**NB!** The problem set consists of two pages

## Problem 2.1

Functions of a complex variable can be written in the general form $f(z) = u(x, y)+iv(x, y)$. Find $u(x, y)$ and $v(x, y)$ for the following functions

a) $\frac{-i+2z}{2+iz}$

b) $e^{iz}$

## Problem 2.2 (Derivatives)

Use the definition of $df/dz$ (Eq.(2.1.) in Boas) to show that

$$\frac{d}{dz}\left[f(z)g(z)\right] = g(z)\frac{df}{dz} + f(z)\frac{dg}{dz} \tag{1}$$

i.e. the product rule holds in the same form as for functions of real variables.

## Problem 2.3 (Cauchy-Riemann conditions)

Find the Cauchy-Riemann conditions in polar coordinates.
*Hint:* Like for cartesian coordinates, the derivation is based on demanding that the derivative $df/dz$ has to be unique, independent of $\Delta z$. In this case, let $\Delta z$ be along the radial and tangential direction, respectively.

## Problem 2.4 (Harmonic functions)

Consider the function

$$u(x, y) = \frac{y}{(1-x)^2 + y^2} \tag{2}$$

a) Show that $u(x, y)$ is harmonic, i.e. satisfies Laplace's equation $\nabla^2 u = 0$.

b) Use the Cauchy-Riemann equations to find its harmonic conjugate $v(x, y)$. Express $f = u + iv$ in terms of $z$.

c) Double check that $v(x, y)$ satisfies Laplace's equation.

## Problem 2.5 (Cauchy's theorem and integral formula)

Use Cauchy's theorem or integral formula to evaluate the following integrals:

a) $\oint_\Gamma \frac{\sin z dz}{2z-\pi}$ where $\Gamma$ is the circle $|z| = 3$

b) $\oint_\Gamma \frac{\sin z dz}{2z-\pi}$ where $\Gamma$ is the circle $|z| = 1$

c) $\oint_\Gamma \frac{\sin 2z dz}{6z-\pi}$ where $\Gamma$ is the circle $|z| = 1$

d) $\oint_\Gamma \frac{e^{2z} dz}{z-\ln 2}$ where $\Gamma$ is the square with vertices $\pm 2$, $\pm 2i$

## Extra problems (Recommended but need not be handed in)

Problems **2.17.**25, 28 and 32 in Boas