FYS 3120 Classical Mechanics and Electrodynamics
Spring semester 2015

Problem Set 11

Problem 11.1

Three point charges, two with charge $+q$ and one with charge $-q$ are positioned in the sequence $(+q, -q, +q)$ along the x-axis. The distance between neighboring charges are equal to d, and the middle charge is placed at the origin $x = 0$.

a) Find an expression for the full Coulomb potential of the three charges in the x, y-plane and make a contour plot of the potential (which shows equipotential lines of the potential).

b) Determine the monopole moment (total charge) q_{tot}, dipole moment $\mathbf{p}$, and the quadrupole moment $Q_{\mathbf{n}}$ of the charge distribution, where $\mathbf{n}$ demotes a unit vector in the x, y-plane.

c) Find the corresponding three contributions to the Coulomb potential, and make a contour plot of the sum of the three contributions. Compare this plot with that of the full potential in a).

Problem 11.2

A Lambda particle (Λ) has energy $E = 3GeV (= 3000MeV)$ and velocity $\mathbf{v}_\Lambda$ along the x-axis in the laboratory frame S_L. The mass of Λ is $m_\Lambda = 1116MeV/c^2$.

a) In its rest frame S_Λ, the Λ particle has (average) life time $\tau_\Lambda \approx 2.6 \times 10^{-10}s$. What is the corresponding life time in the laboratory frame S_L? How far does the Λ particle travel in the laboratory frame S_L, if we assume that it lives exactly the time τ_Λ in its rest frame S_Λ?

b) The Λ particle decays to a nucleon N and a pion (π-meson) π. They have masses $m_N = 940MeV$ and $m_\pi = 140MeV$, respectively. The velocity $(\mathbf{v}_\pi)_\Lambda$ of the pion makes an angle $45°$ with the x-axis in the rest frame S_Λ of Λ. Find the velocity $(\mathbf{v}_\pi)_L$ of the pion and velocity $(\mathbf{v}_N)_L$ of the nucleon in the laboratory frame S_L. More precisely, find the x- and y-components of the vectors $(\mathbf{v}_\pi)_L$ and $(\mathbf{v}_N)_L$ and the angles $(\theta_\pi)_L$ and $(\theta_N)_L$ they make with the x-axis in the laboratory frame S_L.

Problem 10.3 (Exam 2013)

A straight rod is moving along the x-axis of an inertial reference frame S. The two endpoints A and B follow hyperbolic space-time trajectories, described the following time dependent x-coordinates in S,

$$x_A = c\sqrt{t^2 + c^2/a^2}\,, \quad x_B = c\sqrt{t^2 + c^2/b^2} \tag{1}$$

c is the speed of light, and a and b are positive constants, with $b < a$.

a) A second inertial frame S' moves along the x-axis with velocity v relative to S. The coordinates of the two reference frames are chosen to coincide at the space-time point $x = t = 0$.

Show that the motion of A and B, when expressed in terms of the coordinates of S', has precisely the same form as in S,

$$x'_A = c\sqrt{t'^2 + c^2/a^2}\,, \quad x'_B = c\sqrt{t'^2 + c^2/b^2} \tag{2}$$

(To demonstrate this it may be convenient to rewrite the above relations in terms of the squared coordinates x^2 and t^2.)

b) At time $t = 0$ the frame S is an instantaneous rest frame of both A and B. Show this and find the distance between A and B measured in S at this moment. The same results are valid for the reference frame S' at time $t' = 0$.

Based on this we may conclude that for any point on the space-time trajectory of A, the instantaneous inertial rest frame of A is a rest frame also for B. Furthermore the distance between A and B, when measured in the instantaneous inertial rest frame, is constant. Explain these conclusions.

c) Use the above results to show that the proper accelerations of the A and B are constants, and give the values of these.

d) At a given instant $t = 0$ a light signal with frequency ν_0 is sent from A and is subsequently received at B. What is the velocity of B (measured in S) when the signal is received, and what is the frequency of the signal, measured at B? (To answer the last question it may be convenient to use the relation between frequency and four-momentum for a photon sent from A to B.)