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Abstract—*The availability of devices that can be used to track moving objects has increased dramatically leading to a great growth in movement data from almost every application domain. Therefore, there has been an increasing interest in proposing new methodologies for indexing, classifying, clustering, querying and measuring similarity between moving objects' data. One of the main functions for a wide range of application domains is to measure the similarity between two moving objects' trajectories. In this paper, we present a comparative study between widely used trajectory similarity measures observing the advantages and disadvantages of these measures.*

Keywords— trajectory; similarity; moving objects

I. INTRODUCTION

The availability of devices that can be used to track moving objects such as GPS devices has increased dramatically leading to a great growth in movement data. Moving Objects can be animals, cars, hurricanes, sports players, suspected terrorists, mobile users and data packets.

Moving objects can be spatial moving objects (e.g., cars changing their location with time), or nonspatial moving objects (e.g., time series). A moving object's data are generated at a large scale and rate from almost every application domain, e.g., daily changes of stock market, diverse readings obtained from sensor networks and position updates of moving objects in location-based services. Therefore, there has been an increasing interest in proposing new methodologies for indexing, classifying, clustering, querying and measuring similarity among moving objects' data [1].

One of the main functions for a wide range of application domains is to measure the similarity between two moving objects' trajectories [2, 3]. This is desirable for similarity-based retrieval, classification, clustering and other querying and mining tasks over moving objects' data. For example, analyzing trajectories of customers in a supermarket to find similar movement patterns for a better management of products. Another example is finding frequent migration patterns of a migrating group of birds. In surveillance systems, it is possible to find suspicious object movements and rare trajectory patterns. Other domains such as stock and data analysis, recommender systems of travelling routes , recommender systems for friends based on their interests, visited locations, likes, etc. and future prediction of phenomena such as storms, hurricanes and earthquakes. Finding similar trajectory data is one main aspect needed in all these domains.

There are many trajectory distance measures that assume certain similarity definitions, and then propose efficient techniques for similarity matching based on these definitions. In the next section, we explain widely used similarity measures and compare between them showing the advantages and disadvantages of each measure.

II. SIMILARITY MEASURES

Moving objects are objects that change their value with time [4] and tracking them gives rise to a sequence of points in time and space, called a trajectory [5]. A trajectory holds the continuous motion history of the moving object [2]. However, due to the limitation of location positioning devices (e.g., GPS devices), a trajectory T in real world is a sequence of positions $[v_1, v_2, \dots, v_m]$ observed at discrete time instances $[t_1, t_2, \dots, t_m]$ so 'T' can be expressed as follows $T=[(v_1, t_1),(v_2, t_2), \dots,(v_m, t_m)]$ where m is the length of T [6].

The distance (inverse of similarity) between two moving objects is measured as the cost of transforming one object into another using a distance measure/function [7]. The existing movement similarity measures can be classified into two classes as shown in Fig. 1: (1) spatial similarity that focuses on finding trajectories with similar geometric shapes and ignore the temporal dimension; and (2) spatio-temporal similarity that takes into account both the spatial and the temporal dimensions of movement data.

We briefly review in this section some of the trajectory similarity measures to provide a background of the commonly used similarity measures. Notations used through paper is described in Table I.

TABLE I. NOTATIONS USED AND THEIR DESCRIPTION

Notation	Description
R, S	Trajectory $(r_1, \dots, r_m)$ and $(s_1, \dots, s_m)$.
m, n	Length of R and S respectively
Rest(R), Rest(S)	R and S with the first element removed
$r_i(r_{i,x}, r_{i,y})$, $s_i(s_{i,x}, s_{i,y})$	i^{th} element of R and S respectively which is represented by x and y coordinates (i.e.,lon, lat)
$dist(r_i,s_j)$	The distance between two elements r_i and s_j

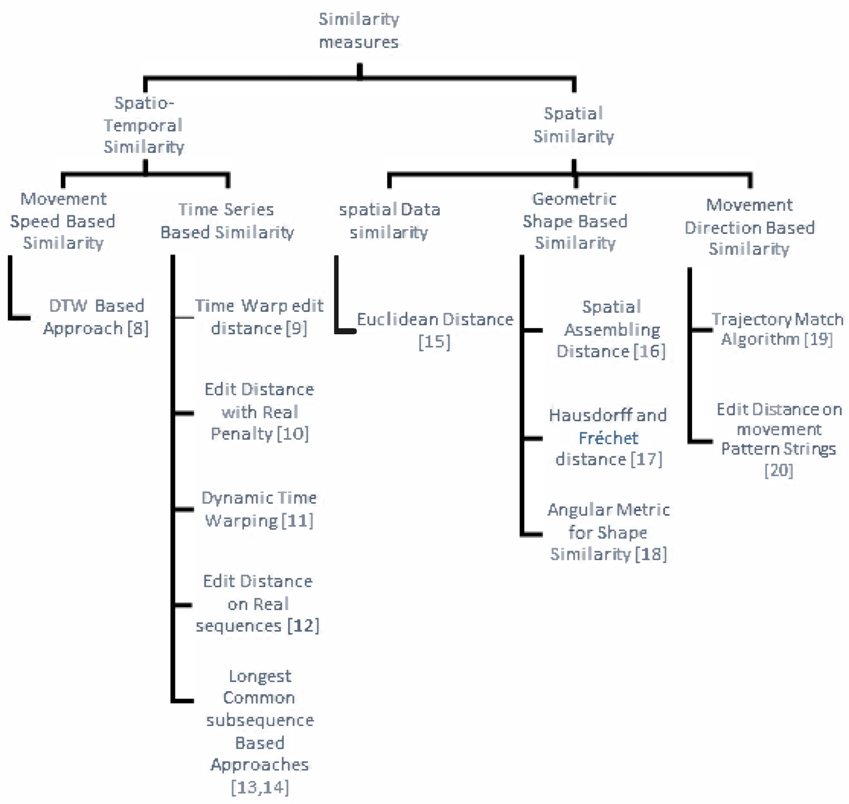

Fig. 1. *Classification of Similarity Measures.*

A. *Spatio-temporal similarity measures*

As mentioned earlier, spatio-temporal similarity measures take both the time and spatial dimensions into consideration while measuring similarity between trajectories. Measuring distance between trajectories can be based on aligning ¡th element of the first trajectory with the ¡th element in the second, while others allows local time shifting for better similarity. Local time shifting occurs when one element of a trajectory is shifted along the time axis to match an element in another trajectory even when the two matched elements appear in different positions in the trajectories. The following subsections shows different types of spatio-temporal similarity measures. Some measured similarity based on movement parameters such as speed while others used the raw representation of data.

1) Movement speed similarity using dynamic time warping based approach [8]

In this work, author proposed to separate between positional information and temporal information so motion of a moving object is represented by both speed and path curves. They followed the warping approach where data points at different positions in the sequences may be closer to each other than the ones in the same position. Warping approach repeats and shift elements for better sequence matching. After constructing path and speed curves, the dynamic time warping method was used to measure similarity between two objects' motion. Due to separation between positional and temporal information, the user can select path, speed, or both of them to measure similarity.This approach takes into account local time shifting and allows finding similarity between trajectories of different lengths but it is not robust to noise.

2) Time series based similarity

A time series representation of a trajectory is a sequence of values sampled at specific time as mentioned earlier. Time series data can be normalized (reducing range of data) to allow the distance between trajectories to be invariant to spatial scaling and shifting. Usually this type of similarity measures is associated with a distance function. This fuction can be metric or nonmetric. A distance function d(r,s) is said to be metric if it satisfies the following criterias:

- *Uniqueness: d(r, s) = 0 iff r=s.*
- *Symmetry: d(r, s) = d(s, r).*
- *Nonnegativity: d(r, s) > 0.*
- *Triangle Inequality: d(x, y) + d(x, z) >= d(y, z).*

To cope with local time shifting, two time series are aligned for better similarity through minimal number of delete,insert and match operations. Deletion from one time series means insertion in the other one. The inserted element is called the gap element. The insert and delete operations are penalized with a gap penalty constant.

a) Dynamic time warping [11]

*Dynamic Time warping (*DTW) is a nonmetric similarity measure that finds similar patterns between trajectories. It recursively search all point combinations between two trajectories for the one with minimal cost. It allows similar shapes to match even if they are not aligned in the time axis in other words, it allows a sequence to stretch or shrink in order to get better match with another one as shown in Fig. 2. DTW allows finding similarity between trajectories of different lengths and with the existence of local time shifting but it is not robust to noise. DTW is defined in (1).

$$\mathrm{DTW}(R,S) = \begin{cases} 0 & \text{if } m = n = 0 \\ \infty & \text{if } m = 0 \text{ or } n = 0 \\ \mathrm{dist}(r_1, s_1) + \min\begin{Bmatrix} \mathrm{DTW}(\mathrm{Rest}(R), \mathrm{Rest}(S)), \\ \mathrm{DTW}(\mathrm{Rest}(R), S), \\ \mathrm{DTW}(R, \mathrm{Rest}(S)) \end{Bmatrix} & \text{otherwise} \end{cases} \tag{1}$$

Where $dist(r_1,s_1) = |r_1 - s_1|$.

Fig. 2. *Two similar time series but not aligned in time axis [11].*

b) Time warp edit distance [9]

Time warp edit distance (TWED) is a metric similarity measure that allows similar shapes to match even if they are not aligned in the time axis as shown in Fig. 2. It defines three operations for alignment delete from R, delete from S and match operations. One of the differences between TWED and other similarity measures is the use of time stamp differences between compared samples as part of the local matching costs. It is based on two parameters, Stiffness parameter (γ) that limits elasticity of the measure (penalty over time) and Lambda parameter (λ) where λ is the Gap Penalty of deletion. It takes into account local time shifting and allows finding similarity between trajectories of different lengths and it is not robust to noise. TWED is defined in (2).

$$TWED_{\lambda,\gamma}(R_1^m, S_1^n) = \begin{cases} TWED_{\lambda,\gamma}(R_1^{m-1}, S_1^n) + d_{lp}(r_m, r_{m-1}) + \gamma . d_{lp}(t_{r_m}, t_{r_{m-1}}) + \lambda \\ \quad delete - R \\ TWED_{\lambda,\gamma}(R_1^{m-1}, S_1^{n-1}) + d_{lp}(r_m, s) + \gamma . d_{lp}(t_{r_m}, t_{s_n}) + \\ \quad d_{lp}(r_{m-1}, s_{n-1}) + \gamma . d_{lp}(t_{r_{m-1}}, t_{s_{n-1}}) \\ \quad Match \\ TWED_{\lambda,\gamma}(R_1^m, S_1^{n-1}) + d_{lp}(s_{n-1}, s_n) + \gamma . d_{lp}(t_{s_n}, t_{s_{n-1}}) + \lambda \\ \quad delete - S \end{cases} \tag{2}$$

Where d_{lp} can be either L_1-*Norm(p=1)* which is called manhattan distance or L_2-*Norm(p=2)* which is called euclidean distance and it is computed using (3).

$$L_p - \mathrm{norm}\ (R, S) = \sqrt[p]{\textstyle\sum_{i=1}^{N} |r_i - s_i|^p} \tag{3}$$

c) Edit distance with real penalty [10]

Edit distance with real penalty (ERP) is a metric similarity measure based on edit distance (ED). ERP uses L_1-norm as the distance measure. In addition, ERP distance is defined on normalized trajectory data. ERP takes into account local time shifting and allows finding similarity between trajectories of different lengths but it is not robust to noise. ERP is defined in (4).

$$ERP(R_1^m, S_1^n) = \begin{cases} \sum_{1}^{n} dist(s_i, g) & if\ m = 0 \\ \sum_{1}^{m} dist(r_i, g) & if\ n = 0 \\ \min\begin{Bmatrix} ERP(Rest(R), Rest(S)) + dist(r_1, s_1), \\ ERP(Rest(R), S) + dist(r_1, g), \\ ERP(R, Rest(S)) + dist(s_1, g) \end{Bmatrix} & otherwise \end{cases} \tag{4}$$

Where $dist(r_1,s_1) = |r_1 - s_1|$, $dist(r_1,g) = |r_1 - g|$, $dist(s_1,g) = |s_1 - g|$ and g is the gap constant of edit distance and it uses real distance between elements as the penalty to handle local time shifting.

d) Edit distance on real sequences[12]

Edit distance on real sequences (EDR) is another nonmetric edit distance (ED) based trajectory similarity measure. EDR uses a threshold ε to detect sample points matching and it also uses normalized trajectory data. EDR is robust to noise and allows finding similarity between trajectories of different lengths and it takes into account local time shifting. EDR between two trajectories R and S is the number of insert, delete or replace operations needed to chage R into S and it is defined in (5).

$$\mathrm{EDR}(R_1^m, S_1^n) = \begin{cases} n & \text{if } m = 0 \\ m & \text{if } n = 0 \\ \min\begin{Bmatrix} \mathrm{EDR}(\mathrm{Rest}(R), \mathrm{Rest}(S)) + \mathrm{subcost}, \\ \mathrm{EDR}(\mathrm{Rest}(R), S) + 1, \\ \mathrm{EDR}(R, \mathrm{Rest}(S)) + 1 \end{Bmatrix} & \text{otherwise} \end{cases} \tag{5}$$

A pair of trajectory element vectors ($r_{i,x}$, $r_{i,y}$) and ($s_{i,x}$, $s_{i,y}$) are said to match if and only if $|r_{i,x} - s_{i,x}| \le \varepsilon$ and $|r_{i,y} - s_{i,y}| \le \varepsilon$ where ε is a matching threshold. Cost of insert,delete or replace is assumed to be 1. Subcost = 0 if (r_1, s_1) matches and subcost =1 otherwise.

e) Longest common subsequence based approaches [13,14]

Longest common subsequences (LCSS) is a nonmetric similarity measure that was mainly proposed to overcome the effect of noise on measuring similarity. It uses a threshold ε to detect elements' matching. It allows finding similarity between trajectories with different lengths and takes into account local time shifting. LCSS is defined in (6).

$$\mathrm{LCSS}(R_1^m, S_1^n) = \begin{cases} 0 & \text{if } m = 0 \text{ or } n = 0 \\ \mathrm{LCSS}(\mathrm{Rest}(R), \mathrm{Rest}(S)) + 1 & \text{if } |r_{1,x} - s_{1,x}| \le \varepsilon \;\&\; |r_{1,y} - s_{1,y}| \le \varepsilon \\ \max\left\{ \begin{array}{l} \mathrm{LCSS}(\mathrm{Rest}(R), S) \\ \mathrm{LCSS}(R, \mathrm{Rest}(S)) \end{array} \right\} & \text{otherwise} \end{cases} \quad (6)$$

Where ε is the matching threshold which determines whether two elements match or not. It limited distance between two elements to 0 and 1.

B. Spatial Similarity Measures

Spatial similarity measures takes into account the spatial dimension only while ignoring the time dimension. It has two categories, similarity based on real data values while the other is based on the geometric shape of the trajectory as shown in the following subsections.

1) Spatial Data similarity: Euclidean Distance [15]

Euclidean distance (ED) is a metric similarity measure, also known as L_2-norm, that assumes that the *ith* point in one sequence is aligned with the *ith* point in the other as shown in Fig. 3. It neither takes into account local time shifting nor allows finding similarity between trajectories with different lengths and it is not robust to noise. ED is defined in (7).

$$\mathrm{Eu}(R_1^n, S_1^n) = \sqrt{\sum_{i=1}^{n} \mathrm{dist}(r_i, s_i)} \quad (7)$$

where $\mathrm{dist}(r_i, s_i) = (r_{i,x} - s_{i,x})^2 + (r_{i,y} - s_{i,y})^2$.

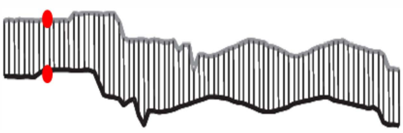

Fig. 3. *Two aligned sequences [11].*

2) Geometric shape based similarity

a) Spatial assembling distance [16]

Some distance measures such as Euclidean are sensitive to the time dimension warps or amplitude dimension warps (data items are not aligned). To reduce the effect of time warps, distances such as DTW, LCSS and EDR have been proposed. Amplitude and time warps can cause two trajectories to be more similar while they are not as shown in Fig. 4 . In Spatial Assembling Distance (SPADe), it took into account both warps in time and amplitude dimensions while measuring similarity. It converts trajectories to a set of local patterns (lp) then a matching matrix is built using a distance function between lp and lp' of trajectory R and S respectively. A large number of LPMs will be formed if R and S are similar in shapes. It is a nonmetric measure, is robust to noise, takes into account local time shifting and allows finding similarity between trajectories of different lengths.

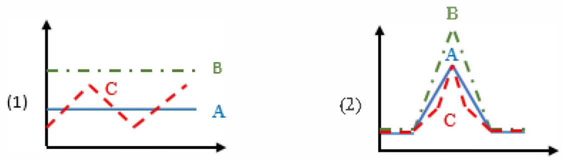

Fig. 4. *Sequence A and B are much more similar in shapes than A and C in the two cases. However, due to amplitude shifting in (1), and scaling in (2), the warping distances of A and B in the two cases will be larger than those of A and C.*

b) Hausdorff and Fréchet distance [17]

Hausdroff distance is a metric measure that measures how close shape A of trajectory R is close to shape B of trajectory S. It considers for any point on shape 'A' the distance to the closest point on shape 'B' and to maximize over all these values. Example of Hausdroff distance is shown in Fig. 5.

Fréchet distance is a nonmetric measure that takes into account both the location and ordering of the points along curves of two trajectories shapes. A popular example for illustrating Fréchet distance between two curves is the minimum length of a leash required to connect a dog and its owner walking on two separate and different paths (curves) continuously with varing speeds as shown in Fig. 6.

They do not take into account time dimension shifting, are not robust to noise and two curves can have different Lengths (number of vertics).

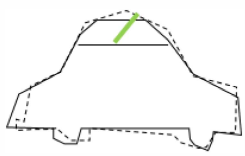

Fig. 5. *The thick bold line represents the hausdroff distnace between the two shapes.*

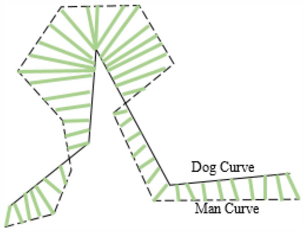

Fig. 6. *Thick bold lines represents Fréchet distance between points where man and dog were located at the same time.*

c) Angular metric for shape similarity [18]

Angular metric for shape similarity (AMSS) is a nonmetric similarity measure that treats a time series as a vector sequence for better handling of spatial variation as shown in Fig. 7. In addition, AMSS calculates similarity between two vector sequences based on vectors' directions, not on the actual locations of the data points in the d-dimensional space. It takes into account time shifting and allows finding similarity between trajectories of different lengths. It is defined in (8).

$$\mathrm{AMSS}(R_m, S_n) = \max \begin{cases} \mathrm{AMSS}(R_{m-1}, S_{n-1}) + 2.\,\mathrm{sim}(r_n, s_n) \\ \mathrm{AMSS}(R_{m-2}, S_{n-1}) + 2.\,\mathrm{sim}(r_{m-1}, s_n) + \mathrm{sim}(r_m, s_n) \\ \mathrm{AMSS}(R_{m-1}, S_{n-2}) + 2.\,\mathrm{sim}(r_m, s_{n-1}) + \mathrm{sim}(r_m, s_n) \end{cases} \quad (8)$$

Where Rm and Sn are introduced to denote vector subsequences ($r_1, \ldots, r_n$) and ($s_1, \ldots, s_m$), respectively. AMSS(R_m, S_n) for n = 0 or m = 0 is defined as −∞. sim(r_m,s_n) is defined in (9).

$$\mathrm{sim}(r_m, s_n) = \begin{cases} 0 & \text{if } \theta > \frac{\pi}{2} \\ \cos\theta \left(= \frac{r_m \cdot s_n}{|r_m||s_n|}\right) & \text{otherwise} \end{cases} \quad (9)$$

Where θ is the angle between qn and cm as illustrated in Fig. 8. The similarity is set to 0 for θ greater than π/2 to limit the influence of two vectors with widely different directions.

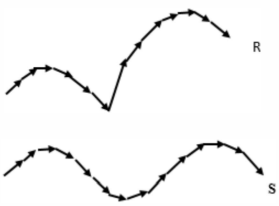

Fig. 7. *Two Trajectories' (R,S) time series as a set of vectors.*

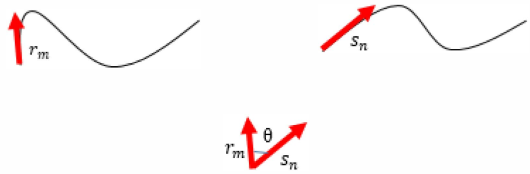

Fig. 8. *Similarity between two vectors.*

3) Movement Direction Similarity

In Direction similarity instead of using raw data, it is processed to generate direction data. Then similarity is based on how closer the direction profile of the trajectories are.

a) Edit Distance on movement Pattern Strings [20]

Using raw representation to compare moving objects' data will miss the trajectories with similar movement but different spatial rotation, shifting, or scaling factors. As shown in Fig. 9, TB can be derived from TA by scaling its x and y positions by a factor of 2, while TC is translated from TB by shifting its x and y positions by 1. When measuring similarity using raw representation, the trajectories will not be similar although they have the same motion.

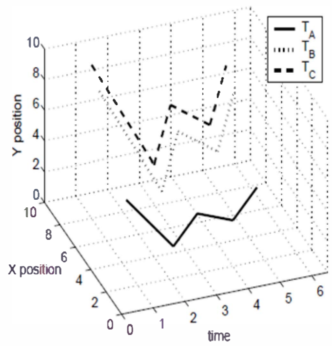

Fig. 9. *Example illustrates the effect of shifting, scaling and rotation on similarity between three trajectories having the same motion where TA = [(3:5; 4:5); (1:5; 2:5); (2:5; 3:5); (2; 3);(3; 4)], TB = [(7; 9); (3; 5); (5; 7); (4; 6); (6; 8)] and TC = [(8; 10);(4; 6); (6; 8); (5; 7); (7; 9)] [20].*

Thus, instead of directly using raw representation, moving objects are represented by a sequence of (movement direction, distance ratio) pairs. Then the new sequence is converted into a moving pattern string(MPS) and euclidean distance is used to measure similairty between MPSs of the moving objects. This approach is robust to noise and allows finding similarity between trajectories of different lengths but it does not take into account local time shifting.

b) TrajectoryMatch and moving spatio-temporal relation match algorithm [19]

This similarity measure is based on spatial directional and topological relations as shown in Table II. The trajectory of a moving object can be described by a set of motions using directional relations for each trajectory element. A trajectory element is defined as a set of three element (S, DR, I) where S is the displacement value from the element to its next element, DR is the heading direction of the element and I is the time instance of the element occurrence. It also used the topological relatios to describe relations between two trajectories by building a moving spatio-temporal Relation (MST). For example (DJ, LT, I), an element in the first trajectory is disjoint and Left to an element in the second trajectory at time instance I.

A proposed similarity approach is used to measure similarity between two moving object representations or between relations of two moving objects to another one (MST). This approach does not take into account local time shifting and two sequences have to be of the same length.

TABLE II. DIRECTIONAL AND TOPOLOGICAL RELATIONS

Directional Relations	Topological Relations
Ai North (NT) Aj Ai East (ET) Aj Ai Northwest (NW) Aj Ai Northeast (NE) Aj Ai South (ST) Aj Ai west (WT) Aj Ai Southwest (SW) Aj Ai Southeast (SE) Aj	Ai equal (EQ) Aj Ai Inside (IN) Aj Ai Contain (CT) Aj Ai Cover (CV) Aj Ai Covered by (CB) Aj Ai Overlap (OL) Aj Ai Touch (TC) Aj Ai Disjoint (DJ) Aj

III. CONCLUSION AND RESEARCH GAPS

Measuring similarity between moving objects' trajectories is an important aspect in many application domains such as stock analysis, recommender systems and prediction analysis. It was classified into spatial and spatio-temporal similarity measures. We briefly reviewed some of the most commonly used ones with their advantages and disadvantages with respect to noise, Local time shifting, length of trajectories, computation cost and whether the distance measure is metric or not as shown in Table III.

All discussed measures define implicitly the meaning of similarity in their technique implementation. Some of them were based on movement parameters such as speed and direction while others used the raw data representation or a combination of both. All of them restrict the user to the defined meaning. So, one of the research gaps is the need for a generic trajectory similarity measure and a corresponding operator in one of the moving objects databases that allows user to choose what is the similarity meaning from his perspective.

TABLE III. COMPARISON BETWEEN SIMILARITY MEASURES

S-Measure	S-Meaning	RTN	LTS	DL	IS	CC
ED	Raw representation	×	×	×	✓	O(n)
LCSS	Raw representation	✓	✓	✓	×	O(n*m)
DTW	Raw representation	×	✓	✓	×	O(n*m)
TWED	Raw representation	×	✓	✓	✓	O(n*m)
ERP	Raw representation	×	✓	✓	✓	O(n*m)
EDR	Raw representation	✓	✓	✓	×	O(n*m)
DTW based Approach	Movement Speed and path	×	✓	✓	×	O(n*m)
Trajectory Match	Movement Direction	×	×	×	×	O(n*m)
EDM	Movement Direction	✓	×	✓	×	O(n*m)
Hausdorff	Shape	×	×	✓	✓	O(n log n) where n is the total number of vertices.
Fréchet	Shape	n.a.	×	✓	×	O(nm log (nm)) where n and m are number of edges in two curves.
SPADe	Shape	✓	✓	✓	×	$O\left(\frac{tNn\xi^2}{w}\right)$ where N is the number of query patterns, t is the average number of LPMs detected from one chopped local pattern,ξ is the largest allowed gap and w is the length of the local patterns.
AMSS	Shape and direction	✓	✓	✓	×	O(n*m)

a. *Note: 'n.a.' (not applicable) means that the axiom is satisfied, but it is meaningless for that distance, 'S' means similarity, 'RTN' means robust to noise, 'LTS' means local time shifting, 'DL' means different lengths, 'IS' means is metric and 'CC' means computation cost.*

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