FYS3120 Kristian T[illegible]

Problem 1

a) $\xi = \gamma$

$x = \gamma \cos \omega t - \eta \sin \omega t$

$y = \gamma \sin \omega t + \eta \cos \omega t$

$\dot{x} = \dot{\gamma} \cos \omega t - \gamma \sin(\omega t)\omega - \dot{\eta} \sin(\omega t) - \eta \cos(\omega t)\omega$

$\dot{y} = \dot{\gamma} \sin(\omega t) + \gamma \cos(\omega t)\omega + \dot{\eta} \cos(\omega t) - \eta \omega \sin(\omega t)$

$\dot{\vec{r}}^2 = \dot{x}^2 + \dot{y}^2 = \underbrace{\dot{\gamma}^2 + \gamma^2\omega^2 + \dot{\eta}^2 + \eta^2\omega^2}_{} - 2\eta\cos(\omega t)\omega\dot{\gamma}\cos(\omega t)$

$- 2\eta\omega\sin(\omega t)\dot{\gamma}\sin(\omega t) + 2\gamma\dot{\eta}\omega\sin^2(\omega t)$

$+ 2\gamma\dot{\eta}\omega\cos^2(\omega t)$

from add the obvious $\cos^2 + \sin^2$

$L = K = \frac{1}{2}m|\dot{\vec{r}}|^2 = \frac{1}{2}m(\dot{\gamma}^2 + \dot{\eta}^2 + \omega^2(\gamma^2 + \eta^2) + ~~[illegible]~~ + 2\gamma\dot{\eta}\omega - 2\eta\omega\dot{\gamma})$

b) $\cancel{\frac{\partial L}{\partial \gamma} = m\omega^2\gamma + 2\eta\omega(\dot{\eta} - \dot{\gamma})}$

$\cancel{\frac{\partial L}{\partial \dot{\gamma}} = m\dot{\gamma} - 2\eta\omega\gamma \rightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot{\gamma}} = m\ddot{\gamma} - 2\eta\omega\dot{\gamma}}$

$\cancel{\frac{\partial L}{\partial \eta} = m\omega^2\eta + 2\omega\gamma(\dot{\eta} - \dot{\gamma})}$

$\cancel{\frac{d}{dt}[m\dot{\eta} + 2\eta\omega\gamma] = m\ddot{\eta} + 2\dot{\eta}\omega\gamma}$

b) $\frac{\partial L}{\partial \gamma} = m\omega^2 \gamma + \dot{\eta}\omega m$

$\frac{\partial L}{\partial \dot{\gamma}} = m\dot{\gamma} - \eta\omega m \Rightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot{\gamma}} = m\ddot{\gamma} - \frac{2}{2}\dot{\eta}\omega m$

$\frac{\partial L}{\partial \eta} = m\omega^2 \eta - \frac{2}{2}\omega\dot{\gamma} m$

$\frac{\partial L}{\partial \dot{\eta}} = m\dot{\eta} - m\frac{2}{2}\gamma\omega \Rightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot{\eta}} = m\ddot{\eta} - \frac{2}{2}\dot{\gamma}\omega m$

E.o.m:

$m\ddot{\gamma} - m\frac{2}{2}\dot{\eta}\omega - m\omega^2\gamma - \frac{2}{2}\dot{\eta}\omega m = 0$

$m\ddot{\gamma} - 2m\dot{\eta}\omega - m\omega^2\gamma = 0$

$m\ddot{\eta} - 2m\dot{\gamma}\omega - m\omega^2\eta = 0$

$F_{coriolis} = -2m\vec{\omega} \times \vec{v}$

$F_{centripetal} = \frac{mv^2}{r} = m\vec{\omega} \times (\vec{\omega} \times \vec{v}) = m\omega^2 r$

We see from e.o.m that

$F = m\ddot{\gamma} = ma$

$F_{co} = -2m\dot{\eta}\omega$

$F_{ce} = -m\omega^2\eta$

Problem 2

a) $T = \int_{t_1}^{t_2} dt = \int_{s_1}^{s_2} \frac{ds}{v} = \int_{x_1}^{x_2} \frac{1}{v}\sqrt{dx^2+dy^2}$

$= \int_{x_1}^{x_2} \frac{1}{v}\sqrt{1+y'^2}\,dx = \int_{x_1}^{x_2} \sqrt{\frac{1+y'^2}{-2gy}}\,dx$

$y<0$

b) $L = \sqrt{\frac{1+y'^2}{-2gy}}$

No explicit time-dependence makes H a constant

$\frac{\partial L}{\partial y'} = \frac{1}{2}\left(\frac{1+y'^2}{-2gy}\right)^{-1/2} \frac{y'}{gy}$

$H = \frac{\partial L}{\partial y'} y' - L = \frac{1}{2}\left(\frac{1+y'^2}{-2gy}\right)^{-1/2} \frac{y'^2}{gy} - \left(\frac{1+y'^2}{-2gy}\right)^{1/2}$

$H\left(\frac{1+y'^2}{-2gy}\right)^{1/2} = \frac{1}{2gy}$

$\Rightarrow \frac{1+y'^2}{-2gy} y^2 = \frac{1}{H^2(2g)^2} \Rightarrow (1+y'^2)y = -\frac{1}{H^2 2g} = -k^2$

c) $$\frac{dy}{dx} = \frac{dy}{d\theta}\frac{d\theta}{dx}$$

$$\frac{dx}{d\theta} = \frac{1}{2}k^2(1-\cos\theta) \Rightarrow \frac{d\theta}{dx} = \frac{2}{k^2}\frac{1}{1-\cos\theta}$$

$$\frac{dy}{d\theta} = \frac{1}{2}k^2(-\sin\theta)$$

$$(1+y'^2)y = -k^2$$

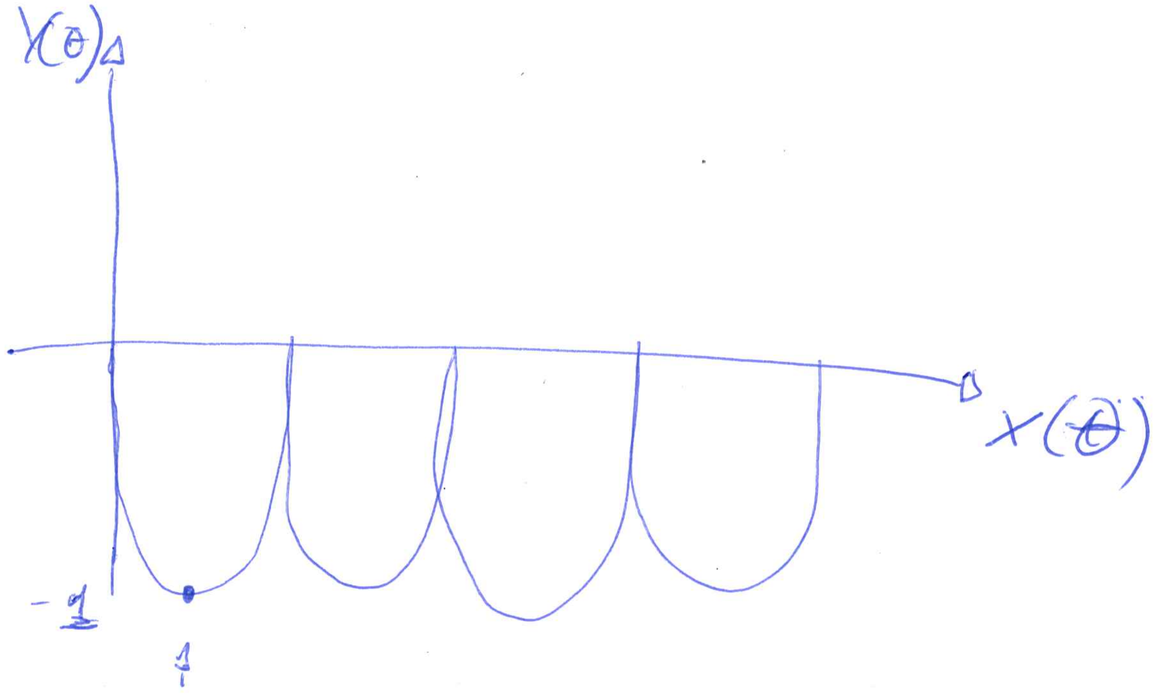
$$\Rightarrow \left(1+\left(\frac{\sin\theta}{\cos\theta-1}\right)^2\right)\cdot\frac{1}{2}k^2(\cos\theta-1) = -k^2$$

$$\Rightarrow (\cos\theta-1) + \frac{\sin^2\theta}{\cos\theta-1} = -2$$

$$\Rightarrow \cos^2\theta - 2\cos\theta + 1 + \sin^2\theta = -2(\cos\theta-1)$$

$$\Rightarrow -2\cos\theta + 2 = -2\cos\theta + 2 \qquad \square$$

When $\theta = 0$, both x and y are zero a)



lowest point $\Leftrightarrow \frac{dy}{dx} = 0$

$\cancel{\frac{dx}{d\theta} = \frac{1}{2}k^2(1-\cos\theta) = 0 \Rightarrow \theta} = \pi$

e) $\frac{dy}{dx} = \frac{dy}{d\theta}\frac{d\theta}{dx} = -\frac{\sin\theta}{1-\cos\theta} = 0 \Rightarrow \cancel{\pi = \theta}$ $\theta = \pi$

$x_B = \frac{1}{2}k^2(\pi - \sin\pi) = \frac{\pi}{2}k^2$

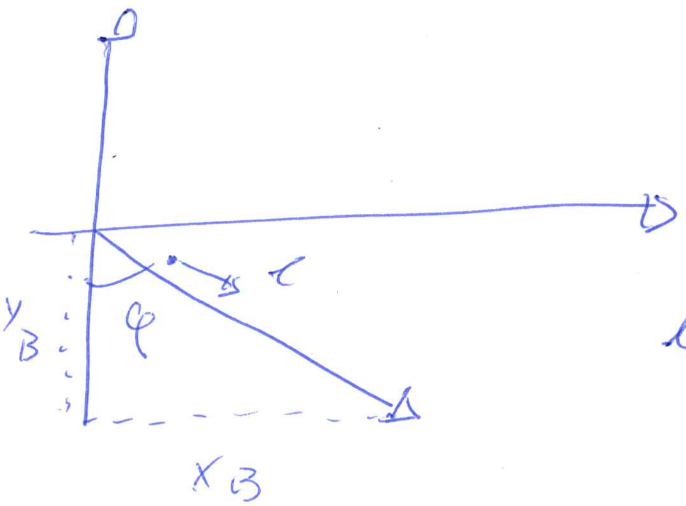
$y_B = \frac{1}{2}k^2(\cos\pi - 1) = -k^2$

$\Rightarrow y_B = -\frac{2}{\pi}x_B$

e) $1 + y'^2 = -\frac{k^2}{y}$

$$\Rightarrow T = \int_0^{x_B} \sqrt{\frac{1+y'^2}{-2gy}}\,dx = \int_0^{\pi} \sqrt{\frac{-k^2}{-2gy^2}}\,(-y)\,d\theta$$

$$= \int_0^{\pi} d\theta\,\frac{k}{\sqrt{2g}} = \frac{k\pi}{\sqrt{2g}}$$



$$l = \sqrt{y_B^2 + x_B^2}$$

$$= \sqrt{\frac{\pi^2}{4}k^4 + k^4}$$

$$= k^2\sqrt{\frac{\pi^2}{4} + 1}$$

$F = ma = mg\cos\varphi \Rightarrow a = g\cos\varphi$

$l = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2l}{a}}$

$$\cos\varphi = \frac{|y_B|}{|l|} = \frac{k^2}{k^2\sqrt{\frac{\pi^2}{4}+1}} = \frac{1}{\sqrt{\frac{\pi}{4}+2}}$$

$$\Rightarrow \quad a = g \frac{1}{\sqrt{\frac{\pi^2}{4} + 1}}$$

$$\Rightarrow \quad t = \sqrt{\frac{2K^2 \sqrt{\frac{\pi^2}{4} + 1}}{g \left(\sqrt{\frac{\pi^2}{4} + 1} \right)^{-1}}}$$

$$= \frac{K\pi}{\sqrt{2g}} \sqrt{\frac{4}{\pi^2} + 1}$$

$$= \sqrt{\frac{4}{\pi^2} + 1}\, T > T$$