Quiz Submissions - Fall 2021 Quiz 03 - With Solutions

Andrew Hou (username: hou116)

Exit Preview

Attempt 1

Written: Sep 14, 2021 18:44 - Sep 14, 2021 18:45

Submission View

Released: Sep 14, 2021 18:00

Question 1 **0 / 1 point**

You are given a task to implement a 6-way mergesort, i.e., divide the array into 6 pieces and merge from those pieces. What is the proper recurrence for such a task?

- $$T(n) = 6T(n/6) + 1$$
- $$T(n) = 6T(n) + 1$$
- ➡ $$T(n) = 6T(n/6) + n$$
- $$T(n) = T(n/6) + n$$

Hide Feedback

We divide into 6 subproblems, each 1/6 the size of the original. Merging takes O(n) - we can have 6 pointers for each subarray and pick the min of the 6 each time, advancing appropriately

Question 2 **0 / 1 point**

You are given a function that, in constant time, when applied to an array of length n, returns

$$\sqrt{n}$$

elements that are possible candidates for the median.

What is the optimal recurrence to find the median of the array using this function?

- $$T(n) = \sqrt{n}T(1) \\ +1$$
- $$T(n) = nT(\sqrt{n}) + 1$$
- $$T(n) = \sqrt{n}T(\sqrt{n}) + 1$$

$$T(n) = T(\sqrt{n}) + 1$$

Question 3 **0 / 1 point**

Assume a sorted array A contains an element that appears n/2 + 1 times. What is the lowest upper bound for an algorithm to determine this element?

$$O(1)$$

$$O(n)$$

$$O(n \log n)$$

$$O(\log n)$$

Question 4 **0 / 1 point**

What is the runtime of the most efficient algorithm to find the last index of the minimum element of a sorted array A?

$$O(\log n)$$

$$O(n - \log n)$$

$$O(1)$$

$$O(n)$$

Question 5 **0 / 1 point**

Bob buys a gold chain of length n, where n =4^k for k>1. As long as the length is larger than 1, he cuts it into two equal halves and gives one of the two halves to Alice.

How many pieces of chain does Alice have at the end?

$$\frac{n}{2}$$

$$\log_4 n$$

$$\log_2 n$$

$$2\log_4 n$$

Question 6 **0 / 1 point**

Order these functions from smallest to largest (1 is smallest)

_____ (2)
$$13n\log n^3$$

_____ (3)
$$16n\log^3 n$$

_____ (1)
$$42n\log\log n$$

_____ (4)
$$2n^2 + 18n\log n + 6^{\log_6 n}$$

Question 7 **0 / 1 point**

Identify the error in this inductive proof:

Let F(n) be the n'th Fibonacci number. We claim that F(n) < 2^n for all n >= 1.

$$(1)\ \text{Base case: } F(1) = 1 < 2^1\text{, so the claim holds for 1}$$

$$(2)\ \text{Inductive hypothesis: Assume the claim holds for all } n > k \geq 2,\ \text{that is } F(k) < 2^k$$

$$\text{We want to show } F(k+1) < 2^{k+1}$$

$$(3)\ F(k+1) = F(k) + F(k-1) < 2^k + 2^{k-1} \quad \text{by IH}$$

$$(4)\ = 2^{k-1}(2+1) < 2^{k-1}\cdot 2^2 = 2^{k+1}$$

Thus, by the principle of mathematical induction the claim holds for all n >= 1

- (3)
- (4)
- (1)
- (2)

Hide Feedback

Since in our proof we apply the inductive hypothesis on both F(k) and F(k-1) simultaneously, we need 2 base cases. Thus, the error is in step 1, where we only prove a single base case for n=1

Question 8 **0 / 1 point**

Solve the recurrence using the Master Theorem (not the extended master theorem), or indicate that it does not apply:

$$T(n) = 3T(n/2) + n^2$$

➡ ◯ $$O(n^2)$$

◯ Master Theorem does not apply

◯ $$O(n^{\log_2 3} \cdot \log n)$$

◯ $$O(n^{\log_2 3})$$

Question 9 **0 / 1 point**

Solve the recurrence using the Master Theorem (not the extended master theorem), or indicate that it does not apply:

$$T(n) = 2T(n/2) + \frac{n}{\log n}$$

◯ $$O(n \log n)$$

◯ $$O(\frac{n}{\log n})$$

➡ ◯ Master Theorem does not apply

◯ $$O(n)$$

▼ Hide Feedback

Non-polynomial difference between f(n) and n^log_b(a)

Question 10 **0 / 1 point**

What is your favorite base case of the Master Theorem?

➡ ◯ Case 1

➡ ◯ Case 2

➡ ◯ Case 3

○ I don't like the Master Theorem :c

Attempt Score:0 / 10 - 0 %

Done