0.1 Størrelser, enheter og prefikser

Objects we can measure and describe by numbers are called **quantities**. A quantity usually consists of both a value and a **unit**. In this section we will look at these four units:

unit	abbreviation	unit for
meter	m	length
gram	g	mass
second	s	time
liter	L	volume

Sometimes we have either very large or very small quantities, like the length around the equator, which is ca. 40 075 000 m! For very large or small numbers it is helpful to use a **prefix**. Using the prefix 'kilo', the length around the equator can be written as ca. 40 075 km, where 'km' is an abbreviation for 'kilometer', and 'kilo' meaning '1 000'. Hence, 1 000 meters is 1 kilometer . The prefixes for the powers of ten with integer exponents ranging from -3 to 3 (except 0) are the following:

prefix	abbreviation	value
kilo	k	1 000
hecto	h	100
deka	da	10
desi	d	0.1
centi	c	0.01
milli	m	0.001

The language box

A (potential) **prefix** and a **unit** constitute a **notation**. For example, 9 km has notation 'km', while 9 m has notation 'm'. These quantities have unequal notations but both have 'm' as unit.

We can create a structured way of changing prefixes by placing the prefixes in a horizontal table with a unit added to the place corresponding to the value 1:

0.1 Converting prefixes

When changing the prefix of a quantity we can use this table:

kilo	hecto	deka	unit	desi	centi	milli

The movement of the decimal separator corresponds to the movement from the original prefix to the new prefix.

Example 1

Write 23,4 mL as an amount of L.

Answer

We set 'L' as the unit, and notice that we must move **three cells to the left** to get from mL to L:

kilo	hecto	deka	L	desi	centi	milli

This means the comma separator is to be moved three places to the left:

$$23.4\,\mathrm{mL} = 0.0234\,\mathrm{L}$$

Example 2

Write 30 hg as an amount of cg.

Answer

We set 'g' as the unit, and notice that we must move **four cells to the right** to get from hg to cg:

kilo	hecto	deka	g	desi	centi	milli

This means the comma separator is to be moved four places to the right:

$$30\,\mathrm{mg} = 300\,000\,\mathrm{cg}$$

Example 3

Write $2.7\,\mathrm{s}$ as an amount of ms.

Answer

We set 's' as the unit, and notice that we must move **three cells to the right** to get from s to ms:

kilo	hecto	deka	s	desi	centi	milli

This means the comma separator is to be moved three places to the right:

$$2.7\,\mathrm{s} = 2\,700\,\mathrm{ms}$$

0.1 Converting prefixes (forklaring)

Changing prefixes corresponds to multiplying/dividing by 10, 100 etc. (see MB).

As our first example, let us write $3{,}452\,\mathrm{km}$ as an amount of meters. We have

$$\begin{aligned} 3.452\,\mathrm{km} &= 3.452 \cdot 1000\,\mathrm{m} \\ &= 3\,452\,\mathrm{m} \end{aligned}$$

As our second example, let us write $47\,\mathrm{mm}$ as an amount of meters. We have

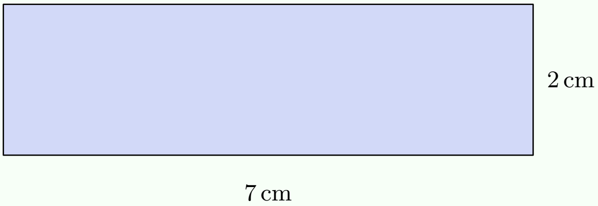
$$\begin{aligned} 47\,\mathrm{mm} &= 47 \cdot \frac{1}{1000}\,\mathrm{m} \\ &= (47 : 1000)\,\mathrm{m} \\ &= 0.047\,\mathrm{m} \end{aligned}$$

0.2 Regning med størrelser

Note: In the examples of this section we use area and volume formulas found in MB.

When performing calculations, notations can be handled in the same way as variables in algebra. [1]. Hence, in the same way as we have $a + a = 2a$, we have $1\,\text{cm} + 1\,\text{cm} = 2\,\text{cm}$, and just like we have $2a \cdot 3a = 6a^2$, we have $2\,\text{cm} \cdot 3\,\text{cm} = 6\,\text{cm}^2$.

Example 1



a) Find the perimeter of the rectangle.

b) Find the area of the rectangle.

Answer

a) The perimeter of the rectangle is

$$7\,\text{cm} + 2\,\text{cm} + 7\,\text{cm} + 2\,\text{cm} = 18\,\text{cm}$$

b) The area of the rectangle is

$$7\,\text{cm} \cdot 2\,\text{cm} = 14\,\text{cm}^2$$

Example 3

A cylinder has radius $4\,\text{m}$ and height $2\,\text{m}$. Find the volume of the cylinder.

Answer

$$\text{base area} = \pi \cdot (4\,\text{cm})^2 = 16\pi\,\text{cm}^2$$

$$\text{volume of the cylinder} = 16\pi\,\text{cm}^2 \cdot 2\,\text{cm} = 32\pi\,\text{cm}^3$$

[1] See MB.

0.3 Proportional quantities

Say it costs[1] 10 kr for 0.5 kg potatoes. If this price is also valid if we buy 0.5 kg additional potatoes, the price for 1 kg potatoes is 20 kr. If this price is valid also if we buy 0,5 kg additional potatoes, the price for 2 kg potatoes is 40 kr. We can put the amount of krones and amount of kilogram potatoes in a table:

kr	10	20	40
kg	0,5	1	2

Moreover, let us divide the price by the weight for each case:

$$\frac{10\,\text{kr}}{0{,}5\,\text{kg}} = 20\,\text{kr}/\text{kg} \qquad \frac{20\,\text{kr}}{1\,\text{kg}} = 20\,\text{kr}/\text{kg} \qquad \frac{40\,\text{kr}}{2} = 20\,\text{kr}/\text{kg}$$

Clearly, the ratio of the price to the weight is the same for all the cases. In that case, we say that the price and the weight are[2] **proportional quantities**. From these two quantities we have also "made" a new quantity with notation[3] 'kr/kg'. This is expelled "krones per kilogram". Therefore, in our example the price per kilogram is 20 kr/kg. Since the price per kilogram is the result of division between two proportional quantities, it is called a **constant of proportionality**.

0.2 Proportional quantities

$$\text{constant of proportionality} = \frac{\text{a quantity}}{\text{another quantity}} \tag{1}$$

Applied mathematics includes a great amount of quantities which are constants of proportionality, and in the definition boxes below you will find a selection of these. Note that all these formulas are identical to equation (1), only with the names of quantities and units changed.

[1] 'kr' is the Norwegian currency

[2] Also see appendix ?? in MB.

[3] We could also write $\frac{\text{kr}}{\text{kg}}$, but in this case it is more convenient to use / as the symbol for division.

0.3 Price per kilo

Price per kilo yields the ratio of a price (in av given currency) to a weight (in kilograms).

$$\text{price per kilo} = \frac{\text{price}}{\text{weight}} \tag{2}$$

Alternatively,

$$\text{price} = \text{price per kilo} \cdot \text{weight} \tag{3}$$

The notation for price per kilo is 'currency/kg'.

0.4 Price per liter

Price per liter yields the ratio of a price (in av given currency) to a volume (in liters).

$$\text{price per liter} = \frac{\text{price}}{\text{volume}} \tag{4}$$

Alternatively,

$$\text{pris} = \text{literpris} \cdot \text{volum} \tag{5}$$

The notation for price per liter is 'currency/L'.

0.5 Speed

Speed yields the ratio of length to time.

$$\text{fart} = \frac{\text{lengde}}{\text{tid}} \tag{6}$$

Alternatively,

$$\text{length} = \text{speed} \cdot \text{time} \tag{7}$$

Common notations for speed are 'km/h' and 'm/s'.

0.6 Tetthet

Density yields the ratio of weight to volume.

$$\text{density} = \frac{\text{weight}}{\text{volume}} \tag{8}$$

Alternatively,

$$\text{weight} = \text{density} \cdot \text{volume} \tag{9}$$

Common notations for density are 'kg/m^3' and 'g/cm^3'

0.7 Effect

Effect yields the ratio of energy to time.

$$\text{effect} = \frac{\text{energy}}{\text{time}} \tag{10}$$

Alternatively,

$$\text{energy} = \text{effect} \cdot \text{time} \tag{11}$$

Common notations for effect are 'J/s' and 'kWh/s'. 'J' is the abbreviation of the energy unit 'Joule'. 'J/s' is the same as 'W', which is the abbreviation of 'Watt'. In 'kWh', 'k' means 'kilo', 'W' means 'Watt' and 'h' means 'hour'.

Note

In (2) - (8), it is assumed that the quantities on the left side of the equations are *constants*, but that is not always the case. If you let a stone fall from a height, it's speed will obviously not remain the same along the way. By dividing the length by the time it took to travel it, you will find *the constant speed the ball would need have if it were to travel the same distance at the identical time.*

0.4 Regning med forskjellige benevninger

When performing calculations with quantities with notations, it is important to ensure that the notations involved are the same.

Example 1

Calculate $5\,\text{km} + 4\,000\,\text{m}$.

Answer

We must either write $5\,\text{km}$ as an amount of 'm' or $4\,000\,\text{m}$ as an amount of 'km' before we can add the quantities. We choose to write $5\,\text{km}$ as an amount of 'm' (see Rule 0.1):

$$5\,\text{km}{=}5\,000\,\text{m}$$

Now

$$\begin{aligned} 5\,\text{km} + 4\,000\,\text{m} &= 5\,000\,\text{m} + 4\,000\,\text{m} \\ &= 9\,000\,\text{m} \end{aligned}$$

Tips

Calculations involving notations can turn out to be a bit cumbersome. If you have ensured that the notations are the same, further calculations can be performed without them. In *Example 1* above, we could have written

$$5\,000 + 4\,000 = 9\,000$$

However, in a final answer the notation is *absolutely necessary*:

$$5\,\text{km} + 4\,000\,\text{m} = 9\,000\,\text{m}$$

Example 2

Use equation (7) to answer the questions.

a) A car drives at 50 km/h. How far will it travel in 3 hours?

b) A car drives at 90 km/h. How far will it travel in 45 minutes?

Answer

a) In equation (7), the speed is now 50, and the time is 3, so

$$\text{length} = 50 \cdot 3 = 150$$

That is, the car will travel 150 km.

b) Here, we have two different notations for time involved; 'hours' ('h') and 'minutes' ('min'). Therefore, we must either write the speed as an amount of 'km/min' or the time as an amount of 'h'. We chose to write 'min' as an amount of 'h'[1]:

$$\begin{aligned} 45\,\text{minutes} &= \frac{45}{60}\,\text{hours} \\ &= \frac{3}{4}\,\text{hours} \end{aligned}$$

In equation (7), the speed is now 90, and the time is $\frac{3}{4}$, so

$$\text{strekning} = 90 \cdot \frac{3}{4} = 67.5$$

That is, the car will travel 67.5 km in 45 minutes.

[1] Recall that 60 min = 1 h.

Example 3

Use equation (2) to answer the questions.

a) 10 kg tomatoes cost 35 kr. what is the price per kilo for the tomatoes?

b) Safran is reckoned to be the world's most expensive spice, with 5 g costing up to 600 kr. In that case, what is the price per kilo for safran?

Answer

a) In equation (2), the price is now 35, and the weight is 10, so

$$\text{price per kilo} = \frac{35}{10} = 3.5$$

That is, tomatoes costs 3.5 kr/kg

b) Here, we have two different units for weight involved; 'kg' and 'g'. We write the amount of 'g' as an amount of 'kg' (see Rule 0.1):

$$5\,\text{g} = 0.005\,\text{kg}$$

In equation (2), the price is now 600, and the weight is 0.005, so

$$\text{price per kilo} = \frac{600}{0{,}005} = 120\,000$$

That is, safran costs 120 000 kr/kg.