CS 381 PSO #1

Week 02

Induction - Basic Steps

1. Base Case (Prove the first step, relatively basic most of the time)
   a. Prove 2^(2*n) > 2^n.
   b. Base Case would be for n = 1: 2^(2*1) > 2^1, which is a true statement.
2. Inductive Hypothesis (Assume true for a single k <= n)
   a. I.H. for problem above is: Assume 2^(2*k) > 2^k.
   b. If strong induction, assume true for all values k <= n.
3. Inductive Step (Prove statement true for n = k + 1)
   a. I.S. for problem above is 2^(2*(k + 1)) = 2^(2*k + 2) = 4*2^(2*k) compared to 2^(k + 1) = 2*2^k. 4 > 2 and 2^(2*k) > 2^k by the I.H., so the the statement is true for n = k + 1.

Induction

Prove that $n! > 2^n$ for all $n \geq 4$

Induction

Prove that $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

Induction

Prove that every positive integer n can be written as a sum of distinct nonnegative integer powers of 2.

Basic log Rules

1. $\log(ab) = \log(a) + \log(b)$
2. $\log(a/b) = \log(a) - \log(b)$
3. $a\log(b) = \log(b^a)$
4. $a^{(\log_a b)} = b$
5. For the rest of this presentation, assume all logs are base 2.

Asymptotic Running Times

State the big-**Ω**,O,**Θ** relationships between the following pair of functions:

$$\sqrt{n} + (\log n)^5 \text{ vs } (\sqrt{2})^{\log n}$$

Asymptotic Running Times

State the big-**Ω**,O,**Θ** relationships between the following pair of functions:

$$8^{\log n} \text{ vs } 2n^3 + n^2(\log n)^4$$

Asymptotic Running Times

$$\text{Show that } (\log n)^{1000} = O(n^{0.0001})$$

Evaluating Loops

How many times is `F` called?

```
for i = 2 to n by i = i^2:
    F(i)
end for
```

Evaluating Loops

How many times is `F` called?

```
for i = 2 to n:
    for j = i to n by sqrt(n):
        F(i, j)
    end for
    F(i, 0)
end for
```