FYS 3120 Classical Mechanics and Electrodynamics
Spring semester 2015

Problem Set 6

Problem 6.1

Two inertial reference frames S and S' are moving with a relative velocity $\mathbf{v}$. The directions of the coordinate axes are chosen so that the coordinate transformation between the reference frames takes the standard form

$$\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma(t - \frac{v}{c^2}x) \end{aligned} \tag{1}$$

with $\gamma = (1 - (v/c)^2)^{-1/2}$.

a) Equivalence between the reference frames S and S', implies that the transformation formula above should be correct if we interchange the the primed and unprimed variables, and only change the sign of the relative velocity v. Show that this is indeed the case by inverting the transformation to express x and t in terms of x' and t'.

b) A moving object is registered with velocity $\mathbf{u}$ relative to reference frame S and velocity $\mathbf{u}'$ relative to S'. With the velocity components defined in the usual way as $u_i = dx_i/dt, u'_i = dx'_i/dt'$, $i = x, y, z$, use the transformation formula (1) to find the corresponding transformation formula for the velocity components.

c) Assume now that the relative velocity of the two reference frames is $v = 0.5c$, and that the object moves, as measured in S', with velocity $u' = 0.8c$ in the direction with angle $45°$ to the x' (and y') axis. What is the absolute value of the velocity, and the direction of the velocity (angle to the x-axis), as measured in S? If we instead of using the Lorentz transformations we had used the Galilean transformations, what had the results been?

Problem 6.2

A thin rigid rod has rest length L_0 (length measured in its rest frame). It moves relative to an inertial reference frame S', so that the midpoint A of the rod has the time dependent coordinates $x'_A = 0, y'_A = ut', z'_A = 0$, with u as the velocity of the rod. In this reference frame the rod is at all times parallel to the x' axis.

a) Let B be one of the end points of the rod. What are the time dependent coordinates of this point measured in S'?

b) The inertial frame S' moves with velocity v along the x-axis relative to another inertial frame S. (The axes of the two frames are parallel.) Find the space coordinates (x, y, z) of the points A and B as functions of the time coordinate t in the reference frame S. (Remember that if the time coordinate t in S is fixed, the time coordinates t'_A and t'_B of the points A and B are not the same.)

c) Show that the the rod is not oriented along the x-axis in S, by calculating ratio $\tan\phi = (y_B - y_A)/(x_B - x_A)$. What is the length of the rod measured in this frame?

Problem 6.1 (Exam 2014)

A small body with mass m and charge q is moving in the horizontal plane (x, y-plane), under influence of a harmonic oscillator potential, $V(r) = \frac{1}{2}m\omega_0^2 r^2$ and a constant magnetic field $\mathbf{B} = B\,\mathbf{k}$, which is directed perpendicular to the plane of the moving particle. The vector potential corresponding to $\mathbf{B}$ can be wrtten as $\mathbf{A} = -\frac{1}{2}\mathbf{r} \times \mathbf{B}$, with $\mathbf{r}$ as the postion vector of the particle.

a) With the polar coordinates of the plane (r, ϕ) used as as generalized coordinates, show that the Lagrangian takes the form

$$L = \frac{1}{2}m(\dot{r}^2 + r^2(\dot{\phi}^2 + \omega_B\dot{\phi} - \omega_0^2)) \tag{2}$$

where we have introduced the cyclotron frequency, $\omega_B = qB/m$.

b) The polar angle ϕ is cyclic. Explain what that means and give the expression for the corresponding conserved quantity, which we lable l. What is the physical interpretation of the quantity? The form of the Lagrangian implies that there is a second constant of motion. Give the expression and physical interpretation of this quantity.

c) Establish Lagrange's equation for the variable r, and use the cyclic property of ϕ to express the equation in the variable r alone.

d) Show that the radial equation has solutions which describe circular motion, and give that radius and angular velocity of the motion as functions of the parameters of the problem.

Show also that it has a solution where the particle performs oscillations about the origin, in a direction which rotates with with time, and find the oscillation and rotation frequencies.

Give a qualitative description of the more general type of motion described by the equation.