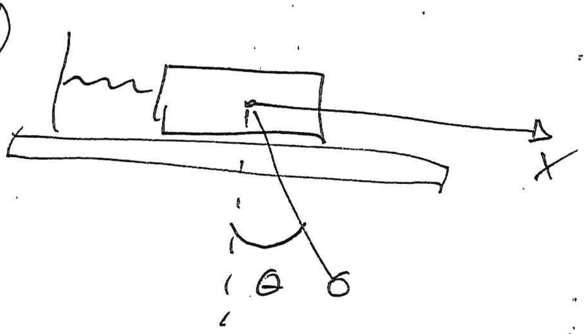
FYS3120 Problem Set 1

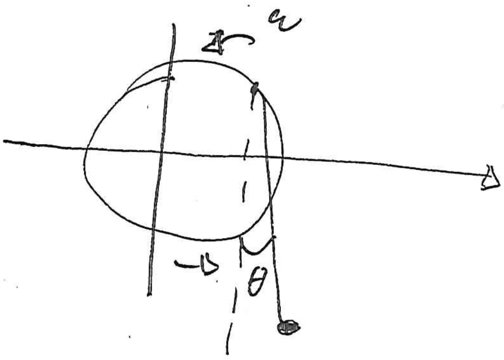
Problem 1

a)



$d = 2$, (x, θ)

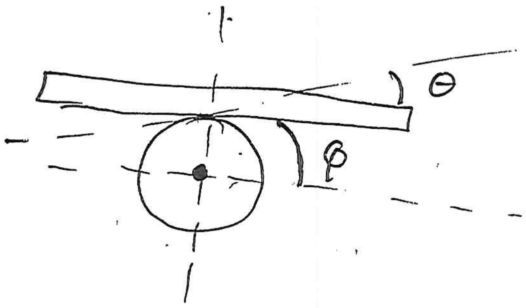
b)



$d = 1$, (θ)

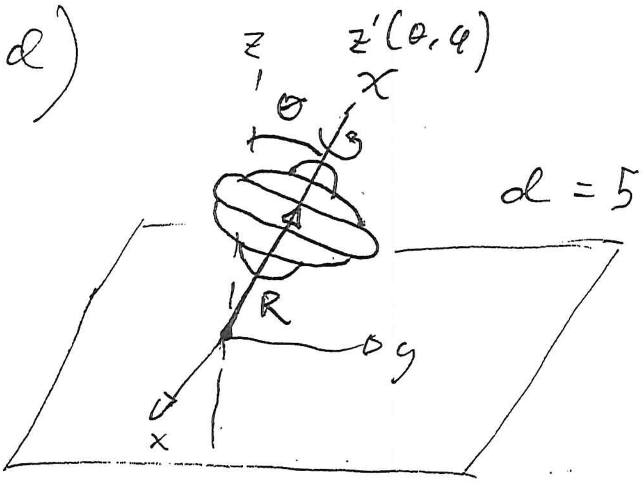
ω is constant.

c)



$d = 2$, (θ, φ)

d)



$d = 5$

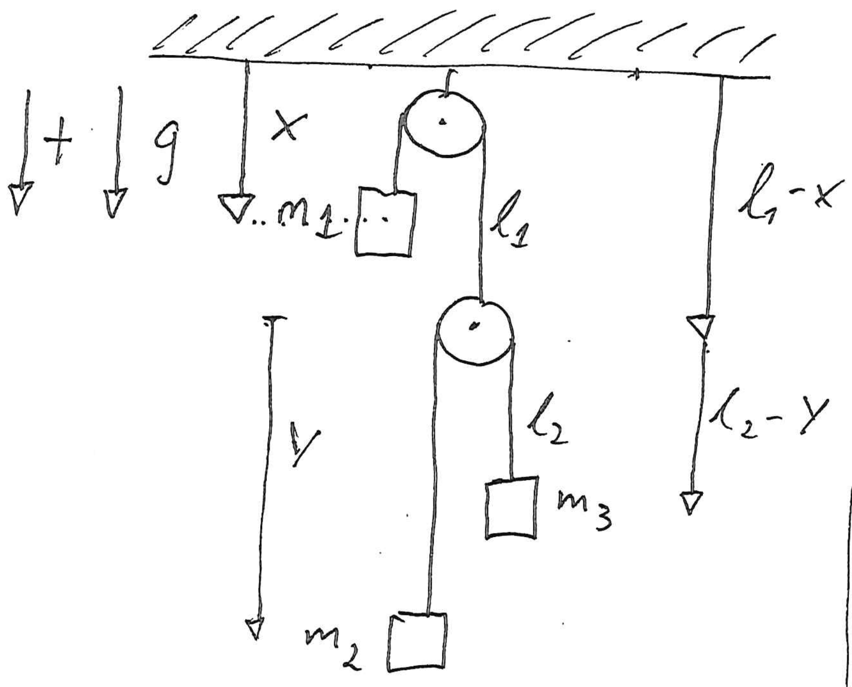
$\vec{R} = x\hat{i} + y\hat{j}$

(The center of mass R does not move in z-direction)

χ is the rotation around its own axis.

θ & φ describes the orientation of the rotation axis in the 3D-space

Problem 2



$m_1 = 4m$

$m_2 = 2m$

$m_3 = m$

$\Rightarrow m_2 > m_3$

$m_1 > m_2 + m_3$

l_1 & l_2 is the total lengths of the ropes

$$K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2$$

$$= \frac{1}{2} m \left(4(\dot{x})^2 + 2(\dot{y} - \dot{x})^2 + (-\dot{x} - \dot{y})^2\right)$$

$$= \frac{1}{2} m \left(7\dot{x}^2 + 3\dot{y}^2 - 2\dot{x}\dot{y}\right)$$

$v_1 = \dot{x}$

$v_2 = -\dot{x} + \dot{y}$

$v_3 = -\dot{x} - \dot{y}$

$$V = m_1 g x_1 + m_2 g x_2 + m_3 g x_3$$

$$= mg\left(4x + 2(l_1 - x + y) + l_1 - x + l_2 - y\right)$$

$$= mg(x + 3l_1 + y + l_2)$$

$x_1 = x$

$x_2 = l_1 - x + y$

$x_3 = l_1 - x + l_2 - y$

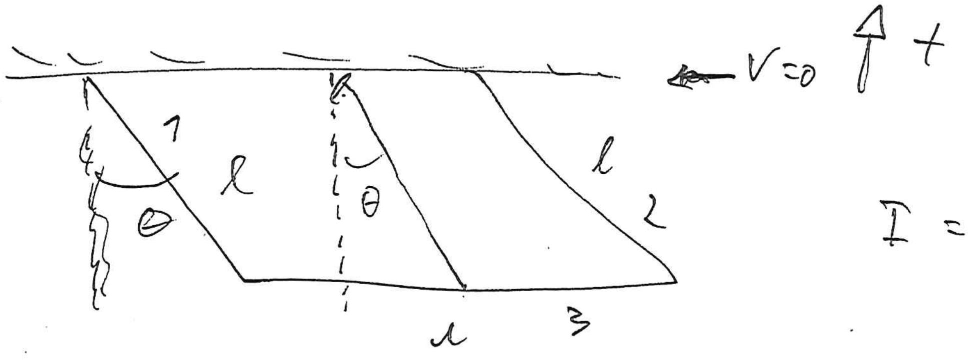
$$L = K - V = \frac{1}{2} m (7\dot{x}^2 + 3\dot{y}^2 - 2\dot{x}\dot{y}) - mg(x + 3l_1 + y + l_2)$$

Degrees of freedom: 2

- we can describe the motion of m_3 using m_2, and describe the motion of the pulley carrying m_2 & m_3 using m_1

Problem 3

- The rods can only move in 2D but because the distance l is fixed which means we only have one degree of freedom and use the angle θ



$I = \frac{1}{3} m l^2$

$K = \frac{1}{2} m v_3^2 + \frac{1}{2} I_1 \dot{\theta}^2 + \frac{1}{2} I_2 \dot{\theta}^2$, $I_1 = I_2 = \frac{1}{3} m l^2$

$\Rightarrow K = \frac{1}{2} m l^2 \dot{\theta}^2 + \frac{1}{2} \cdot \frac{1}{3} m l^2 \dot{\theta}^2 + \frac{1}{2} \cdot \frac{1}{3} m l^2 \dot{\theta}^2$, $v_3^2 = l^2 \dot{\theta}^2$

$= \frac{5}{6} m l^2 \dot{\theta}^2$

$V = mgh_1 + mgh_2 + mgh_3$, where ~~$h_1, h_2, h_3$~~ h_1, h_2, h_3 is the distance from the point of suspension down to the center of mass of each rod. We define the positive direction as in the drawing

$$\Rightarrow V = -mg\left(\frac{l}{2}\cos\theta + \frac{l}{2}\cos\theta + l\cos\theta\right)$$

$$= -2mgl\cos\theta$$

$$\Rightarrow L = K - V = \frac{5}{6}ml^2\dot{\theta}^2 + 2mgl\cos\theta$$

Problem 4

a) One particle moving in three dimension means we have a 3D-coordinatesystem but we can use express one of the coordinates using the two others:

$$3N - m = 3\cdot 1 - 1 = 2$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} = x\hat{i} + y\hat{j} + e^{-(x^2+y^2)}\hat{k}$$

b) $$\delta \vec{r}_i = \sum_{j=1}^{2} \frac{\partial \vec{r}_i}{\partial q_j}\delta q_j = \frac{\partial \vec{r}}{\partial x}\delta x + \frac{\partial \vec{r}}{\partial y}\delta y$$

$$= \delta x \hat{i} + 2x e^{-(x^2+y^2)}\delta x \hat{k} + \delta y \hat{j} - 2y e^{-(x^2+y^2)}\delta y \hat{k}$$

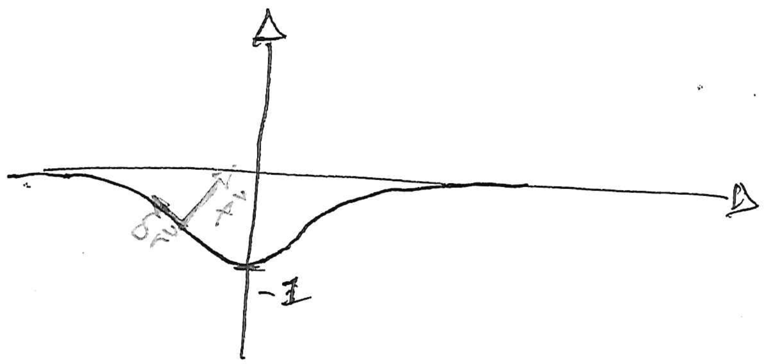
$$= \delta x \hat{i} + \delta y \hat{j} - (2x\delta x + 2y\delta y) e^{-(x^2+y^2)}\hat{k}$$

c) $\vec{f}\cdot\delta\vec{r} = 0$, so $\vec{f}$ is perpendicular to the surface

$$\frac{1}{A}\vec{f} = \nabla(e^{-(x^2+y^2)} + z) = 2x e^{-(x^2+y^2)}\hat{i} + 2y e^{-(x^2+y^2)}\hat{j} + \hat{k}$$

where A is a constant of normalization

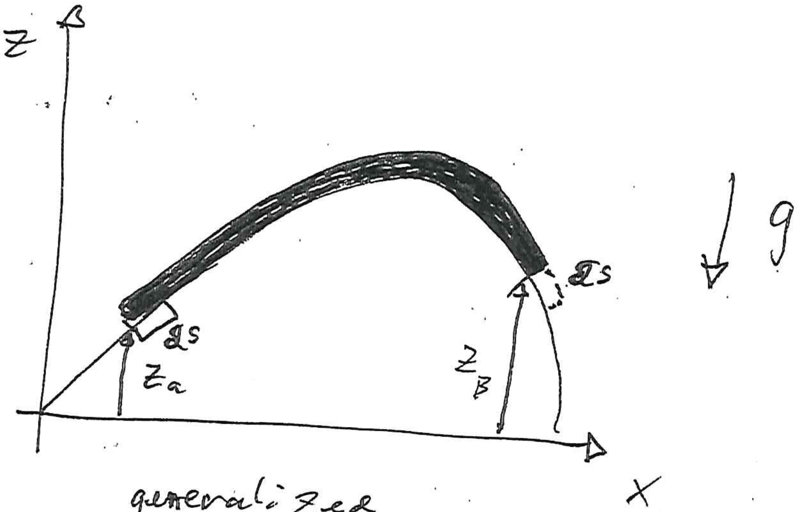
d) $\vec{r}(y=0) = x\hat{i} - e^{-x^2}\hat{k}$



Problem 5

For static equilibrium

$$\frac{dV}{dq} = 0$$



We choose our generalized coordinate system along the surface. We look at the change in Potential dV when we move the chain an infinitecimal distance ds along the surface

$$dV = g\frac{M}{L}ds\,z_A - g\frac{M}{L}ds\,z_B\,,$$ where

M is the total mass of the chain and L is the total length. We divide by ds on both sides and get

$$\frac{dV}{ds} = g\frac{M}{L}(z_A - z_B) = 0,$$ ~~which~~

Hence $z_A = z_B$