FYS 3120 Classical Mechanics and Electrodynamics
Spring semester 2015

Problem Set 10

Problem 10.1

Two photons in the laboratory system have frequencies ν_1 and ν_2. The angle between the propagation directions is θ.

a) Find the total energy and the absolute value of the total momentum of the photons in the laboratory system, expressed in terms of the frequencies ν_1 and ν_2.

b) Find the photons' frequency in the center of mass system.

c) Is it always possible to find a center of mass system for the photons?

Problem 10.2

We send a photon towards an electron at rest.

a) What is the minimum energy of the photon required for the following process to take place

$$\gamma + e^- \to e^- + e^- + e^+ \tag{1}$$

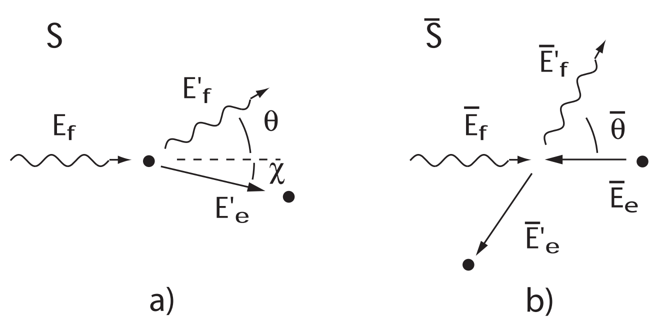
The particles e^- and e^+ have the same mass m_e.

b) Show that the process

$$\gamma \to e^- + e^+ \tag{2}$$

is impossible.

Problem 10.3 (Exam 2005)



A photon with energy $E_f = 100\,\mathrm{keV}$ is scattered on a free electron which is, before the scattering, at rest in the laboratory frame. After the scattering the energy of the photon is E'_f, and the direction of propagation makes an angle θ relative to the direction of the incoming photon. The rest energy of the electron is $E_e = m_e c^2 = 0.51\text{MeV}$, and after the scattering it has an energy which we denote by E'_e. The electron is scattered in a direction which makes an angle χ relative to the direction of the incoming photon.

We examine this process both in the lab frame S (figure a) and in the center of mass system $\bar{S}$ (figure b). In the center of mass system all variables are marked with a "bar", for example with $\bar{E}_f$ as the energy of the incoming photon.

a) What is meant by the center of mass system? Use the transformation formulas for energy and momentum to determine the relative velocity between the lab system and the center of mass system.

b) Explain why the energy of the incoming and outgoing photon is the same in the center of mass system and find this energy.

c) If $\theta = 90°$ what is the energy of the outgoing photon in the lab system? What is the corresponding energy of the outgoing electron?

Problem 10.4

A thin straight conducting cable, oriented along the z axis in an inertial reference frame S, carries a constant current I. The cable is charge neutral.

a) Show, by use of Ampere's law, that the current produces a rotating magnetic field $\mathbf{B} = B(r)\mathbf{e}_\phi$, where (r, ϕ) are polar coordinates in the x, y plane and $\mathbf{e}_\phi$ is a unit vector in the direction of increasing ϕ. Determine the function $B(r)$.

Consider next the same situation in a reference frame S' that moves with velocity v along the z axis.

b) Use the fact that charge and current densities transform under Lorentz transformation as components of a current 4-vector to show that in S' the conducting cable will be charged. Determine the charge per unit length, λ' and the current I' in this reference frame.

c) Use Gauss' and Ampere's laws to determine the electric and magnetic fields, $\mathbf{E}'$ and $\mathbf{B}'$, as functions of the polar coordinates (r', ϕ') in reference frame S'.

d) Show that if the fields in S' are derived from the fields in S by use of the relativistic transformation formulas for $\mathbf{E}$ and $\mathbf{B}$, that gives the same results as found in c).