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Problem 1

a)

$u''(t) + \omega^2 u = f(t), u(0) = I, u'(0) = V, \in (0, T]$

$\frac{u^{n+1} - 2u^n + u^{n-1}}{\Delta t^2} = f^n - \omega^2 u^n$

$\rightarrow u^{n+1} = f^n \Delta t^2 - \Delta t^2 \omega^2 u^n + 2u^n - u^{n-1}$

$\frac{u^1 - u^{-1}}{2\Delta t} = V \rightarrow u^{-1} = u^1 - V 2\Delta t$

$\rightarrow u^1 = \frac{1}{2}(f^0 - \omega^2 u^0)\Delta t^2 + u^0 + V\Delta t$

b)

d = I
c = V
$f = \omega^2(Vt + I)$
$\frac{t^{n+1} - 2t^n + t^{n-1}}{\Delta t^2} = \frac{\Delta t - \Delta t}{\Delta t^2} = 0$
$\frac{d - 2d + d}{\Delta t^2} = 0$

e)

We approximate the ODE by adding up the straight slopes between the mesh points. With a polynomial of higher degree than the ODE, the slope between the points is no longer constant and we can not fullfill the discrete equations.

Exercise 21

c)

Pure vertical:
$y'' = -\frac{\beta}{1-\beta}\left((y-1) - \beta\frac{y-1}{\sqrt{(y-1)^2}}\right) - \beta$
y does not exceed 1, hence $\frac{y-1}{\sqrt{(y-1)^2}} = -1$
$\rightarrow y'' = -\frac{\beta}{1-\beta}y$
which has the a solution $y = y_0 cos(\omega t)$ with $\omega = \sqrt{\frac{\beta}{1-\beta}}$