

Problem 1.1 (Complex power series)

Find the disk of convergence for the following complex power series

a) $\sum_{n=0}^{\infty} n(n+1)(z-2i)^n$

b) $\sum_{n=1}^{\infty} 2^n(z+i-3)^{2n}$

Problem 1.2 (Euler's formula, powers and roots)

a) Express the number $\sqrt{2}e^{5i\pi/4}$ in $x+iy$ form.

b) Express the number $\frac{(1+i)^{48}}{(\sqrt{3}-i)^{25}}$ in $x+iy$ form.

c) Find all values of the root $(8i\sqrt{3}-8)^{1/4}$.

d) Show that the sum of the three cube roots of 8 is zero. Then show that the sum of the n n th roots of *any* complex number is zero.

Problem 1.3 (Elementary functions)

In a)-c), use the definitions of $\sin z$, $\cos z$, $\sinh z$, and $\cosh z$ in terms of exponential functions to show that

a) $\int_0^{2\pi} \sin^2(4x)dx = \pi$.

b) $\sin 2z = 2 \sin z \cos z$.

c) $\cosh^2 z - \sinh^2 z = 1$.

d) Find the $x+iy$ form of $\sin\left(i \ln \frac{1-i}{1+i}\right)$.

e) Find the $x+iy$ form of $(-e)^{i\pi}$.

f) Show that $\tanh^{-1} z = \frac{1}{2} \ln \frac{1+z}{1-z}$

Extra problem (Voluntary but recommended)

Problem 2.17.30 in Boas