

# Posterior Probabilities of Hypotheses

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## First, Review Posterior Probabilities of Hypotheses

Example 1:  $\text{BetaT} = (0.0, 0.1, -0.1, 0.1, -0.1)$

Example 2:  $\text{BetaT} = (0.0, 0.2, -0.2, 0.2, -0.2)$

Example 3:  $\text{BetaT} = (0.0, 0.3, -0.3, 0.3, -0.3)$

Example 4:  $\text{BetaT} = (0.0, -0.4, 0.4, -0.4, 0.4)$

Example 5:  $\text{BetaT} = (0, 0, 0)$

First, Review Posterior Probabilities of  
Hypotheses

# Posterior Probabilities of Hypotheses

- Posterior Probability of model  $H_\ell, \ell \in \{0, 1\}$ :

$$\begin{aligned} P(H_\ell|y, X) &= \frac{\pi_\ell f(y|H_\ell, X)}{\pi_0 f(y|H_0, X) + \pi_1 f(y|H_1, X)} \\ &= \frac{f(y|H_\ell, X)}{f(y|H_0, X) + f(y|H_1, X)} \end{aligned}$$

Since we assume  $\pi_0, \pi_1 = \frac{1}{2}$ .

- $f(y|H_\ell, X) = \phi_{\ell, X}(y)$  is the model evidence, i.e. density of  $N_N(X\mu_\ell, \sigma_\varepsilon^2 I + XV_\ell X^T)$  evaluated at a given  $y$  and design **D** with  $N$  design points.
- $E[P(H_\ell|y, X)|H_r, X]$  may be estimated using MC approximation from simulated responses  $y$  under a chosen hypothesis  $H_r$ .

## Estimate Expected Posterior Probability of a Hypothesis

Estimate the expected posterior probability of hypothesis  $H_\ell$  for  $J$  simulations of  $Y$  under  $H_r$ , given design  $\mathbf{D} = \{x_1, \dots, x_N\}$ :

1. For  $j = 1, \dots, J$ :

1.1 Draw  $y_i^{(j)} | \mathbf{x}_i \sim N(\mathbf{x}_i^T \beta_T, \sigma_\varepsilon^2)$ ,  $\forall \mathbf{x}_i \in \mathbf{D}$ , so  $y^{(j)} \in R^N$ .

1.2  $\forall m = \{0, 1\}$ , calculate model evidences  $f(y | H_m, \mathbf{D})$

1.3 Calculate the posterior probability of  $H_\ell$ ,  $P(H_\ell | y^{(j)}, \mathbf{D})$ , from simulation  $j$

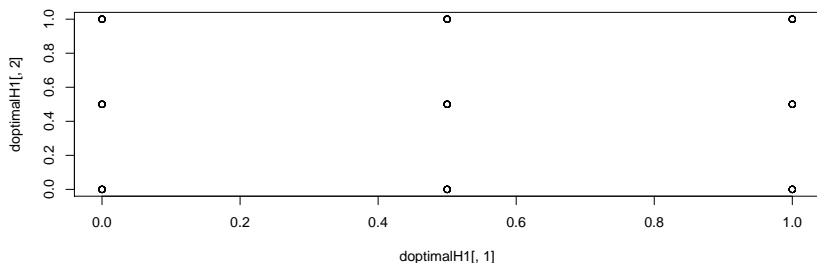
$$P(H_\ell | y^{(j)}, \mathbf{D}) = \frac{f(y^{(j)} | H_\ell, X)}{f(y^{(j)} | H_0, X) + f(y^{(j)} | H_1, X)}$$

2. Average the estimated posterior probabilities of  $H_\ell$  over  $\forall j$  to obtain MC estimate of  $E[P(H_\ell | y, \mathbf{D}) | H_r, \mathbf{D}]$

Note that  $y^{(j)}$  are generated from  $N_N(X\beta_T, \sigma_\varepsilon^2 I)$  and are independent, while the model evidence for  $H_m$  marginalizes out  $\beta$  and evaluates  $y^{(j)}$  using  $f(y | H_m, \mathbf{D})$ , the density of  $N_N(X\mu_m, \sigma_\varepsilon^2 I + XV_m X^T)$ , in which they are no longer assumed to be independent.

## Additional Designs

- ▶ A center design with all points at (0.5, 0.5)
- ▶ A mixture of the center design and the space-filling design with 36 space-filling lattice points and the rest of the points at (0.5, 0.5).
- ▶ D-optimal design for quadratic model

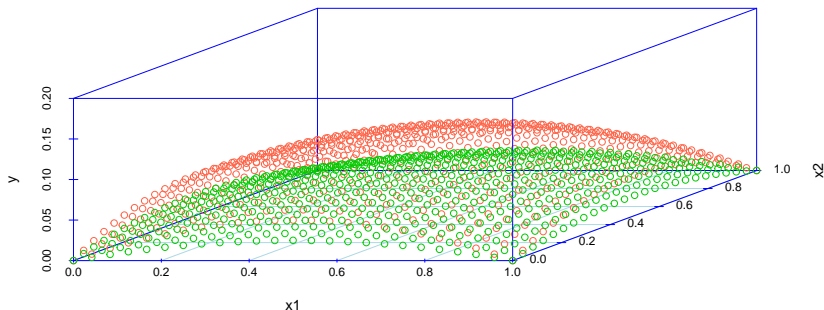


Example 1:  $\text{BetaT} = (0.0, 0.1, -0.1, 0.1, -0.1)$

# Expected Posterior Probabilities, example 1

- ▶  $H_0 : \beta \sim N(\mu_0, V_0)$ 
  - ▶  $y = \beta_0 + x_1\beta_1 + x_2\beta_2 + \varepsilon_i$
  - ▶  $\mu_0 = (0, 0, 0)^T$
- ▶  $H_1 : \beta \sim N(\mu_1, V_1)$ 
  - ▶  $y = \beta_0 + x_1\beta_1 + x_1^2\beta_2 + x_2\beta_3 + x_2^2\beta_4 + \varepsilon_i$
  - ▶  $\mu_1 = (0, -0.2, 0.2, -0.2, 0.2)^T$
- ▶ True Model:  $\beta_T = (0, 0.1, -0.1, 0.1, -0.1)^T$ 
  - ▶  $y = \beta_{0T} + x_1\beta_{1T} + x_1^2\beta_{2T} + x_2\beta_{3T} + x_2^2\beta_{4T} + \varepsilon_i$

Plane vs. Paraboloid





## Expected Posterior Probabilities, example 1

	$E[P(H_0 Y,D) BT]$	$E[P(H_1 Y,D) BT]$	$E[BF_{01} BT]$
1atT,k=1	0.59681	0.40319	4.00232
1atT,k=4	0.58552	0.41448	5.08085
1atT,k=50	0.55667	0.44333	6.64282
Fast	0.54542	0.45458	6.75271
DOptH0	0.65826	0.34174	1.92654
Space	0.54699	0.45301	19.40618
Random	0.56834	0.43166	9.27737
Center	0.50724	0.49276	1.03319
C+S	0.44832	0.55168	9.98731
DOptH1	0.47406	0.52594	46.44659

# Estimates of Beta: Beta\_LS | H0

	$E[B s_0 BT, H_0]$	$E[B s_1 BT, H_0]$	$E[B s_2 BT, H_0]$	$MSE H_0$
beta_true	0.00000	0.10000	0.10000	0.00000
ls,k=1	0.04015	0.00130	0.00534	0.02262
ls,k=4	0.02946	-0.01313	0.03698	0.01905
ls,k=50	0.03940	-0.01868	0.02607	0.02275
ls,fast	0.05230	0.00558	-0.02182	0.02859
ls,dopt	-0.00452	-0.00264	0.00262	0.02087
ls,space	0.02265	0.00906	0.00826	0.01838
ls,random	0.03761	-0.00762	0.00053	0.02390
ls,center	NA	NA	NA	NA
ls,c+s	0.04377	-0.01412	0.00316	0.02776
ls,doptH1	0.01752	-0.00564	0.00108	0.02176

# Estimates of Beta: Beta\_n | H0

	$E[B_{n0} BT, H_0]$	$E[B_{n1} BT, H_0]$	$E[B_{n2} BT, H_0]$	$MSE H_0$
beta_true	0.00000	0.10000	0.10000	0.00000
pm,k=1	0.03704	0.00436	0.00777	0.02038
pm,k=4	0.02879	-0.00778	0.03246	0.01780
pm,k=50	0.03623	-0.01163	0.02476	0.02044
pm,fast	0.04665	0.00849	-0.01441	0.02505
pm,dopt	-0.00439	-0.00262	0.00244	0.02081
pm,space	0.02221	0.00924	0.00851	0.01804
pm,random	0.03459	-0.00449	0.00257	0.02235
pm,center	0.03331	0.01666	0.01666	0.01506
pm,c+s	0.03862	-0.00769	0.00627	0.02389
pm,doptH1	0.01677	-0.00486	0.00147	0.02141

# Estimates of Beta: Beta\_LS | H1

	E[Bls0 BT,H1]	E[Bls1 BT,H1]	E[Bls2 BT,H1]	E[Bls3 BT,H1]	E[Bls4 BT,H1]	MSE H1
beta_true	0.00000	0.10000	-0.10000	0.10000	-0.10000	0.00000
ls,k=1	0.00281	0.13904	-0.13755	0.04996	-0.04463	0.05523
ls,k=4	0.01385	-0.00568	-0.00811	0.10451	-0.06720	0.04811
ls,k=50	-0.00939	0.04090	-0.06283	0.19982	-0.16999	0.10679
ls,fast	0.01473	0.07926	-0.07353	0.09257	-0.11442	0.04683
ls,dopt	NA	NA	NA	NA	NA	NA
ls,space	-0.01648	0.15492	-0.14586	0.12654	-0.11828	0.02829
ls,random	0.01076	0.06357	-0.06601	0.06492	-0.06148	0.02659
ls,center	NA	NA	NA	NA	NA	NA
ls,c+s	0.00499	0.04891	-0.06303	0.12656	-0.12340	0.01822
ls,doptH1	0.00376	0.08774	-0.09380	0.07860	-0.07794	0.00579

# Estimates of Beta: Beta\_n | H1

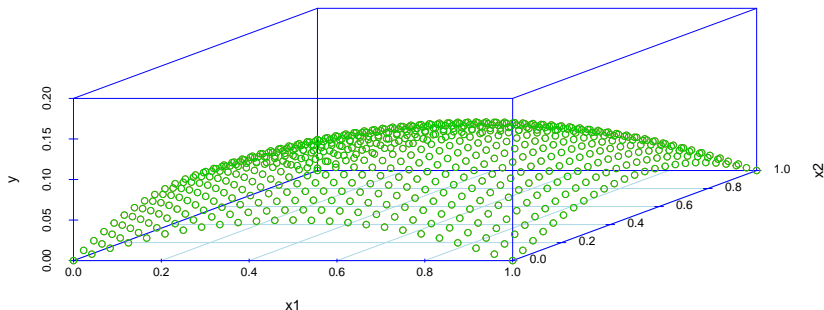
	E[Bn0 BT,H1]	E[Bn1 BT,H1]	E[Bn2 BT,H1]	E[Bn3 BT,H1]	E[Bn4 BT,H1]	MSE H1
beta_true	0.00000	0.10000	-0.10000	0.10000	-0.10000	0.00000
pm,k=1	-0.03444	0.19270	-0.19392	0.18505	-0.18110	0.03397
pm,k=4	-0.04138	0.16900	-0.18201	0.20138	-0.16825	0.02893
pm,k=50	-0.03671	0.17067	-0.19152	0.20850	-0.18350	0.03515
pm,fast	-0.02384	0.18559	-0.17894	0.17573	-0.19543	0.03073
pm,dopt	-0.00439	0.19866	-0.20134	0.20124	-0.19876	0.04051
pm,space	-0.02923	0.18855	-0.17980	0.18062	-0.17190	0.02886
pm,random	-0.03407	0.17744	-0.17042	0.17635	-0.16724	0.02348
pm,center	-0.03040	0.18480	-0.20760	0.18480	-0.20760	0.03852
pm,c+s	-0.02907	0.15427	-0.16097	0.17110	-0.16451	0.01889
doptH1	-0.00940	0.15342	-0.15734	0.15175	-0.14886	0.01229

Example 2:  $\text{BetaT} = (0.0, 0.2, -0.2, 0.2, -0.2)$

## Expected Posterior Probabilities, example 2

- ▶  $H_0 : \beta \sim N(\mu_0, V_0)$ 
  - ▶  $y = \beta_0 + x_1\beta_1 + x_2\beta_2 + \varepsilon_i$
  - ▶  $\mu_0 = (0, 0, 0)^T$
- ▶  $H_1 : \beta \sim N(\mu_1, V_1)$ 
  - ▶  $y = \beta_0 + x_1\beta_1 + x_1^2\beta_2 + x_2\beta_3 + x_2^2\beta_4 + \varepsilon_i$
  - ▶  $\mu_1 = (0, -0.2, 0.2, -0.2, 0.2)^T$
- ▶ True Model:  $\beta_T = (0, 0.1, -0.1, 0.1, -0.1)^T$ 
  - ▶  $y = \beta_{0T} + x_1\beta_{1T} + x_1^2\beta_{2T} + x_2\beta_{3T} + x_2^2\beta_{4T} + \varepsilon_i$

Plane vs. Paraboloid



## Expected Posterior Probabilities, example 2

	$E[P(H_0 Y,D) BT]$	$E[P(H_1 Y,D) BT]$	$E[BF_{01} BT]$
1atT,k=1	0.27105	0.72895	0.61174
1atT,k=4	0.24143	0.75857	0.58337
1atT,k=50	0.22110	0.77890	0.87618
Fast	0.17772	0.82228	0.49243
DOptH0	0.65826	0.34174	1.92654
Space	0.04498	0.95502	0.07953
Random	0.11167	0.88833	0.22117
Center	0.34993	0.65007	0.53980
C+S	0.01124	0.98876	0.01230
DOptH1	0.00997	0.99003	0.01432



## Estimates of Beta: Beta\_LS | H0

	$E[B s_0 BT, H_0]$	$E[B s_1 BT, H_0]$	$E[B s_2 BT, H_0]$	$MSE H_0$
beta_true	0.00000	0.20000	0.20000	0.00000
ls,k=1	0.08080	0.00153	0.00585	0.08592
ls,k=4	0.07100	-0.01171	0.03601	0.07817
ls,k=50	0.08128	-0.01617	0.02319	0.08625
ls,fast	0.09234	0.00522	-0.02163	0.09768
ls,dopt	-0.00452	-0.00264	0.00262	0.08087
ls,space	0.05228	0.00906	0.00826	0.07714
ls,random	0.07984	-0.01594	-0.00409	0.09566
ls,center	NA	NA	NA	NA
ls,c+s	0.08537	-0.01412	0.00316	0.09533
ls,doptH1	0.03354	-0.00515	0.00156	0.08309

# Estimates of Beta: Beta\_n | H0

	$E[Bn0 BT, H0]$	$E[Bn1 BT, H0]$	$E[Bn2 BT, H0]$	$MSE H0$
beta_true	0.00000	0.20000	0.20000	0.00000
pm,k=1	0.07405	0.00784	0.01160	0.07925
pm,k=4	0.06636	-0.00308	0.03539	0.07354
pm,k=50	0.07394	-0.00591	0.02623	0.07907
pm,fast	0.08328	0.01124	-0.01125	0.08861
pm,dopt	-0.00439	-0.00262	0.00244	0.08085
pm,space	0.05031	0.01050	0.00977	0.07557
pm,random	0.07314	-0.00936	0.00086	0.08959
pm,center	0.06643	0.03321	0.03321	0.06011
pm,c+s	0.07622	-0.00408	0.00988	0.08561
pm,doptH1	0.03224	-0.00397	0.00236	0.08213

# Estimates of Beta: Beta\_LS | H1

	E[Bls0 BT,H1]	E[Bls1 BT,H1]	E[Bls2 BT,H1]	E[Bls3 BT,H1]	E[Bls4 BT,H1]	MSE H1
beta_true	0.00000	0.20000	-0.20000	0.20000	-0.20000	0.00000
ls,k=1	0.00281	0.23904	-0.23755	0.14996	-0.14463	0.05523
ls,k=4	0.01385	0.09432	-0.10811	0.20451	-0.16720	0.04811
ls,k=50	-0.00939	0.14090	-0.16283	0.29982	-0.26999	0.10679
ls,fast	0.01473	0.17926	-0.17353	0.19257	-0.21442	0.04683
ls,dopt	NA	NA	NA	NA	NA	NA
ls,space	-0.01648	0.25492	-0.24586	0.22654	-0.21828	0.02829
ls,random	0.01076	0.16357	-0.16601	0.16492	-0.16148	0.02659
ls,center	NA	NA	NA	NA	NA	NA
ls,c+s	0.00499	0.14891	-0.16303	0.22656	-0.22340	0.01822
ls,doptH1	0.00376	0.18774	-0.19380	0.17860	-0.17794	0.00579

# Estimates of Beta: Beta\_n | H1

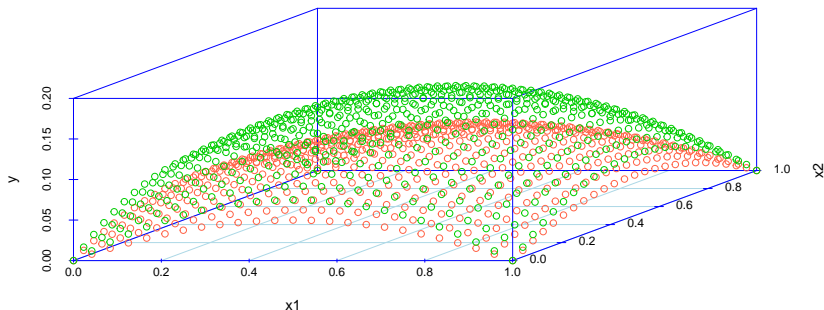
	E[Bn0 BT,H1]	E[Bn1 BT,H1]	E[Bn2 BT,H1]	E[Bn3 BT,H1]	E[Bn4 BT,H1]	MSE H1
beta_true	0.00000	0.20000	-0.20000	0.20000	-0.20000	0.00000
pm,k=1	0.00034	0.20539	-0.20539	0.19683	-0.19116	0.00171
pm,k=4	-0.00640	0.18352	-0.19462	0.21471	-0.18137	0.00170
pm,k=50	-0.00151	0.18496	-0.20285	0.22125	-0.19749	0.00238
pm,fast	0.00941	0.20083	-0.19421	0.19214	-0.21163	0.00208
pm,dopt	-0.00439	0.19866	-0.20134	0.20124	-0.19876	0.00051
pm,space	-0.00712	0.21730	-0.21028	0.20937	-0.20238	0.00267
pm,random	-0.00176	0.19413	-0.19349	0.19570	-0.19049	0.00120
pm,center	0.00019	0.20009	-0.19995	0.20009	-0.19995	0.00006
pm,c+s	0.00116	0.18666	-0.19746	0.20348	-0.20101	0.00237
pm,doptH1	0.00182	0.19349	-0.19915	0.19182	-0.19067	0.00120

Example 3:  $\text{BetaT} = (0.0, 0.3, -0.3, 0.3, -0.3)$

## Expected Posterior Probabilities, example 3

- ▶  $H_0 : \beta \sim N(\mu_0, V_0)$ 
  - ▶  $y = \beta_0 + x_1\beta_1 + x_2\beta_2 + \varepsilon_i$
  - ▶  $\mu_0 = (0, 0, 0)^T$
- ▶  $H_1 : \beta \sim N(\mu_1, V_1)$ 
  - ▶  $y = \beta_0 + x_1\beta_1 + x_1^2\beta_2 + x_2\beta_3 + x_2^2\beta_4 + \varepsilon_i$
  - ▶  $\mu_1 = (0, -0.2, 0.2, -0.2, 0.2)^T$
- ▶ True Model:  $\beta_T = (0, 0.3, -0.3, 0.3, -0.3)^T$ 
  - ▶  $y = \beta_{0T} + x_1\beta_{1T} + x_1^2\beta_{2T} + x_2\beta_{3T} + x_2^2\beta_{4T} + \varepsilon_i$

Plane vs. Paraboloid



## Expected Posterior Probabilities, example 3

	$E[P(H_0 Y,D) BT]$	$E[P(H_1 Y,D) BT]$	$E[BF_{01} BT]$
1atT,k=1	0.06809	0.93191	0.08627
1atT,k=4	0.04873	0.95127	0.05963
1atT,k=50	0.05318	0.94682	0.10521
Fast	0.02470	0.97530	0.02960
DOptH0	0.65826	0.34174	1.92654
Space	0.00012	0.99988	0.00012
Random	0.00313	0.99687	0.00322
Center	0.21536	0.78464	0.27497
C+S	0.00000	1.00000	0.00000
DOptH1	0.00000	1.00000	0.00000

## Estimates of Beta: Beta\_LS | H0

	$E[B s_0 BT, H_0]$	$E[B s_1 BT, H_0]$	$E[B s_2 BT, H_0]$	$MSE H_0$
beta_true	0.00000	0.30000	0.30000	0.00000
ls,k=1	0.12144	0.00176	0.00636	0.19222
ls,k=4	0.11255	-0.01029	0.03505	0.18056
ls,k=50	0.12317	-0.01366	0.02030	0.19343
ls,fast	0.13238	0.00486	-0.02144	0.21005
ls,dopt	-0.00452	-0.00264	0.00262	0.18087
ls,space	0.08191	0.00906	0.00826	0.17765
ls,random	0.12207	-0.02426	-0.00870	0.21635
ls,center	NA	NA	NA	NA
ls,c+s	0.12697	-0.01412	0.00316	0.20635
doptH1	0.04956	-0.00467	0.00205	0.18456



# Estimates of Beta: Beta\_n | H0

	$E[B_n0 BT, H0]$	$E[B_n1 BT, H0]$	$E[B_n2 BT, H0]$	$MSE H0$
beta_true	0.00000	0.30000	0.30000	0.00000
pm,k=1	0.11105	0.01131	0.01543	0.17800
pm,k=4	0.10394	0.00162	0.03833	0.16910
pm,k=50	0.11165	-0.00019	0.02769	0.17774
pm,fast	0.11991	0.01398	-0.00810	0.19252
pm,dopt	-0.00439	-0.00262	0.00244	0.18088
pm,space	0.07840	0.01176	0.01102	0.17368
pm,random	0.11170	-0.01423	-0.00085	0.20248
pm,center	0.09954	0.04977	0.04977	0.13520
pm,c+s	0.11383	-0.00046	0.01350	0.18733
pm,doptH1	0.04771	-0.00309	0.00324	0.18263

# Estimates of Beta: Beta\_LS | H1

	$E[\text{Bls0} \text{BT}, \text{H1}]$	$E[\text{Bls1} \text{BT}, \text{H1}]$	$E[\text{Bls2} \text{BT}, \text{H1}]$	$E[\text{Bls3} \text{BT}, \text{H1}]$	$E[\text{Bls4} \text{BT}, \text{H1}]$	$\text{MSE} \text{H1}$
beta_true	0.00000	0.30000	-0.30000	0.30000	-0.30000	0.00000
ls,k=1	0.00281	0.33904	-0.33755	0.24996	-0.24463	0.05523
ls,k=4	0.01385	0.19432	-0.20811	0.30451	-0.26720	0.04811
ls,k=50	-0.00939	0.24090	-0.26283	0.39982	-0.36999	0.10679
ls,fast	0.01473	0.27926	-0.27353	0.29257	-0.31442	0.04683
ls,dopt	NA	NA	NA	NA	NA	NA
ls,space	-0.01648	0.35492	-0.34586	0.32654	-0.31828	0.02829
ls,random	0.01076	0.26357	-0.26601	0.26492	-0.26148	0.02659
ls,center	NA	NA	NA	NA	NA	NA
ls,c+s	0.00499	0.24891	-0.26303	0.32656	-0.32340	0.01822
ls,doptH1	0.00376	0.28774	-0.29380	0.27860	-0.27794	0.00579

# Estimates of Beta: Beta\_n | H1

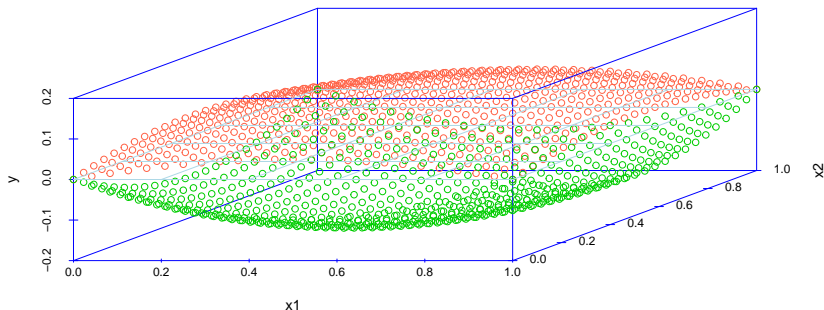
	$E[B_n0 BT,H1]$	$E[B_n1 BT,H1]$	$E[B_n2 BT,H1]$	$E[B_n3 BT,H1]$	$E[B_n4 BT,H1]$	$MSE H1$
beta_true	0.00000	0.30000	-0.30000	0.30000	-0.30000	0.00000
pm,k=1	0.03513	0.21808	-0.21686	0.20862	-0.20122	0.03453
pm,k=4	0.02859	0.19803	-0.20723	0.22805	-0.19448	0.03693
pm,k=50	0.03369	0.19925	-0.21418	0.23400	-0.21148	0.03252
pm,fast	0.04265	0.21607	-0.20948	0.20856	-0.22782	0.03239
pm,dopt	-0.00439	0.19866	-0.20134	0.20124	-0.19876	0.04051
pm,space	0.01499	0.24604	-0.24076	0.23811	-0.23286	0.01711
pm,random	0.03054	0.21083	-0.21656	0.21504	-0.21375	0.03151
pm,center	0.03077	0.21538	-0.19231	0.21538	-0.19231	0.03852
pm,c+s	0.03139	0.21904	-0.23396	0.23586	-0.23750	0.02209
pm,doptH1	0.01304	0.23355	-0.24096	0.23188	-0.23248	0.01827

Example 4:  $\text{BetaT} = (0.0, -0.4, 0.4, -0.4, 0.4)$

## Expected Posterior Probabilities, example 4

- ▶  $H_0 : \beta \sim N(\mu_0, V_0)$ 
  - ▶  $y = \beta_0 + x_1\beta_1 + x_2\beta_2 + \varepsilon_i$
  - ▶  $\mu_0 = (0, 0, 0)^T$
- ▶  $H_1 : \beta \sim N(\mu_1, V_1)$ 
  - ▶  $y = \beta_0 + x_1\beta_1 + x_1^2\beta_2 + x_2\beta_3 + x_2^2\beta_4 + \varepsilon_i$
  - ▶  $\mu_1 = (0, -0.2, 0.2, -0.2, 0.2)^T$
- ▶ True Model:  $\beta_T = (0, -0.4, 0.4, -0.4, 0.4)^T$ 
  - ▶  $y = \beta_{0T} + x_1\beta_{1T} + x_1^2\beta_{2T} + x_2\beta_{3T} + x_2^2\beta_{4T} + \varepsilon_i$

Plane vs. Paraboloid



## Expected Posterior Probabilities, example 4

	$E[P(H_0 Y,D) BT]$	$E[P(H_1 Y,D) BT]$	$E[BF_{01} BT]$
1atT,k=1	0.99979	0.00021	1.416959e+04
1atT,k=4	0.99992	0.00008	4.574658e+04
1atT,k=50	0.99988	0.00012	3.813961e+04
Fast	0.99997	0.00003	1.958643e+05
DOpt	0.65826	0.34174	1.926540e+00
Space	1.00000	0.00000	1.767159e+07
Random	1.00000	0.00000	7.795890e+05
Center	0.94759	0.05241	1.815886e+01
C+S	1.00000	0.00000	2.991799e+07
NA	1.00000	0.00000	8.990292e+06

## Estimates of Beta: Beta\_LS | H0

	$E[B s_0 BT, H_0]$	$E[B s_1 BT, H_0]$	$E[B s_2 BT, H_0]$	$MSE H_0$
beta_true	0.00000	-0.40000	-0.40000	0.00000
ls,k=1	-0.16308	0.00016	0.00279	0.35127
ls,k=4	-0.17827	-0.02025	0.04180	0.37259
ls,k=50	-0.17002	-0.03124	0.04050	0.36058
ls,fast	-0.14789	0.00736	-0.02275	0.33223
ls,dopt	-0.00452	-0.00264	0.00262	0.32085
ls,space	-0.12549	0.00906	0.00826	0.35094
ls,random	-0.17355	0.03400	0.02360	0.39891
ls,center	NA	NA	NA	NA
ls,c+s	-0.16423	-0.01412	0.00316	0.34186
ls,doptH1	-0.06258	-0.00806	-0.00135	0.31695

# Estimates of Beta: Beta\_n | H0

	$E[B_n0 BT, H_0]$	$E[B_n1 BT, H_0]$	$E[B_n2 BT, H_0]$	$MSE H_0$
beta_true	0.00000	-0.40000	-0.40000	0.00000
pm,k=1	-0.14797	-0.01301	-0.01139	0.32403
pm,k=4	-0.15908	-0.03126	0.01777	0.33660
pm,k=50	-0.15230	-0.04023	0.01746	0.32791
pm,fast	-0.13649	-0.00525	-0.03019	0.31263
pm,dopt	-0.00439	-0.00262	0.00244	0.32062
pm,space	-0.11825	0.00297	0.00223	0.33910
pm,random	-0.15819	0.01986	0.01112	0.37107
pm,center	-0.13225	-0.06612	-0.06612	0.24050
pm,c+s	-0.14942	-0.02577	-0.01181	0.31507
pm,doptH1	-0.06058	-0.00928	-0.00295	0.31441



# Estimates of Beta: Beta\_LS | H1

	E[Bls0 BT,H1]	E[Bls1 BT,H1]	E[Bls2 BT,H1]	E[Bls3 BT,H1]	E[Bls4 BT,H1]	MSE H1
beta_true	0.00000	-0.40000	0.40000	-0.40000	0.40000	0.00000
ls,k=1	0.00281	-0.36096	0.36245	-0.45004	0.45537	0.05523
ls,k=4	0.01385	-0.50568	0.49189	-0.39549	0.43280	0.04811
ls,k=50	-0.00939	-0.45910	0.43717	-0.30018	0.33001	0.10679
ls,fast	0.01473	-0.42074	0.42647	-0.40743	0.38558	0.04683
ls,dopt	NA	NA	NA	NA	NA	NA
ls,space	-0.01648	-0.34508	0.35414	-0.37346	0.38172	0.02829
ls,random	0.01076	-0.43643	0.43399	-0.43508	0.43852	0.02659
ls,center	NA	NA	NA	NA	NA	NA
ls,c+s	0.00499	-0.45109	0.43697	-0.37344	0.37660	0.01822
ls,doptH1	0.00376	-0.41226	0.40620	-0.42140	0.42206	0.00579

# Estimates of Beta: Beta\_n | H1

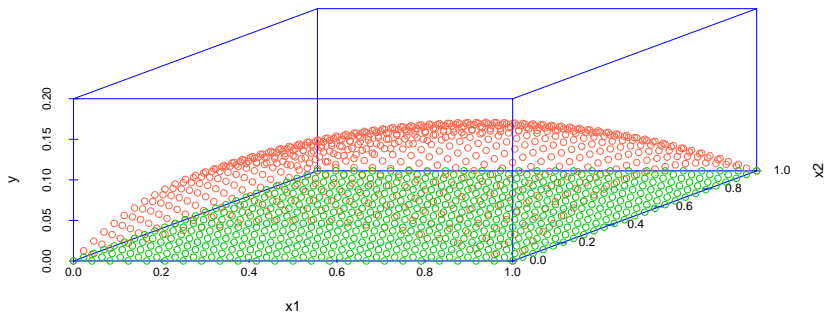
	E[Bn0 BT,H1]	E[Bn1 BT,H1]	E[Bn2 BT,H1]	E[Bn3 BT,H1]	E[Bn4 BT,H1]	MSE H1
beta_true	0.00000	-0.40000	0.40000	-0.40000	0.40000	0.00000
pm,k=1	-0.20837	0.12924	-0.13657	0.12613	-0.13079	1.17153
pm,k=4	-0.21631	0.09641	-0.11895	0.13471	-0.10267	1.10191
pm,k=50	-0.21272	0.09924	-0.13485	0.14476	-0.11356	1.14274
pm,fast	-0.19005	0.10941	-0.10258	0.09365	-0.11445	1.05832
pm,dopt	-0.00439	0.19866	-0.20134	0.20124	-0.19876	1.44051
pm,space	-0.13980	0.04484	-0.02740	0.03691	-0.01950	0.76909
pm,random	-0.19559	0.09397	-0.05505	0.07961	-0.05098	0.92375
pm,center	-0.18330	0.10835	-0.24583	0.10835	-0.24583	1.38468
pm,c+s	-0.18020	-0.00764	0.02150	0.00918	0.01796	0.64523
pm,doptH1	-0.06550	-0.04690	0.05173	-0.04857	0.06021	0.49022

Example 5:  $\text{BetaT} = (0, 0, 0)$

# Expected Posterior Probabilities, example 5

- ▶  $H_0 : \beta \sim N(\mu_0, V_0)$ 
  - ▶  $y = \beta_0 + x_1\beta_1 + x_2\beta_2 + \varepsilon_i$
  - ▶  $\mu_0 = (0, 0, 0)^T$
- ▶  $H_1 : \beta \sim N(\mu_1, V_1)$ 
  - ▶  $y = \beta_0 + x_1\beta_1 + x_1^2\beta_2 + x_2\beta_3 + x_2^2\beta_4 + \varepsilon_i$
  - ▶  $\mu_1 = (0, -0.2, 0.2, -0.2, 0.2)^T$
- ▶ True Model:  $\beta_T = (0, 0, 0)^T$ 
  - ▶  $y = \beta_{0T} + x_1\beta_{1T} + x_2\beta_{2T} + \varepsilon_i$

Plane vs. Paraboloid



## Expected Posterior Probabilities, example 5

	$E[P(H_0 Y,D) BT]$	$E[P(H_1 Y,D) BT]$	$E[BF_{01} BT]$
1atT,k=1	0.85876	0.14124	24.15041
1atT,k=4	0.86820	0.13180	39.44015
1atT,k=50	0.85158	0.14842	45.78680
Fast	0.86541	0.13459	76.50011
DOptH0	0.65826	0.34174	1.92654
Space	0.95144	0.04856	1831.47025
Random	0.93213	0.06787	237.92666
Center	0.65750	0.34250	1.92815
C+S	0.95304	0.04696	2150.59760
DoptH1	0.98300	0.01700	19864.80940