FYS 3120 Classical Mechanics and Electrodynamics
Spring semester 2015

Problem Set 14

Problem 14.1

The figure shows a straight antenna of length $2a$ lying along the z-axis with its center at the origin. We assume that the charge of the antenna is at all times located at the endpoints. The current in the antenna (between the charged end points) is given by $I = I_0 \sin \omega t$ where ω and I_0 are constants. The antenna is electrical neutral at time $t = 0$. The point where the field is evaluated is given by the position vector $\mathbf{r}$ and in spherical coordinates (r, θ, ϕ).

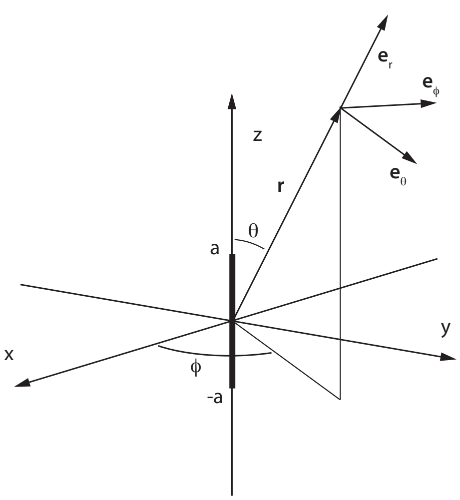


Figure 1:

a) Show that the antenna's electrical dipole moment at time t is given by $\mathbf{p}(t) = \frac{2aI_0}{\omega}(1 - \cos \omega t)\mathbf{k}$, where $\mathbf{k}$ is the unit vector in the z-direction.

We will in the following assume that the radiation from the antenna can be treated as electrical dipole radiation.

b) Find the components of the radiation fields $\mathbf{B}$ and $\mathbf{E}$ along the directions of $\mathbf{e}_r$, $\mathbf{e}_\theta$ and $\mathbf{e}_\phi$ in the point (r, θ, ϕ) at time t.

c) Show that the time average of the total radiated power in all directions kan be written as $\bar{P} = \frac{1}{2}RI_0^2$ and find R (the radiation resistance). What is the time average of the total power consumed by the antenna if it has an 'ordinary' resistance R_0 as well?

d) Find R for an antenna of length $2a = 5$ cm which is conducting a current with frequency $f = 150$ MHz. What is the time average of the total radiated power when $I_0 = 30$ A?

Problem 14.2 (Exam 2008)

An electron (with charge e and mass m) is moving with constant speed in a circle under the influence of a constant magnetic field $\mathbf{B}_0$. The magnetic field is directed along the z axis while the motion of the electron takes place in the x, y plane. We assume the motion of the electron to be non-relativistic.

Since the electron is accelerated it will radiate electromagnetic energy and thereby loose kinetic energy when no energy is added to the particle.

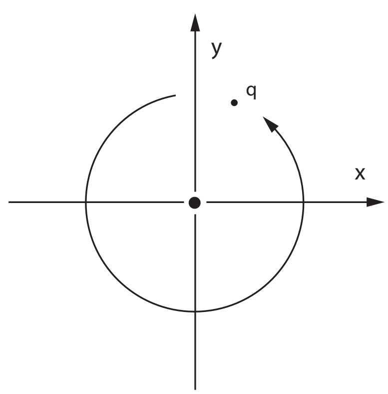
a) By use of Larmor's radiation formula, find an expression for the radiated energy per unit time expressed in terms of the radius r of the electron orbit and the cyclotron frequency $\omega = -eB_0/m$.

b) Show that the radius of the electron orbit is slowly reduced with an exponential form of the time dependence, $r = r_0\, e^{-\lambda t}$, and determine λ.

c) The electromagnetic fields produced by the moving charge are essentially electric dipole radiation fields. What is the electric dipole moment of the circulating electron? Give the expressions for the radiation fields $\mathbf{E}(z, t)$ and $\mathbf{B}(z, t)$ on the z axis far from the electron. Show that they correspond to a propagating wave, with direction away from the electron, and determine the form of polarization of the wave.

Expressions found in the formula collection of the course may be useful for this problem.

Problem 14.3 (Exam 2005)



Consider a simple classical model of the hydrogen atom, where the negatively charged electron moves in a circular orbit about the positively charged nucleus. The electron has a small mass $m_e = 9.1 \cdot 10^{-31} kg$ and charge $q = -1.60 \cdot 10^{-19} C$. The nucleus (proton) is much heavier and we consider it at all time to be at rest. Its charge is equally large as that of the electron, but with opposite sign. The initial radius of the orbit we assume equal to the Bohr radius $a_0 = 0.53 \cdot 10^{-10} m$. The plane of the orbit we identify with the x, y-plane, with the nucleus at rest at the origin, as shown in the figure. At first we disregard the effects of radiation and assume the electron moves only under influence of the Coulomb field of the nucleus. The electron and the nucleus er treated as point particles. We disregard effects of the particles intrinsic spin, and assume non-relatiivistic description is adequate.

The permittivity of vacuum is $\epsilon_0 = 8.85 \cdot 10^{-12} F/m$ (or $8.85 \cdot 10^{-12} C^2 N^{-1} m^{-2}$), the permeability of vacuum is $\mu_0 = 4\pi \cdot 10^{-7} N/A^2$, and the speed of light is $c = 3.00 \cdot 10^8 m/s$

a) Determine the angular frequency ω of the circulating electron.

b) Use Larmor's radiation formula to determine the radiated power from the electron.

c) Conservation of energy implies that the radiated energy leads to reduction in the electron's kinetic and potential energy. Assume the energy loss leads to a slow reduction in the radius r of the electron orbit (slow relative to the orbital velocity). Find an expression for the time derivative of the radius for $r = a_0$, and use this to estimate the life time of the classical atom.