Telecommunication Network Optimization

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1 Introduction

Highways, telephone lines, electric power systems, computer chips, water delivery systems, and rail lines: these physical networks, and many others, are familiar to all of us. In each of these problem settings, we often wish to send some good(s) (vehicles, messages, electricity, or water) from one point to another, typically as efficiently as possible (e.g., in term of cost, time, reliability, etc.).

In this case study, we will learn more about telecommunication networks and the way their design can be optimized.

A telecommunication network is a collection of terminal nodes (origin / destination) which are connected so as to enable telecommunication between the terminals. The transmission links connect the nodes together. The nodes use circuit switching, message switching or packet switching to pass the signal through the correct links and nodes to reach the correct destination terminal.

Among different types of telecommunication networks, Digital Data Service (DDS) is a high-quality digital transport service in the telecommunications industry using permanent network connections and dedicated transmission facilities. In this use case, you will discover a particular DDS network design problem that is encountered by a lot of major telecommunication companies.

The input elements of the problem include a set of end offices, a set of digital hubs, and a set of customer locations that are geographically distributed on a plane. Each customer location is connected directly to its own designated end office, which in turn needs to be connected to exactly one selected hub. Then the selected hubs must be connected by a ring. The ring topology is widely used in communication network design to provide reliability. Each hub has a fixed cost for being chosen and each link has a connection cost for being included in the solution. The objective is to design such a network at minimum cost. Figure 1 shows a real scenario of a small ring-based DDS network.

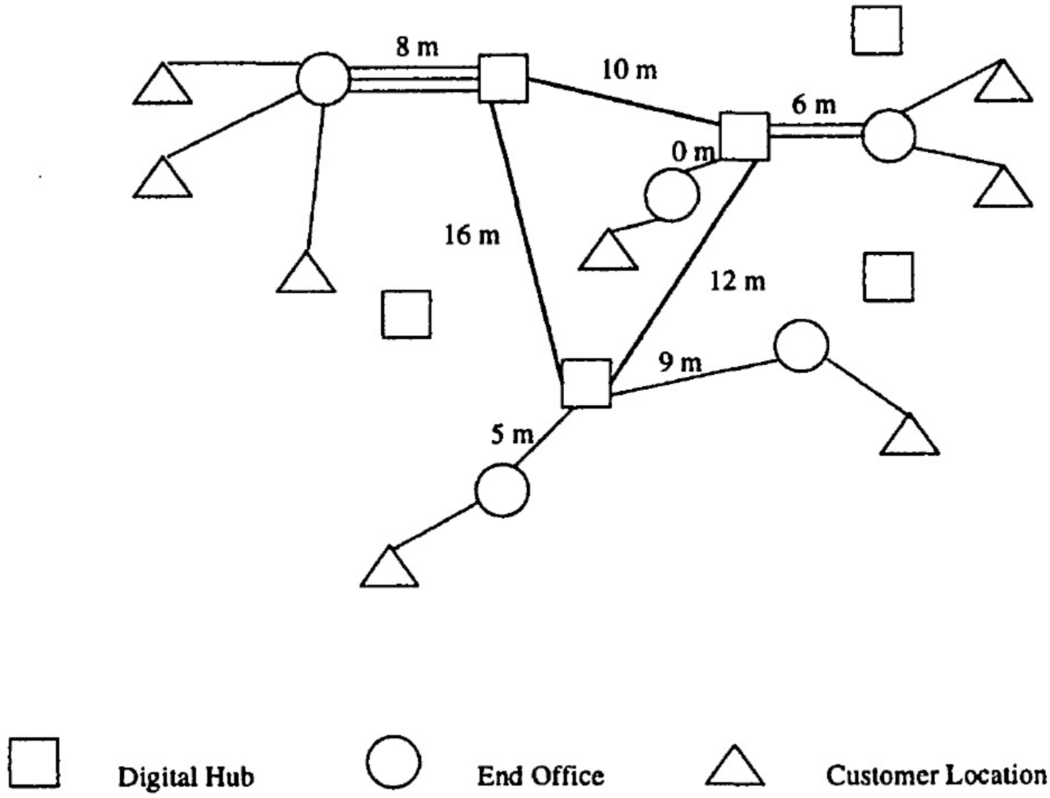

Figure 1: A ring-based DDS network.

The three main decisions faced by the network designers are as follows:

- Select a subset of digital hubs among all potential hubs and connect them via a ring (a travelling salesman tour over the selected hubs).
- Connect each end office to a selected hub (so that the original customer locations can communicate to each other).
- Allocate the customers to end offices.

The objective of the design is to minimize its total cost. In order to see how the design cost of the telecommunication network of Figure 1 is calculated, you are highly recommended to study the scientific article provided in the website of the course even before starting the next section.

2 Problem statement

NOTE: *Before starting to discover the problem, please note that the problem of this practical work is mostly similar to the one in the scientific paper with a slight modification that requires your complete attention to model the problem. You will discover that the objective function will contain some extra terms, and you will also encounter some extra constraints that need to be modeled.*

Consider a DDS telecommunication network that contains a set of customer nodes that have communications demands. These demands are handled and satisfied through a set of end offices nodes and digital hubs. The aim is to design a network by *allocating* the customers to end offices, *selecting* a set of digital hubs, *allocating* the end offices to the selected digital hubs, and finally *connecting* the digital hubs as a ring. The network must have the *minimum design cost*.

2.1 Assumptions

You are provided with the following information/assumptions that MUST be incorporated/respected in your mathematical model:

- The location of all customers, end offices and digital hubs is fixed during the design phase
- Each customer should be allocated at most to one end office
- Each end office should be allocated to only one digital hub
- There is a fixed cost for selecting a digital hub in the network
- There is a connection cost to connect each customer to an end office
- There is a connection cost to connect each end office to a digital hub
- Each end office has a maximum capacity on the number of customers that it can serve
- Each digital hub node has a maximum capacity on the number of customers that it can serve
- The network between digital hubs must be a ring

2.2 Sets and parameters

Table 1 presents the sets and parameters that are required to develop the mathematical formulation of the problem.

Table 1: List of sets and parameters.

Sets:	
C	Set of customers
M	Set of end offices
N	Set of digital hubs
$i \in C$	Index of customers ($i \in \{1, 2, \cdots, \|C\|\}$)
$j \in M$	Index of end offices ($j \in \{1, 2, \cdots, \|M\|\}$)
$k, m \in N$	Indices of digital hubs ($k, m \in \{1, 2, \cdots, \|N\|\}$)
Parameters:	
h_{ij}	Cost of allocating customer i to end office j
c_{jk}	Cost of allocating end office j to digital hub k
g_{km}	Cost of connecting two digital hubs k and m
f_k	Fixed cost of selecting digital hub k
α	Minimum percentage of customers that should be served
U_j^{max}	Capacity of end office j on the number of customers to serve
V_k^{max}	Capacity of digital hub k on the number of customers to serve

2.3 Decision variables

The main decisions in this problem are as follows:

- Locating (using) a set of digital hubs in the network,
- Allocating customers to the end offices,
- Allocating end offices to the digital hubs,
- Connecting the digital hubs together as a ring.

Therefore, the decision variables are defined as:

$X_{ij} = 1$ if and only if customer i is allocated to end office j; 0 otherwise.

$Y_{jk} = 1$ if and only if end office j is allocated to digital hub k; 0 otherwise.

$Z_{km} = 1$ if and only if digital hub k is connected to digital hub m ($k \neq m$); 0 otherwise.

$L_k = 1$ if and only if digital hub k is located; 0 otherwise.

3 Problem formulation

In this section, you are expected to formulate the problem and provide a mathematical model. In this regard, you are guided through a set of consecutive steps to build the model.

Step 1: Objective function development

In this step, you formulate different terms of the objective function. In this problem, we aim at minimizing the *Total Cost* of the network. The *Total Cost* is the sum of *Total Allocation Cost*, *Total Location Cost*, and *Total Connection Cost*. These costs are defined as follows:

- *Total Allocation Cost* is the sum of the costs for allocating customers to the end offices PLUS the sum of the costs for allocating the end offices to the digital hubs
- *Total Selection Cost* is the sum of the costs for locating (selecting) digital hubs
- *Total Connection Cost* is the sum of the costs for connecting digital hubs

Therefore, we have:

Objective Function Z = MINIMIZE Total Cost

Total Cost = Total Allocation Cost + Total Location Cost + Total Connection Cost

Based on the provided parameters and the decision variables, formulate each part of the objective function in the following table.

Fill the following table ...

Item (Total...)		**Formulation**
Total Allocation Cost	=	
Total Location Cost	=	
Total Connection Cost	=	

Finally, the objective function Z is:

Write your answer in the following box ...

Step 2: Constraints

In this section, you are expected to develop the mathematical formulation of the constraints. Each constraint is explained below and you just need to provide the mathematical formulations.

Constraint 1: Single allocation for customers

As mentioned in the assumptions, each customer must be allocated to at most one end office.

Write your answer in the following box ...

Constraint 2: Single allocation for end offices

As mentioned in the assumptions, each end office must be allocated to exactly one digital hub.

Write your answer in the following box ...

Constraint 3: Allocation of end offices to located digital hubs

An end office can be allocated to a digital hub if and only if that digital hub has been already located.

Write your answer in the following box ...

Constraint 4: Ring (tour) structure over located digital hubs

In this constraint, we guarantee that the degree of each **selected** digital hub is equal to two. The degree of a digital hub is defined as the number of edges (connection links) incident to that digital hub. Hence, in this case, there is one entering link and one exiting link.

Write your answer in the following box ...

Constraint 5: Subtour elimination

In this constraint we ensure that all located digital hubs are connected by exactly one ring. To this end, we consider all possible subsets of **located** digital hubs with size greater than three, and we guarantee, for each subset, that the number of links (edges) that connect the digital hubs is lower than or equal to the size of the subset minus one.

Write your answer in the following box ...

Constraint 6: End office capacity constraint

Each end office can accept a limited number of customers. Therefore, the number of customers allocated to a specific end office must be less than or equal to the capacity of that end office.

Write your answer in the following box ...

Constraint 7: Digital hub capacity constraint

Each digital hub can accept a limited number of customers (indirectly). Therefore, the number of customers that have been indirectly (through end offices) allocated to a specific digital hub must be less than or equal to the capacity of that digital hub.

Write your answer in the following box ...

Constraint 8: A ring must have at least three digital hubs

This constraint forces the model to locate/use at least (or more than) 3 digital hubs in the network in order to keep its reliability.

Write your answer in the following box ...

Constraint 9: Covering constraints

This constraint forces the model to serve a given percentage of customers.

Write your answer in the following box ...

Constraint 10: Domain of variables

These constraints provide the domain of the decision variables, e.g., binary, integer, real, etc.

Write your answer in the following box ...

4 Numerical experiment: Small-sized instance

In this section, you are expected to solve a small-sized instance and to find the optimal solution. You are expected to write the proposed model in PuLP, a free open source software written in Python used to describe optimisation problems as mathematical models.

You are provided with the data for a small-sized telecommunication network with 15 potential customers, 8 end offices and 6 digital hubs. These data can be downloaded from the website of your course.

After solving the problem using PuLP, please report the objective function value and draw the network below .

Write your answer in the following box ...

Optimal objective function value = ...

Draw your network here: ...