## Problem 3.1 (Generalized Cauchy integral formula)

Derive the *generalised* Cauchy integral formula,

$$f^{(n)}(z) = \frac{n!}{2\pi i} \oint_\Gamma \frac{f(w)dw}{(w-z)^{n+1}} \tag{1}$$

where $f^{(n)}$ denotes the $n$'th derivative. Here, $f$ is analytic on and inside the simple, closed contour $\Gamma$, and $z$ is any point inside the contour. (Hint: Differentiate the Cauchy integral formula an appropriate number of times). Use the result of a) to evaluate the integral

$$\oint_\Gamma \frac{\sin 2z\, dz}{(6z-\pi)^3} \tag{2}$$

where $\Gamma$ is the circle $|z| = 2$

## Problem 3.2 (Laurent series)

Determine the Laurent series (about the origin) of the function

$$f(z) = \frac{z-1}{z^2(z-2)} \tag{3}$$

a) Valid for $|z| < 2$
b) Valid for $|z| > 2$
c) What is the residue of $f$ at the origin?

## Problem 3.3 (Singularities)

For each of the following functions, say whether the indicated point is regular (removable singularity), an essential singularity, or a pole (if so, of what order):
a) $\frac{\sin z}{3z}$, $z = 0$

b) $\frac{\cos z}{z^4}$, $z = 0$

c) $\frac{z^3-1}{(z-1)^3}$, $z = 1$. Hint: Use partial fraction decomposition

d) $\frac{e^z}{z-1}$, $z = 1$.

## Extra problems (Recommended but need not be handed in)

Problems **14.3.**20 and 23, as well as **14.4.**8 and 11 in Boas.