The Fractal Structure of Treelines in High Mountains

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ABSTRACT The occurrence of tree species in high mountains usually ends abruptly at some elevation. This is the treeline. The appearance of treeline structures in such areas is a topic of interest to ecologists and climate scientists. These treeline structures exhibit a characteristic mainland-island structure, where the tree cover is continuous at lower elevations (‘mainland’) and fragmented at higher ones (‘islands’). Theoretical predictions from percolation theory using simulation-based results obtained by studying two widely known types of models (the static Uniform/Gradient Random Map model and the dynamic Uniform/Gradient Contact Process model) suggest that the hull of the ‘mainland’ should be a fractal with dimension 7/4. This research project aims to test this hypothesis using satellite images of treelines and to develop simulation software that appropriately presents the gradient models as well.

INDEX TERMS Treeline structure, percolation theory, fractal, gradient contact process

1. INTRODUCTION

The occurrence of tree species in high mountains usually ends abruptly at some elevation. This is the treeline. The appearance of treeline structures in such areas is a topic of interest to ecologists and climate scientists. These treeline structures exhibit a characteristic mainland-island structure, where the tree cover is continuous at lower elevations (‘mainland’) and fragmented at higher ones (‘islands’). Theoretical predictions from percolation theory using simulation-based results obtained by studying two widely known types of models (the static Uniform/Gradient Random Map model and the dynamic Uniform/Gradient Contact Process model) suggest that the hull of the ‘mainland’ should be a fractal with dimension 7/4. This research project aims to test this hypothesis using satellite images of treelines and to develop simulation software that appropriately presents the gradient models as well.

1. RELATED WORK

Several studies have investigated the transition from connected to fragmented vegetation across environmental gradients and scaling laws in ecotone geometry (Gastner et al., 2009). Recent research has also suggested that percolation theory predicts some universal features in range margins across environmental gradients (Juhász & Oborny, 2020). However, there is a lack of research that uses satellite images to analyse the fractal structure of treelines and its implications for species border delineation and climate change detection. Additionally, the existing simulation software of this population model in finite space does not capture environmental gradients

1. METHOD
   1. General description

Satellite images of treelines will be obtained from publicly available datasets after careful consideration and thorough discussion with a cartographer specialist. A custom algorithm based on the existing literature in the field was implemented to calculate the fractal dimension of the treeline structures. The potential limitations of the data and methods is being cautiously addressed. Additionally, simulation software was developed (involving visualization of population dynamics models in finite space, implementation of algorithms for detecting the so-called ‘giant component’ of the population, and for the delineation of its ‘hull’) that captures some environmental gradients and appropriately presents gradient models. The results of the simulation software will be compared with those obtained from real-life ecological data.

The so-called finite-size effect in analysing tree species distribution on a bounded square lattice refers to the influence of the lattice's limited size on observed spatial patterns and population dynamics. It introduces boundary effects that can alter colonization patterns and spatial interactions. As a common attempt to address it, toroidal boundary conditions are employed to mitigate these effects and simulate an infinite lattice. Understanding the finite size effect is vital for the accurate interpretation of results.

To be able to effectively test our hypothesis and to maintain the possibility of extending this research topic in the future we have decided to use the following methodology:

1. Programming language: Python (desktop application);
2. Build a compact, yet easily extendible framework for the research topic;
3. Using our python program and multiple process variables we will test the hypothesis.
4. DESCRIPTION OF THE IMPLEMENTATION
   1. Software design(?)

TODO

* 1. Simulation of the Popularization Models

The analysis of tree species distribution on a bounded square lattice provides valuable insights into ecosystem dynamics, biodiversity patterns, and conservation strategies.

* + 1. Homogeneous Random Map

For the sake of clarity, we begin with describing the simplest possible model, the Homogeneous Random Map model, which is not implemented in our project, but contributes to a more thorough understanding of the more complex models implemented.

The homogeneous random map model assumes that the distribution of tree species is random and uniform, without specific spatial patterns or dependencies. To apply this model, we consider a square lattice with rows and columns   
(), representing the discrete sampling units within the study area. Each lattice point corresponds to a sampling unit where a tree species can be observed or recorded.

The model begins by randomly assigning 1% of the lattice cells as occupied (seeded with tree species).

At each Monte Carlo step of the model, random lattice cells are selected uniformly, one after the other, and their status is updated based only on the colonization and extinction rates. If the selected cell is already occupied, it has a probability  
 to become unoccupied (extinct), representing the extinction rate. On the other hand, if the cell is unoccupied, it has a probability   
 to become occupied (colonized), representing the colonization rate.

By iteratively updating the lattice cells according to these colonization and extinction rates, the model captures the dynamics of tree species distribution, considering both the expansion and contraction of occupied cells over time.

The resulting distribution of tree species on the bounded square lattice provides insights into the spatial patterns and population dynamics of the tree species under investigation. Statistical analysis can be performed on the simulated data to study aspects such as species richness, clustering, or the effects of different colonization and extinction rates on the observed patterns.

It is important to acknowledge that the homogeneous random map model on a bounded square lattice has its limitations. While it provides a rather simplified representation of tree species distribution, it offers a starting point for understanding the impact of colonization and extinction processes on the spatial patterns of tree species on the lattice. For a comprehensive analysis of tree species spatial patterns, to account for additional factors and complexities, more advanced models incorporating factors like environmental gradients or species interactions may be considered.

In conclusion, the homogeneous random map model with colonization and extinction rates on a bounded square lattice enables the study of tree species distribution dynamics. By considering the probabilities of colonization and extinction at each step, the model provides insights into the expansion and contraction of tree species populations and contributes to the understanding of spatial patterns and ecological processes involved in tree species distribution.

* + 1. Homogeneous Contact Process

In this project, we utilize the homogeneous contact process model with toroidal boundary conditions to study the spatial distribution of tree species on a bounded square lattice. The homogeneous contact process model extends the previous model by incorporating the colonization probability based on the number of occupied neighbouring cells, while toroidal boundary conditions address the finite size effect associated with the bounded lattice.

To begin, we randomly assign 1% of the lattice cells as occupied, representing the initial presence of tree species. At each Monte Carlo step of the model, random lattice cells are uniformly selected, one after the other, for updating. If the selected cell is already occupied, it has a probability  
 to become unoccupied (extinct), representing the extinction rate. However, if the cell is unoccupied, the colonization occurs with a probability of , where is the colonization rate and is the number of occupied neighbouring cells (in general, we consider two sells neighbours if they share an edge; more on the boundaries in the next paragraph). This colonization probability accounts for a factor of species interaction, namely the influence of neighbouring occupied cells on the colonization process. This is a special case of the idea of considering two occupied sites connected and hence belonging to the same vegetation patch if and only if the species can move from one site to the other without stepping on a vacant site in between. Various step lengths were tested in [1], our approach is clearly the step length of 1.

To tackle the finite-size effect and create an environment that mimics an infinite lattice, we employ toroidal boundary conditions. By applying toroidal boundary conditions, the lattice wraps around at the edges, creating a torus-like shape. This means that cells on opposite edges are considered neighbours, effectively connecting the lattice, and removing the boundary effects. The toroidal approach allows the colonization and interaction processes to occur seamlessly across the lattice as if it were infinitely extended.

By incorporating the toroidal boundary conditions, the homogeneous contact process model captures the spatial interactions, clustering tendencies, and colonization dynamics of tree species on the bounded square lattice. This approach helps minimize biases caused by the finite size of the lattice and facilitates the exploration of tree species distribution patterns that would be observed in an infinitely extended lattice.

Statistical analysis can be conducted on the simulated data to investigate various aspects of tree species distribution, such as species richness, spatial clustering, and correlations between colonization rates and the surrounding occupied cells. The results obtained from the model with toroidal boundary conditions provide valuable insights into the spatial patterns and population dynamics of tree species within the bounded square lattice.

However, it's important to acknowledge that while the toroidal approach mitigates finite-size effects, it may introduce wrapping interactions and altered edge effects. The appropriateness of toroidal boundary conditions should be carefully considered in the specific context and results should be validated accordingly.

In conclusion, the homogeneous contact process model with toroidal boundary conditions offers a mathematical framework for analysing tree species distribution on a bounded square lattice while addressing the finite size effect. This approach improves the realism of the tree species distribution analysis and allows for a more accurate representation of their spatial patterns within the bounded square lattice.

* + 1. Gradient Random Map

We also employ the gradient random map model to study the spatial distribution of tree species on a bounded square lattice, considering the influence of an environmental (spatial) gradient, e.g. altitude, on colonization and extinction dynamics.

The gradient random map model extends the previous homogeneous random map model by incorporating gradient-dependent colonization and extinction probabilities. In this model, the probability of colonization, denoted as , and the probability of extinction, denoted as , become functions of the gradient variable , which represents a continuous spatial variation.

To begin, we again randomly assign 1% of the lattice cells as occupied, representing the initial presence of tree species. At each Monte Carlo step of the model, random lattice cells are uniformly selected, one after the other, for updating. The updating process takes into account the gradient-dependent colonization and extinction probabilities. If the selected cell is already occupied, it will go extinct with a probability of . If the cell is unoccupied, the colonization occurs with a probability of , where and represent the colonization and extinction rate influenced by the gradient variable . The colonization probability is higher in areas with more favourable environmental conditions. Similarly, the extinction probability is higher in areas with less favourable environmental conditions.

By incorporating gradient-dependent colonization and extinction probabilities, the gradient random map model captures the spatial dynamics of tree species in response to environmental variations. It allows for the exploration of how species colonization and persistence are influenced by the gradient variable , providing insights into the relationship between environmental gradients and tree species distribution patterns.

In our simulations, we studied the dynamics of the model with the functions and , and being constants. However, it is important to note that the gradient random map model assumes a continuous gradient variable and functions and that appropriately capture the underlying ecological processes. These functions should be carefully selected and validated based on ecological knowledge and data availability.

* + 1. Gradient Contact Process

Finally, we utilize the gradient contact process model with a tube-like boundary to study the spatial distribution of tree species on a bounded square lattice, incorporating gradient-dependent colonization and extinction probabilities.

The gradient contact process model extends the homogeneous contact process by incorporating gradient-dependent colonization and extinction probabilities. In this model, the probability of extinction, denoted as , and the probability of colonization, denoted as , also become functions of the gradient variable , which represents a continuous spatial variation.

To initiate the model, as usual, we randomly assign 1% of the lattice cells as occupied, representing the initial presence of tree species. At each Monte Carlo step, random lattice cells are uniformly selected, one by one, for updating, considering the gradient-dependent colonization and extinction probabilities.

If the cell is unoccupied, the colonization occurs with a probability of , where represents the colonization rate influenced by the gradient variable , and is the number of occupied neighbouring cells. The colonization probability considers both the suitability of the local environment, as indicated by the gradient value at that location, and the presence of neighbouring occupied cells.

On the other hand, if the selected cell is already occupied, it has a probability of to become unoccupied, reflecting the extinction rate influenced by the gradient variable . The extinction probability varies based on the gradient value at that location, considering the suitability of the local environment.

To tackle the finite-size effect associated with the bounded lattice, instead of the traditional toroidal boundary, we implement a tube-like boundary approach. Instead of connecting the opposite edges, we duplicate the last and first columns on the vertical edges, creating a tube-like structure. This boundary setup allows colonization, extinction, and spatial interactions to occur seamlessly within the lattice, mimicking an infinitely extended environment and minimizing the finite-size effects.

By incorporating gradient-dependent colonization and extinction probabilities and utilizing the tube-like boundary conditions, the gradient contact process model captures the spatial dynamics of tree species in response to environmental gradients within the bounded square lattice. It enables the investigation of how the gradient variable influences colonization patterns, extinction probabilities, and resulting tree species distribution.

In our simulations, we studied the dynamics of the model with the functions and , and being constants. However, it is again crucial to note that the gradient contact process model assumes appropriate functions and that accurately represent the underlying ecological processes. These functions should be attentively selected and validated based on ecological knowledge and available data.

* 1. Detection of the Giant Component

As already mentioned before, we consider two occupied sites connected and hence belonging to the same vegetation patch if and only if the species can move from one site to the other without stepping on a vacant site in between. Various step lengths can be studied, but in this project, we focus on the step length of 1.

We want to determine which sites are connected and hence belong to the same patch (i.e. percolation cluster). The largest patch is usually the one that connects the highest number of sites (cells) in the region of a low density to the region of a higher density. If the gradient points along the horizontal -direction from sparse to full vegetation cover, this patch will span from the bottom to the top of the lattice, a property that can be used later in delineating the hull. The probability of two distinct large patches in the densest region is negligible [1] in a sufficiently large lattice. Therefore, we call the largest patch the “connected patch”, or “giant component”. All the other patches we call “fragments”.

There are two (and eventually a lot more) algorithms that can be used to detect the giant component, which we will describe here.

The detection can be made using the Hoshen-Kopelman algorithm which is a simple and efficient algorithm for labelling clusters on a grid, where the grid is a regular network of cells, with the cells being either occupied or unoccupied. This algorithm is based on a well-known union-finding algorithm. In this algorithm, we scan through a grid looking for occupied cells and labelling them with cluster labels. The scanning process is called a raster scan. The algorithm begins with scanning the grid cell by cell and checking whether the cell is occupied or not. If the cell is occupied, then it must be labelled with a cluster label. This cluster label is assigned based on the neighbours of that cell. (For this we are going to use Union-Find Algorithm which is explained in the next paragraph.) If the cell doesn’t have any occupied neighbours, then a new label is assigned to the cell.

The Union-Find algorithm is a simple method for computing equivalence classes. Calling the function returns whether items and are members of the same equivalence class. Because equivalence relations are transitive, all the items equivalent to are equivalent to all the items equivalent to . Thus, for any item , there is a set of items which are all equivalent to (called the equivalence class). A second function returns a representative member of the equivalence class to which belongs.

During the raster scan of the grid, whenever an occupied cell is encountered, neighbouring cells are scanned to check whether any of them have already been scanned. If we find already scanned neighbours, the operation is performed, to specify that these neighbouring cells are in fact members of the same equivalence class. Then the operation is performed to find a representative member of that equivalence class with which the current cell will be labelled. On the other hand, if the current cell has no neighbours, it is assigned a new, previously unused, label. The entire grid is processed in this way.

The other algorithm, which is actually used in our project, is the well-known DFS algorithm, where we consider the cells of the lattice as the vertices of a graph, the neighbouring cells being connected in the graph with an edge.

* 1. Delineation of the Hull

TODO

* 1. Approximation of Fractal Dimension
     1. Box Counting

TODO

* + 1. Correlation Dimension

TODO

* 1. Analyzing real-life ecological data

TODO

1. EXPERIMENTS

TODO

1. EVALUATION AND VISUALIZATION OF RESULTS
   1. Model performance comparison

TODO

* 1. Result analysis

TODO.

1. CONCLUSION AND DISCUSSION

TODO.

This research project aims to test the hypothesis that the hull of the treeline mainland in high mountains is a fractal with dimension 7/4, using satellite imagery. By developing a feasible method for characterizing the fractal structure of treelines, this project has the potential to contribute to the precise delineation of species borders and the detection of population shifts due to climate change. The project will also involve the development of simulation software that includes environmental gradients to test the fractal result on simulated data. With the guidance of our supervisor and her team, we are confident that this project will make a valuable contribution to the field of plant ecology and theoretical biology.

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