0.1 The equal sign, amounts, and number lines

The equal sign

As the name implies, the *equal sign* $=$ refers to things that are the same. In what sense some things are the same is a philosophical question and initially we are bound to this: What equality $=$ points to must be understood by the context in which the sign is used. With this understanding of $=$ we can study some basic properties of our numbers and then later return to more precise meanings of the sign.

> **The language box**
>
> Common ways of expressing $=$ is
>
> - ”equals”
> - ”is the same as”

Amounts and number lines

There are many ways a number can be defined, however, in this book we shall stick to two ways of interpreting a number; a number as an *amount* and a number as a *placement on a line*. All representations of numbers rely on the understanding of 0 and 1.

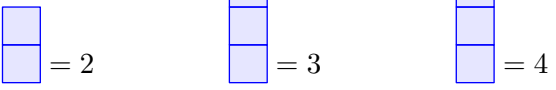
Numbers as amounts

Talking about an amount, the number 0 is[1] connected to ”nothing”. A figure showing nothing will therefore equal 0:

$$= 0$$

1 we’ll draw like a box:

In this way, other numbers are defined by how many one-boxes (ones/units) we have:



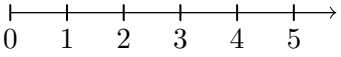
[1] In *Chapter* **??** we’ll see that there are other interpretations of 0.

Numbers as placements on a line

When placing numbers on a line, 0 is our starting point:

Now we place 1 a set length to the right of 0:

Other numbers are now defined by how many one-lengths (ones/units) we are away from 0:



Positive integers

We'll soon see that numbers do not necessarily have to be a *whole* amount of ones, but those which *are* have their own name:

0.1 Positive integers

Numbers which are a whole amount of ones are called *positive*[1] *integers*. The positive integers are

$$1, 2, 3, 4, 5 \text{ and so on.}$$

Positive integers are also called *natural numbers*.

What about 0?

Some authors also include 0 in the definition of natural numbers. This is in some cases beneficial, in others not.

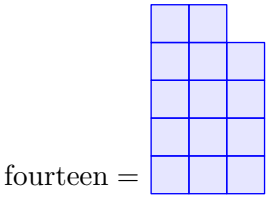
[1] We'll see what the word *positive* refers to in chapter *chapter* **??**.

0.2 Numbers, digits and value

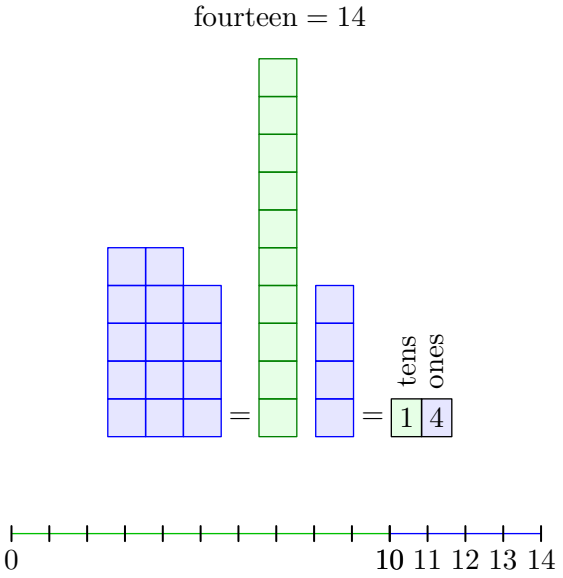
Our numbers consist of the *digits* $0, 1, 2, 3, 4, 5, 6, 7, 8$ and 9 along with their *positions*. The digits and their positions defines[1] the *value* of numbers.

Integers larger then 10

Let's, as an example, write the number *fourteen* by our digits.



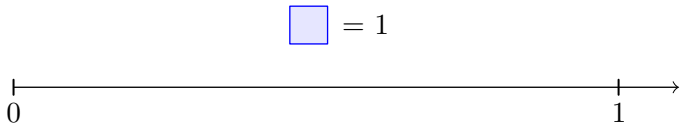
We can now make a group of 10 ones, then we also have 4 ones. By this, we write fourteen as



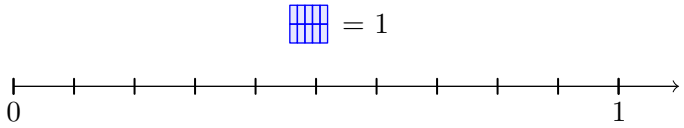
[1] Later on, we'll also see that *signs* have an impact on a numbers value (see *Chapter ??*).

Decimal numbers

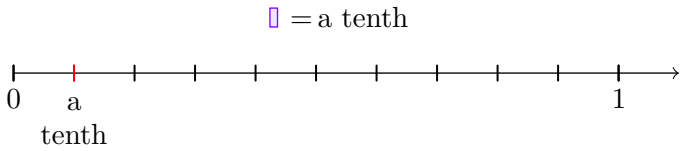
Sometimes we don't have a whole amount of ones, and this brings about the need to divide "ones" into smaller pieces. Let's start off by drawing a one:



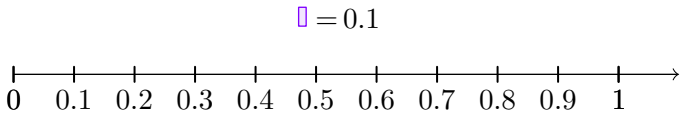
Now we divide our one into 10 smaller pieces:



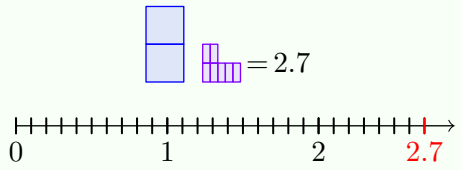
Since we have divided 1 into 10 pieces, we name one such piece *a tenth*:



We indicate tenths by using the *decimal mark*: .



Example



The language box

In a lot of countries, a comma is used in place of the period for the decimal mark.

3,5 (*other*)

3.5 (*English*)

Base-10 positional notation

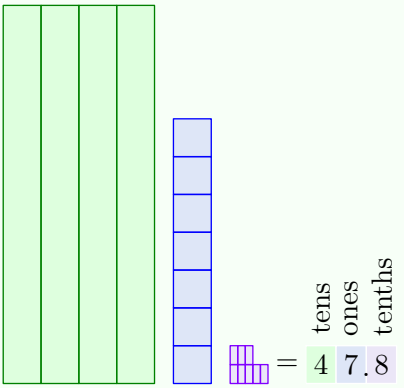
So far, we have seen how we can express the value of a number by placing digits according to the amount of tens, ones and tenths. The pattern continues:

0.2 Base-10 positional notation

The value of a number is given by the digits $0, 1, 2, 3, 4, 5, 6, 7, 8$ and 9 and their position. In respect to the digit indicating ones,

- digits to the left indicate amounts of tens, hundreds, thousands etc.
- digits to the left indicate amounts of tenths, hundredths, thousandths etc.

Example 1



Example 2

0.3 Even and odd numbers

Inegers with 0, 2, 4, 6 or 8 on the ones place are called *even numbers* .

Integers with 1, 3, 5, 7 or 9 on the ones place are called *odd numbers* .

Example

The first ten (positive) even numbers are

0, 2, 4, 6, 8, 10, 12, 14, 16, og 18

The first ten (positive) odd numbers are

1, 3, 5, 7, 9, 11, 13, 15, 17, og 19

0.3 Coordinate systems

Two number lines can be put together to form a *coordinate system*. In that case we place one number line *horizontally* and one *vertically*. A position in a coordinate system is called a *point*.

In fact, there are many types of coordinate systems, but we'll use the *cartesian coordinate system*. It is named after the French mathematician and philosopher, René Descartes.

A point is written as two numbers inside a bracket. We shall call these two numbers the *first coordinate* and the *second coordinate*.

- The first coordinate tells how many units to move along the horizontal axis.
- The second coordinate tells how many units to move along the vertical axis.

In the figure, the points $(2, 3)$, $(5, 1)$ and $(0, 0)$ are shown. The point where the axes intersect, $(0, 0)$, is called *origo*.