Posterior Probabilities of Hypotheses

Kristyn Pantoja

9/13/2019

First, Review Posterior Probabilities of Hypotheses

Example 1: BetaT = (0.0, 0.1, -0.1, 0.1, -0.1)

Example 2: BetaT = (0.0, 0.2, -0.2, 0.2, -0.2)

Example 3: BetaT = (0.0, 0.3, -0.3, 0.3, -0.3)

Example 4: BetaT = (0.0, -0.4, 0.4, -0.4, 0.4)

Example 5: BetaT = (0, 0, 0)

First, Review Posterior Probabilities of Hypotheses

Posterior Probabilities of Hypotheses

- Posterior Probability of model $H_\ell, \ell \in \{0,1\}$:

$$P(H_\ell|y,X) = \frac{\pi_\ell f(y|H_\ell,X)}{\pi_0 f(y|H_0,X) + \pi_1 f(y|H_1,X)}$$
$$= \frac{f(y|H_\ell,X)}{f(y|H_0,X) + f(y|H_1,X)}$$

  Since we assume $\pi_0, \pi_1 = \frac{1}{2}$.
- $f(y|H_\ell,X) = \phi_{\ell,X}(y)$ is the model evidence, i.e. density of $N_N(X\mu_\ell, \sigma_\varepsilon^2 I + XV_\ell X^T)$ evaluated at a given y and design **D** with N design points.
- $E[P(H_\ell|y,X)|H_r,X]$ may be estimated using MC approximation from simulated responses y under a chosen hypothesis H_r.

Estimate Expected Posterior Probability of a Hypothesis

Estimate the expected posterior probability of hypothesis H_ℓ for J simulations of Y under H_r, given design $\mathbf{D} = \{x_1, ..., x_N\}$:

1. For $j = 1, \ldots, J$:
   1.1 Draw $y_i^{(j)}|\mathbf{x}_i \sim N(\mathbf{x}_i^T \beta_T, \sigma_\varepsilon^2)$, $\forall \mathbf{x}_i \in \mathbf{D}$, so $y^{(j)} \in R^N$.
   1.2 $\forall m = \{0, 1\}$, calculate model evidences $f(y|H_m, \mathbf{D})$
   1.3 Calculate the posterior probability of H_ℓ, $P(H_\ell|y^{(j)}, \mathbf{D})$, from simulation j

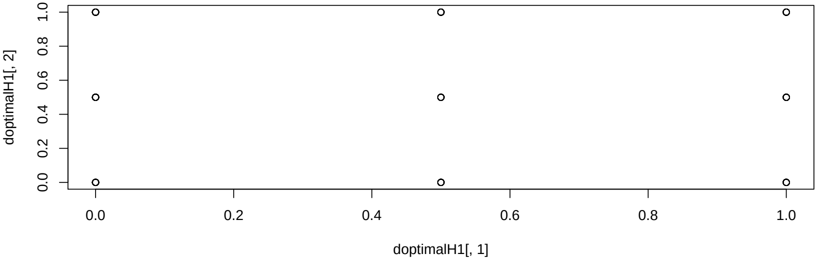
$$P(H_\ell|y^{(j)}, \mathbf{D}) = \frac{f(y^{(j)}|H_\ell, X)}{f(y^{(j)}|H_0, X) + f(y^{(j)}|H_1, X)}$$

2. Average the estimated posterior probabilities of H_ℓ over $\forall j$ to obtain MC estimate of $E[P(H_\ell|y, \mathbf{D})|H_r, \mathbf{D}]$

Note that $y^{(j)}$ are generated from $N_N(X\beta_T, \sigma_\varepsilon^2 I)$ and are independent, while the model evidence for H_m marginalizes out β and evaluates $y^{(j)}$ using $f(y|H_m, \mathbf{D})$, the density of $N_N(X\mu_m, \sigma_\varepsilon^2 I + XV_m X^T)$, in which they are no longer assumed to be independent.

Additional Designs

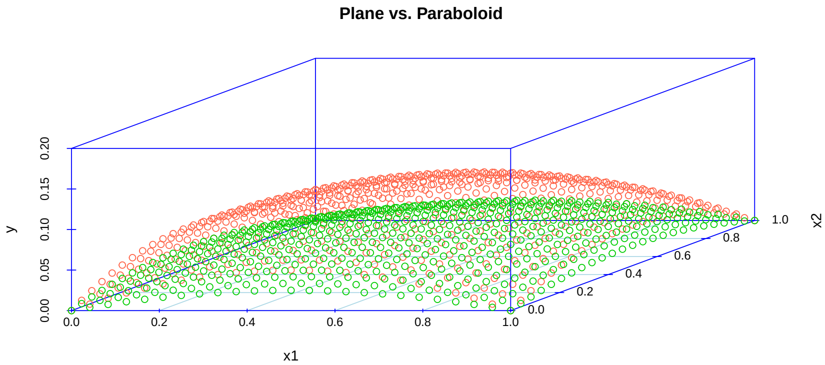
- A center design with all points at (0.5, 0.5)
- A mixture of the center design and the space-filling design with 36 space-filling lattice points and the rest of the points at (0.5, 0.5).
- D-optimal design for quadratic model



Example 1: BetaT = (0.0, 0.1, -0.1, 0.1, -0.1)

Expected Posterior Probabilities, example 1

- $H_0 : \beta \sim N(\mu_0, V_0)$
  - $y = \beta_0 + x_1\beta_1 + x_2\beta_2 + \varepsilon_i$
  - $\mu_0 = (0, 0, 0)^T$
- $H_1 : \beta \sim N(\mu_1, V_1)$
  - $y = \beta_0 + x_1\beta_1 + x_1^2\beta_2 + x_2\beta_3 + x_2^2\beta_4 + \varepsilon_i$
  - $\mu_1 = (0, -0.2, 0.2, -0.2, 0.2)^T$
- True Model: $\beta_T = (0, 0.1, -0.1, 0.1, -0.1)^T$
  - $y = \beta_{0T} + x_1\beta_{T1} + x_1^2\beta_{T2} + x_2\beta_{T3} + x_2^2\beta_{T4} + \varepsilon_i$



Expected Posterior Probabilities, example 1

	E[P(H0|Y,D)|BT]	E[P(H1|Y,D)|BT]	E[BF01|BT]
1atT,k=1	0.59681	0.40319	4.00232
1atT,k=4	0.58552	0.41448	5.08085
1atT,k=50	0.55667	0.44333	6.64282
Fast	0.54542	0.45458	6.75271
DOptH0	0.65826	0.34174	1.92654
Space	0.54699	0.45301	19.40618
Random	0.56834	0.43166	9.27737
Center	0.50724	0.49276	1.03319
C+S	0.44832	0.55168	9.98731
DOptH1	0.47406	0.52594	46.44659

Estimates of Beta: Beta_LS | H0

	E[Bls0|BT,H0]	E[Bls1|BT,H0]	E[Bls2|BT,H0]	MSE|H0
beta_true	0.00000	0.10000	0.10000	0.00000
ls,k=1	0.04082	0.00121	-0.00090	0.02479
ls,k=4	0.03679	0.01124	-0.00178	0.02273
ls,k=50	0.04129	0.00673	-0.00466	0.02431
ls,fast	0.03539	0.00109	0.00428	0.02229
ls,dopt	-0.00164	0.00046	0.00146	0.02019
ls,space	0.02732	0.00402	-0.00005	0.02117
ls,random	0.04128	-0.01029	-0.00001	0.02608
ls,center	NA	NA	NA	NA
ls,c+s	0.04214	-0.00354	0.00308	0.02459
ls,doptH1	0.01853	0.00132	-0.00173	0.02104

Estimates of Beta: Beta_n | H0

	E[Bn0|BT,H0]	E[Bn1|BT,H0]	E[Bn2|BT,H0]	MSE|H0
beta_true	0.00000	0.10000	0.10000	0.00000
pm,k=1	0.03712	0.00429	0.00270	0.02192
pm,k=4	0.03404	0.01228	0.00196	0.02029
pm,k=50	0.03738	0.00909	-0.00002	0.02139
pm,fast	0.03279	0.00364	0.00626	0.02048
pm,dopt	-0.00156	0.00041	0.00138	0.02016
pm,space	0.02607	0.00482	0.00112	0.02046
pm,random	0.03782	-0.00663	0.00229	0.02392
pm,center	0.03235	0.01617	0.01617	0.01512
pm,c+s	0.03806	0.00080	0.00615	0.02173
pm,doptH1	0.01786	0.00174	-0.00114	0.02073

Estimates of Beta: Beta_LS | H1

	E[Bls0|BT,H1]	E[Bls1|BT,H1]	E[Bls2|BT,H1]	E[Bls3|BT,H1]	E[Bls4|BT,H1]	MSE|H1
beta_true	0.00000	0.10000	-0.10000	0.10000	-0.10000	0.00000
ls,k=1	0.00909	0.09042	-0.08929	0.06527	-0.06645	0.07023
ls,k=4	-0.00320	0.10430	-0.09445	0.09723	-0.09808	0.09022
ls,k=50	0.00338	0.08968	-0.08526	0.09374	-0.09574	0.09939
ls,fast	0.00434	0.09590	-0.09438	0.06454	-0.06053	0.06601
ls,dopt	NA	NA	NA	NA	NA	NA
ls,space	0.00058	0.09027	-0.08625	0.09419	-0.09424	0.03196
ls,random	0.00324	0.09467	-0.09783	0.08756	-0.08348	0.06089
ls,center	NA	NA	NA	NA	NA	NA
ls,c+s	0.00317	0.09758	-0.10113	0.08932	-0.08624	0.04658
ls,doptH1	0.00082	0.11254	-0.11175	0.10705	-0.10932	0.01730

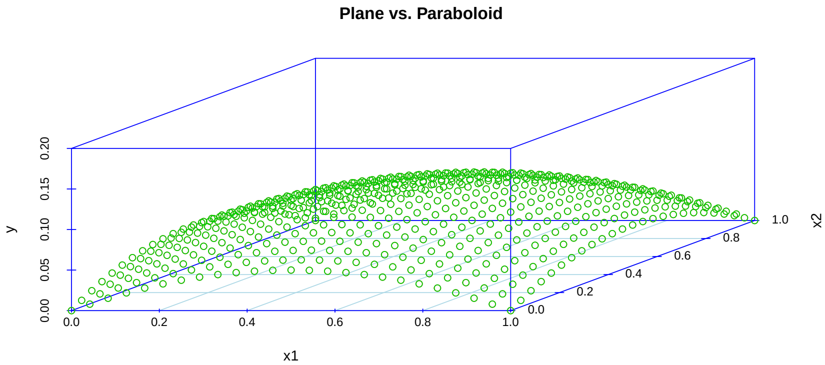
Estimates of Beta: Beta_n | H1

	E[Bn0|BT,H1]	E[Bn1|BT,H1]	E[Bn2|BT,H1]	E[Bn3|BT,H1]	E[Bn4|BT,H1]	MSE|H1
beta_true	0.00000	0.10000	-0.10000	0.10000	-0.10000	0.00000
pm,k=1	-0.03431	0.18750	-0.18772	0.18486	-0.18715	0.03323
pm,k=4	-0.03804	0.18936	-0.18271	0.18568	-0.18685	0.03327
pm,k=50	-0.03523	0.18648	-0.18537	0.18608	-0.18637	0.03304
pm,fast	-0.03612	0.18495	-0.18378	0.18008	-0.17582	0.02972
pm,dopt	-0.00157	0.20021	-0.19979	0.20070	-0.19930	0.04035
pm,space	-0.02343	0.16987	-0.16405	0.16957	-0.16778	0.02198
pm,random	-0.03235	0.18163	-0.17733	0.18025	-0.17183	0.02803
pm,center	-0.03129	0.18436	-0.20782	0.18436	-0.20782	0.03848
pm,c+s	-0.02935	0.16511	-0.16385	0.16648	-0.15891	0.01974
doptH1	-0.00930	0.16519	-0.16288	0.16218	-0.16274	0.01910

Example 2: BetaT $=$ (0.0, 0.2, -0.2, 0.2, -0.2)

Expected Posterior Probabilities, example 2

- $H_0 : \beta \sim N(\mu_0, V_0)$
  - $y = \beta_0 + x_1\beta_1 + x_2\beta_2 + \varepsilon_i$
  - $\mu_0 = (0, 0, 0)^T$
- $H_1 : \beta \sim N(\mu_1, V_1)$
  - $y = \beta_0 + x_1\beta_1 + x_1^2\beta_2 + x_2\beta_3 + x_2^2\beta_4 + \varepsilon_i$
  - $\mu_1 = (0, -0.2, 0.2, -0.2, 0.2)^T$
- True Model: $\beta_T = (0, 0.1, -0.1, 0.1, -0.1)^T$
  - $y = \beta_{0T} + x_1\beta_{T1} + x_1^2\beta_{T2} + x_2\beta_{T3} + x_2^2\beta_{T4} + \varepsilon_i$



Expected Posterior Probabilities, example 2

	E[P(H0|Y,D)|BT]	E[P(H1|Y,D)|BT]	E[BF01|BT]
1atT,k=1	0.27105	0.72895	0.61174
1atT,k=4	0.24143	0.75857	0.58337
1atT,k=50	0.22110	0.77890	0.87618
Fast	0.17772	0.82228	0.49243
DOptH0	0.65826	0.34174	1.92654
Space	0.04498	0.95502	0.07953
Random	0.11167	0.88833	0.22117
Center	0.34993	0.65007	0.53980
C+S	0.01124	0.98876	0.01230
DOptH1	0.00997	0.99003	0.01432

Estimates of Beta: Beta_LS | H0

	E[Bls0|BT,H0]	E[Bls1|BT,H0]	E[Bls2|BT,H0]	MSE|H0
beta_true	0.00000	0.20000	0.20000	0.00000
ls,k=1	0.08146	0.00144	-0.00038	0.08940
ls,k=4	0.07833	0.01266	-0.00274	0.08548
ls,k=50	0.08317	0.00924	-0.00754	0.08933
ls,fast	0.07543	0.00073	0.00447	0.08572
ls,dopt	-0.00164	0.00046	0.00146	0.07981
ls,space	0.05695	0.00402	-0.00005	0.08287
ls,random	0.08352	-0.01861	-0.00462	0.09885
ls,center	NA	NA	NA	NA
ls,c+s	0.08374	-0.00354	0.00308	0.08992
ls,doptH1	0.03455	0.00181	-0.00125	0.08158

Estimates of Beta: Beta_n | H0

	E[Bn0|BT,H0]	E[Bn1|BT,H0]	E[Bn2|BT,H0]	MSE|H0
beta_true	0.00000	0.20000	0.20000	0.00000
pm,k=1	0.07412	0.00776	0.00653	0.08180
pm,k=4	0.07161	0.01698	0.00490	0.07852
pm,k=50	0.07509	0.01481	0.00144	0.08108
pm,fast	0.06942	0.00639	0.00942	0.07996
pm,dopt	-0.00156	0.00041	0.00138	0.07981
pm,space	0.05416	0.00608	0.00238	0.08053
pm,random	0.07638	-0.01150	0.00058	0.09190
pm,center	0.06546	0.03273	0.03273	0.06026
pm,c+s	0.07566	0.00441	0.00976	0.08180
pm,doptH1	0.03333	0.00262	-0.00026	0.08070

Estimates of Beta: Beta_LS | H1

	E[Bls0|BT,H1]	E[Bls1|BT,H1]	E[Bls2|BT,H1]	E[Bls3|BT,H1]	E[Bls4|BT,H1]	MSE|H1
beta_true	0.00000	0.20000	-0.20000	0.20000	-0.20000	0.00000
ls,k=1	0.00909	0.19042	-0.18929	0.16527	-0.16645	0.07023
ls,k=4	-0.00320	0.20430	-0.19445	0.19723	-0.19808	0.09022
ls,k=50	0.00338	0.18968	-0.18526	0.19374	-0.19574	0.09939
ls,fast	0.00434	0.19590	-0.19438	0.16454	-0.16053	0.06601
ls,dopt	NA	NA	NA	NA	NA	NA
ls,space	0.00058	0.19027	-0.18625	0.19419	-0.19424	0.03196
ls,random	0.00324	0.19467	-0.19783	0.18756	-0.18348	0.06089
ls,center	NA	NA	NA	NA	NA	NA
ls,c+s	0.00317	0.19758	-0.20113	0.18932	-0.18624	0.04658
ls,doptH1	0.00082	0.21254	-0.21175	0.20705	-0.20932	0.01730

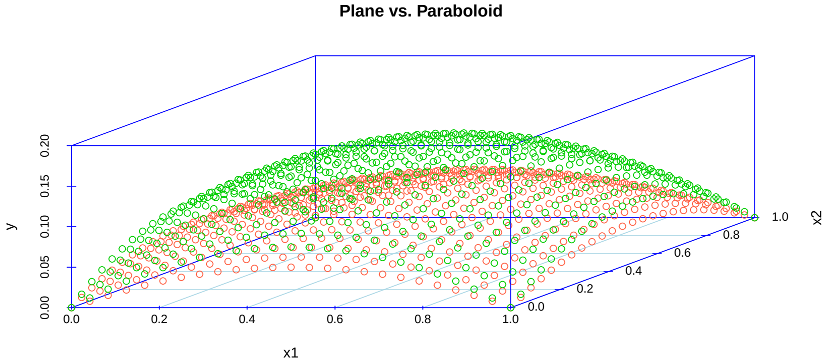
Estimates of Beta: Beta_n | H1

	E[Bn0|BT,H1]	E[Bn1|BT,H1]	E[Bn2|BT,H1]	E[Bn3|BT,H1]	E[Bn4|BT,H1]	MSE|H1
beta_true	0.00000	0.20000	-0.20000	0.20000	-0.20000	0.00000
pm,k=1	0.00048	0.20019	-0.19919	0.19665	-0.19722	0.00192
pm,k=4	-0.00305	0.20388	-0.19532	0.19901	-0.19997	0.00216
pm,k=50	-0.00003	0.20077	-0.19670	0.19883	-0.20036	0.00218
pm,fast	-0.00288	0.20019	-0.19905	0.19649	-0.19201	0.00210
pm,dopt	-0.00157	0.20021	-0.19979	0.20070	-0.19930	0.00035
pm,space	-0.00131	0.19862	-0.19453	0.19831	-0.19826	0.00305
pm,random	-0.00005	0.19833	-0.20040	0.19960	-0.19508	0.00277
pm,center	-0.00071	0.19965	-0.20018	0.19965	-0.20018	0.00002
pm,c+s	0.00088	0.19750	-0.20034	0.19886	-0.19541	0.00270
pm,doptH1	0.00191	0.20525	-0.20470	0.20224	-0.20455	0.00309

Example 3: BetaT = (0.0, 0.3, -0.3, 0.3, -0.3)

Expected Posterior Probabilities, example 3

- $H_0 : \beta \sim N(\mu_0, V_0)$
  - $y = \beta_0 + x_1\beta_1 + x_2\beta_2 + \varepsilon_i$
  - $\mu_0 = (0, 0, 0)^T$
- $H_1 : \beta \sim N(\mu_1, V_1)$
  - $y = \beta_0 + x_1\beta_1 + x_1^2\beta_2 + x_2\beta_3 + x_2^2\beta_4 + \varepsilon_i$
  - $\mu_1 = (0, -0.2, 0.2, -0.2, 0.2)^T$
- True Model: $\beta_T = (0, 0.3, -0.3, 0.3, -0.3)^T$
  - $y = \beta_{0T} + x_1\beta_{T1} + x_1^2\beta_{T2} + x_2\beta_{T3} + x_2^2\beta_{T4} + \varepsilon_i$



Expected Posterior Probabilities, example 3

	E[P(H0|Y,D)|BT]	E[P(H1|Y,D)|BT]	E[BF01|BT]
1atT,k=1	0.06809	0.93191	0.08627
1atT,k=4	0.04873	0.95127	0.05963
1atT,k=50	0.05318	0.94682	0.10521
Fast	0.02470	0.97530	0.02960
DOptH0	0.65826	0.34174	1.92654
Space	0.00012	0.99988	0.00012
Random	0.00313	0.99687	0.00322
Center	0.21536	0.78464	0.27497
C+S	0.00000	1.00000	0.00000
DOptH1	0.00000	1.00000	0.00000

Estimates of Beta: Beta_LS | H0

	E[Bls0|BT,H0]	E[Bls1|BT,H0]	E[Bls2|BT,H0]	MSE|H0
beta_true	0.00000	0.30000	0.30000	0.00000
ls,k=1	0.12211	0.00167	0.00013	0.19702
ls,k=4	0.11988	0.01409	-0.00371	0.19150
ls,k=50	0.12505	0.01175	-0.01043	0.19804
ls,fast	0.11547	0.00038	0.00465	0.19243
ls,dopt	-0.00164	0.00046	0.00146	0.17942
ls,space	0.08658	0.00402	-0.00005	0.18633
ls,random	0.12575	-0.02694	-0.00924	0.22054
ls,center	NA	NA	NA	NA
ls,c+s	0.12534	-0.00354	0.00308	0.19871
doptH1	0.05057	0.00229	-0.00076	0.18225

Estimates of Beta: Beta_n | H0

	E[Bn0|BT,H0]	E[Bn1|BT,H0]	E[Bn2|BT,H0]	MSE|H0
beta_true	0.00000	0.30000	0.30000	0.00000
pm,k=1	0.11112	0.01124	0.01036	0.18154
pm,k=4	0.10918	0.02167	0.00784	0.17657
pm,k=50	0.11280	0.02053	0.00290	0.18082
pm,fast	0.10605	0.00914	0.01258	0.17979
pm,dopt	-0.00156	0.00041	0.00138	0.17945
pm,space	0.08226	0.00734	0.00364	0.18119
pm,random	0.11493	-0.01637	-0.00113	0.20555
pm,center	0.09857	0.04929	0.04929	0.13545
pm,c+s	0.11327	0.00803	0.01338	0.18186
pm,doptH1	0.04880	0.00350	0.00063	0.18044

Estimates of Beta: Beta_LS | H1

	E[Bls0|BT,H1]	E[Bls1|BT,H1]	E[Bls2|BT,H1]	E[Bls3|BT,H1]	E[Bls4|BT,H1]	MSE|H1
beta_true	0.00000	0.30000	-0.30000	0.30000	-0.30000	0.00000
ls,k=1	0.00909	0.29042	-0.28929	0.26527	-0.26645	0.07023
ls,k=4	-0.00320	0.30430	-0.29445	0.29723	-0.29808	0.09022
ls,k=50	0.00338	0.28968	-0.28526	0.29374	-0.29574	0.09939
ls,fast	0.00434	0.29590	-0.29438	0.26454	-0.26053	0.06601
ls,dopt	NA	NA	NA	NA	NA	NA
ls,space	0.00058	0.29027	-0.28625	0.29419	-0.29424	0.03196
ls,random	0.00324	0.29467	-0.29783	0.28756	-0.28348	0.06089
ls,center	NA	NA	NA	NA	NA	NA
ls,c+s	0.00317	0.29758	-0.30113	0.28932	-0.28624	0.04658
ls,doptH1	0.00082	0.31254	-0.31175	0.30705	-0.30932	0.01730

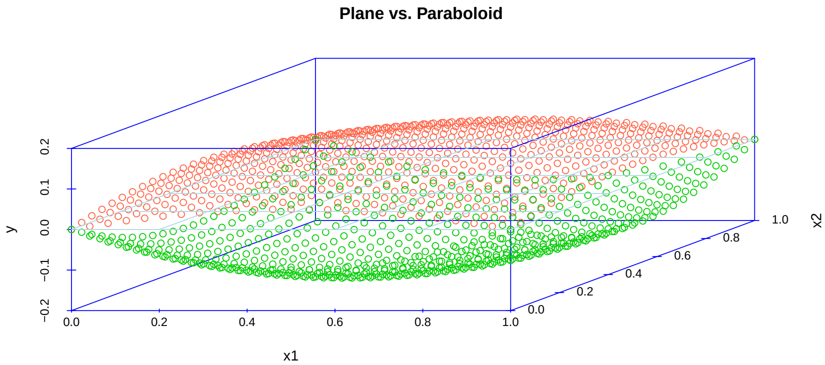
Estimates of Beta: Beta_n | H1

	E[Bn0|BT,H1]	E[Bn1|BT,H1]	E[Bn2|BT,H1]	E[Bn3|BT,H1]	E[Bn4|BT,H1]	MSE|H1
beta_true	0.00000	0.30000	-0.30000	0.30000	-0.30000	0.00000
pm,k=1	0.03526	0.21288	-0.21066	0.20843	-0.20728	0.03570
pm,k=4	0.03194	0.21839	-0.20794	0.21235	-0.21308	0.03350
pm,k=50	0.03517	0.21505	-0.20804	0.21158	-0.21435	0.03423
pm,fast	0.03037	0.21542	-0.21432	0.21291	-0.20821	0.03344
pm,dopt	-0.00157	0.20021	-0.19979	0.20070	-0.19930	0.04035
pm,space	0.02080	0.22736	-0.22501	0.22706	-0.22874	0.02475
pm,random	0.03226	0.21502	-0.22348	0.21894	-0.21833	0.03010
pm,center	0.02987	0.21494	-0.19253	0.21494	-0.19253	0.03848
pm,c+s	0.03111	0.22988	-0.23684	0.23125	-0.23190	0.02191
pm,doptH1	0.01313	0.24532	-0.24651	0.24231	-0.24637	0.01524

Example 4: BetaT = (0.0, -0.4, 0.4, -0.4, 0.4)

Expected Posterior Probabilities, example 4

- $H_0 : \beta \sim N(\mu_0, V_0)$
  - $y = \beta_0 + x_1\beta_1 + x_2\beta_2 + \varepsilon_i$
  - $\mu_0 = (0, 0, 0)^T$
- $H_1 : \beta \sim N(\mu_1, V_1)$
  - $y = \beta_0 + x_1\beta_1 + x_1^2\beta_2 + x_2\beta_3 + x_2^2\beta_4 + \varepsilon_i$
  - $\mu_1 = (0, -0.2, 0.2, -0.2, 0.2)^T$
- True Model: $\beta_T = (0, -0.4, 0.4, -0.4, 0.4)^T$
  - $y = \beta_{0T} + x_1\beta_{T1} + x_1^2\beta_{T2} + x_2\beta_{T3} + x_2^2\beta_{T4} + \varepsilon_i$



Expected Posterior Probabilities, example 4

	E[P(H0|Y,D)|BT]	E[P(H1|Y,D)|BT]	E[BF01|BT]
1atT,k=1	0.99979	0.00021	1.416959e+04
1atT,k=4	0.99992	0.00008	4.574658e+04
1atT,k=50	0.99988	0.00012	3.813961e+04
Fast	0.99997	0.00003	1.958643e+05
DOpt	0.65826	0.34174	1.926540e+00
Space	1.00000	0.00000	1.767159e+07
Random	1.00000	0.00000	7.795890e+05
Center	0.94759	0.05241	1.815886e+01
C+S	1.00000	0.00000	2.991799e+07
NA	1.00000	0.00000	8.990292e+06

Estimates of Beta: Beta_LS | H0

	E[Bls0|BT,H0]	E[Bls1|BT,H0]	E[Bls2|BT,H0]	MSE|H0
beta_true	0.00000	-0.40000	-0.40000	0.00000
ls,k=1	-0.16242	0.00007	-0.00345	0.34687
ls,k=4	-0.17094	0.00412	0.00305	0.35812
ls,k=50	-0.16813	-0.00583	0.00977	0.35450
ls,fast	-0.16480	0.00288	0.00334	0.35424
ls,dopt	-0.00164	0.00046	0.00146	0.32211
ls,space	-0.12083	0.00402	-0.00005	0.33899
ls,random	-0.16988	0.03133	0.02306	0.39609
ls,center	NA	NA	NA	NA
ls,c+s	-0.16586	-0.00354	0.00308	0.34986
ls,doptH1	-0.06157	-0.00111	-0.00416	0.32020

Estimates of Beta: Beta_n | H0

	E[Bn0|BT,H0]	E[Bn1|BT,H0]	E[Bn2|BT,H0]	MSE|H0
beta_true	0.00000	-0.40000	-0.40000	0.00000
pm,k=1	-0.14789	-0.01308	-0.01646	0.32060
pm,k=4	-0.15383	-0.01120	-0.01272	0.32664
pm,k=50	-0.15116	-0.01950	-0.00733	0.32354
pm,fast	-0.15035	-0.01010	-0.00952	0.32844
pm,dopt	-0.00156	0.00041	0.00138	0.32195
pm,space	-0.11439	-0.00145	-0.00515	0.32877
pm,random	-0.15496	0.01772	0.01084	0.36886
pm,center	-0.13322	-0.06661	-0.06661	0.24007
pm,c+s	-0.14998	-0.01728	-0.01193	0.32119
pm,doptH1	-0.05950	-0.00268	-0.00556	0.31751

Estimates of Beta: Beta_LS | H1

	E[Bls0|BT,H1]	E[Bls1|BT,H1]	E[Bls2|BT,H1]	E[Bls3|BT,H1]	E[Bls4|BT,H1]	MSE|H1
beta_true	0.00000	-0.40000	0.40000	-0.40000	0.40000	0.00000
ls,k=1	0.00909	-0.40958	0.41071	-0.43473	0.43355	0.07023
ls,k=4	-0.00320	-0.39570	0.40555	-0.40277	0.40192	0.09022
ls,k=50	0.00338	-0.41032	0.41474	-0.40626	0.40426	0.09939
ls,fast	0.00434	-0.40410	0.40562	-0.43546	0.43947	0.06601
ls,dopt	NA	NA	NA	NA	NA	NA
ls,space	0.00058	-0.40973	0.41375	-0.40581	0.40576	0.03196
ls,random	0.00324	-0.40533	0.40217	-0.41244	0.41652	0.06089
ls,center	NA	NA	NA	NA	NA	NA
ls,c+s	0.00317	-0.40242	0.39887	-0.41068	0.41376	0.04658
ls,doptH1	0.00082	-0.38746	0.38825	-0.39295	0.39068	0.01730

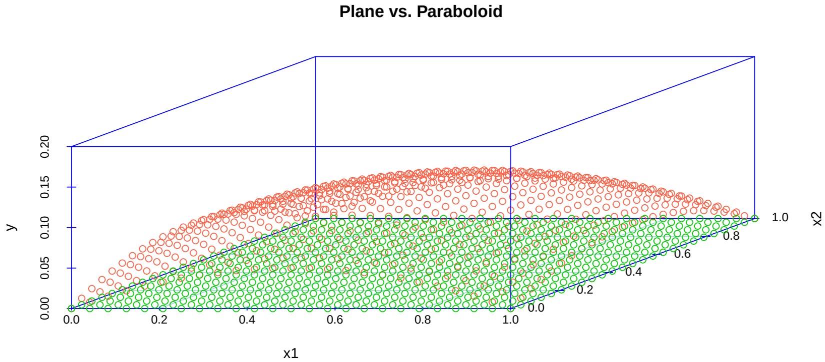
Estimates of Beta: Beta_n | H1

	E[Bn0|BT,H1]	E[Bn1|BT,H1]	E[Bn2|BT,H1]	E[Bn3|BT,H1]	E[Bn4|BT,H1]	MSE|H1
beta_true	0.00000	-0.40000	0.40000	-0.40000	0.40000	0.00000
pm,k=1	-0.20823	0.12404	-0.13038	0.12594	-0.13684	1.16600
pm,k=4	-0.21297	0.11677	-0.11966	0.11901	-0.12127	1.12565
pm,k=50	-0.21124	0.11504	-0.12870	0.12234	-0.11642	1.13110
pm,fast	-0.20233	0.10876	-0.10743	0.09800	-0.09484	1.05215
pm,dopt	-0.00157	0.20021	-0.19979	0.20070	-0.19930	1.44035
pm,space	-0.13400	0.02616	-0.01165	0.02585	-0.01538	0.72593
pm,random	-0.19388	0.09816	-0.06196	0.08351	-0.05557	0.94322
pm,center	-0.18419	0.10790	-0.24605	0.10790	-0.24605	1.38463
pm,c+s	-0.18049	0.00320	0.01863	0.00456	0.02356	0.64864
pm,doptH1	-0.06540	-0.03513	0.04618	-0.03815	0.04632	0.52163

Example 5: BetaT $= (0, 0, 0)$

Expected Posterior Probabilities, example 5

- $H_0 : \beta \sim N(\mu_0, V_0)$
  - $y = \beta_0 + x_1\beta_1 + x_2\beta_2 + \varepsilon_i$
  - $\mu_0 = (0, 0, 0)^T$
- $H_1 : \beta \sim N(\mu_1, V_1)$
  - $y = \beta_0 + x_1\beta_1 + x_1^2\beta_2 + x_2\beta_3 + x_2^2\beta_4 + \varepsilon_i$
  - $\mu_1 = (0, -0.2, 0.2, -0.2, 0.2)^T$
- True Model: $\beta_T = (0, 0, 0)^T$
  - $y = \beta_{0T} + x_1\beta_{T1} + x_2\beta_{T2} + \varepsilon_i$



Expected Posterior Probabilities, example 5

	E[P(H0|Y,D)|BT]	E[P(H1|Y,D)|BT]	E[BF01|BT]
1atT,k=1	0.85876	0.14124	24.15041
1atT,k=4	0.86820	0.13180	39.44015
1atT,k=50	0.85158	0.14842	45.78680
Fast	0.86541	0.13459	76.50011
DOptH0	0.65826	0.34174	1.92654
Space	0.95144	0.04856	1831.47025
Random	0.93213	0.06787	237.92666
Center	0.65750	0.34250	1.92815
C+S	0.95304	0.04696	2150.59760
DoptH1	0.98300	0.01700	19864.80940