

2D Linear Model Selection

Kristyn Pantoja

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Example 1

Compare Example 1 MEDs to Other Designs

Example 2

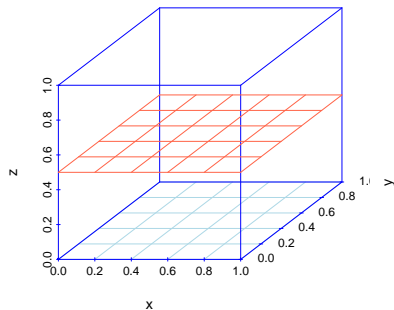
Compare Example 2 MEDs to Other Designs

See Behavior of Candidate Sets in Example 2 Fast MED, $N = 300$

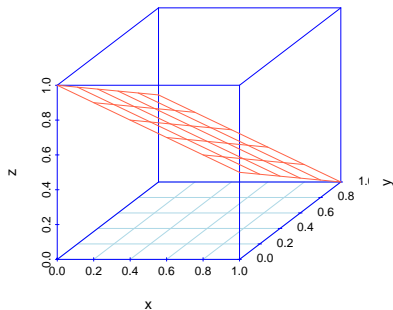
Experiments

Here I consider two examples in which two different models are compared:

Two Planes



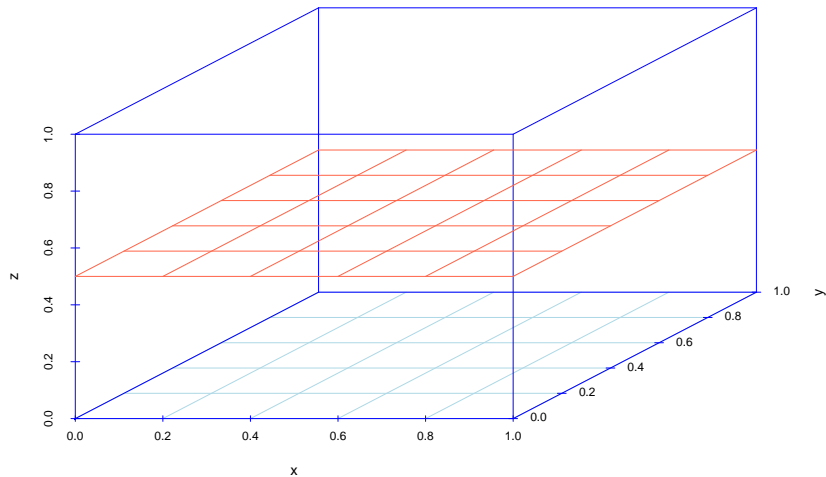
Two Intersecting Planes



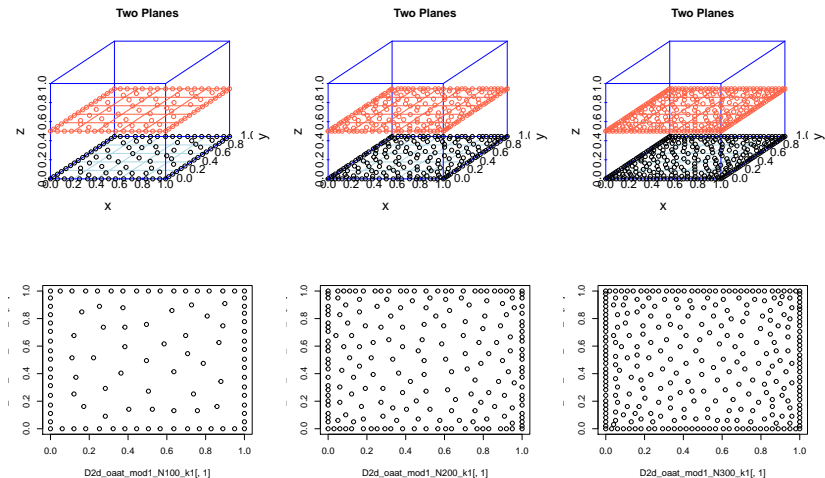
Example 1

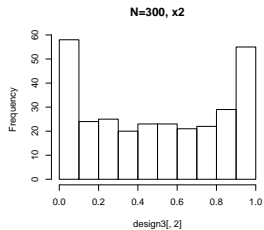
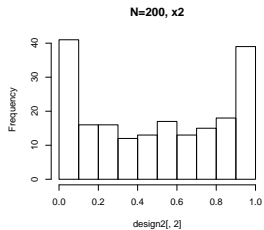
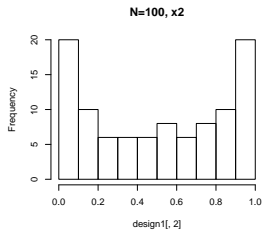
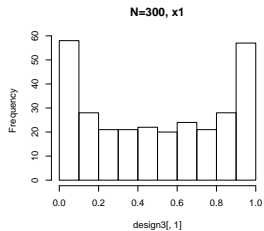
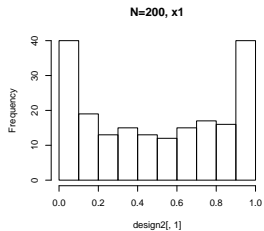
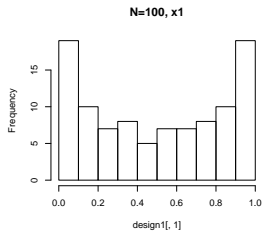
Example 1

Two Planes



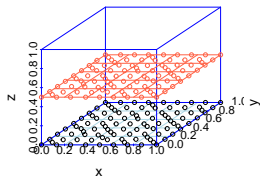
One-at-a-Time Algorithm, $k = 1$, $N = 100, 200, 300$



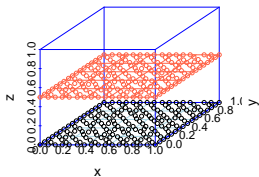


One-at-a-Time Algorithm, $k = 4$, $N = 100, 200, 300$

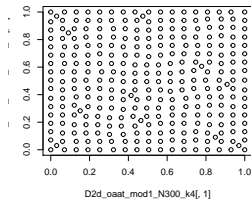
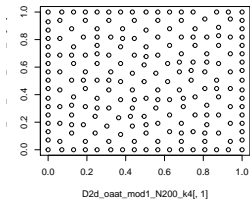
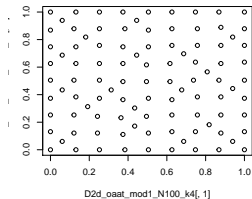
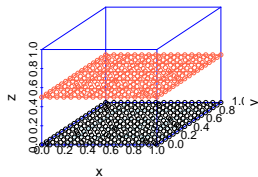
Two Planes

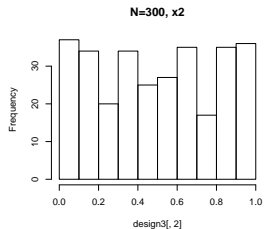
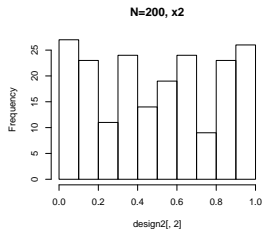
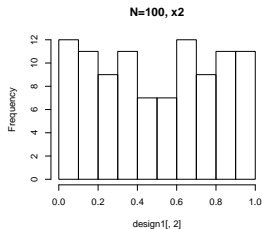
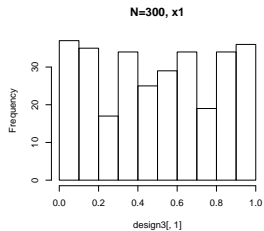
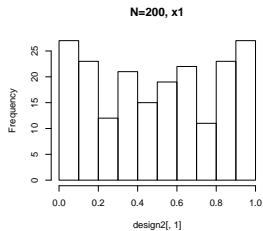
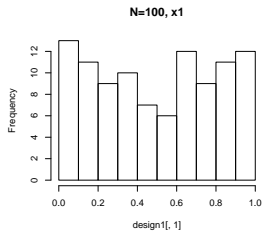


Two Planes



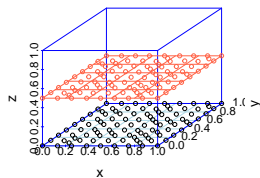
Two Planes



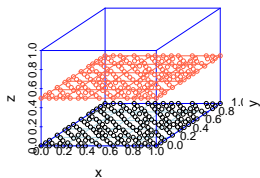


One-at-a-Time Algorithm, $k = 50$, $N = 100, 200, 300$

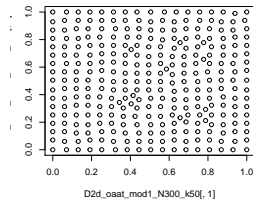
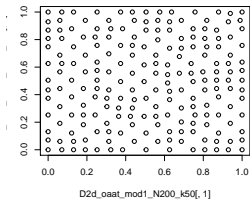
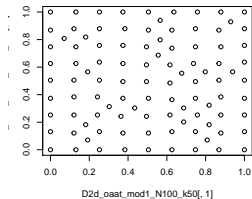
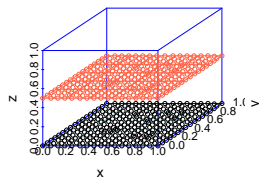
Two Planes

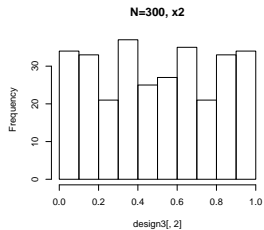
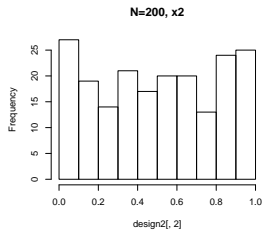
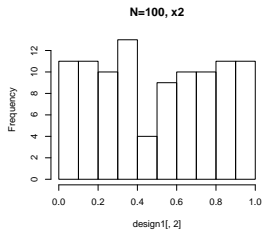
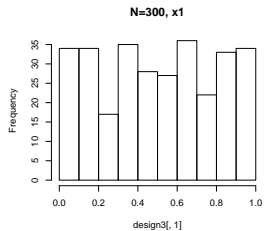
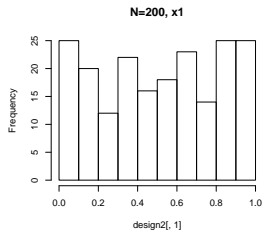
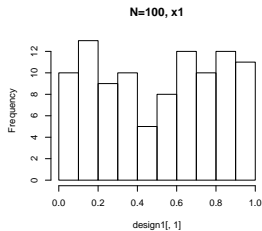


Two Planes



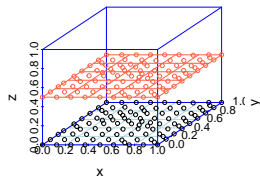
Two Planes



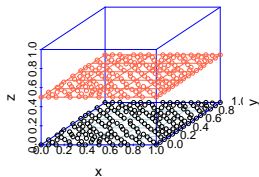


Fast Algorithm, $S = 5$, $N = 100, 200, 300$

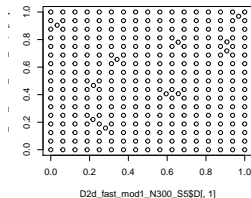
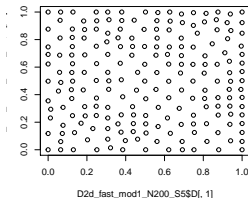
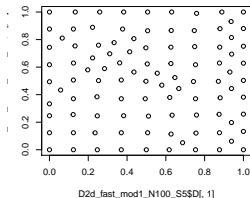
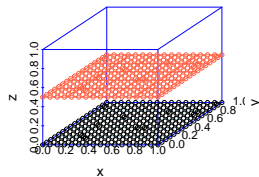
Two Planes

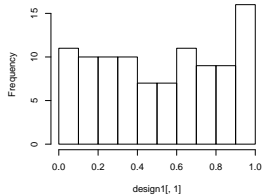
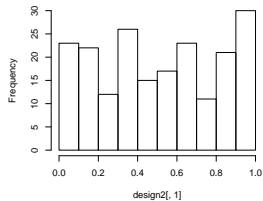
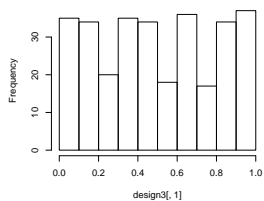
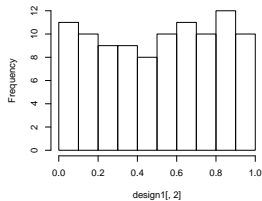
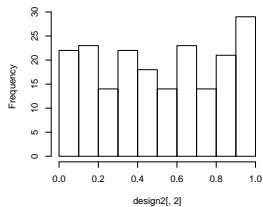
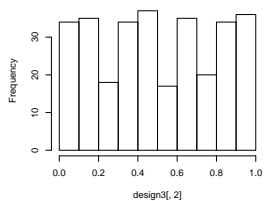


Two Planes



Two Planes



N=100, x1**N=200, x1****N=300, x1****N=100, x2****N=200, x2****N=300, x2**

Example 1 Evaluations, $N = 100$

	1atT,k=1	1atT,k=4	1atT,k=50	Fast,S=5
TPE $\times 10^{-3}$	13.903	14.892	15.103	15.079
Fast Crit	24.946	14.7	14.7	16.255
1atT Crit (k=4)	61.838	48.316	48.542	48.966
$E[P(H_0 Y,D) H_0,D]$	1	0.999	0.998	0.999
$E[P(H_1 Y,D) H_0,D]$	0.000245	0.000955	0.00162	0.00081
$E[BF_{01} H_0,D]$	$2.88e+14$	$4.85e+13$	$8.84e+13$	$9.94e+12$
$E[P(H_0 Y,D) H_1,D]$	0.00011	$2.1e-05$	0.000131	0.000286
$E[P(H_1 Y,D) H_1,D]$	1	1	1	1
$E[BF_{01} H_1,D]$	0.000111	$2.1e-05$	0.000132	0.000293
$V[B_0 Y,X]$	0.000791	0.000885	0.000902	0.000928
$V[B_1 Y,X]$	0.0013	0.00152	0.00156	0.0015
$V[B_2 Y,X]$	0.00128	0.00154	0.00156	0.00159

Example 1 Evaluations, $N = 200$

	1atT,k=1	1atT,k=4	1atT,k=50	Fast,S=5
TPE $\times 10^{-3}$	59.56	62.211	63.801	63.648
Fast Crit	62.366	24.462	20.789	23.261
1atT Crit (k=4)	124.21	84.365	85.319	85.682
$E[P(H_0 Y,D) H_0,D]$	0.996	0.998	0.999	1
$E[P(H_1 Y,D) H_0,D]$	0.00383	0.00179	0.000633	6.04e-05
$E[BF_{01} H_0,D]$	1.99e+17	1.92e+17	6.42e+16	4.65e+16
$E[P(H_0 Y,D) H_1,D]$	3.8e-05	0.000227	6.24e-05	2.93e-05
$E[P(H_1 Y,D) H_1,D]$	1	1	1	1
$E[BF_{01} H_1,D]$	3.8e-05	0.000232	6.27e-05	2.94e-05
$V[B_0 Y,X]$	0.000487	0.000548	0.000574	0.000578
$V[B_1 Y,X]$	0.000796	0.00094	0.000982	0.000969
$V[B_2 Y,X]$	0.000797	0.00095	0.000973	0.000977

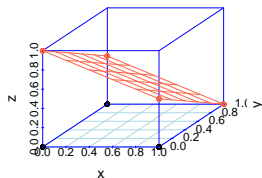
Example 1 Evaluations, $N = 300$

	1atT,k=1	1atT,k=4	1atT,k=50	Fast,S=5
TPE $\times 10^{-3}$	137.15	146.52	148.88	147.7
Fast Crit	62.366	29.4	29.4	30.363
1atT Crit (k=4)	156.82	112.4	112.79	112.97
$E[P(H_0 Y,D) H_0,D]$	1	1	1	1
$E[P(H_1 Y,D) H_0,D]$	$3.11e-06$	$6.13e-07$	$6.6e-07$	$9.95e-07$
$E[BF_{01} H_0,D]$	$8.05e+14$	$2.84e+14$	$2e+14$	$2.24e+14$
$E[P(H_0 Y,D) H_1,D]$	$7.07e-06$	$6.2e-05$	0.00034	$1.9e-05$
$E[P(H_1 Y,D) H_1,D]$	1	1	1	1
$E[BF_{01} H_1,D]$	$7.08e-06$	$6.24e-05$	0.00035	$1.9e-05$
$V[B_0 Y,X]$	0.000354	0.000418	0.00043	0.000422
$V[B_1 Y,X]$	0.000579	0.000723	0.000743	0.000729
$V[B_2 Y,X]$	0.000583	0.000717	0.000742	0.000733

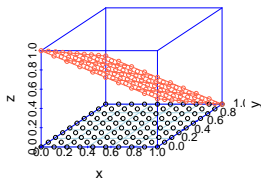
Compare Example 1 MEDs to Other Designs

Other Designs, $N = 100$

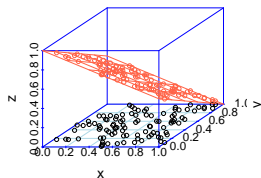
D-Optimal



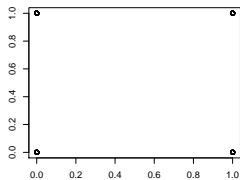
Space-Filling



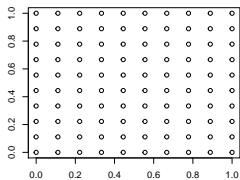
Random



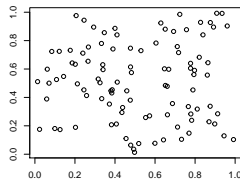
D-Optimal

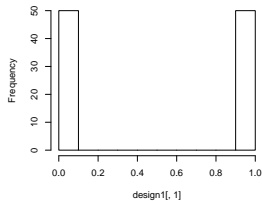
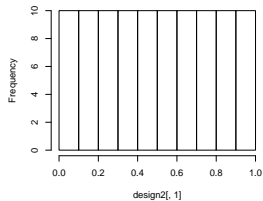
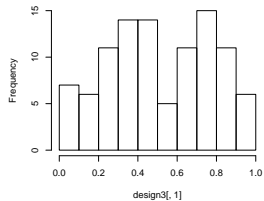
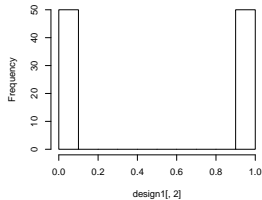
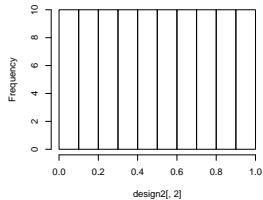
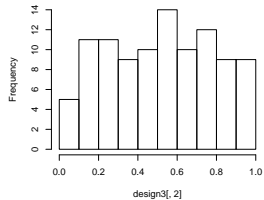


Space-Filling



Random



N=100, x1**N=200, x1****N=300, x1****N=100, x2****N=200, x2****N=300, x2**

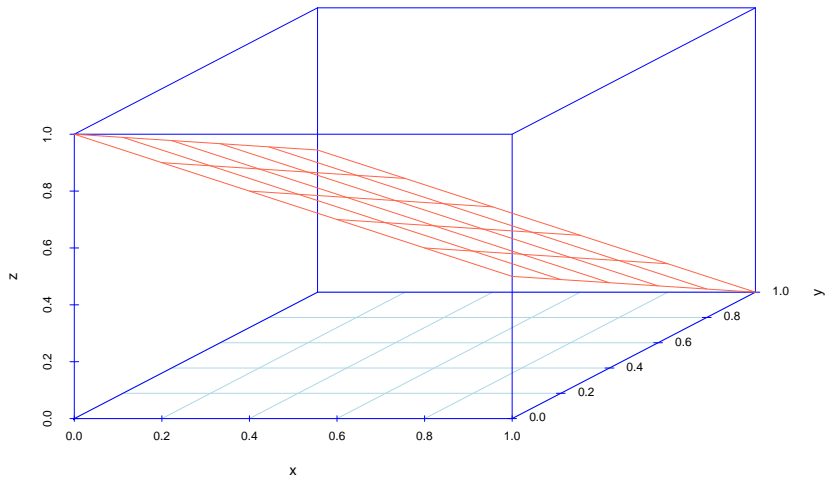
Example 1 Evaluations for Other Designs, $N = 100$

	1atT,k=1	1atT,k=4	1atT,k=50	Fast,S=5	D-Opt	Space	Random
TPEx10e-3	13.903	14.892	15.103	15.079	Inf	14.778	19.295
Fast Crit	24.946	14.7	14.7	16.255	Inf	11.339	129.35
1atT Crit (k=4)	61.838	48.316	48.542	48.966	Inf	45.241	158.09
$E[P(H_0 Y,D) H_0,D]$	1	0.999	0.998	0.999	1	1	1
$E[P(H_1 Y,D) H_0,D]$	0.000245	0.000955	0.00162	0.00081	2.36e-05	0.000436	0.000234
$E[BF_{01} H_0,D]$	2.88e+14	4.85e+13	8.84e+13	9.94e+12	3.35e+14	1.22e+14	1.71e+13
$E[P(H_0 Y,D) H_1,D]$	0.00011	2.1e-05	0.000131	0.000286	5.62e-05	0.000819	0.00109
$E[P(H_1 Y,D) H_1,D]$	1	1	1	1	1	0.999	0.999
$E[BF_{01} H_1,D]$	0.000111	2.1e-05	0.000132	0.000293	5.63e-05	0.000854	0.00121
$V[B_0 Y,X]$	0.000791	0.000885	0.000902	0.000928	0.000588	0.000884	0.00106
$V[B_1 Y,X]$	0.0013	0.00152	0.00156	0.0015	0.000803	0.00153	0.00189
$V[B_2 Y,X]$	0.00128	0.00154	0.00156	0.00159	0.000803	0.00153	0.00186

Example 2

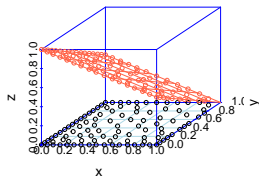
Example 2

Two Intersecting Planes

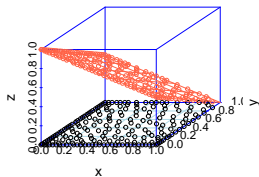


One-at-a-Time Algorithm, $k = 1$, $N = 100, 200, 300$

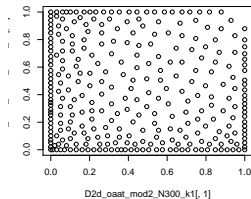
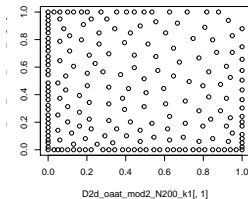
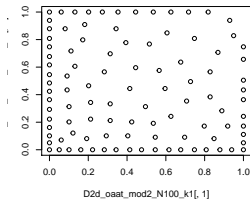
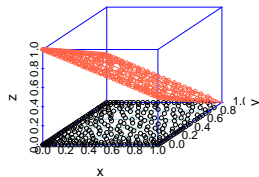
Two Intersecting Planes

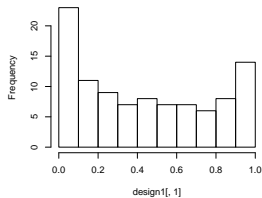
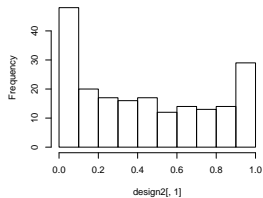
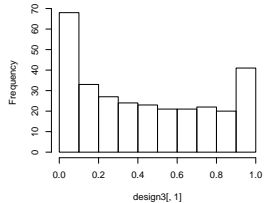
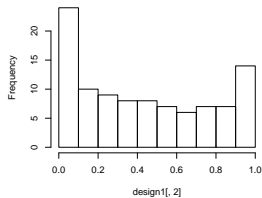
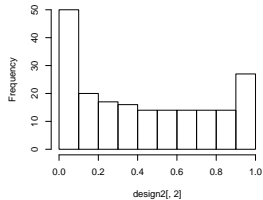
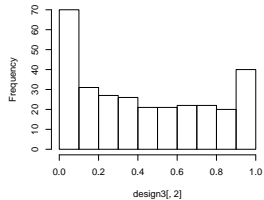


Two Intersecting Planes



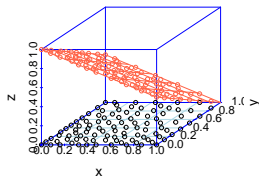
Two Intersecting Planes



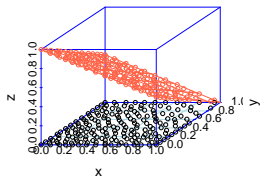
N=100, x1**N=200, x1****N=300, x1****N=100, x2****N=200, x2****N=300, x2**

One-at-a-Time Algorithm, $k = 4$, $N = 100, 200, 300$

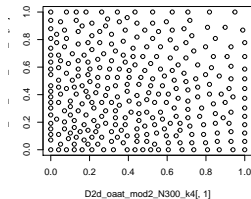
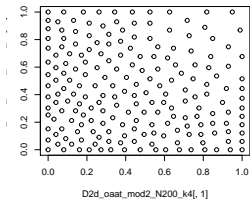
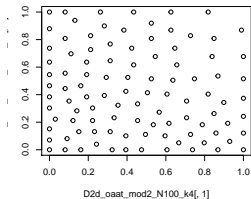
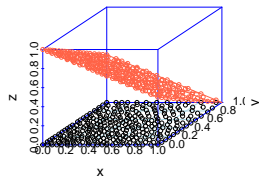
Two Intersecting Planes

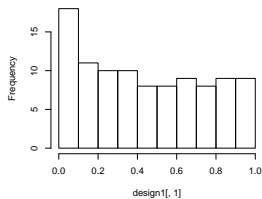
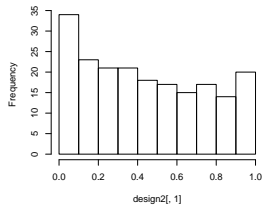
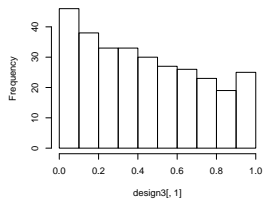
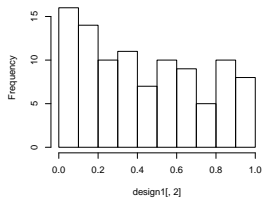
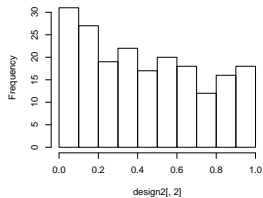
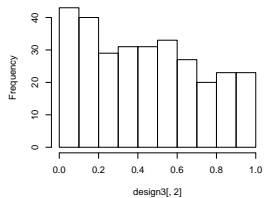


Two Intersecting Planes



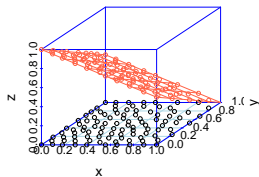
Two Intersecting Planes



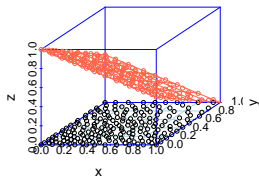
N=100, x1**N=200, x1****N=300, x1****N=100, x2****N=200, x2****N=300, x2**

One-at-a-Time Algorithm, $k = 50$, $N = 100, 200, 300$

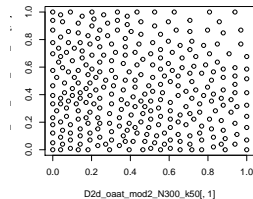
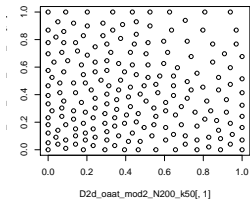
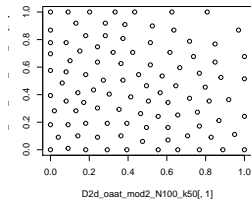
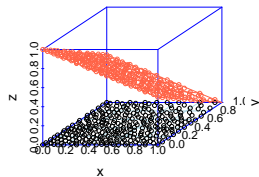
Two Intersecting Planes

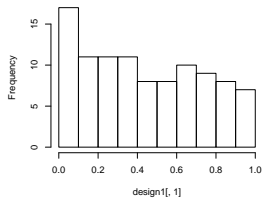
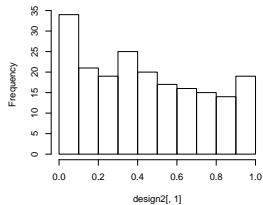
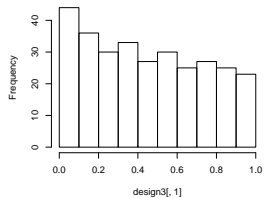
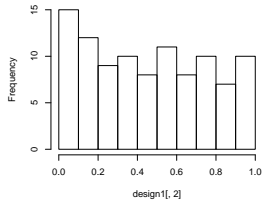
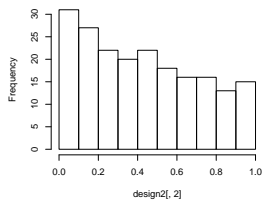
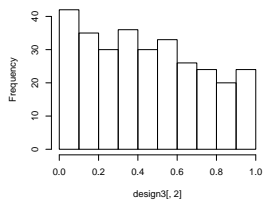


Two Intersecting Planes



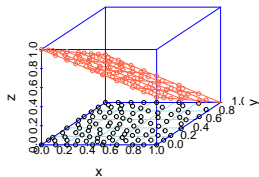
Two Intersecting Planes



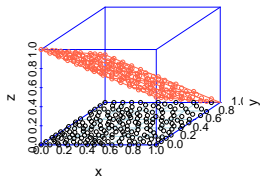
N=100, x1**N=200, x1****N=300, x1****N=100, x2****N=200, x2****N=300, x2**

Fast Algorithm, $S = 5$, $N = 100, 200, 300$

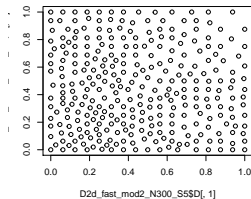
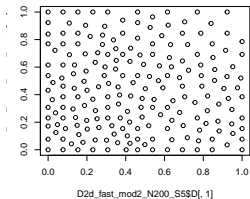
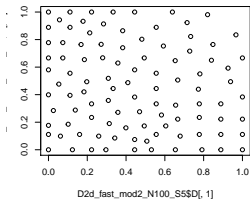
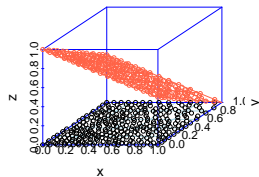
Two Intersecting Planes

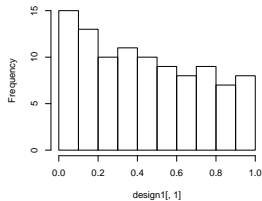
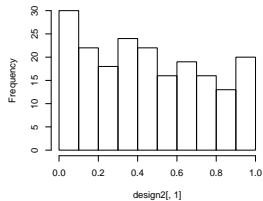
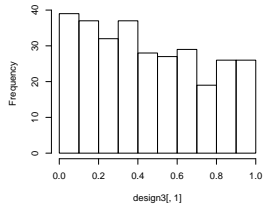
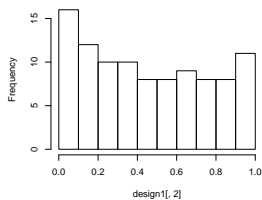
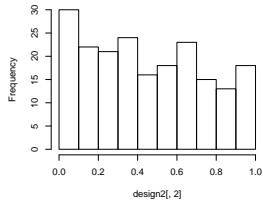
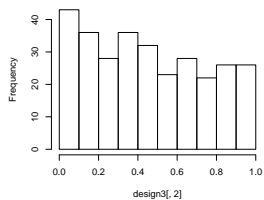


Two Intersecting Planes



Two Intersecting Planes



N=100, x1**N=200, x1****N=300, x1****N=100, x2****N=200, x2****N=300, x2**

Example 2 Evaluations, $N = 100$

	1atT,k=1	1atT,k=4	1atT,k=50	Fast,S=5
TPEx10e-3	14.364	15.186	15.641	15.355
Fast Crit	25.163	16.395	15.294	17.351
1atT Crit (k=4)	56.53	48.13	48.564	49.618
$E[P(H0 Y,D) H0,D]$	1	1	1	1
$E[P(H1 Y,D) H0,D]$	9.04e-34	2.78e-27	1.29e-28	1.75e-27
$E[BF01 H0,D]$	3.23e+55	6.91e+51	5.49e+49	1.74e+50
$E[P(H0 Y,D) H1,D]$	9.68e-29	2.73e-25	3.79e-25	1.42e-27
$E[P(H1 Y,D) H1,D]$	1	1	1	1
$E[BF01 H1,D]$	9.68e-29	2.73e-25	3.79e-25	1.42e-27
$V[B0 Y,X]$	0.000723	0.000788	0.000847	0.000838
$V[B1 Y,X]$	0.0014	0.00159	0.00169	0.00164
$V[B2 Y,X]$	0.0014	0.00162	0.00162	0.00156

Example 2 Evaluations, $N = 200$

	1atT,k=1	1atT,k=4	1atT,k=50	Fast,S=5
TPE $\times 10^{-3}$	61.195	64.639	66.003	66.335
Fast Crit	62.579	24.731	21.455	23.745
1atT Crit (k=4)	122.19	84.472	84.789	85.837
$E[P(H_0 Y,D) H_0,D]$	1	1	1	1
$E[P(H_1 Y,D) H_0,D]$	7.58e-36	1.05e-31	1.59e-28	6.53e-30
$E[BF_{01} H_0,D]$	6.88e+64	2.66e+62	1.52e+66	2.88e+63
$E[P(H_0 Y,D) H_1,D]$	2.78e-36	5.23e-35	2.08e-34	3.89e-35
$E[P(H_1 Y,D) H_1,D]$	1	1	1	1
$E[BF_{01} H_1,D]$	2.78e-36	5.23e-35	2.08e-34	3.89e-35
$V[B_0 Y,X]$	0.000441	0.000516	0.000504	0.000525
$V[B_1 Y,X]$	0.000847	0.00101	0.00104	0.00105
$V[B_2 Y,X]$	0.000847	0.00102	0.00108	0.00106

Example 2 Evaluations, $N = 300$

	1atT,k=1	1atT,k=4	1atT,k=50	Fast,S=5
TPEx10e-3	141.25	151.39	153.49	153.1
Fast Crit	99.084	30.068	27.986	29.829
1atT Crit (k=4)	159	116.17	117.71	118.09
$E[P(H0 Y,D) H0,D]$	1	1	1	1
$E[P(H1 Y,D) H0,D]$	3.65e-40	4.53e-43	6.44e-41	6.52e-42
$E[BF01 H0,D]$	1.69e+67	9.77e+69	3.12e+67	7.37e+64
$E[P(H0 Y,D) H1,D]$	9.64e-44	3.18e-40	5.84e-41	3.6e-40
$E[P(H1 Y,D) H1,D]$	1	1	1	1
$E[BF01 H1,D]$	9.64e-44	3.18e-40	5.84e-41	3.6e-40
$V[B0 Y,X]$	0.000323	0.000372	0.000394	0.000393
$V[B1 Y,X]$	0.000622	0.000776	0.000782	0.000788
$V[B2 Y,X]$	0.000623	0.000778	0.000796	0.000762

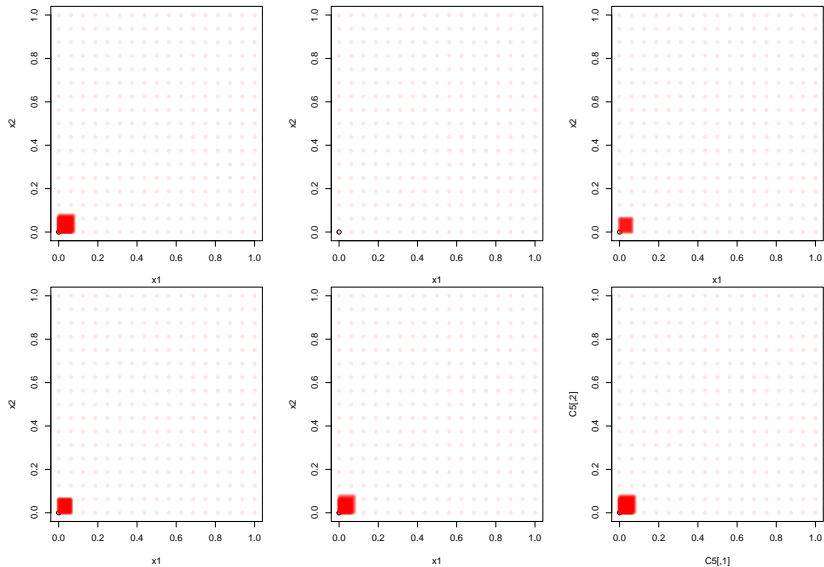
Compare Example 2 MEDs to Other Designs

Example 2 Evaluations for Other Designs, $N = 100$

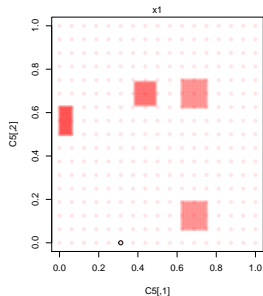
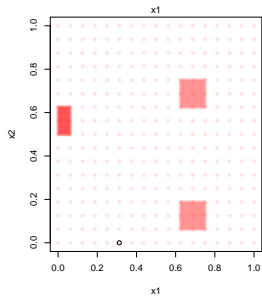
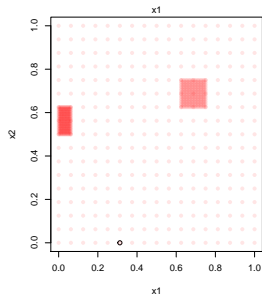
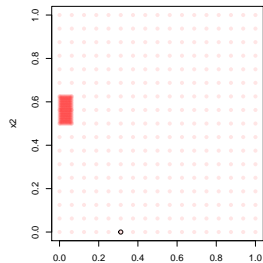
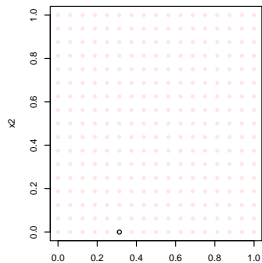
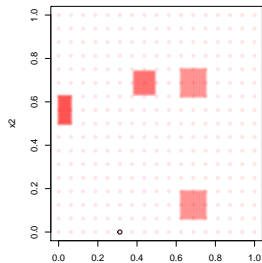
	1atT,k=1	1atT,k=4	1atT,k=50	Fast,S=5	D-Opt	Space	Random
TPEx10e-3	14.364	15.186	15.641	15.355	Inf	Inf	20.425
Fast Crit	25.163	16.395	15.294	17.351	Inf	Inf	122.86
1atT Crit (k=4)	56.53	48.13	48.564	49.618	Inf	Inf	165.77
$E[P(H_0 Y,D) H_0,D]$	1	1	1	1	1	1	1
$E[P(H_1 Y,D) H_0,D]$	9.04e-34	2.78e-27	1.29e-28	1.75e-27	1.68e-40	4.65e-28	1.21e-23
$E[BF_{01} H_0,D]$	3.23e+55	6.91e+51	5.49e+49	1.74e+50	1.45e+62	3.79e+50	1.13e+44
$E[P(H_0 Y,D) H_1,D]$	9.68e-29	2.73e-25	3.79e-25	1.42e-27	1.17e-31	8.96e-23	4.94e-18
$E[P(H_1 Y,D) H_1,D]$	1	1	1	1	1	1	1
$E[BF_{01} H_1,D]$	9.68e-29	2.73e-25	3.79e-25	1.42e-27	1.17e-31	8.96e-23	4.94e-18
$V[B_0 Y,X]$	0.000723	0.000788	0.000847	0.000838	0.000588	0.000884	0.00106
$V[B_1 Y,X]$	0.0014	0.00159	0.00169	0.00164	0.000803	0.00153	0.00189
$V[B_2 Y,X]$	0.0014	0.00162	0.00162	0.00156	0.000803	0.00153	0.00186

See Behavior of Candidate Sets in Example 2
Fast MED, $N = 300$

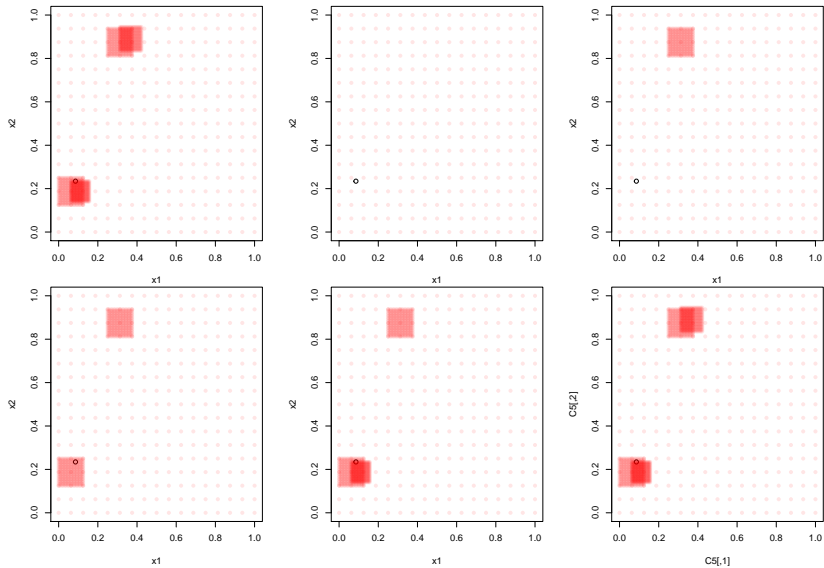
Candidates for Design Point indexed at 1



Candidates for Design Point indexed at 10



Candidates for Design Point indexed at 100



Candidates for Design Point indexed at 300

