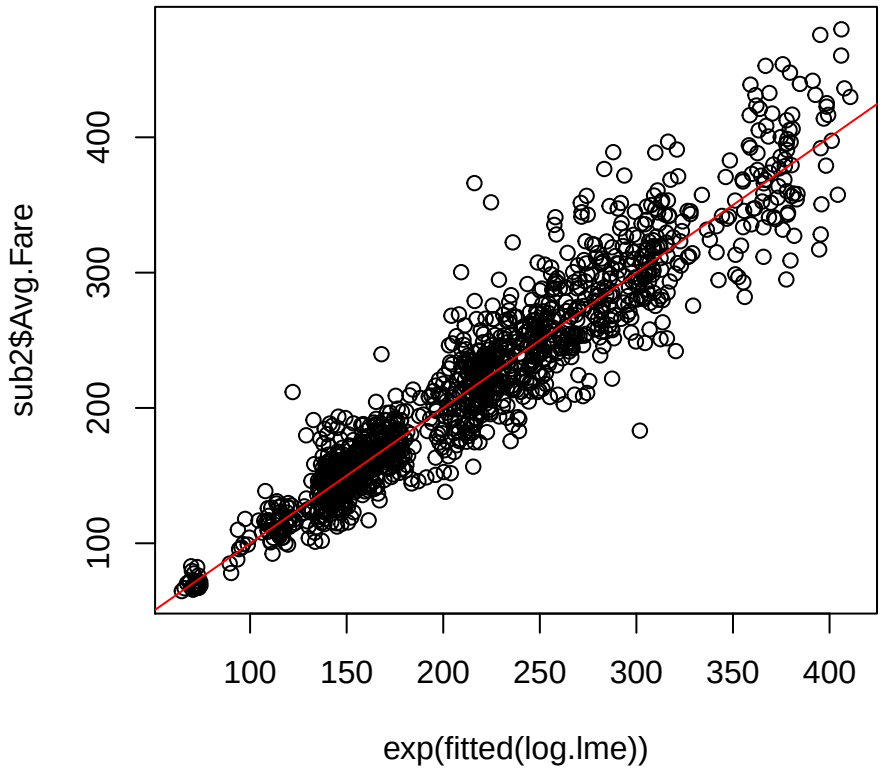
```
plot(exp(fitted(log.lme)), sub2$Avg.Fare)
abline(a = 0, b = 1, col = 'red')
```




```
summary(log.lme)
```

```
## Linear mixed-effects model fit by REML
##  Data: sub2
##         AIC       BIC   logLik
##   -1654.593 -1618.429 834.2964
##
## Random effects:
##  Formula: ~1 | Route
##         (Intercept) Residual
## StdDev:   0.3362234 0.111256
##
## Fixed effects: log(Avg.Fare) ~ Quarter + I(Quarter^2) + Avg.Pass + I(Avg.Pass^2)
##                   Value  Std.Error   DF   t-value p-value
## (Intercept)    5.385867 0.04982064 1246 108.10514       0
## Quarter        0.015383 0.00172770 1246   8.90384       0
## I(Quarter^2)  -0.000568 0.00006217 1246  -9.12952       0
```

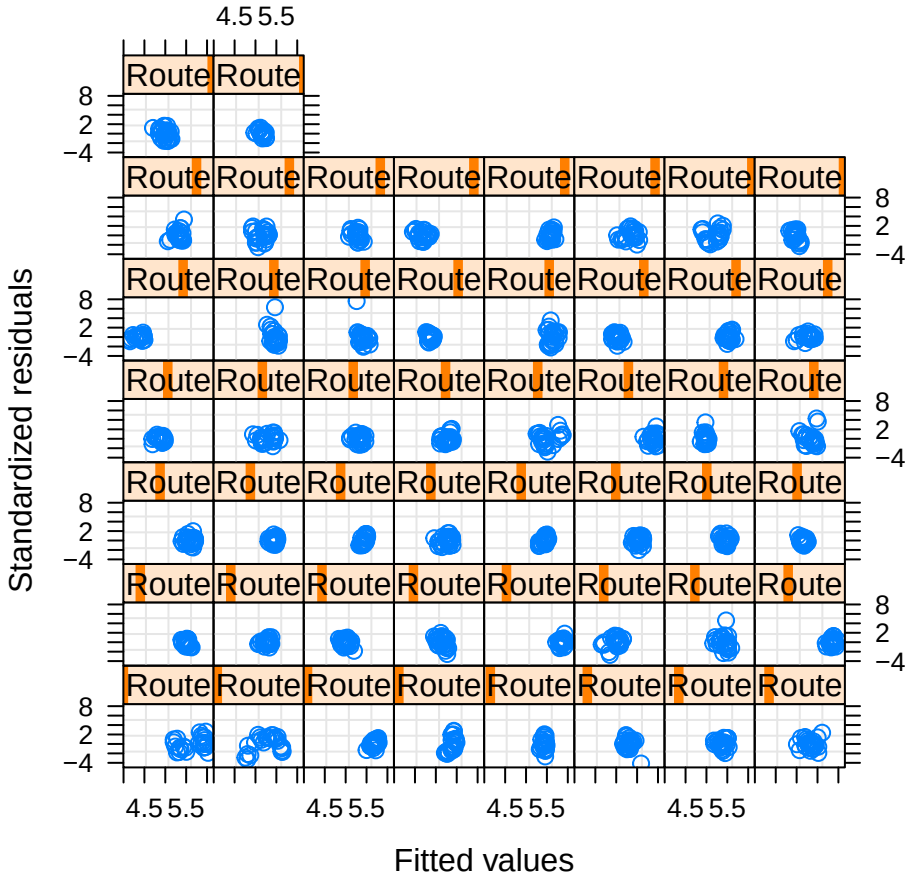
```
## Avg.Pass      -0.001869 0.00016305 1246 -11.46461       0
## I(Avg.Pass^2)  0.000001 0.00000010 1246   6.68229       0
##  Correlation:
##               (Intr) Quartr I(Q^2) Avg.Ps
## Quarter       -0.151
## I(Quarter^2)   0.124 -0.971
## Avg.Pass      -0.214 -0.130  0.146
## I(Avg.Pass^2)  0.156  0.142 -0.142 -0.852
##
## Standardized Within-Group Residuals:
##         Min          Q1         Med          Q3         Max
## -4.48583786 -0.60389065 -0.01403567  0.60718032  4.95395712
##
## Number of Observations: 1300
## Number of Groups: 50
```

```
log.lme.rand <- lme(log(Avg.Fare) ~ Quarter + I(Quarter^2)+ log(Avg.Pass),
               data = rand.samp, random = ~1|Route)
```

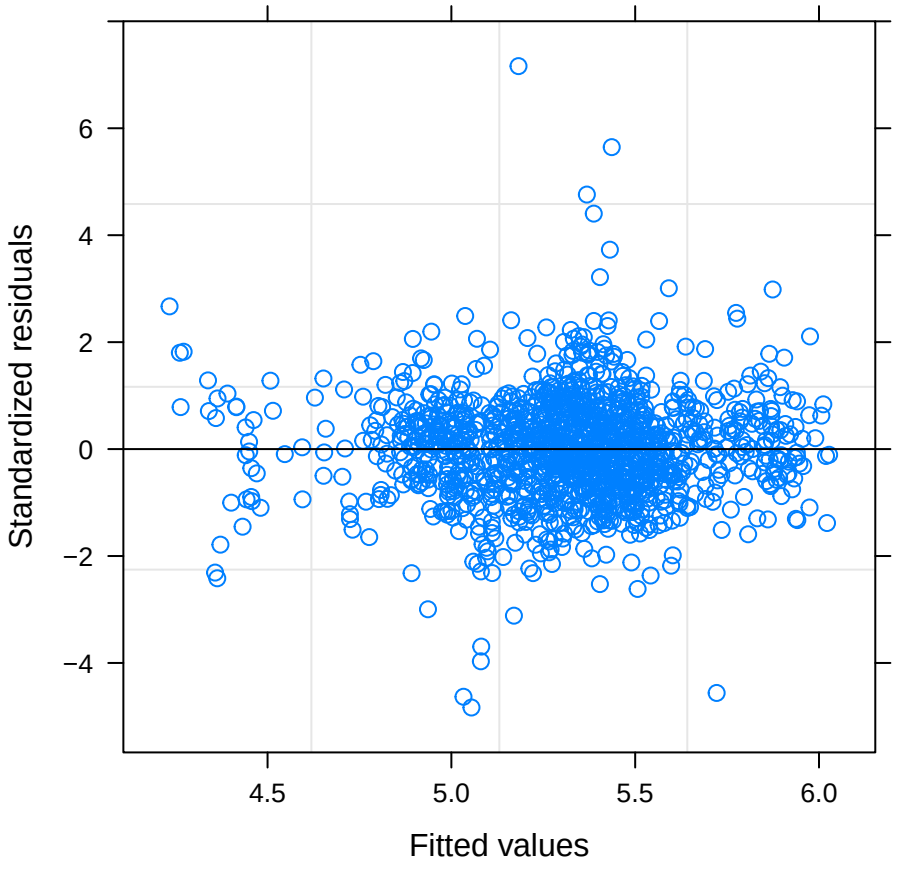
```
fit.sl <- lme(log(Avg.Fare) ~ Quarter + I(Quarter^2)+ log(Avg.Pass),
              random = ~ Quarter| Route,
              data=rand.samp)
exp(summary(log.lme.rand)$coef$fixed)
```

```
##   (Intercept)       Quarter  I(Quarter^2) log(Avg.Pass)
##   680.6808284     1.0288228     0.9989474     0.7349510
```

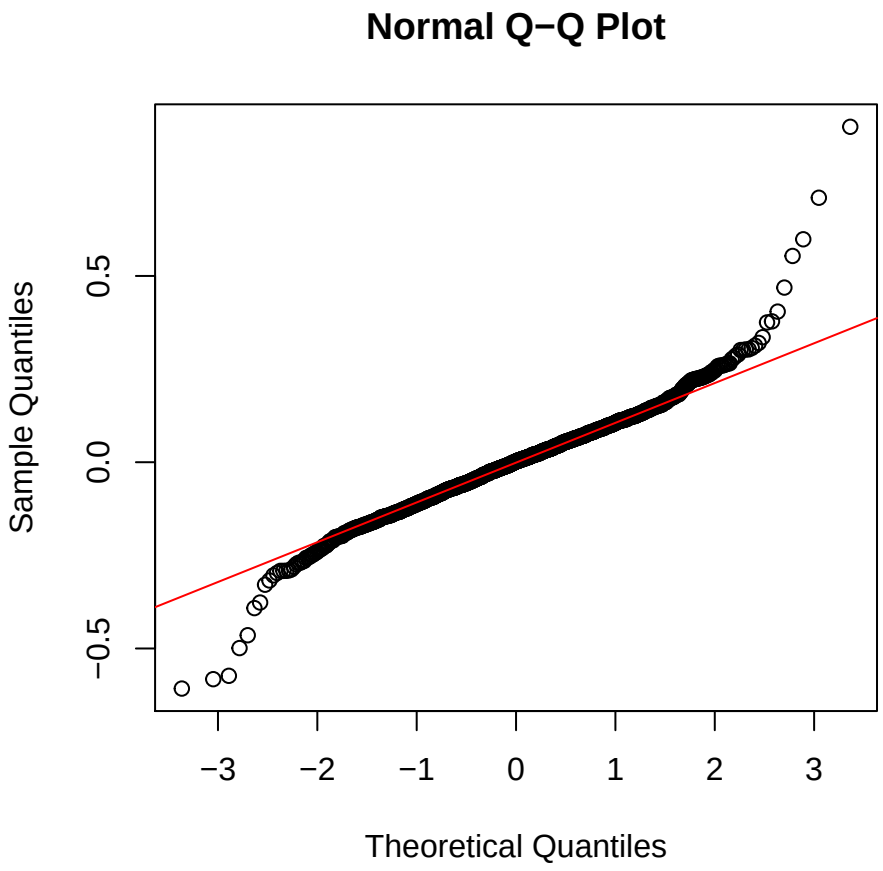
```
plot(fit.sl, resid(., type="p")~fitted(.) | Route)
```




```
plot(log.lme.rand)
```




```
qqnorm(resid(log.lme.rand)); qqline(resid(log.lme.rand), col = "red")
```

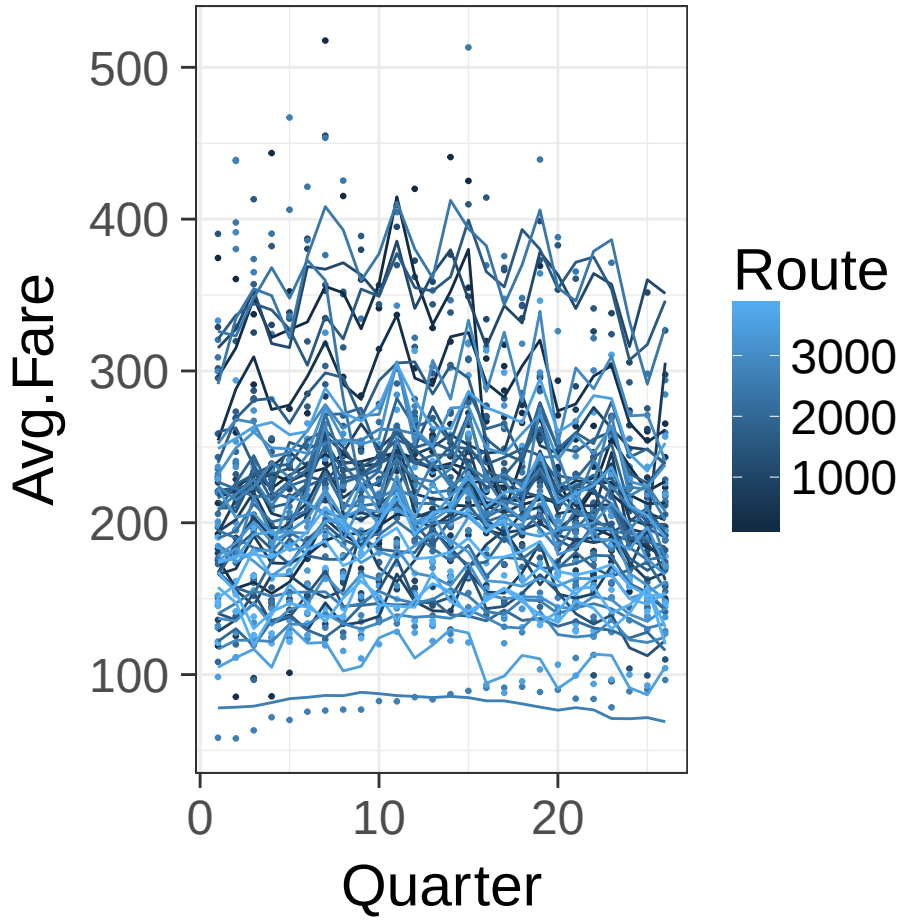



```
summary(log.lme.rand)
```

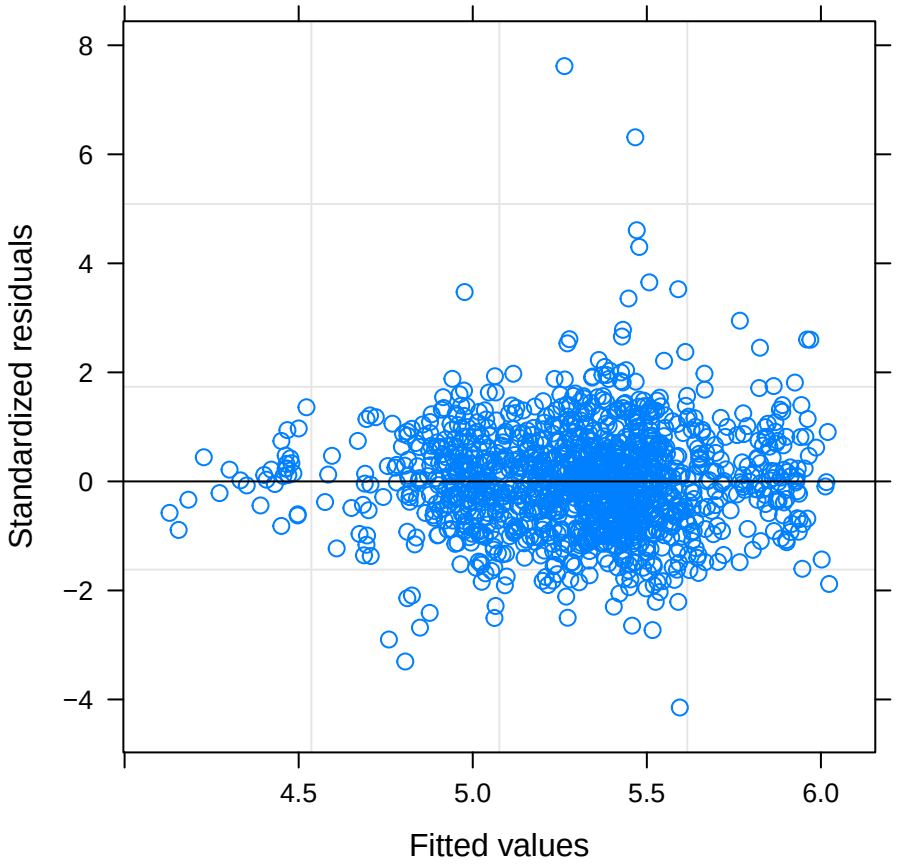
```
## Linear mixed-effects model fit by REML
##  Data: rand.samp
##         AIC       BIC   logLik
##   -1372.053 -1341.051 692.0266
##
## Random effects:
##  Formula: ~1 | Route
##         (Intercept)  Residual
## StdDev:   0.4084697 0.1257121
##
## Fixed effects: log(Avg.Fare) ~ Quarter + I(Quarter^2) + log(Avg.Pass)
##                   Value  Std.Error   DF   t-value p-value
## (Intercept)    6.523094 0.07961112 1247  81.93696       0
## Quarter        0.028415 0.00196298 1247  14.47556       0
## I(Quarter^2)  -0.001053 0.00007012 1247 -15.01924       0
## log(Avg.Pass) -0.307951 0.01283578 1247 -23.99165       0
##  Correlation:
##               (Intr) Quartr I(Q^2)
```

```
## Quarter       -0.005
## I(Quarter^2)   0.016 -0.971
## log(Avg.Pass) -0.673 -0.177  0.137
##
## Standardized Within-Group Residuals:
##         Min          Q1         Med          Q3         Max
## -4.83315749 -0.58187602  0.01569318  0.56413565  7.16186635
##
## Number of Observations: 1300
## Number of Groups: 50
```

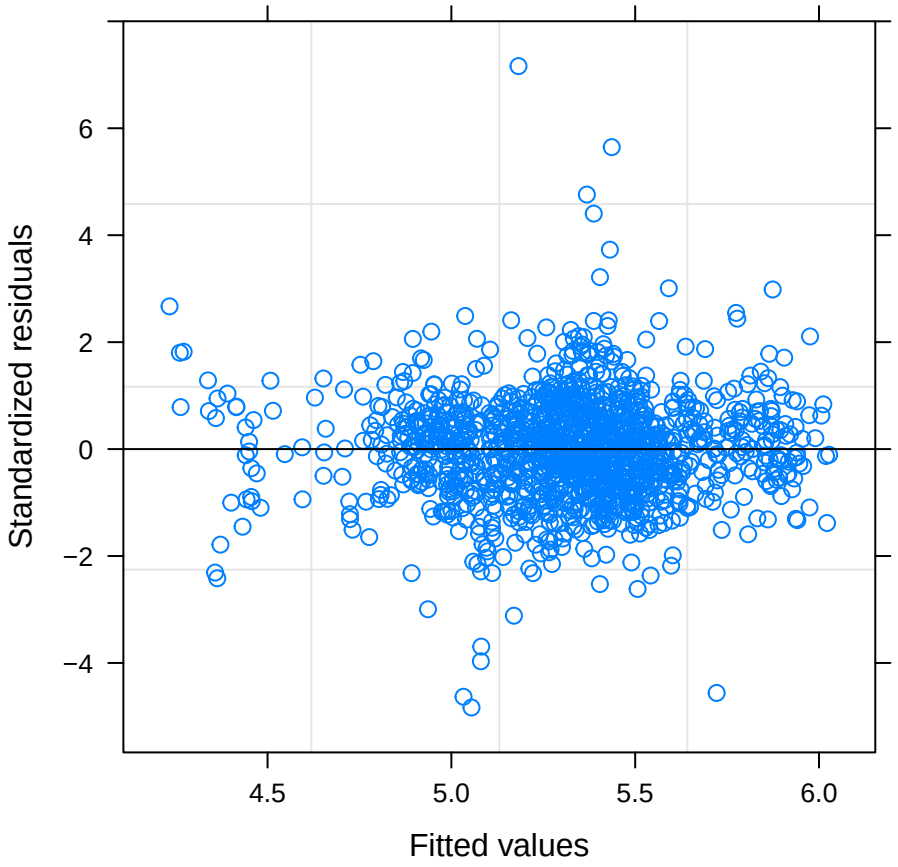
```
p <- ggplot(rand.samp, aes(x=Quarter, y=Avg.Fare, color = Route )) +
  geom_point(size=.5) +
  geom_line(aes(y=exp(fitted(log.lme.rand)), group=Route), size = .5) +
  theme_bw(base_size=22)
print(p)
```




```
plot(fit.sl)
```




```
plot(log.lme.rand)
```


Standardized residuals
6
4
2
0
−2
−4
4.5
5.0
5.5
6.0
Fitted values