt datasettet

$$2 \quad 5 \quad 9 \quad 7 \quad 7$$

Da har vi at

$$\text{gjennomsnitt} = \frac{2+5+9+7+7}{5} = 6$$

Og videre er

$$\begin{aligned}\text{variansen} &= \frac{(2-6)^2+(5-6)^2+(9-6)^2+(7-6)^2+(7-6)^2}{5} \\ &= 5\end{aligned}$$

Da er standardavviket $= \sqrt{5} \approx 2{,}23$.

Survey 1

(Utregning utelatt)

Variansen er 754,01. Standardavviket er $\sqrt{754{,}01} \approx 27{,}46$

Survey 2

Gjennomsnittet fant vi på side 9. Vi utvider frekvenstabellen vår fra side 3:

antall epler	frekvens	frekvens · kvadrert avvik
0	2	$2 \cdot \left(0 - \frac{73}{15}\right)^2$
1	2	$2 \cdot \left(1 - \frac{73}{15}\right)^2$
4	3	$3 \cdot \left(4 - \frac{73}{15}\right)^2$
5	2	$2 \cdot \left(5 - \frac{73}{15}\right)^2$
6	2	$2 \cdot \left(6 - \frac{73}{15}\right)^2$
7	1	$1 \cdot \left(7 - \frac{73}{15}\right)^2$
8	2	$2 \cdot \left(8 - \frac{73}{15}\right)^2$
14	1	$1 \cdot \left(9 - \frac{73}{15}\right)^2$
sum	15	$189{,}7\overline{3}$

Altså er variansen

$$\frac{189{,}7\overline{3}}{15} \approx 12{,}65$$

Da er standardavviket $\sqrt{12{,}65} \approx 3.57$

Survey 4

(Utregning utelatt)

- For mobiler totalt er variansen 17 781,25 og standardavviket ca. 133,4.
- For mobiler uten smartfunksjoner er variansen $318\,848.\overline{3}$ og standardavviket ca. 17,87
- For mobiler med smartfunksjoner er variansen $245\,847.91\overline{6}$ og standardavviket ca. 495,83.

Hvorfor innebærer variansen kvadrering?

La oss se hva som skjer hvis vi gjentar utregningen fra *Eksempel* på side 18, men uten å kvadrere:

$$\frac{(2-6)+(5-6)+(9-6)+(7-6)+(7-6)}{5} = \frac{2+5+9+7+7}{5} - 6 \quad (1)$$

Men brøken $\frac{2+5+9+7+7}{5}$ er jo per definisjon gjennomsnittet til datasettet, og dermed blir uttrykket over lik 0. Dette vil gjelde for alle datasett, så i denne sammenhengen gir ikke tallet 0 noen ytterligere informasjon. Om vi derimot kvadrerer avvikene, unngår vi et uttrykk som alltid blir lik 0.