Posterior Probability of H1 ~ SigmaSq01

Kristyn Pantoja

9/23/2019

1 Dimension

1D Example 1

1D Example 2

1D Example 3

Some Overarching Conclusions

2 Dimensions

2D Example 1

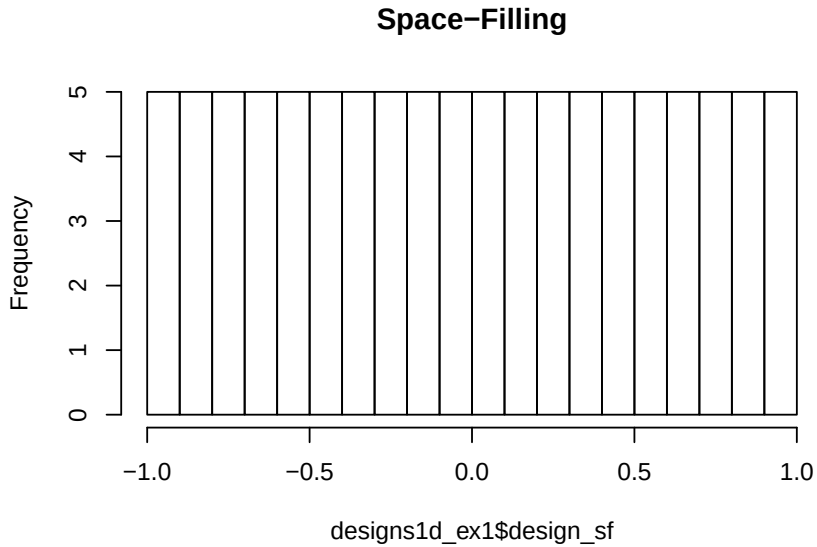
2D Example 2

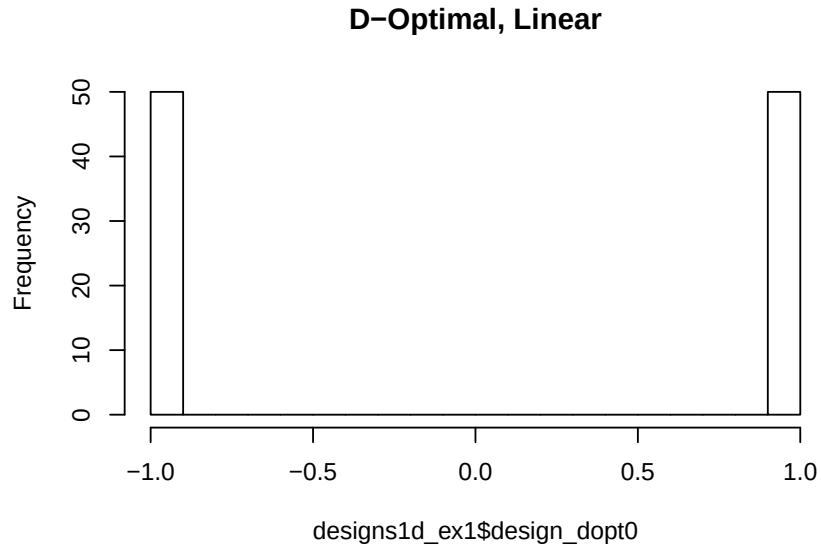
Summary and Some General Observations

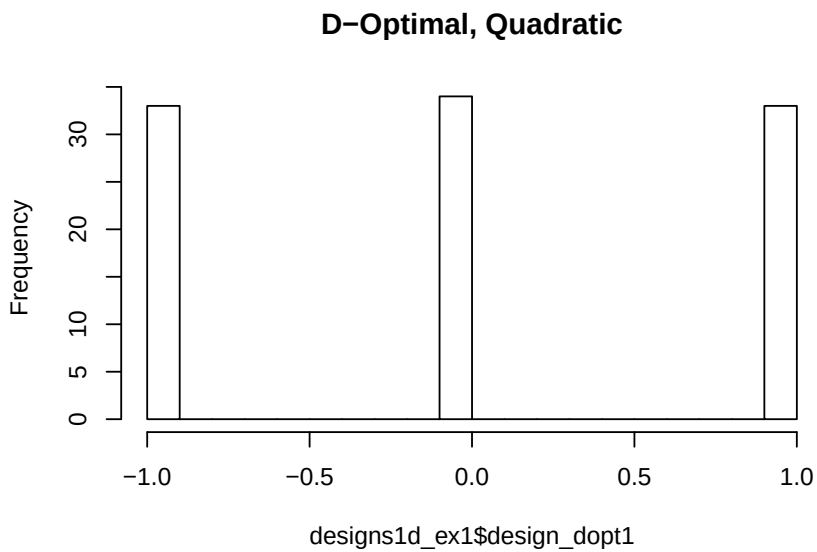
- In these slides, I look at different pairs of sharp hypotheses for generating MED, and how they evaluate based on
  - $E[P(H_1|Y, D)|\beta_T]$
  - $MSE(\beta_n|Y, D, H_1, \beta_T)$
  - $Var[\beta_n|Y, D, H_1]$
- From looking at plots of $E[P(H_1|Y, D)]$ for different values of σ^2, i.e. error variance, it does not seem that the relationships between the designs changes (i.e. the MEDs result in higher values of $E[P(H_1|Y, D)]$ than space-filling design for across all tested values of σ^2)
- It also does not seem like the relationships between the designs in $E[P(H_1|Y, D)]$ evaluations changes for different values of ν^2 (prior variances, i.e. `sigmasq01`), either, **unless** $\beta_T \neq \mu_1$.
- For this reason, all evaluations of $MSE(\beta_n|Y, D)$ and $Var[\beta_n|Y, D, H_1]$ use $\sigma^2 = 1$.

1 Dimension

Non-MED Designs





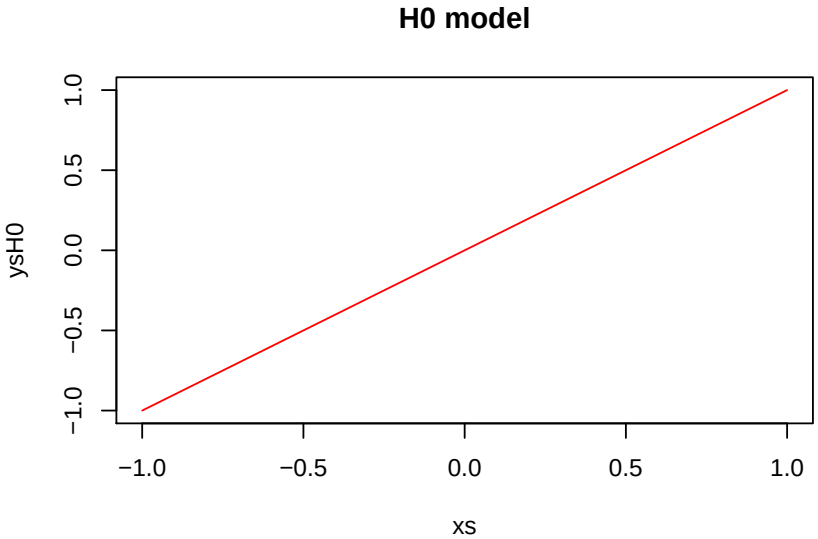


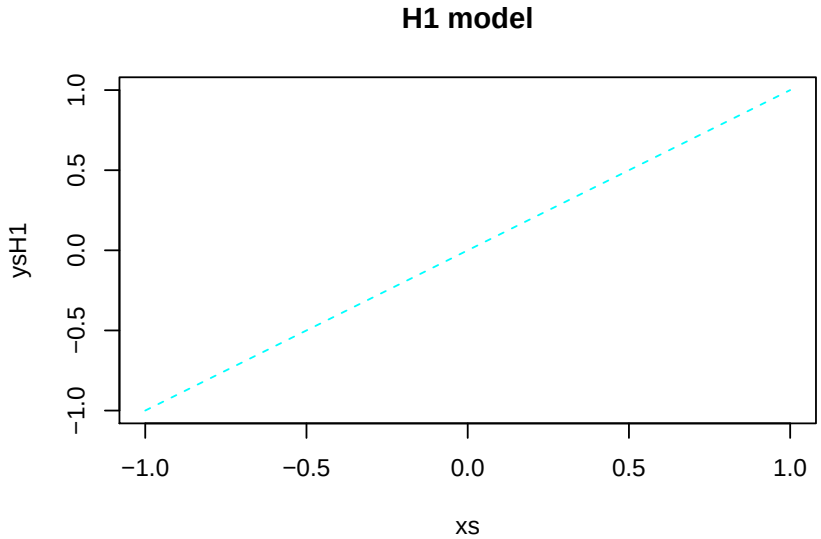
1D Example 1

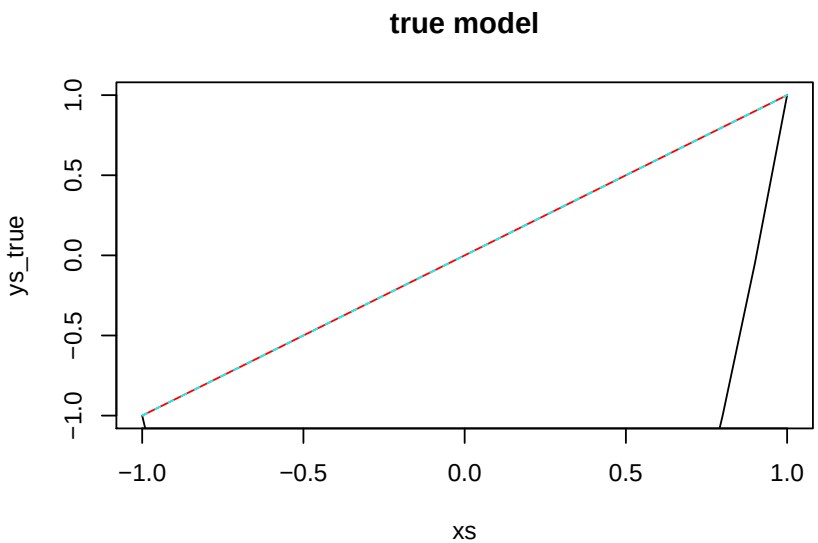
1D Example 1 Parameters

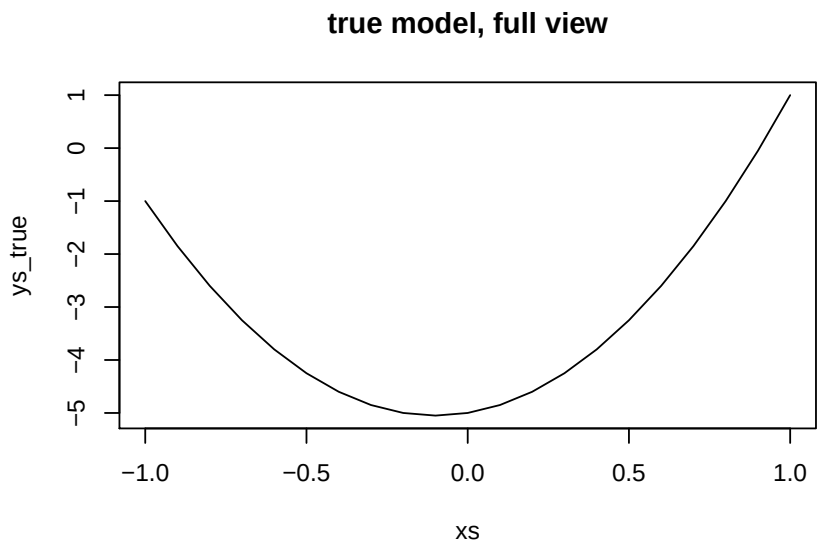
```
# Priors
sigmasq01 = 0.01
mu0 = c(0,1)
V0 = diag(rep(sigmasq01,length(mu0)))
mu1 = c(0,1,0)
V1 = diag(rep(sigmasq01,length(mu1)))
sigmasq = 1
betaT = c(-5, 1, 5)
```

1D Example 1 Plots

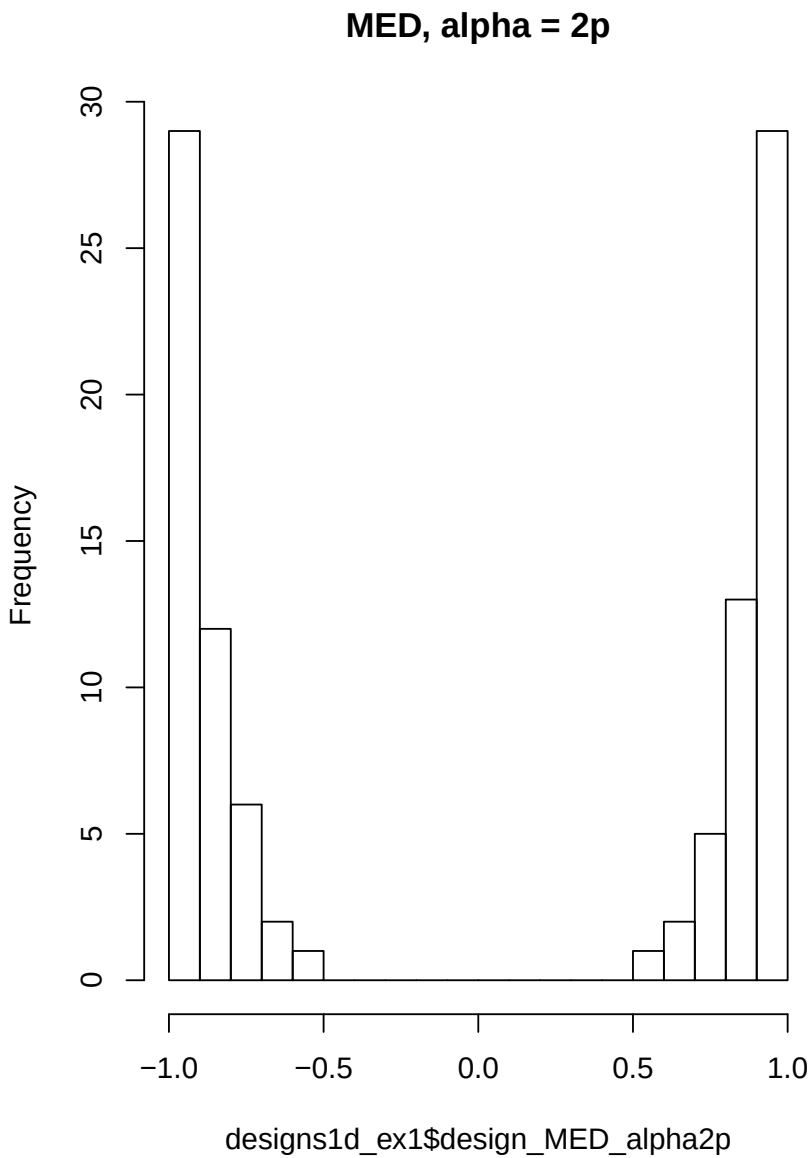


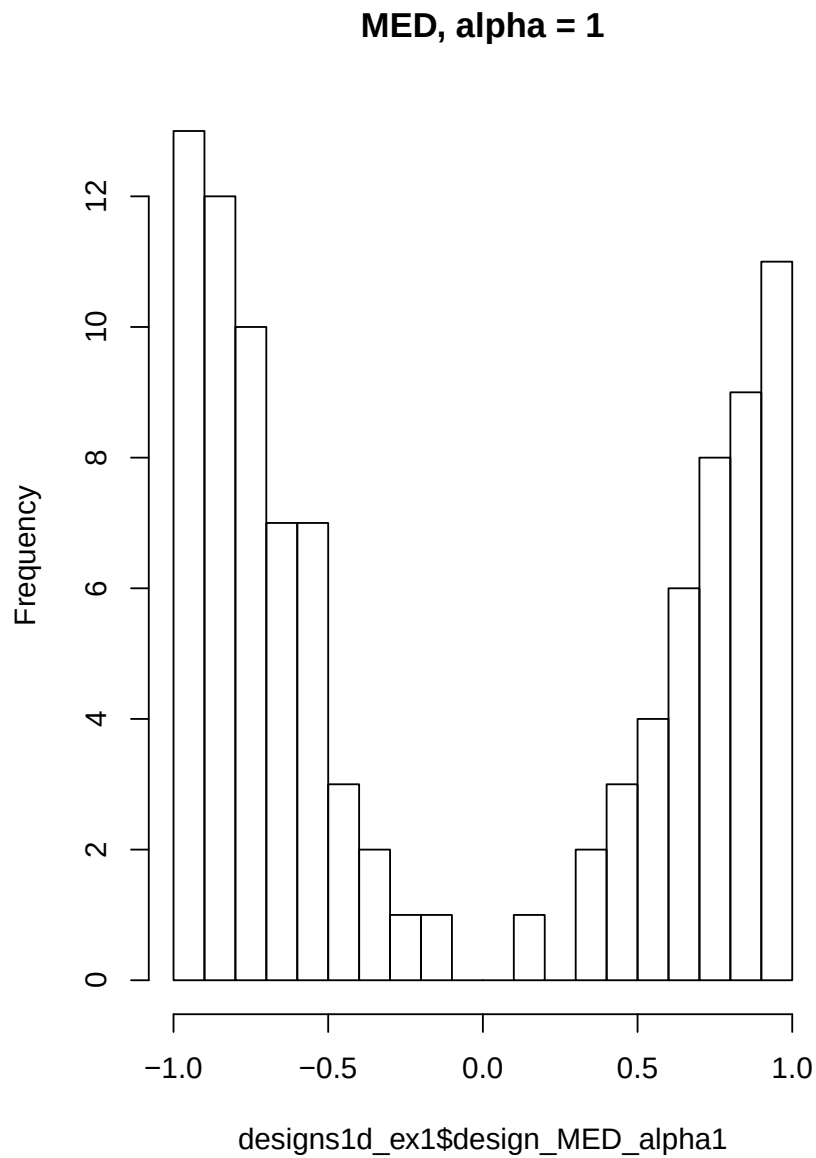




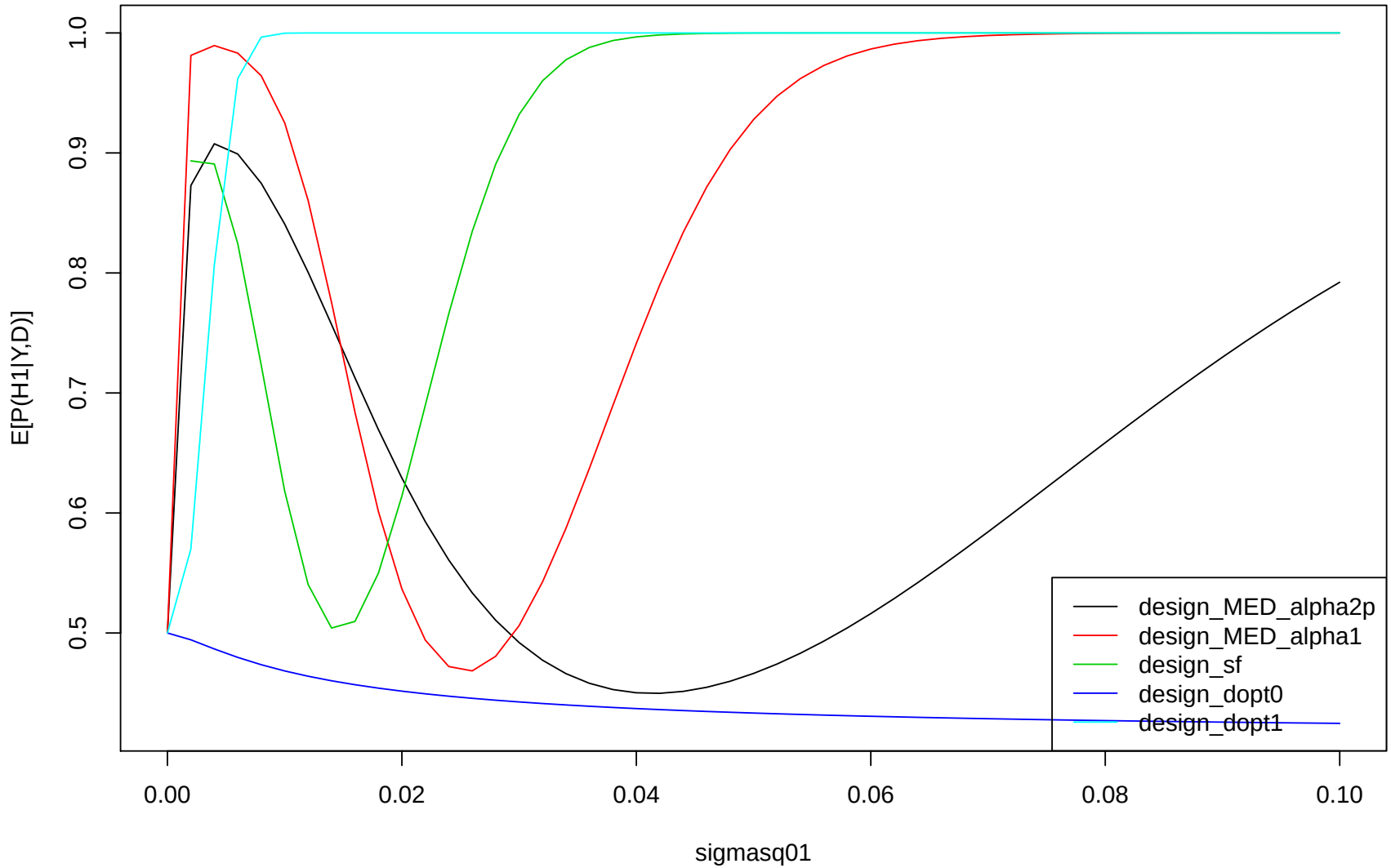


1D Example 1 MEDs



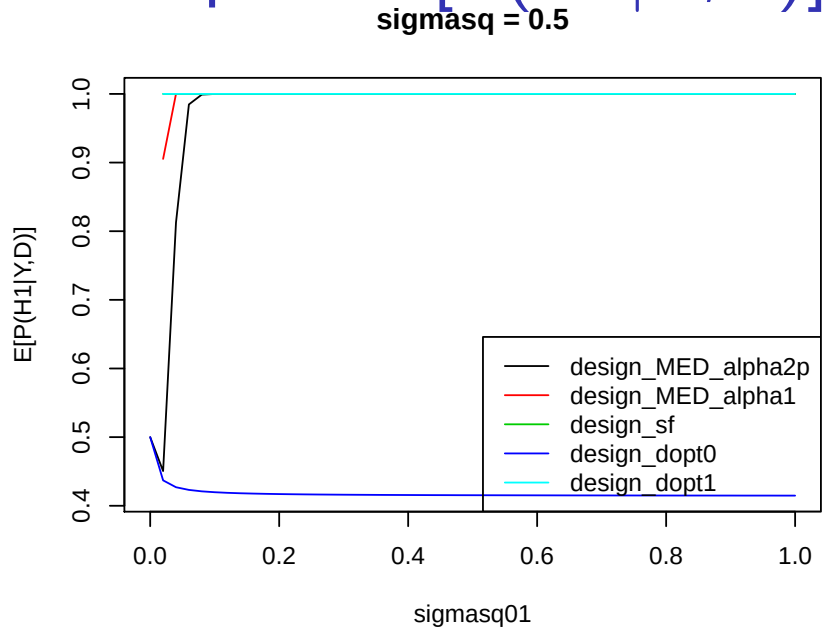


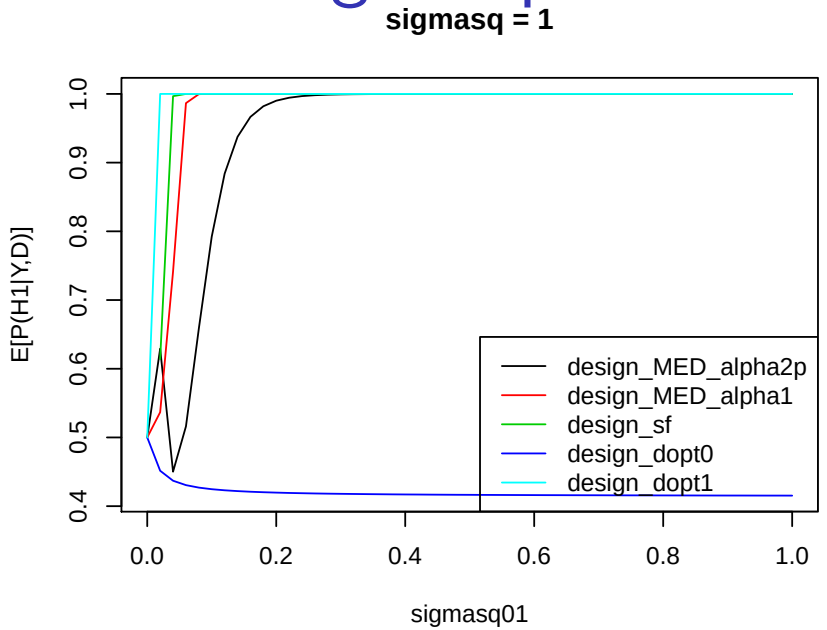
1D Example 1 E[P(H1|Y,D)]

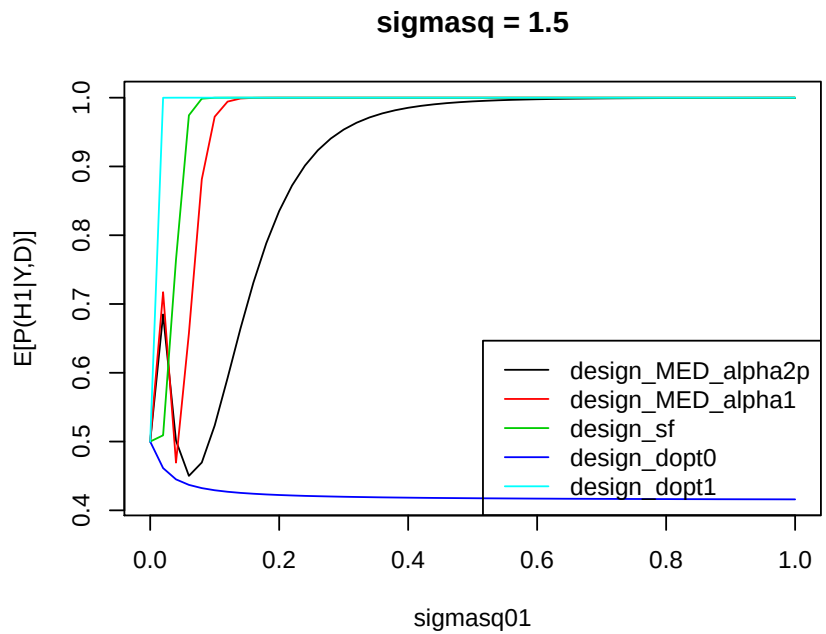


- For $E[P(H_1|Y, D)]$, it seems that when we are very sure about the priors (even though they are wrong), i.e. when the prior variances are assumed to be very small, the MEDs perform better than the space-filling design and also the D-optimal design for quadratic model:
  - When `sigmasq01 = 0.005`, it appears that $E[P(H_1|Y, D_{MED}, \alpha = 1)] > E[P(H_1|Y, D_{MED}, \alpha = 2p)] > E[P(H_1|Y, D_{MED}, \alpha = 2p)] > E[P(H_1|Y, D_{space})] > E[P(H_1|Y, D_{D-Opt1})] > E[P(H_1|Y, D_{D-Opt0})]$
  - When `sigmasq01` gets larger, though, the quadratic D-optimal design and the space-filling design favor H_1 more.

1D Example 1 E[P(H1|Y,D)] Different sigmasq







- Here, I'm just trying to see the affect of `sigmasq`, the error variance, relative to `sigma01`, the prior variance on β. For posterior probabilities, there does not seem to be much of a difference in how the designs are related.
- It seems that the only effect `sigmasq` has on $E[P(H_1|Y, D)]$ is to increase the rate at which the limiting value of $E[P(H_1|Y, D)]$ is approached.

MSE(Bn|Y,D,H1, BT)

- `sigmasq01 = 0.01`

	design_MED_alpha2p	design_MED_alpha1	design_sf	design_dopt0	design_dopt1
MSE(B0)	21.13918	16.17305	11.32378	25.00111	16.17546
MSE(B1)	0.00248	0.00513	0.00189	0.00250	0.00240
MSE(B2)	26.76344	27.14943	25.86551	25.00111	21.26302

- `sigmasq01 = 0.5`

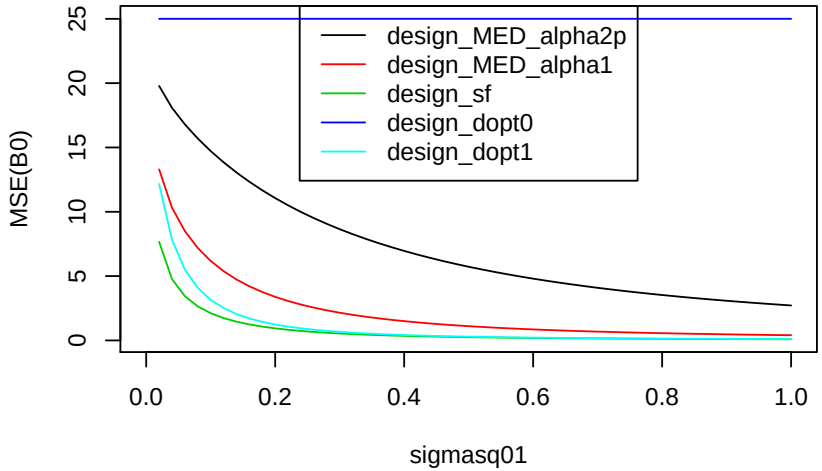
	design_MED_alpha2p	design_MED_alpha1	design_sf	design_dopt0	design_dopt1
MSE(B0)	5.72716	1.10788	0.25268	25.00245	0.29341
MSE(B1)	0.01176	0.01635	0.02623	0.00961	0.01427
MSE(B2)	8.32825	2.76610	1.42037	25.00245	0.45911

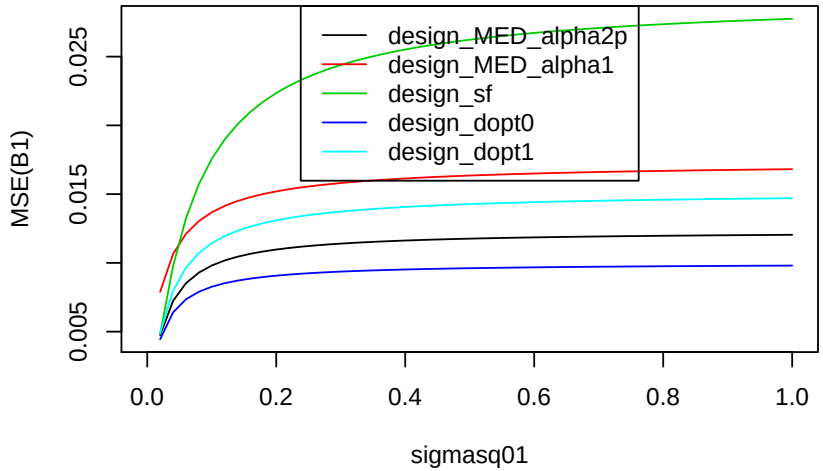
- `sigmasq01 = 1`

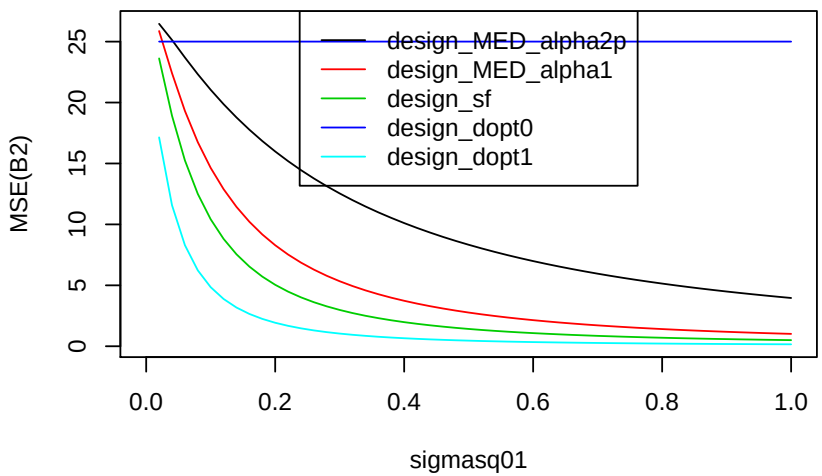
	design_MED_alpha2p	design_MED_alpha1	design_sf	design_dopt0	design_dopt1
MSE(B0)	2.71932	0.40526	0.09045	25.00248	0.10234
MSE(B1)	0.01204	0.01681	0.02775	0.00980	0.01470
MSE(B2)	3.96243	1.01522	0.50204	25.00248	0.15933

- To calculate $MSE(\beta_n)$ for each design, I assume `sigmasq = 1` and tested different values of prior variance, `sigmasq01`.
- It seems that space-filling design does worse, and the D-optimal designs do better, when `sigmasq01` gets larger - I think this makes sense, since the D-optimal designs have replication to decrease variance in predictions.
- It also looks like, when `sigmasq01 = 0.5` or `sigmasq01 = 1`, the MEDs do in-between the space-filling and linear D-optimal design, in terms of performance. Is this a good compromise, or just a bad thing?

MSEs

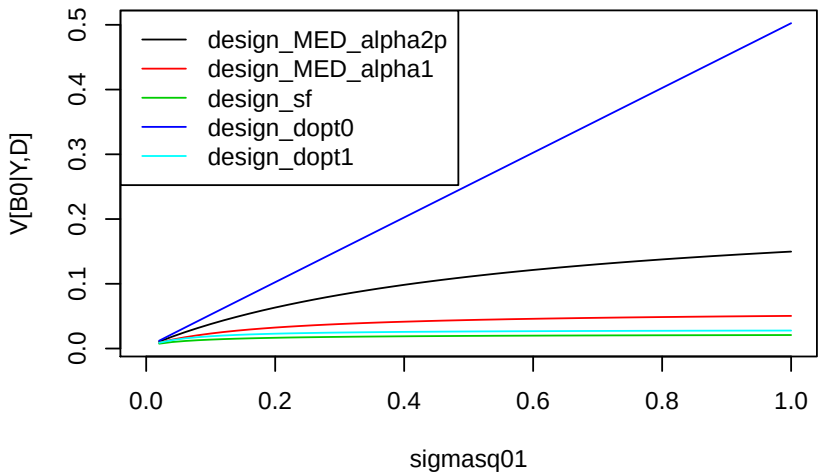


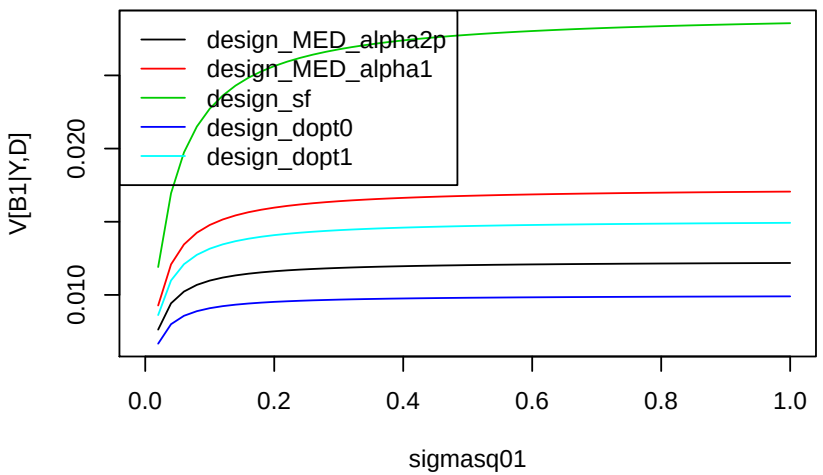


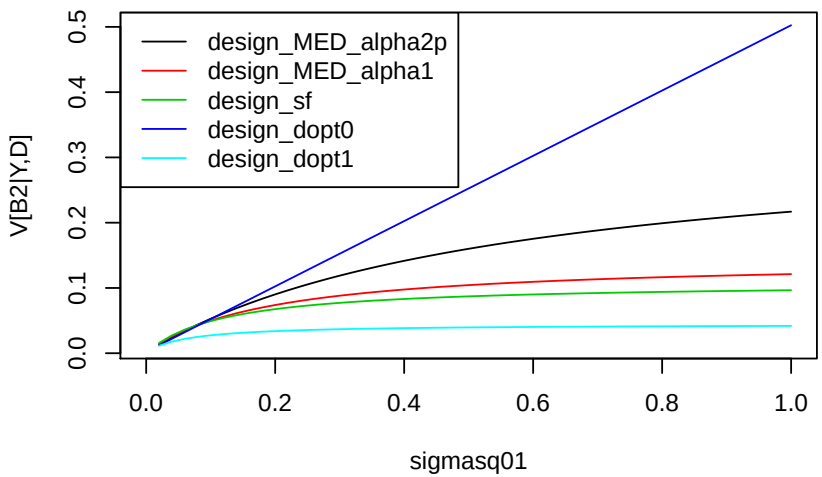


- Only $MSE(\beta_1)$ increases as variance increases, because it matched the prior.

Posterior Variances | H1







- It seems like when the sharp hypothesis gives a wrong prior mean for a parameter, such as for β_0 (the intercept) and β_2 (the quadratic term), the space-filling algorithm and quadratic D-optimal design do better than the MEDs and the linear D-optimal design (which does worst).
- However, when the prior mean matched the true value, as with β_1 (the linear term), the linear D-optimal design performed best and the space-filling design performed worst.
- These speculations might not be correct, though, considering that the computation for $Var[\beta_n|Y, H_1]$ does not rely on the true value of β.

MSE(Bn|Y,D,H1, BT), BT = mu1

Assuming $\beta_T = \mu_1$, i.e. in the case where $MSE(\beta_n|Y, D, H_1, \beta_T) = Var[\beta_n]$

- `sigmasq01 = 0.5`

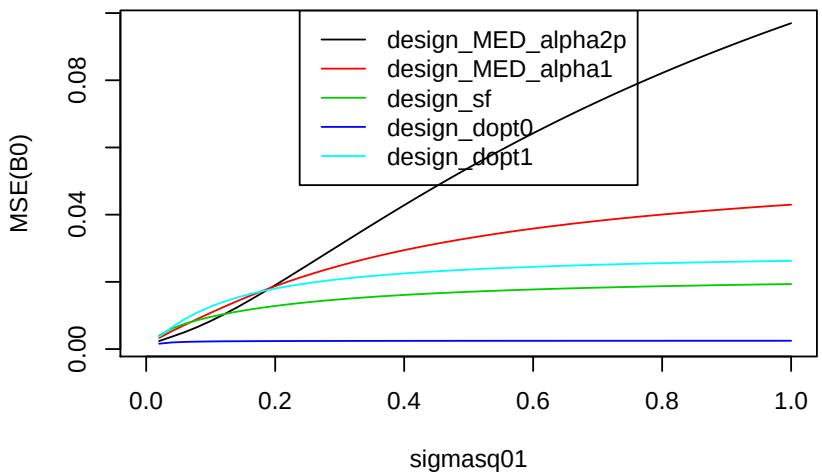
	design_MED_alpha2p	design_MED_alpha1	design_sf	design_dopt0	design_dopt1
MSE(B0)	0.05396	0.03302	0.01704	0.00245	0.02366
MSE(B1)	0.01175	0.01621	0.02623	0.00961	0.01427
MSE(B2)	0.07644	0.07547	0.07027	0.00245	0.03510

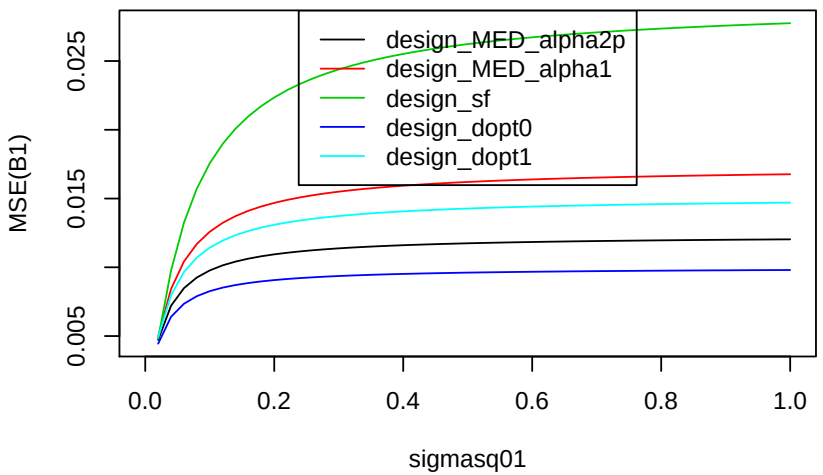
- `sigmasq01 = 1`

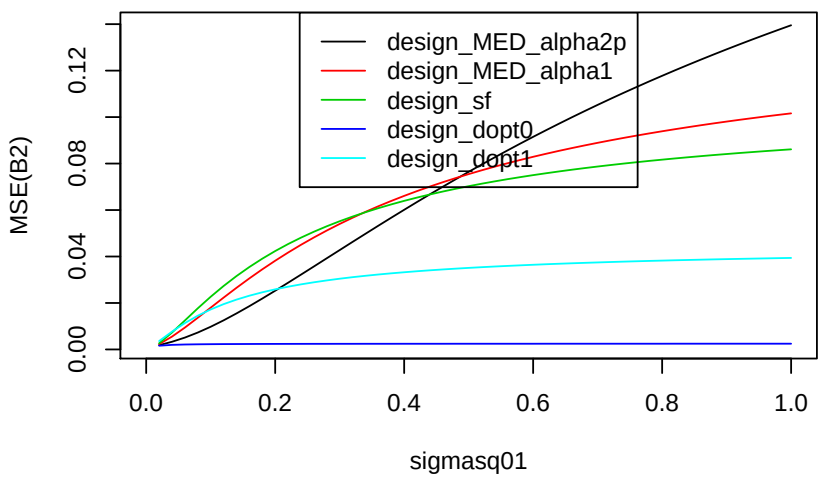
	design_MED_alpha2p	design_MED_alpha1	design_sf	design_dopt0	design_dopt1
MSE(B0)	0.09698	0.04297	0.01935	0.00248	0.02627
MSE(B1)	0.01203	0.01677	0.02775	0.00980	0.01470
MSE(B2)	0.13953	0.10161	0.08613	0.00248	0.03939

- linear D-Optimal design does best, probably because the true model is basically a linear model (since the coefficient for the quadratic term is 0)

MSEs


design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B0)
0.00
0.04
0.08
0.0
0.2
0.4
0.6
0.8
1.0
sigmasq01


design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B1)
0.005
0.015
0.025
0.0
0.2
0.4
0.6
0.8
1.0
sigmasq01


design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B2)
0.00
0.04
0.08
0.12
0.0
0.2
0.4
0.6
0.8
1.0
sigmasq01

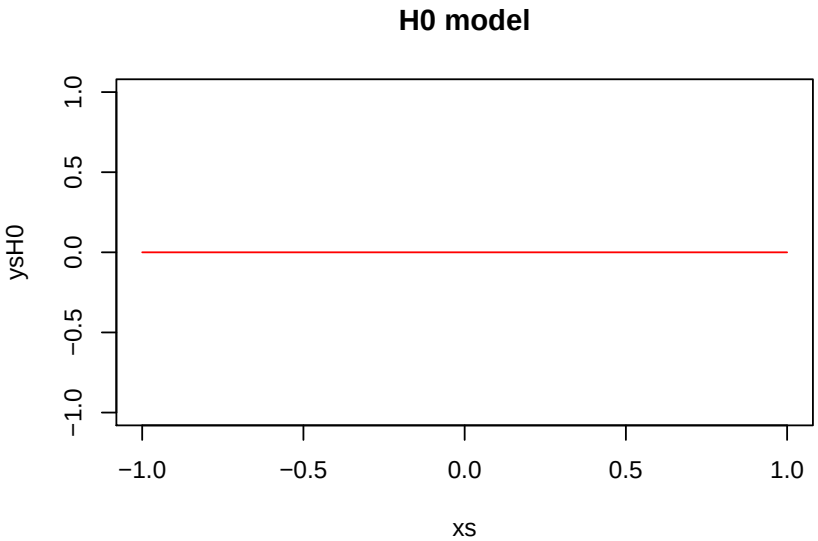
- Here, the space-filling algorithm is better than the MEDs at estimating the intercept and quadratic term's coefficients. This was true when $\beta_T \neq \mu_1$ also, though.

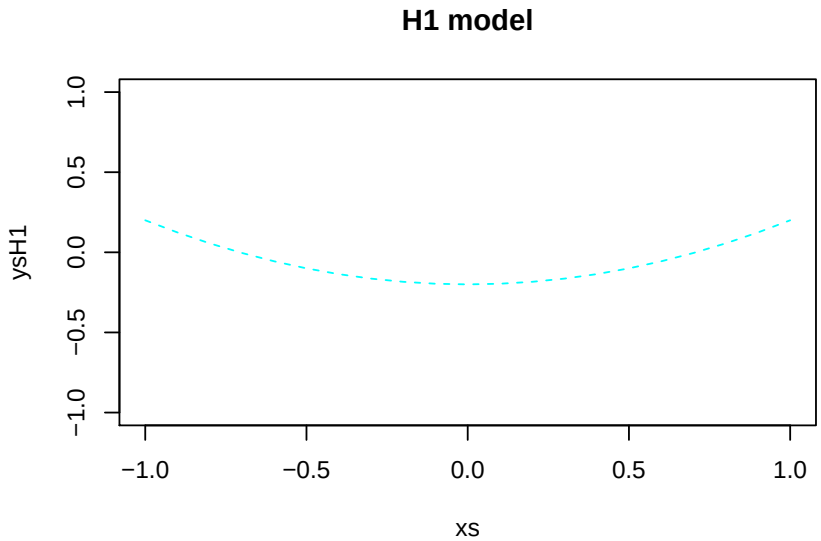
1D Example 2

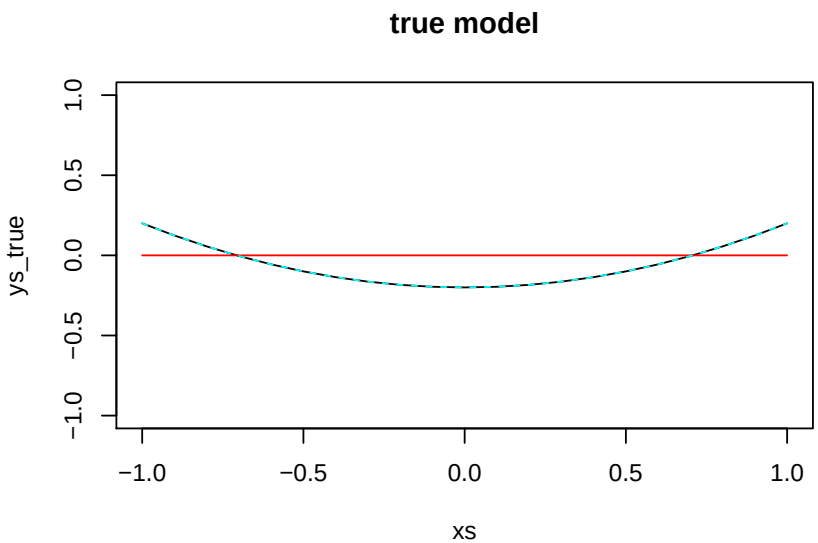
1D Example 2 Parameters

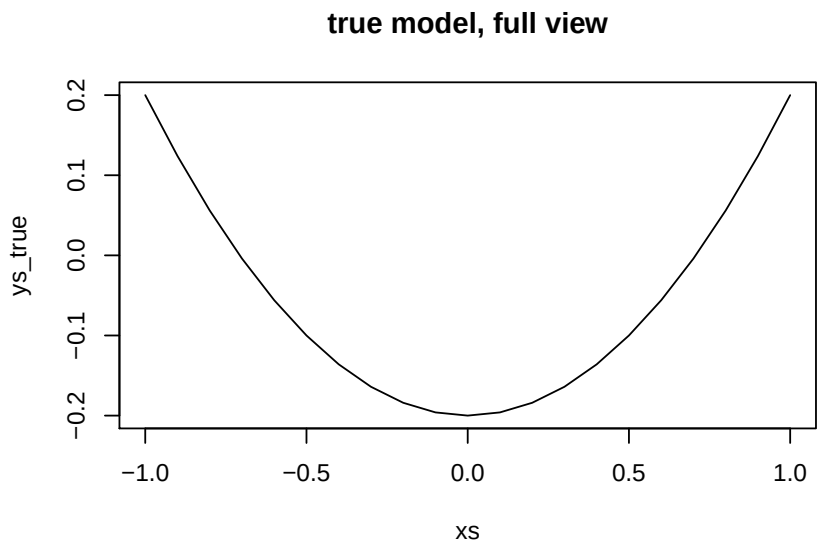
```
# Priors
sigmasq01 = 0.01
mu0 = c(0, 0)
V0 = diag(rep(sigmasq01,length(mu0)))
mu1 = c(-0.2, 0, 0.4)
V1 = diag(rep(sigmasq01,length(mu1)))
sigmasq = 1
betaT = mu1
```

1D Example 2 Plots

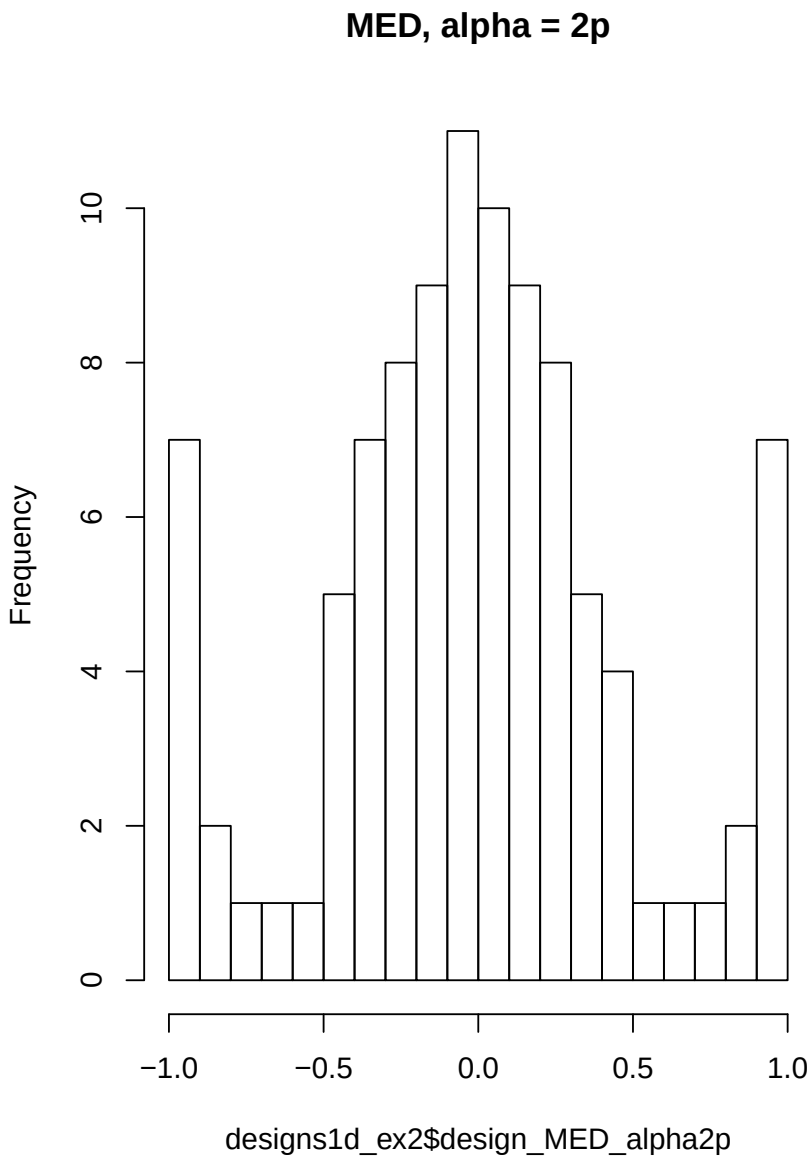


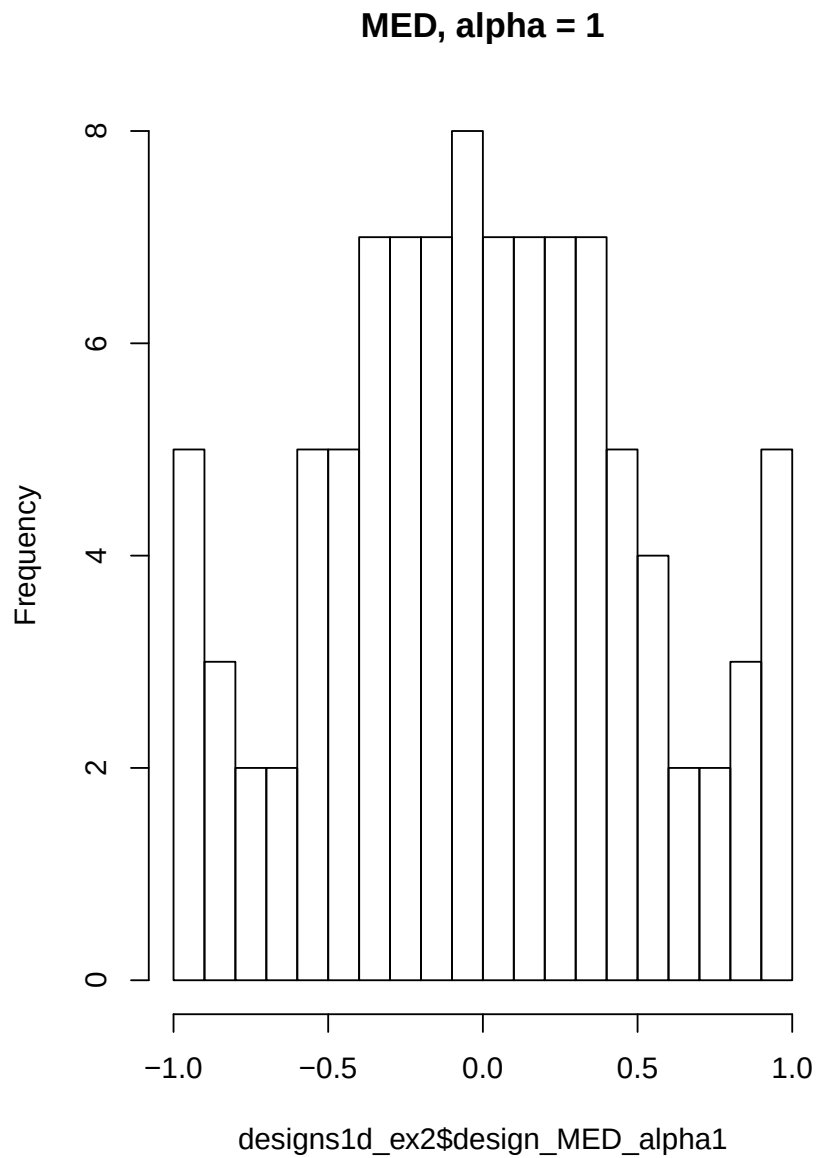




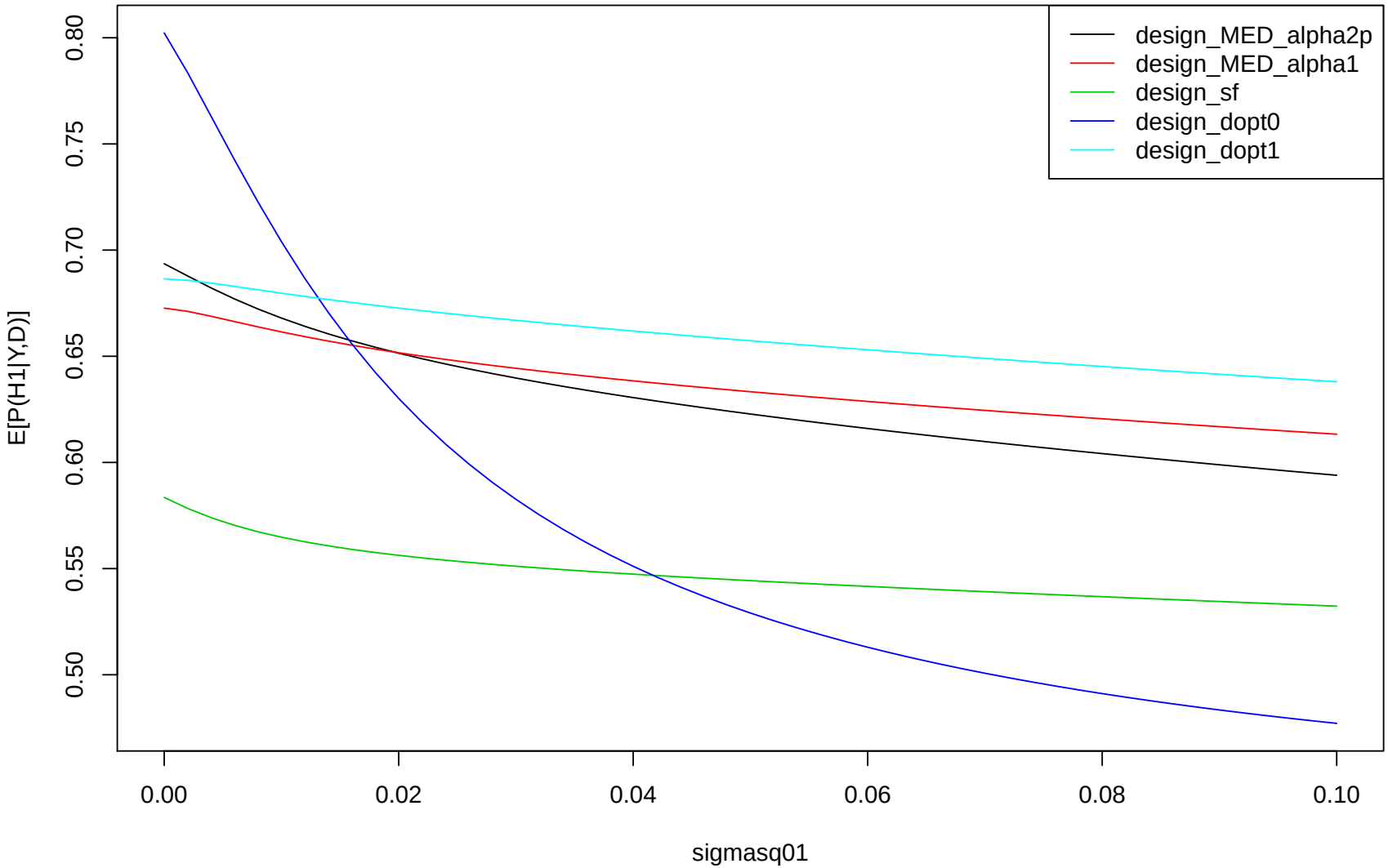


1D Example 2 MEDs



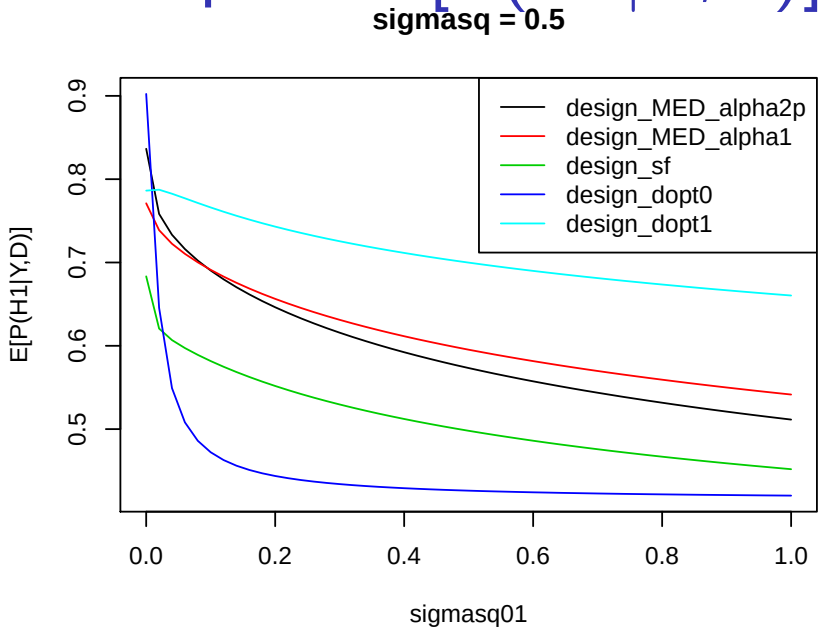


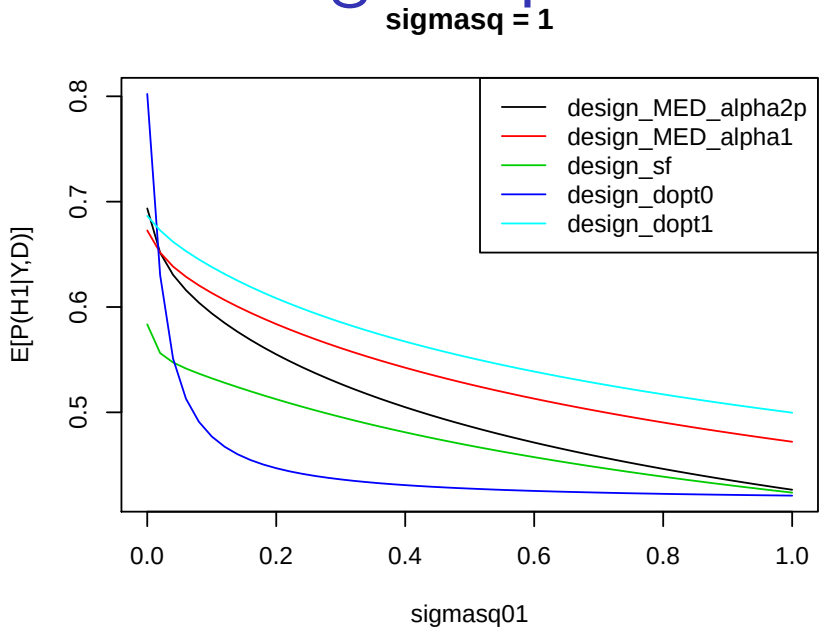
1D Example 2 E[P(H1|Y,D)]

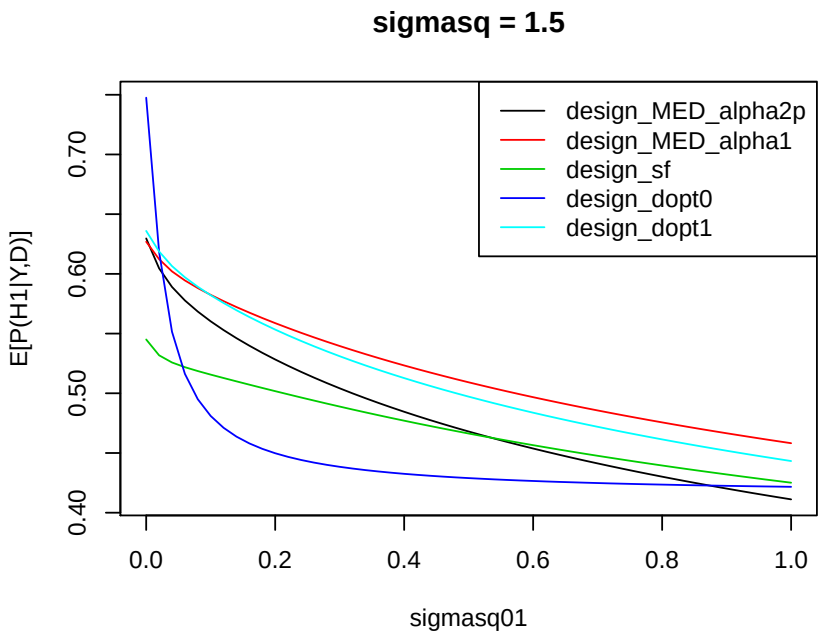


- I'm not sure if this is a great example since the quadratic model is very close to the linear model, and it might be the case that $E[P(H_1|Y, D)|\beta_T]$ favors simpler models.
- Regardless, MEDs tend to perform in-between the space-filling design and the linear D-optimal design.
- It looks like when prior variance is high, though, the MEDs might cross over the space-filling design and actually be better? Might be worth investigating.

1D Example 2 E[P(H1|Y,D)] Different sigmasq







MSE(Bn|Y,D,H1, BT)

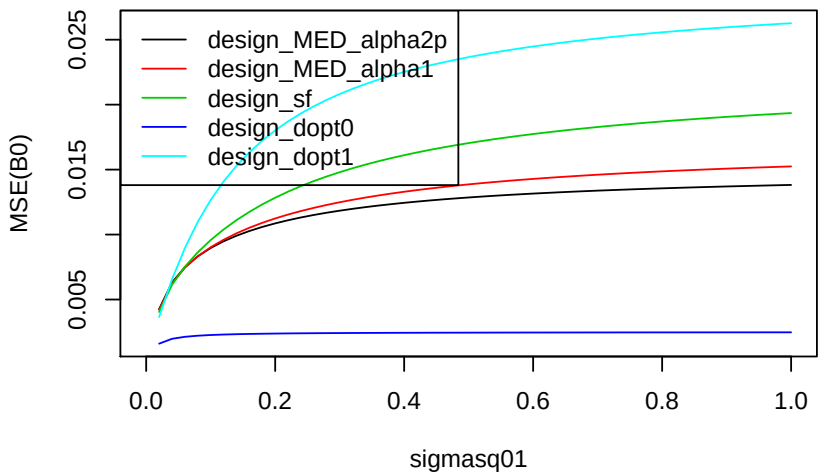
- `sigmasq01 = 0.5`

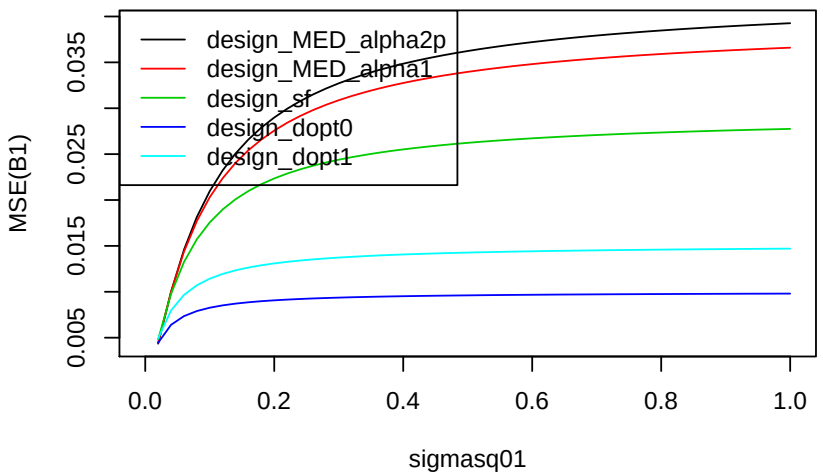
	design_MED_alpha2p	design_MED_alpha1	design_sf	design_dopt0	design_dopt1
MSE(B0)	0.01285	0.01386	0.01704	0.00245	0.02366
MSE(B1)	0.03624	0.03396	0.02623	0.00961	0.01427
MSE(B2)	0.06432	0.07268	0.07027	0.00245	0.03510

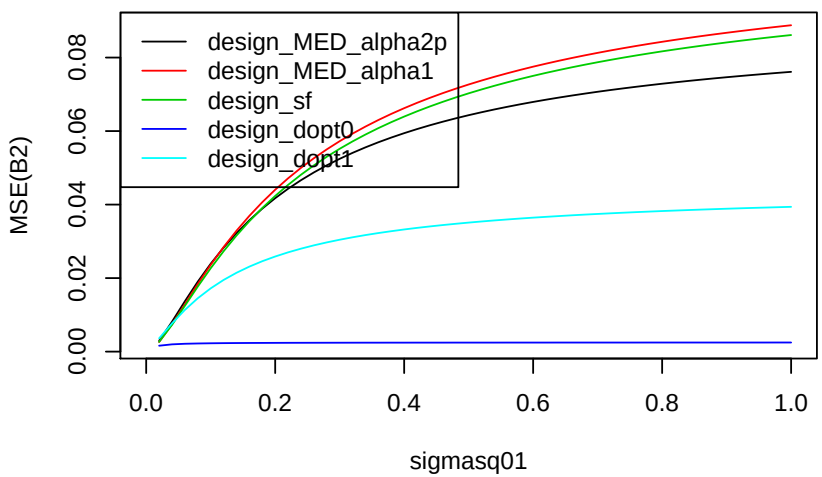
- `sigmasq01 = 1`

	design_MED_alpha2p	design_MED_alpha1	design_sf	design_dopt0	design_dopt1
MSE(B0)	0.01382	0.01525	0.01935	0.00248	0.02627
MSE(B1)	0.03927	0.03660	0.02775	0.00980	0.01470
MSE(B2)	0.07611	0.08879	0.08613	0.00248	0.03939

MSEs

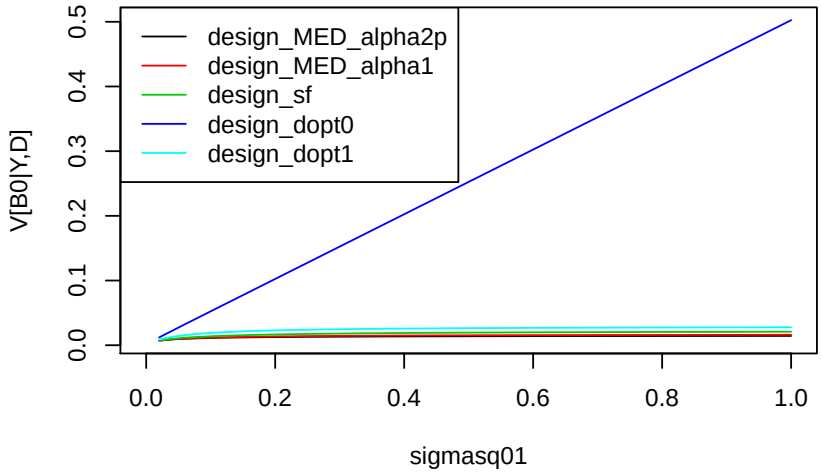

design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B0)
0.005
0.015
0.025
0.0
0.2
0.4
0.6
0.8
1.0
sigmasq01

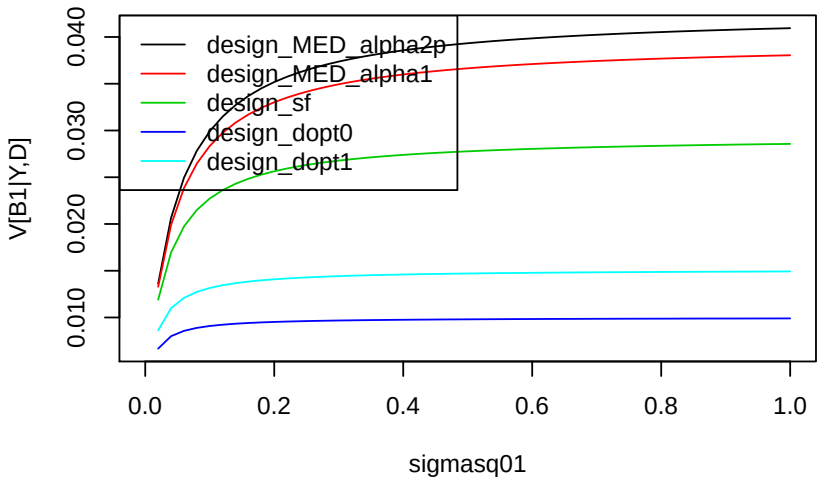

design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B1)
0.005
0.015
0.025
0.035
0.0
0.2
0.4
0.6
0.8
1.0
sigmasq01

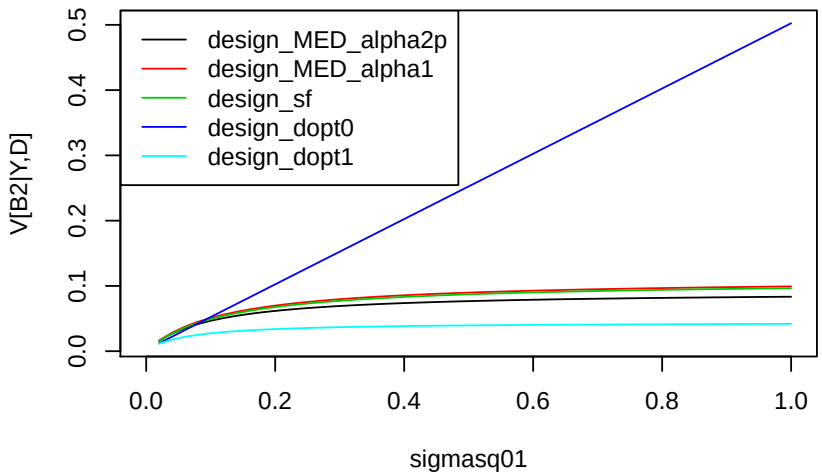

design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B2)
0.00
0.02
0.04
0.06
0.08
0.0
0.2
0.4
0.6
0.8
1.0
sigmasq01

- MEDs do better than or are very comparable to the space-filling design only for the intercept and quadratic term - possibly due to the fact that it puts so many points near 0 (and not so many at -1 and 1, which might be how it would have done better for the linear term)

Posterior Variances | H1







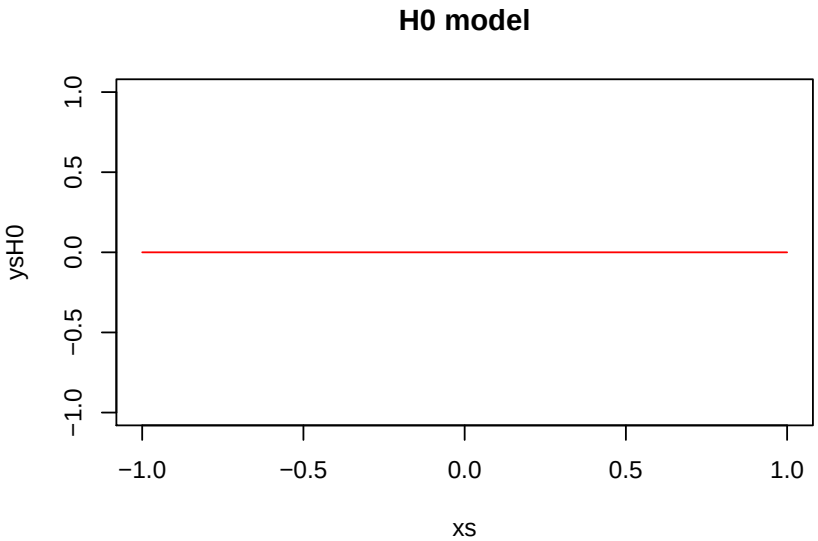
- MED only does better/comparable to the space-filling design for the quadratic term here, though. Why?

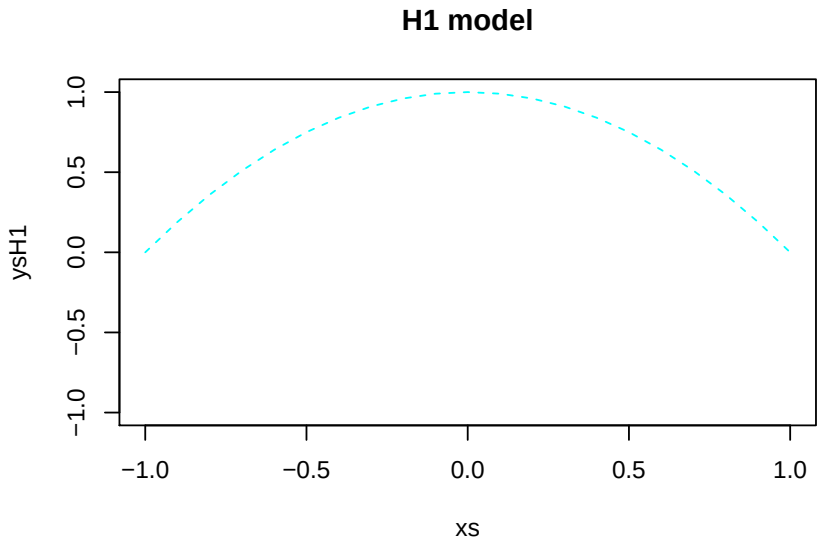
1D Example 3

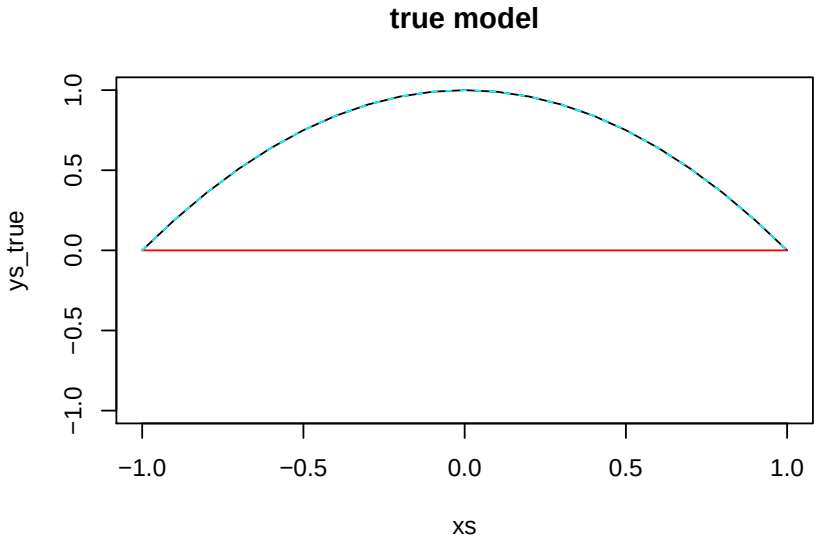
1D Example 3 Parameters

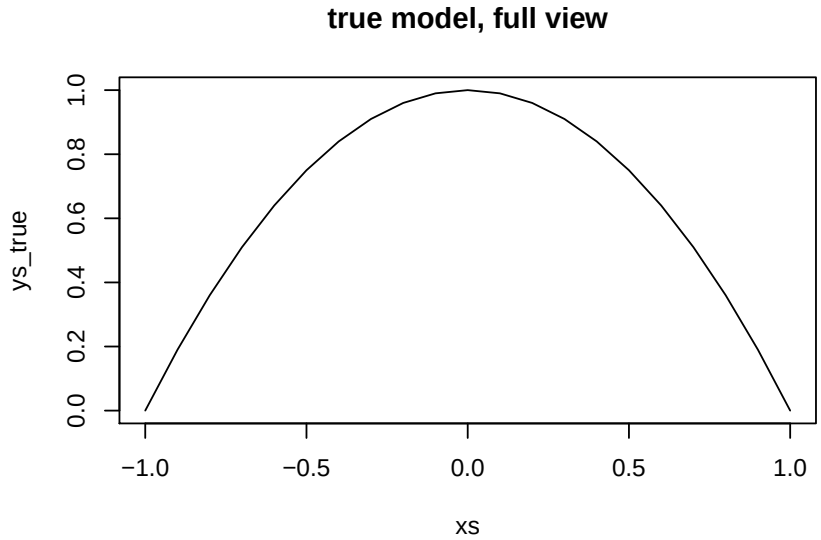
```
# Priors
sigmasq01 = 0.01
mu0 = c(0, 0, 0)
V0 = diag(rep(sigmasq01,length(mu0)))
mu1 = c(2, 0, -1, 0, -1)
V1 = diag(rep(sigmasq01,length(mu1)))
sigmasq = 1
betaT = mu1
```

1D Example 3 Plots

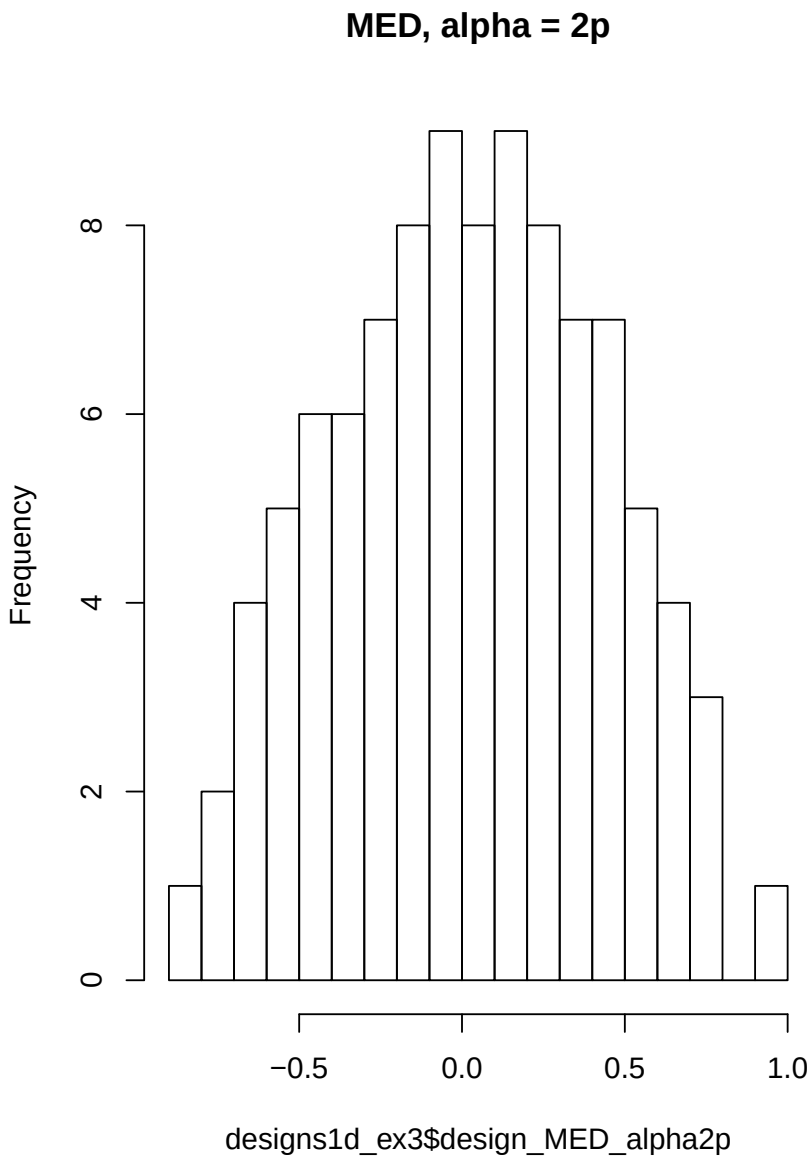


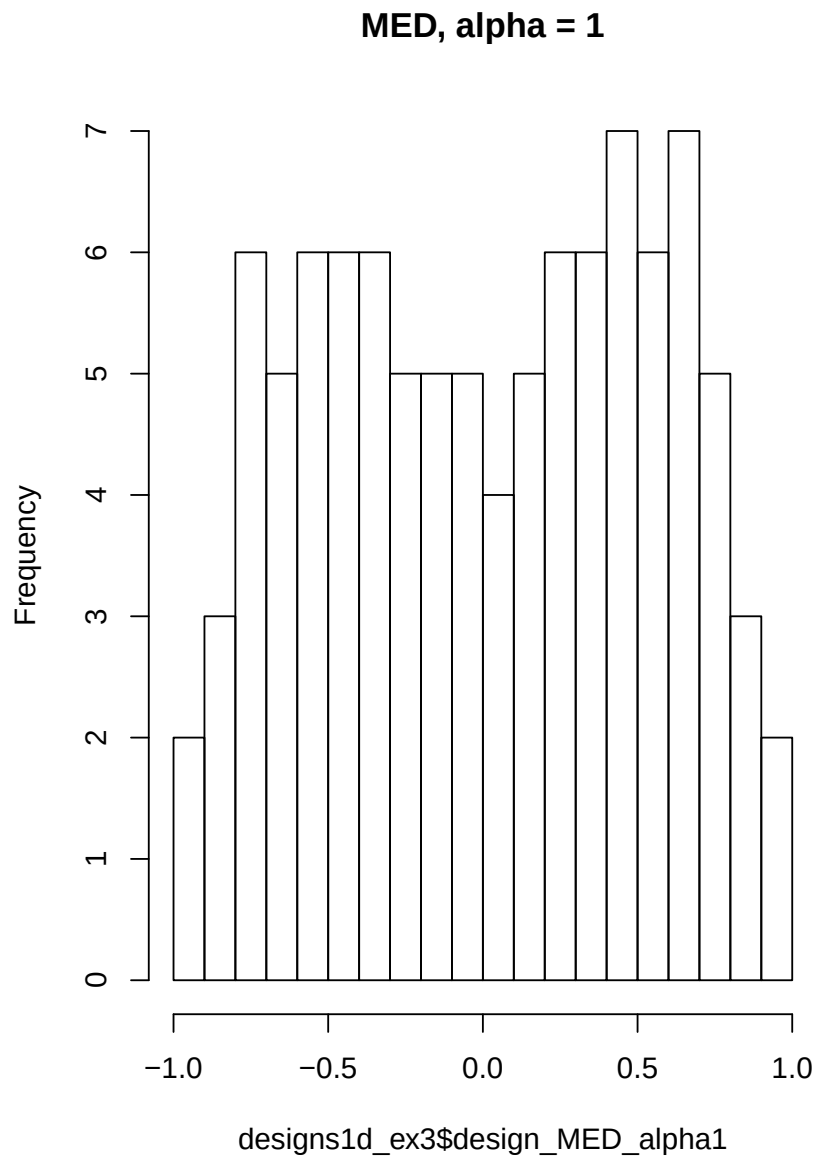




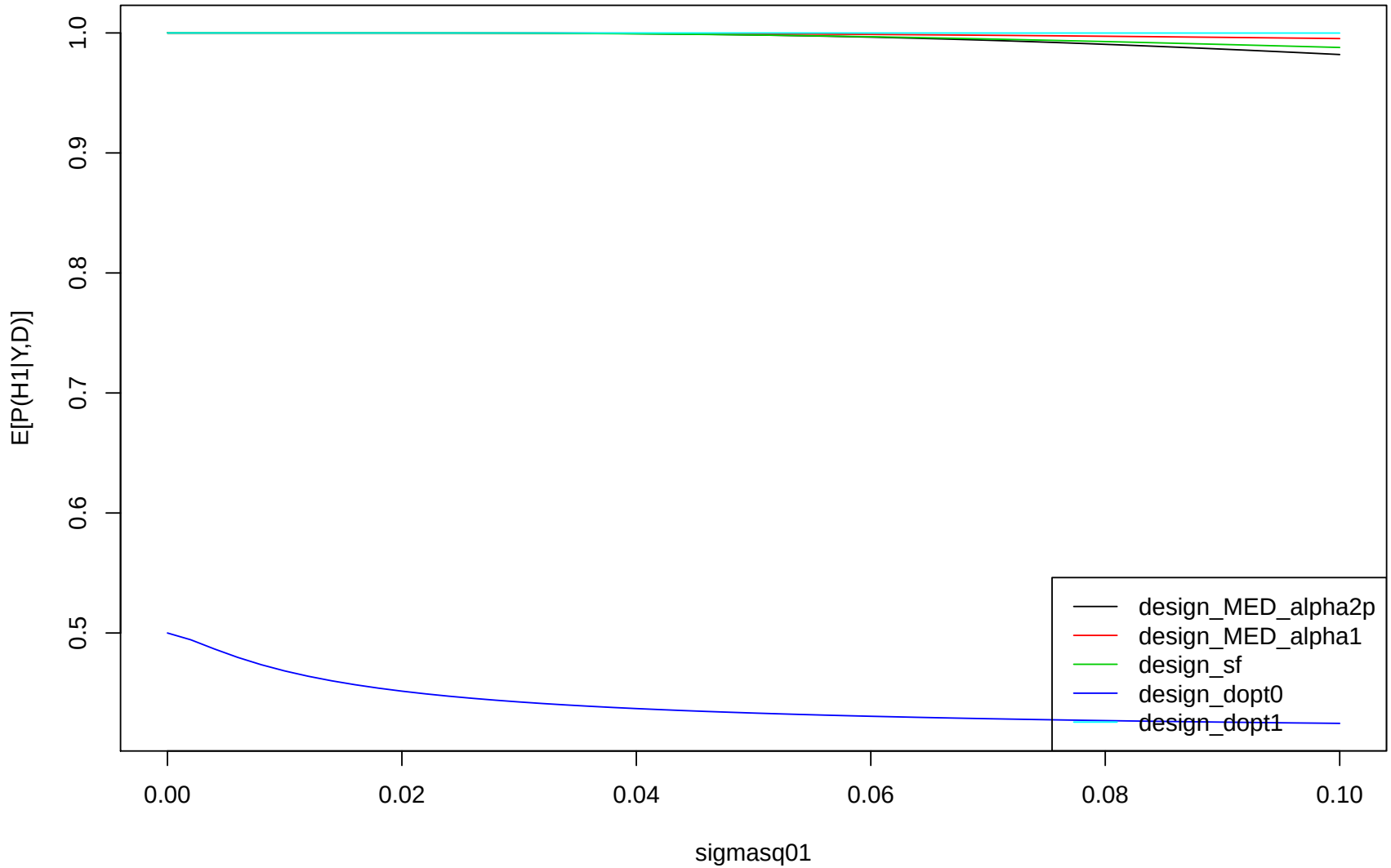


1D Example 3 MEDs

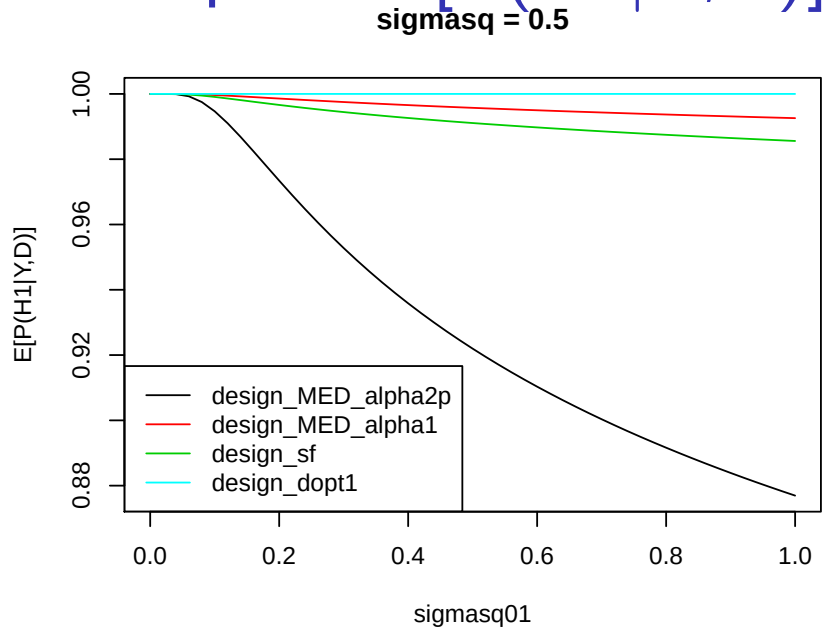


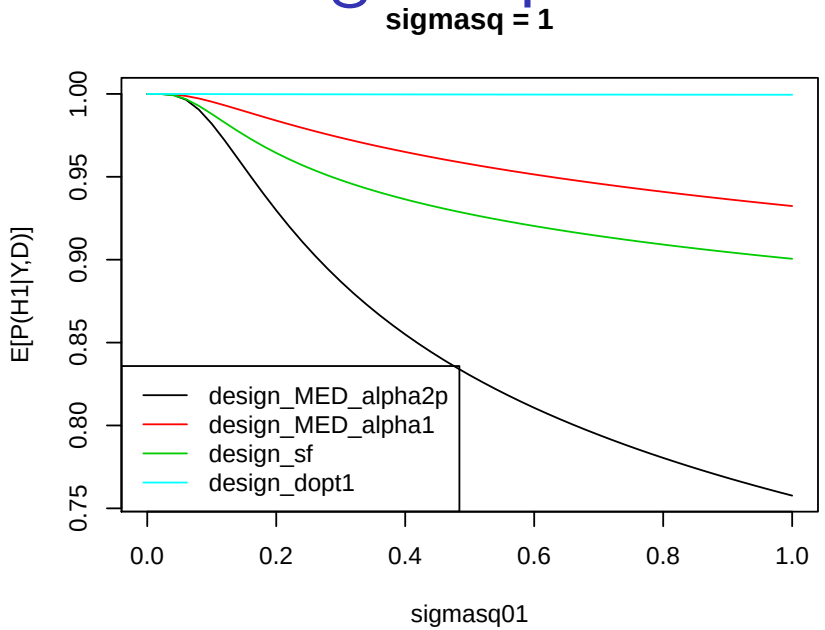


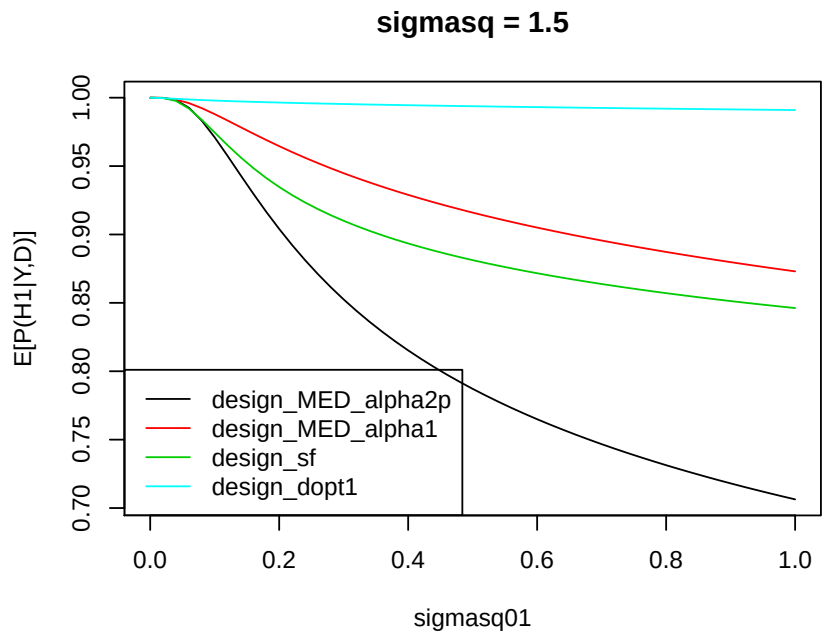
1D Example 3 E[P(H1|Y,D)]



1D Example 3 E[P(H1|Y,D)] Different sigmasq







- I did not include the (blue) line for linear D-optimal design, since it did drastically worse than the others and made the plot hard to read
- It seems here, though, that for these sharp hypotheses, MED (only the one with $\alpha = 2p$, i.e. no exponent on f_D) in-between space-filling and quadratic D-optimal

MSE(Bn|Y,D,H1, BT)

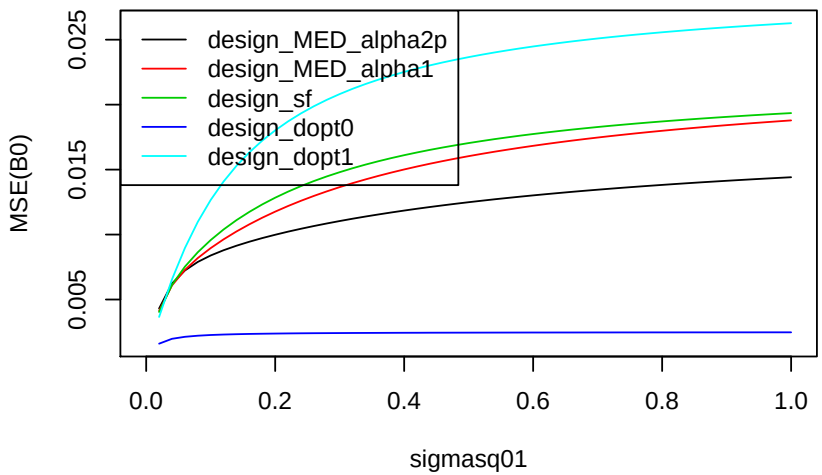
- `sigmasq01 = 0.5`

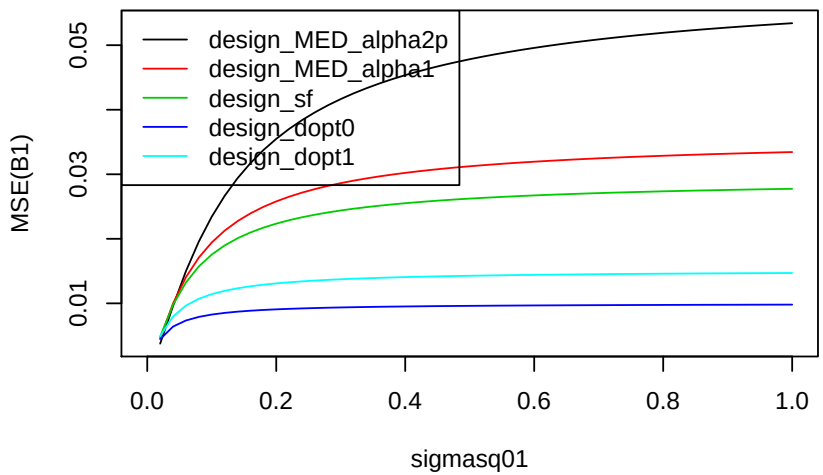
	design_MED_alpha2p	design_MED_alpha1	design_sf	design_dopt0	design_dopt1
MSE(B0)	0.01249	0.01603	0.01704	0.00245	0.02366
MSE(B1)	0.04783	0.03124	0.02623	0.00961	0.01427
MSE(B2)	0.11403	0.09072	0.07027	0.00245	0.03510

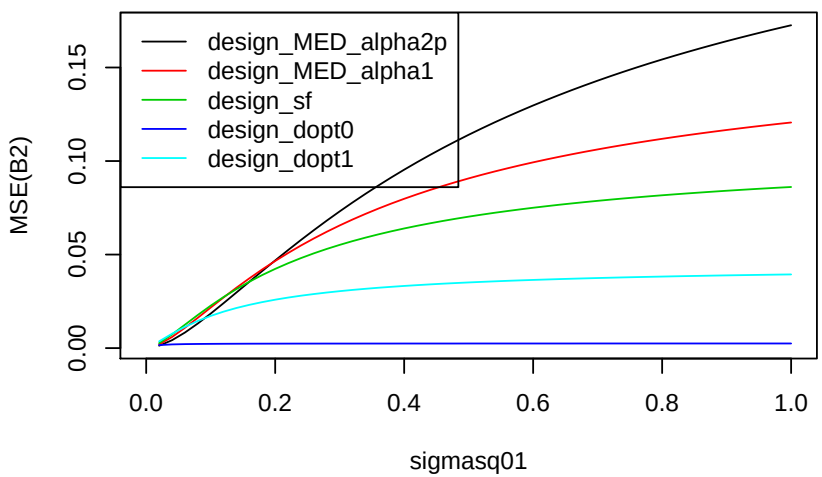
- `sigmasq01 = 1`

	design_MED_alpha2p	design_MED_alpha1	design_sf	design_dopt0	design_dopt1
MSE(B0)	0.01441	0.01879	0.01935	0.00248	0.02627
MSE(B1)	0.05341	0.03344	0.02775	0.00980	0.01470
MSE(B2)	0.17258	0.12060	0.08613	0.00248	0.03939

MSEs

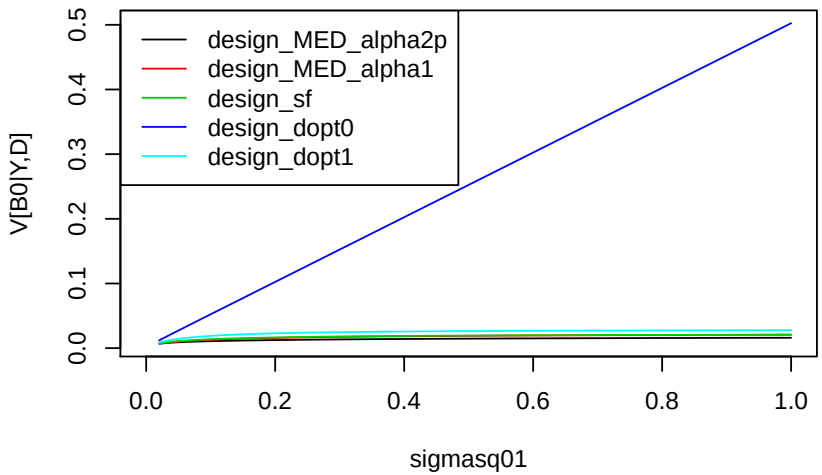

design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B0)
0.005
0.015
0.025
0.0
0.2
0.4
0.6
0.8
1.0
sigmasq01

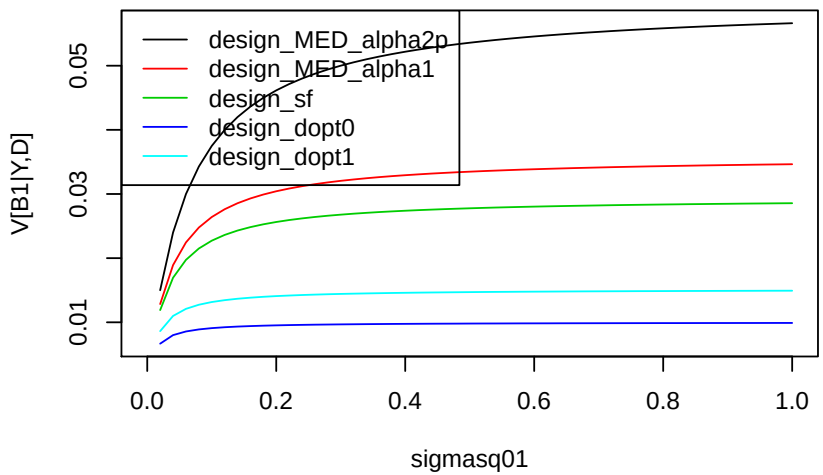

design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B1)
0.01
0.03
0.05
0.0
0.2
0.4
0.6
0.8
1.0
sigmasq01

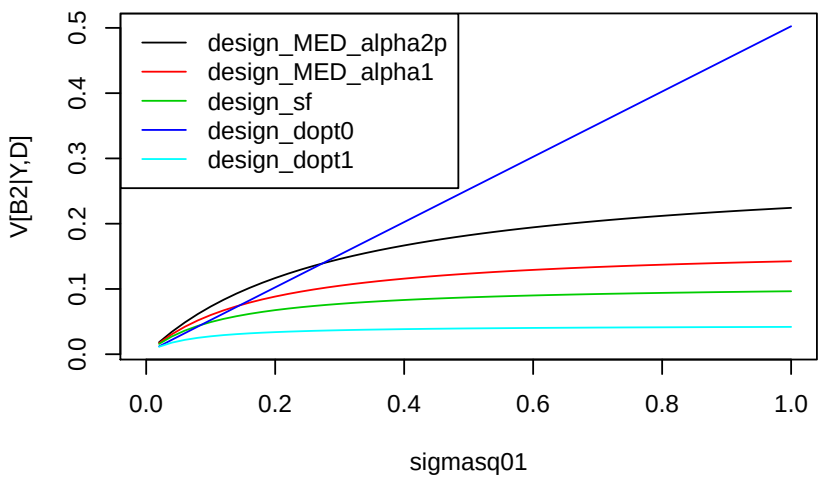

design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B2)
0.00
0.05
0.10
0.15
0.0
0.2
0.4
0.6
0.8
1.0
sigmasq01

- As in the previous example(s), MEDs do better than the space-filling design for the intercept and quadratic terms' coefficients, but not for the linear term's coefficients...
- Similar behavior in $Var[\beta_n | Y, D, H_1]$ in the next slide

Posterior Variances | H1







Some Overarching Conclusions

Thoughts

- Sadly I think I'll need to do some more examples before I can be sure what anything is doing
- Possibly think of a way to use the fact that our design can be used for sharp hypotheses (whereas D-optimal does not, and space-filling has no assumed model at all) to think of a case where it would perform better than D-optimal designs and space-filling design
- It seems to me that D-optimal design is like a space-filling design but with more thoughtful replication, and that's why it does so well?
- Trying to figure out why sometimes both D-optimal designs AND space-filling design do better than MED, when it's supposed to be a compromise between them - this could help explain the behavior that we see in our evaluations.

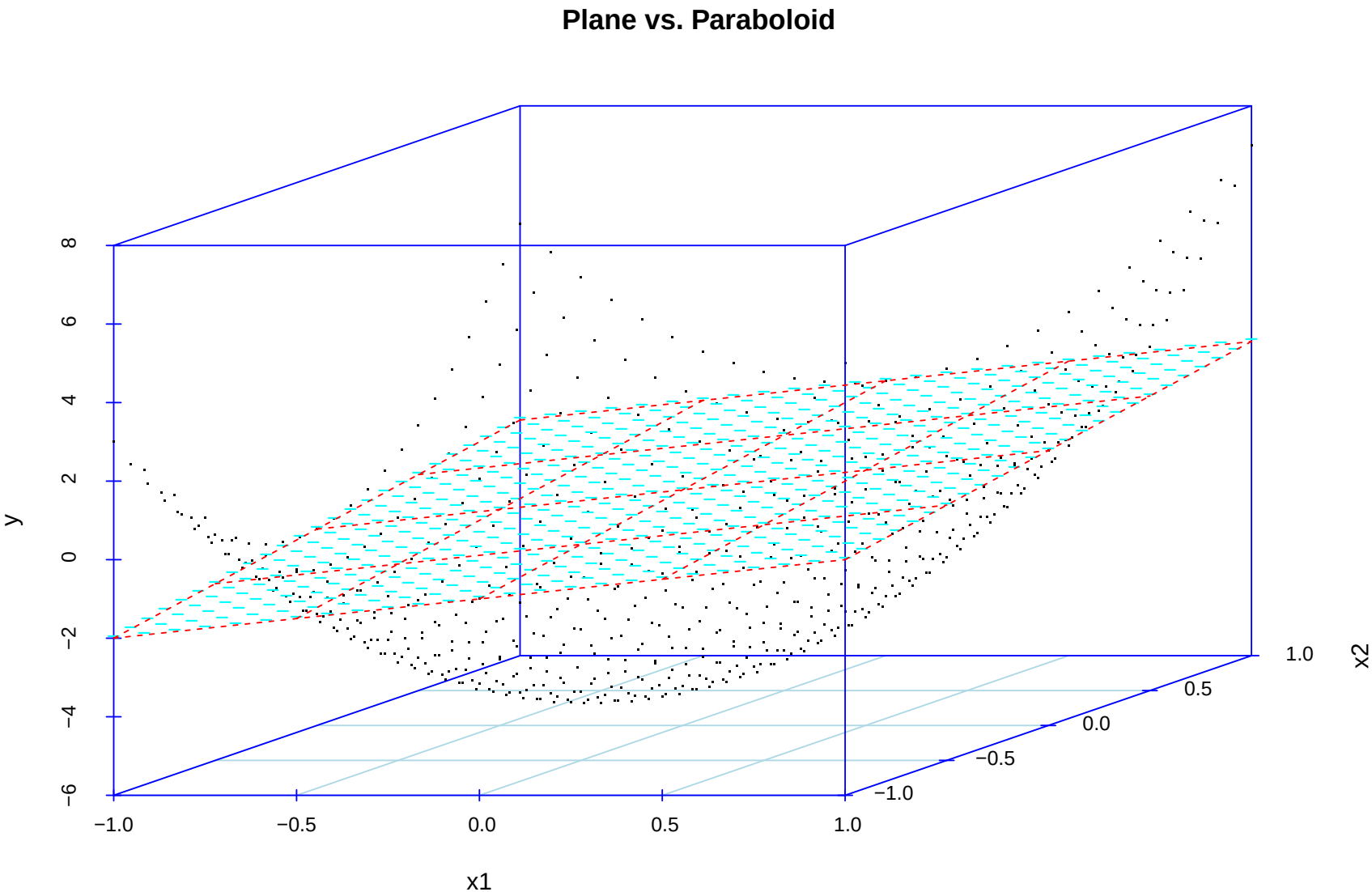
2 Dimensions

2D Example 1

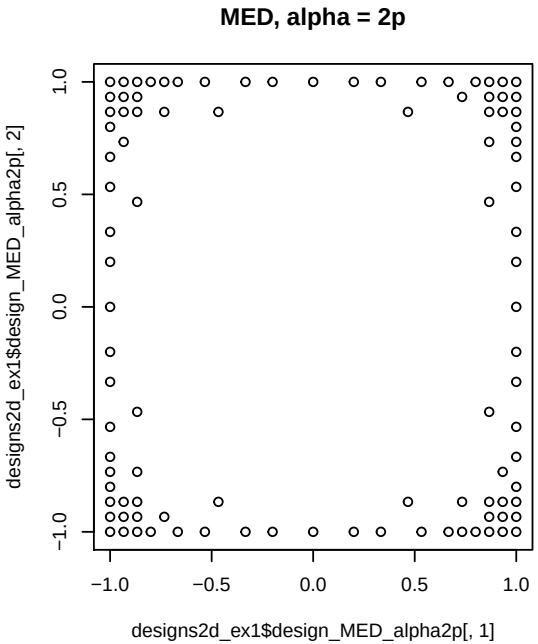
2D Example 1 Parameters

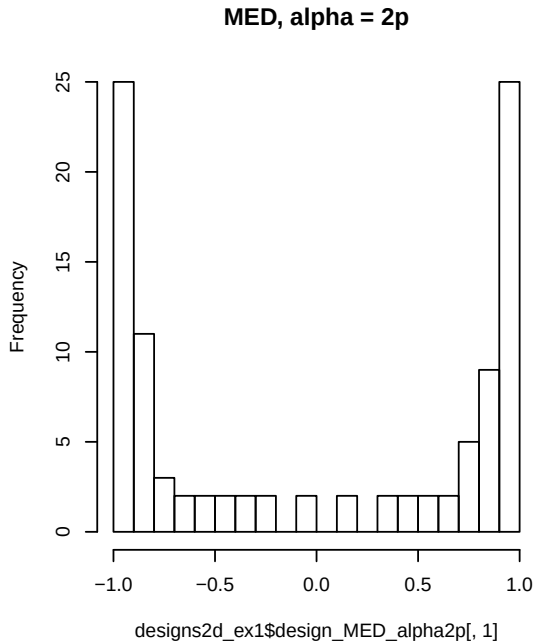
```
# Priors
sigmasq01 = 0.01
mu0 = c(0, 1, 1)
V0 = diag(rep(sigmasq01,length(mu0)))
mu1 = c(0, 1, 0, 1, 0)
V1 = diag(rep(sigmasq01,length(mu1)))
sigmasq = 1
betaT = c(-5, 1, 5, 1, 5)
```

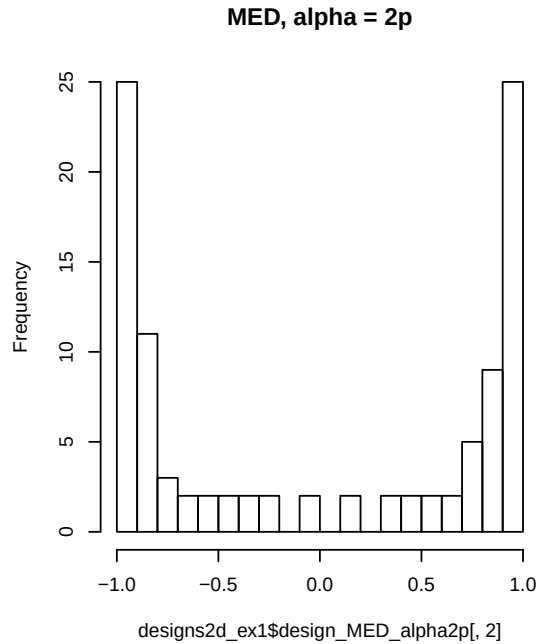
2D Example 1 Plots

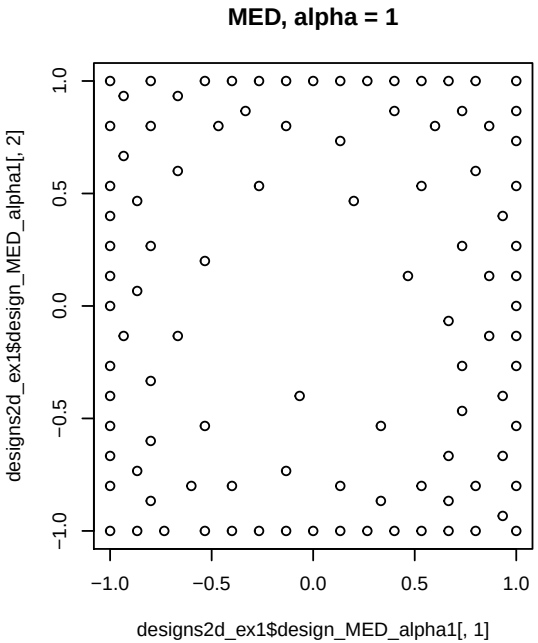


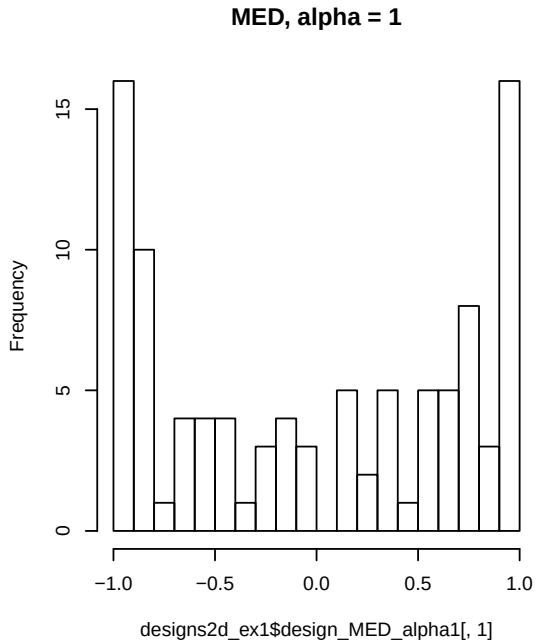
2D Example 1 MEDs

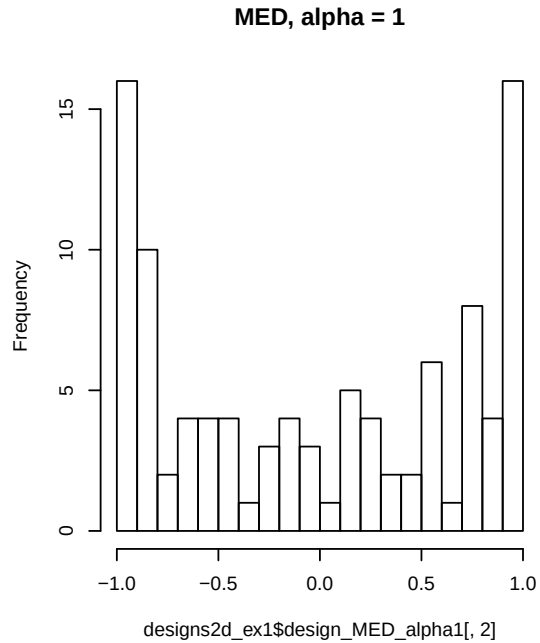




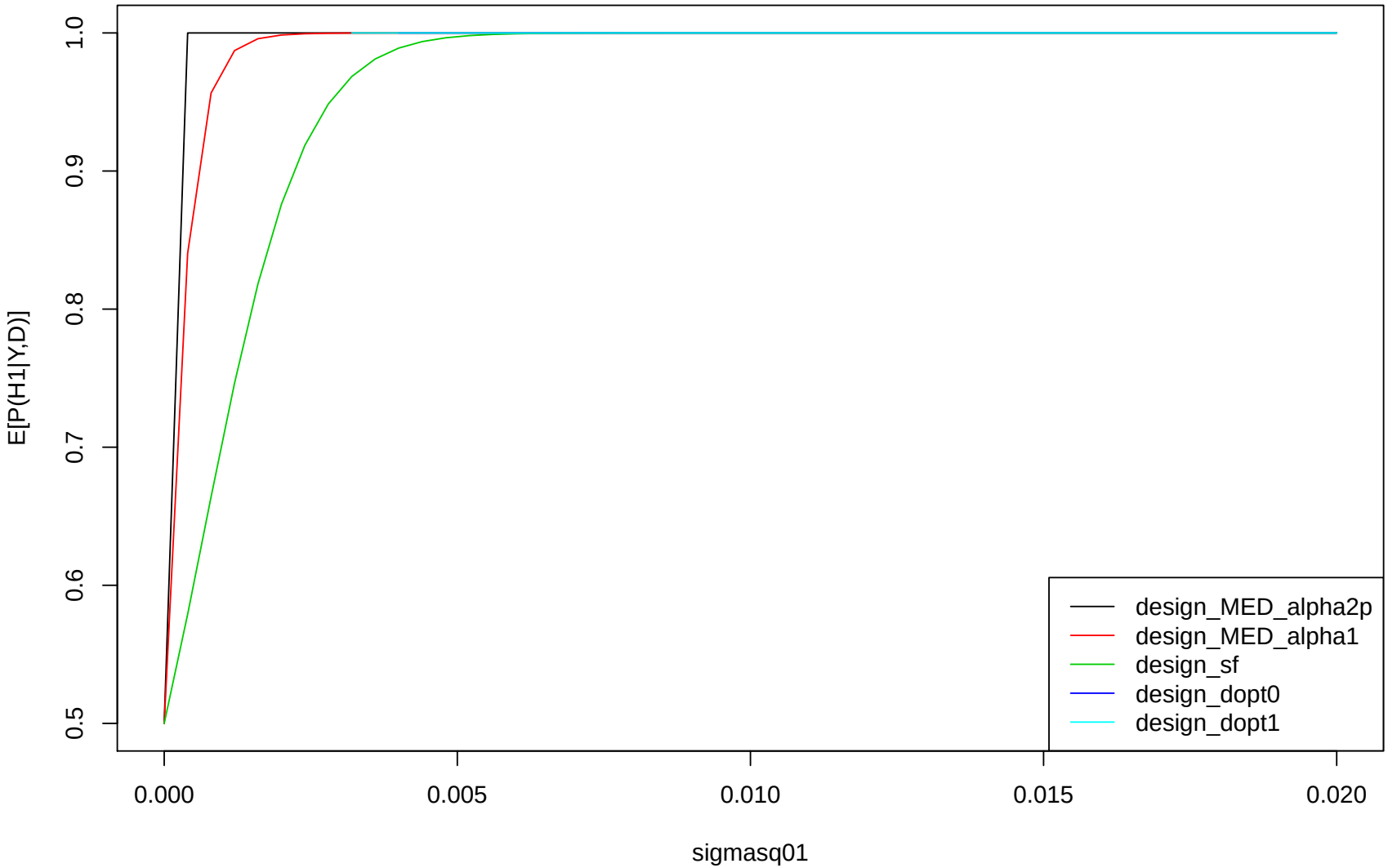




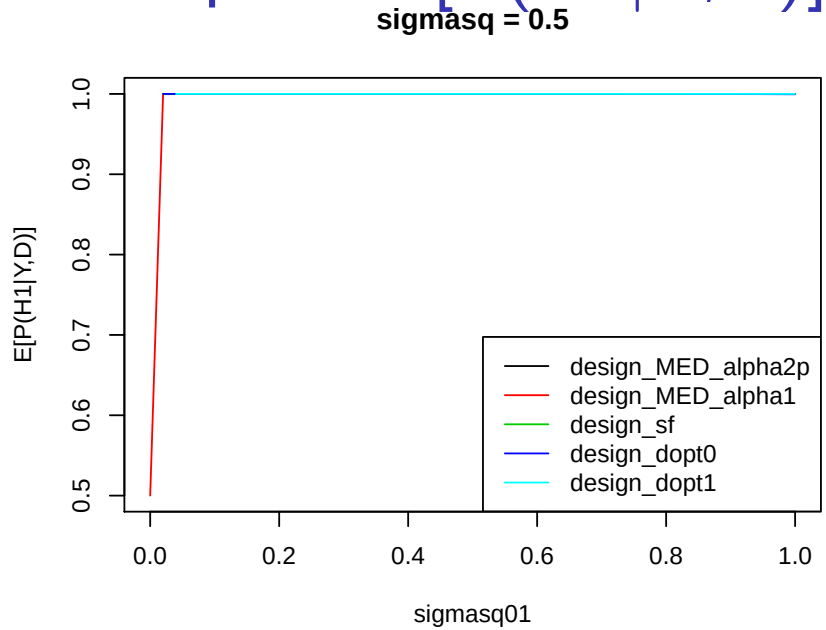


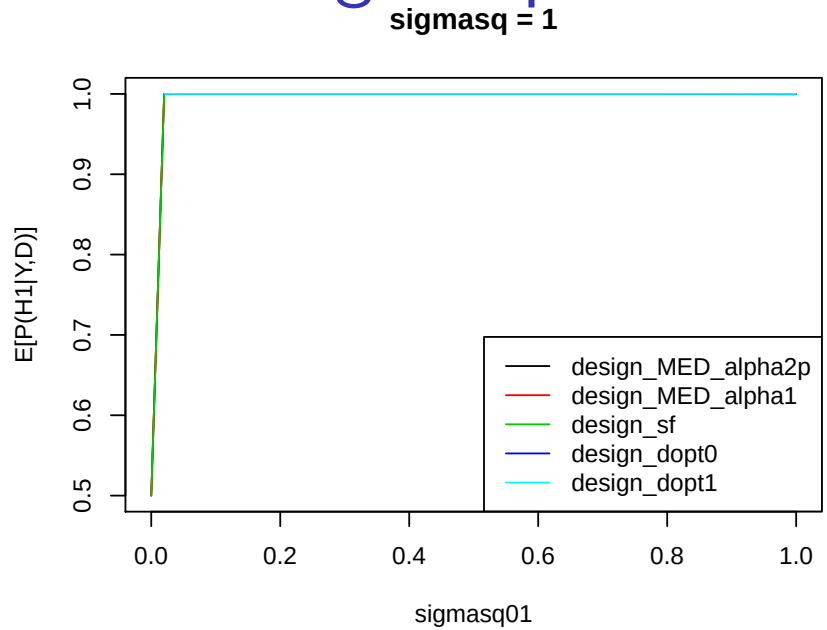


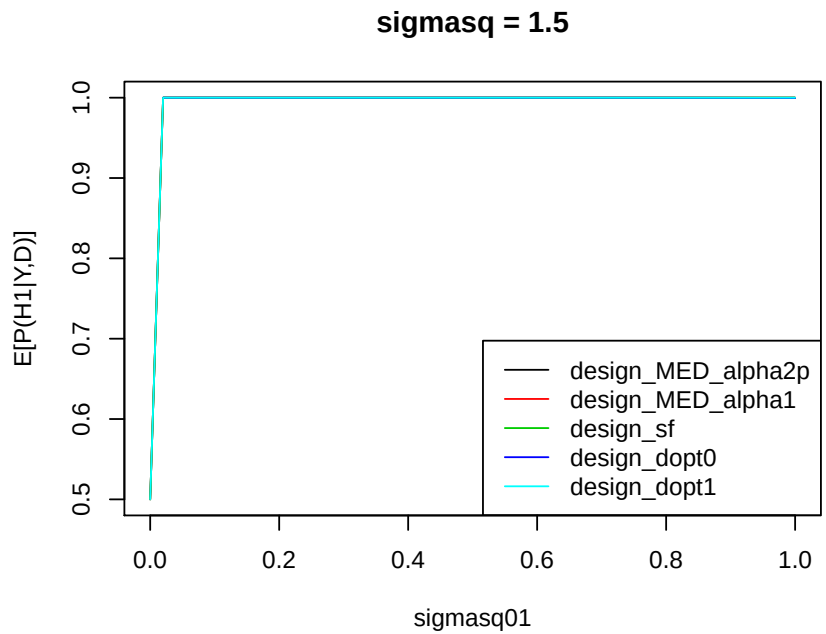
2D Example 1 E[P(H1|Y,D)]



2D Example 1 E[P(H1|Y,D)] Different sigmasq







MSE(Bn|Y,D,H1, BT)

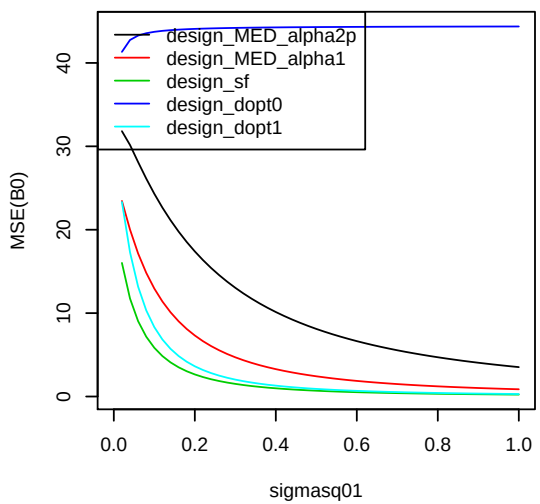
- `sigmasq01 = 0.5`

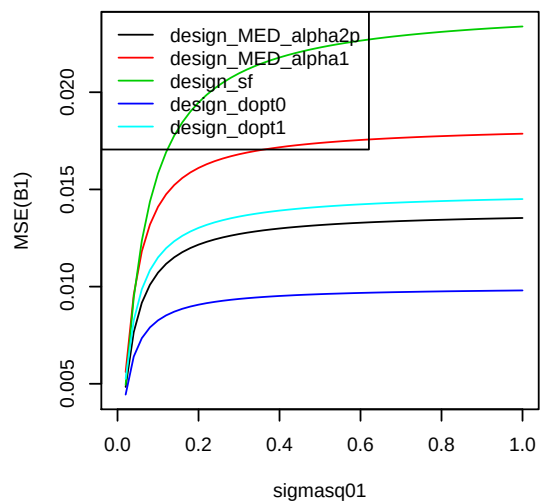
	design_MED_alpha2p	design_MED_alpha1	design_sf	design_dopt0	design_dopt1
MSE(B0)	8.10736	2.42599	0.69502	44.29850	0.91271
MSE(B1)	0.01317	0.01739	0.02230	0.00961	0.01410
MSE(B2)	3.82836	1.94878	0.86813	11.18591	0.44246
MSE(B3)	0.01317	0.01705	0.02230	0.00961	0.01410
MSE(B4)	3.82836	1.81807	0.86813	11.18591	0.44246

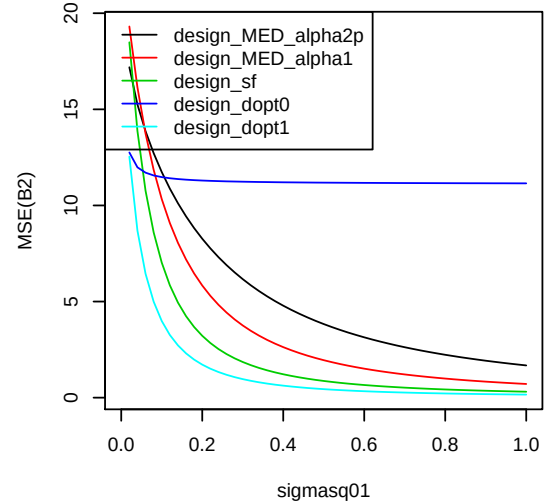
- `sigmasq01 = 1`

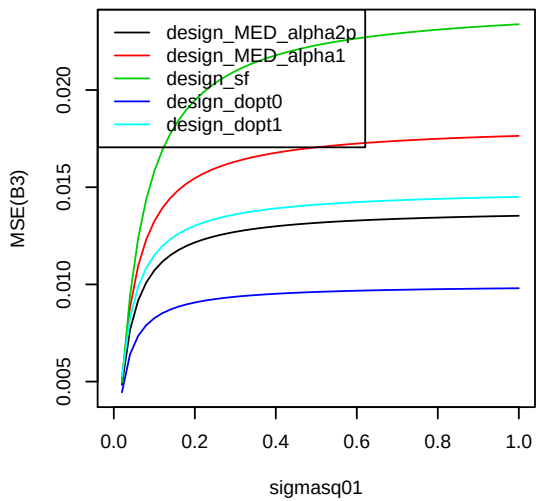
	design_MED_alpha2p	design_MED_alpha1	design_sf	design_dopt0	design_dopt1
MSE(B0)	3.52662	0.86873	0.23084	44.37175	0.30003
MSE(B1)	0.01353	0.01787	0.02338	0.00980	0.01450
MSE(B2)	1.67709	0.71272	0.30937	11.14916	0.15898
MSE(B3)	0.01353	0.01764	0.02338	0.00980	0.01450
MSE(B4)	1.67709	0.66373	0.30937	11.14916	0.15898

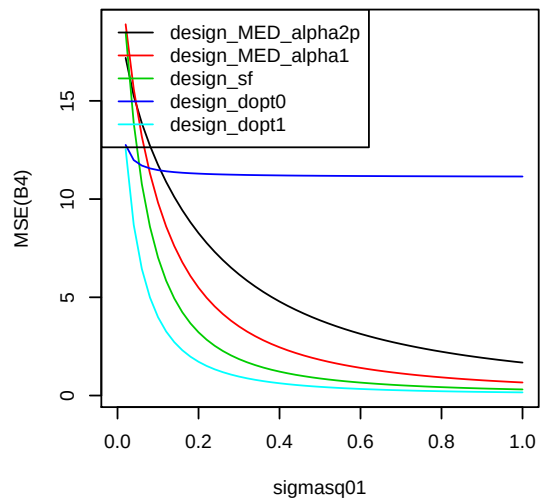
MSEs


design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B0)
sigmasq01

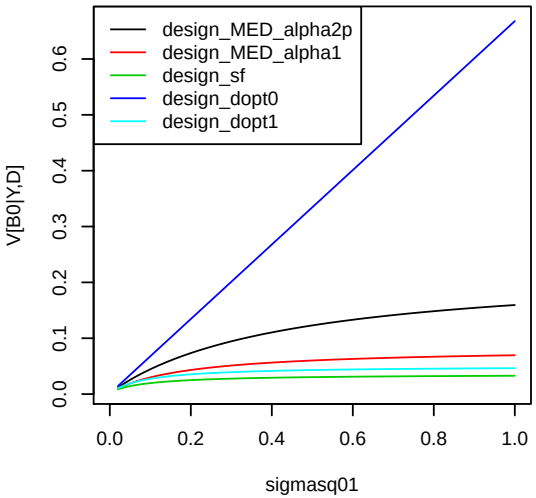

design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B1)
sigmasq01

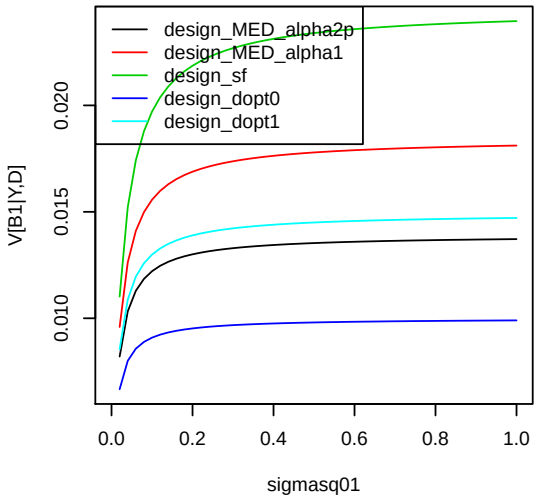

design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B2)
sigmasq01

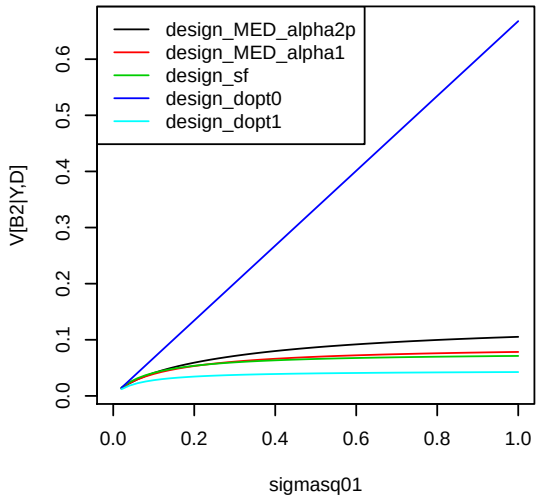

design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B3)
sigmasq01

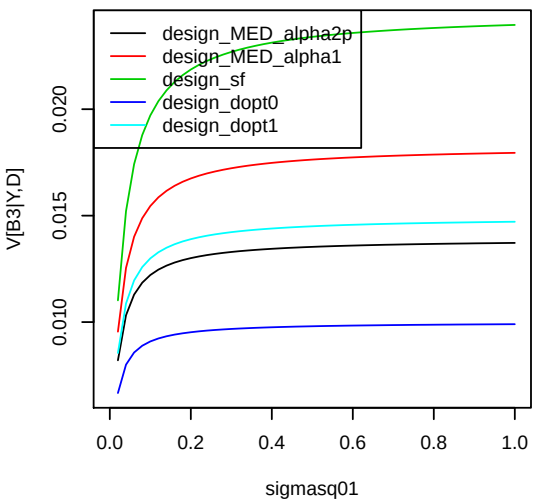

design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B4)
sigmasq01

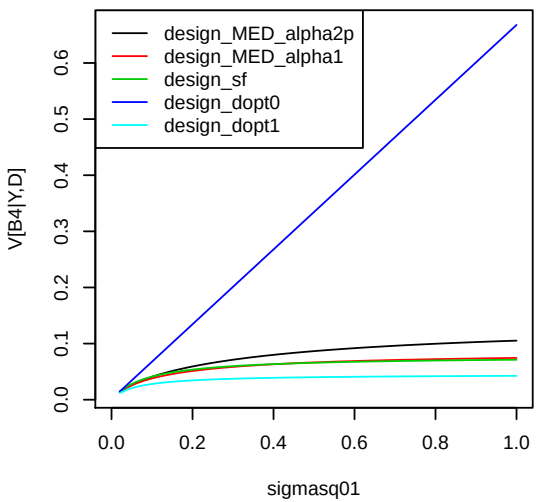
Posterior Variances | H1









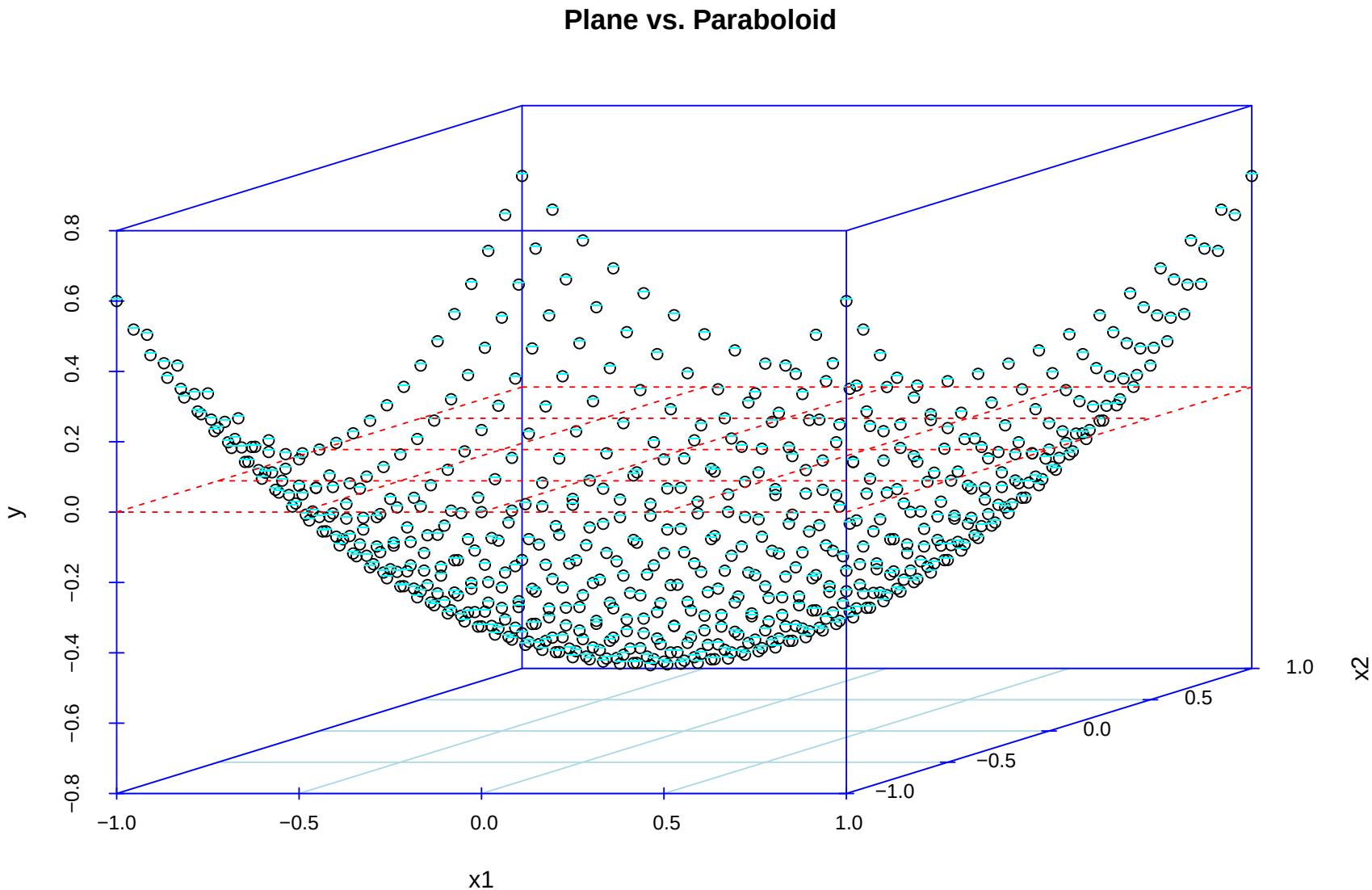


2D Example 2

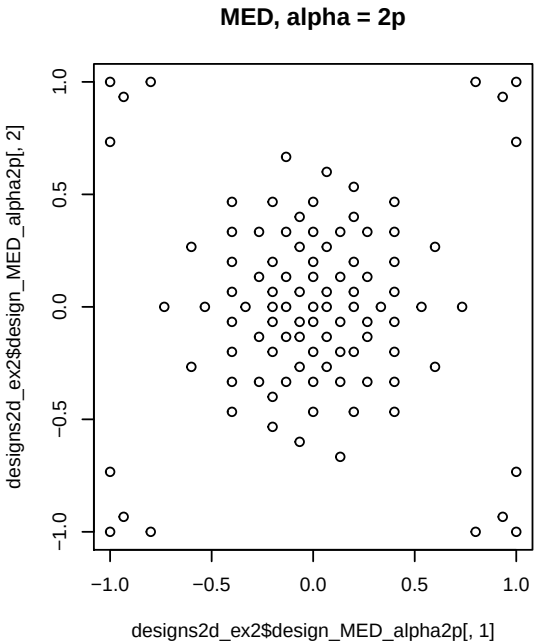
2D Example 2 Parameters

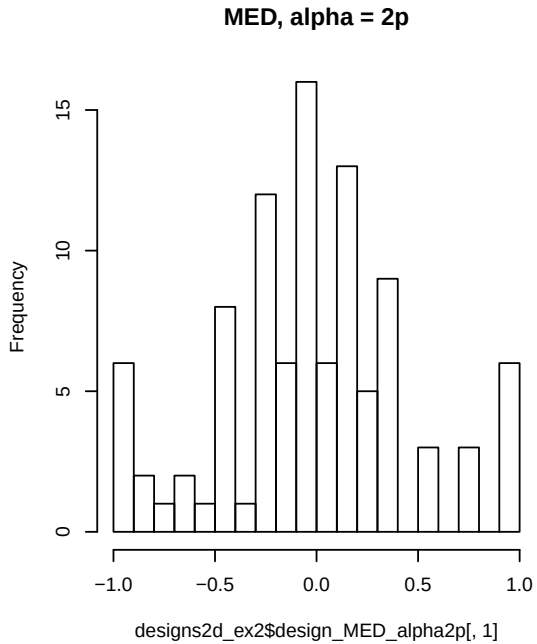
```
# Priors
sigmasq01 = 0.01
mu0 = c(0, 0, 0)
V0 = diag(rep(sigmasq01,length(mu0)))
mu1 = c(-0.6, 0, 0.6, 0, 0.6)
V1 = diag(rep(sigmasq01,length(mu1)))
sigmasq = 1
betaT = mu1
```

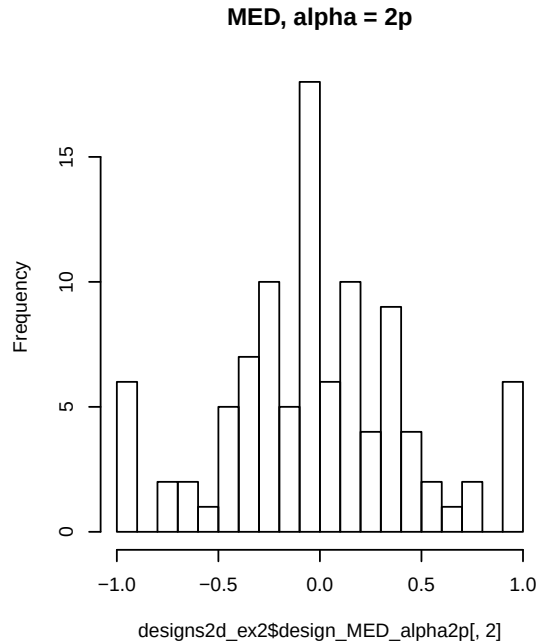
2D Example 2 Plots

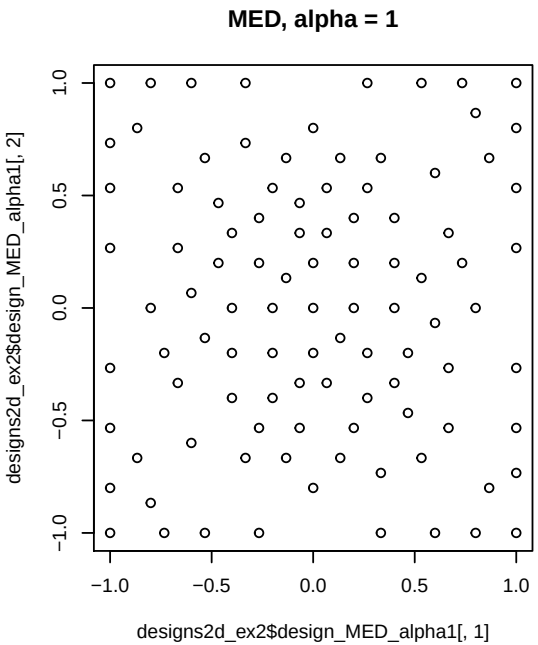


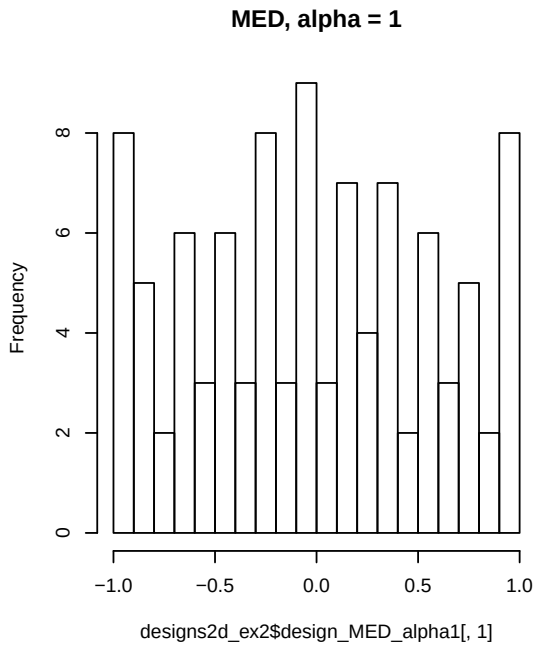
2D Example 2 MEDs

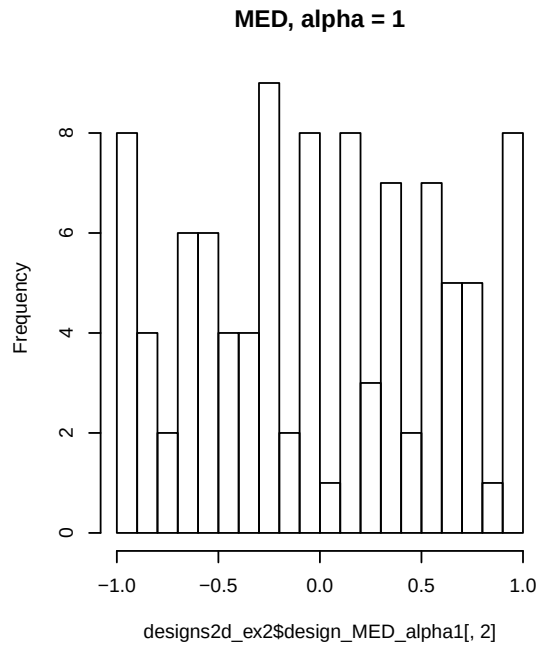




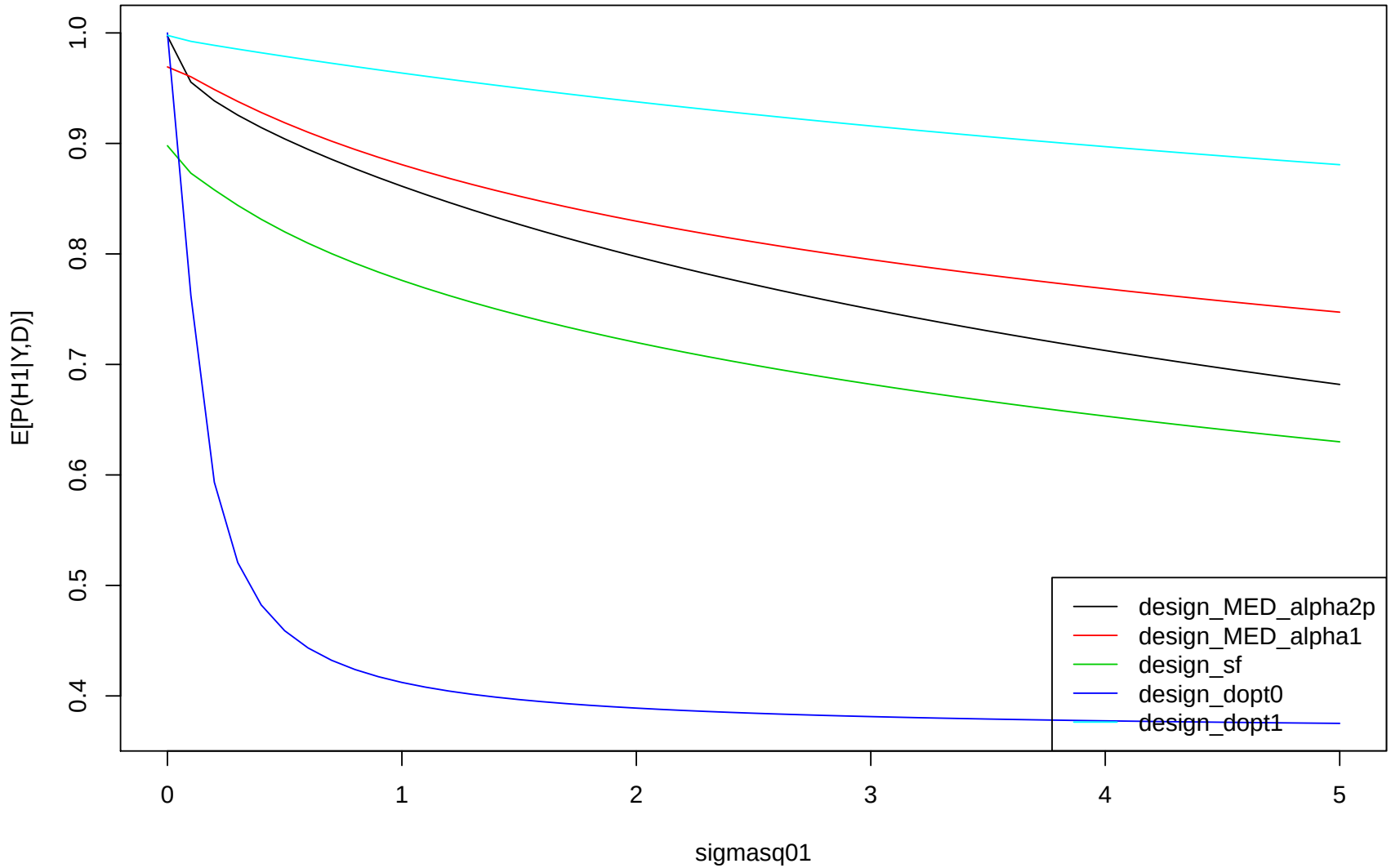




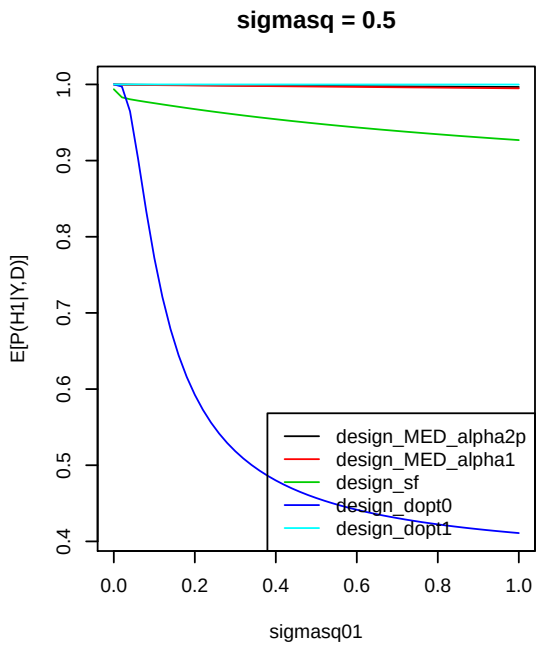


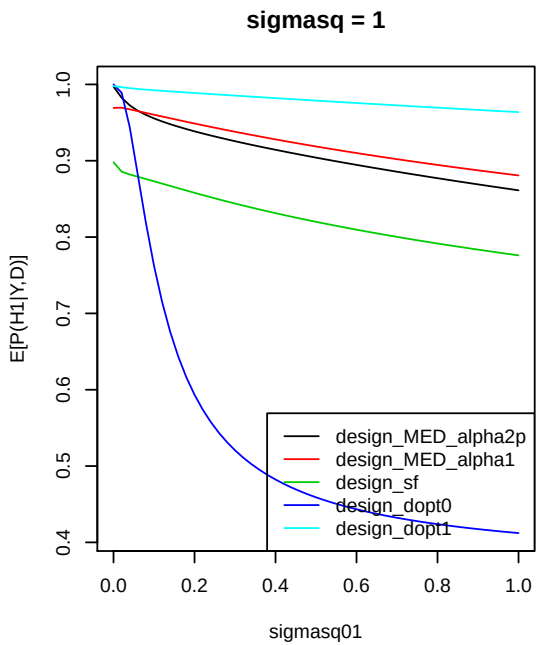


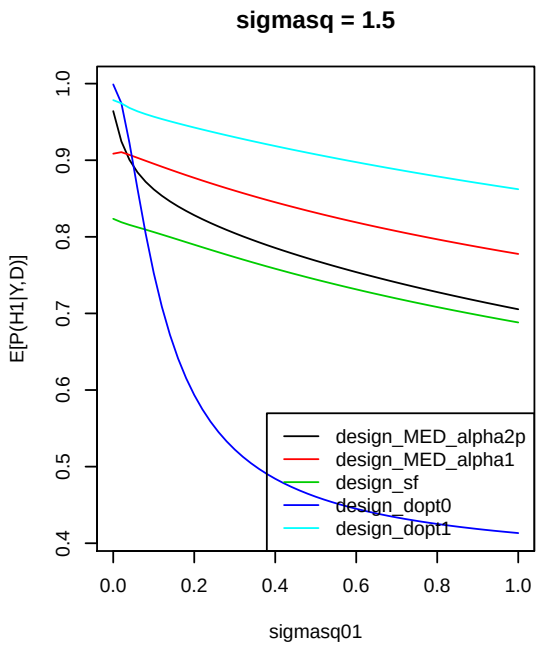
2D Example 2 E[P(H1|Y,D)]



2D Example 2 E[P(H1|Y,D)] Different sigmasq







MSE(Bn|Y,D,H1, BT)

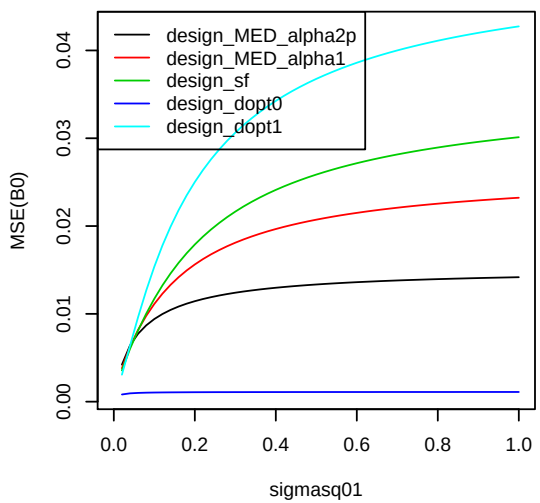
- `sigmasq01 = 0.5`

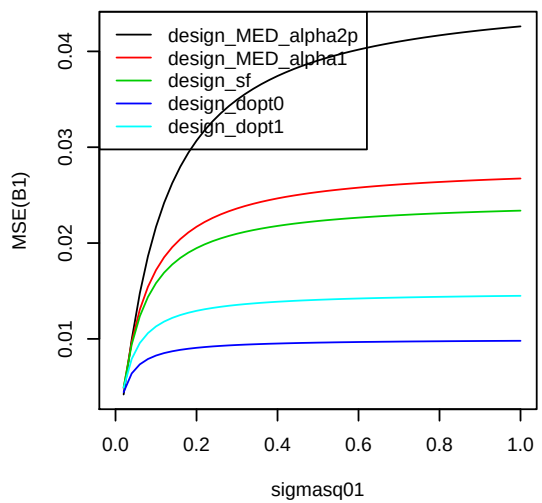
	design_MED_alpha2p	design_MED_alpha1	design_sf	design_dopt0	design_dopt1
MSE(B0)	0.01334	0.02073	0.02586	0.00110	0.03676
MSE(B1)	0.03905	0.02533	0.02230	0.00961	0.01408
MSE(B2)	0.08390	0.06111	0.05582	0.00110	0.03559
MSE(B3)	0.03996	0.02521	0.02230	0.00961	0.01408
MSE(B4)	0.08641	0.06314	0.05582	0.00110	0.03559

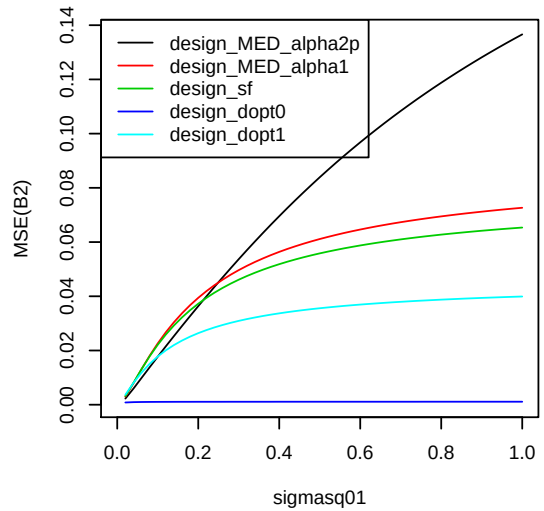
- `sigmasq01 = 1`

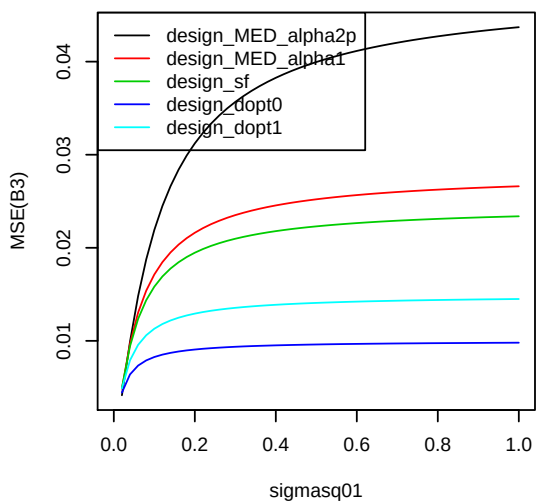
	design_MED_alpha2p	design_MED_alpha1	design_sf	design_dopt0	design_dopt1
MSE(B0)	0.01417	0.02323	0.03012	0.0011	0.04274
MSE(B1)	0.04261	0.02674	0.02338	0.0098	0.01450
MSE(B2)	0.13661	0.07265	0.06535	0.0011	0.03992
MSE(B3)	0.04370	0.02661	0.02338	0.0098	0.01450
MSE(B4)	0.14222	0.07560	0.06535	0.0011	0.03992

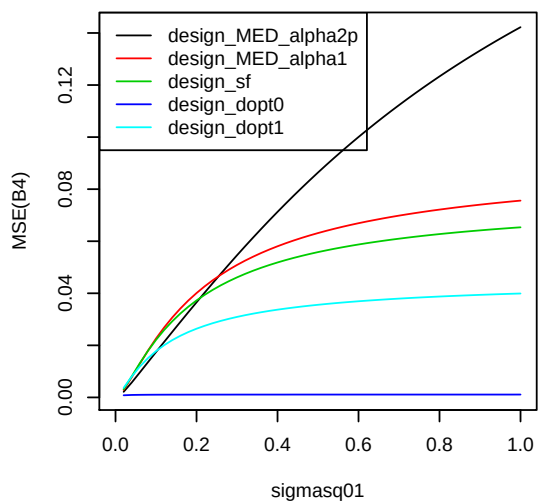
MSEs


design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B0)
sigmasq01

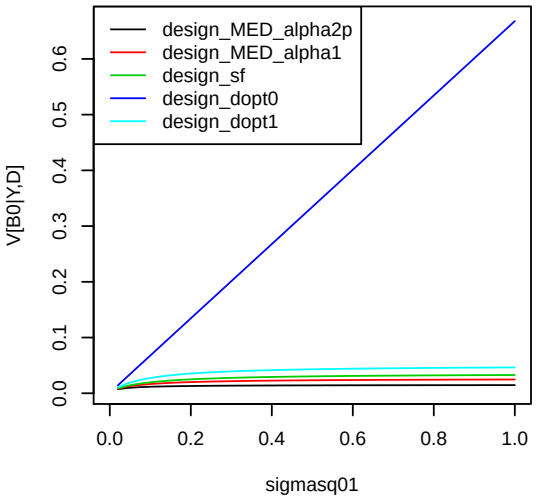

design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B1)
sigmasq01

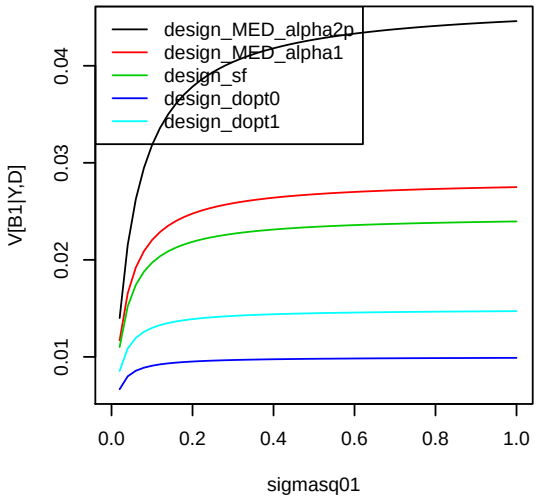

design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B2)
sigmasq01

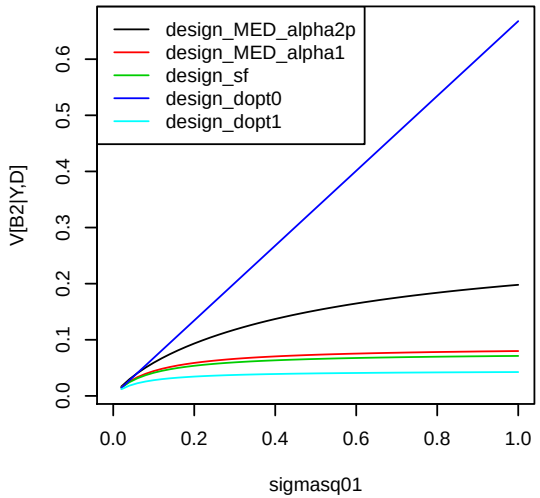

design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B3)
sigmasq01

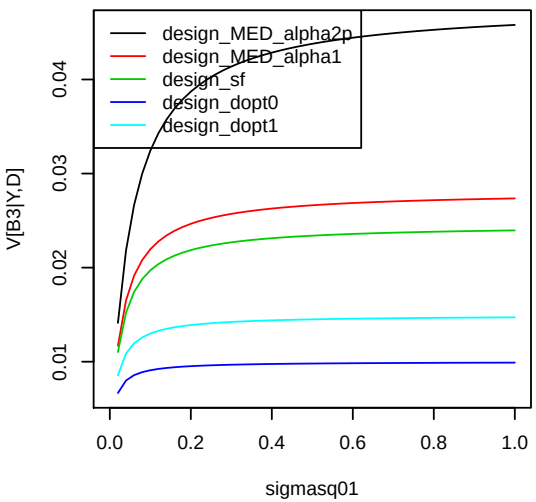

design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B4)
sigmasq01

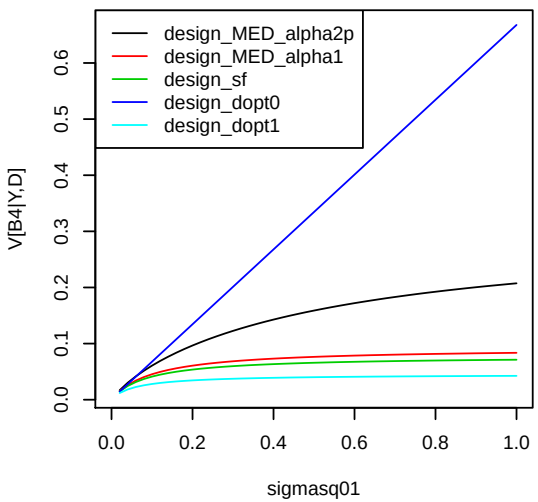
Posterior Variances | H1









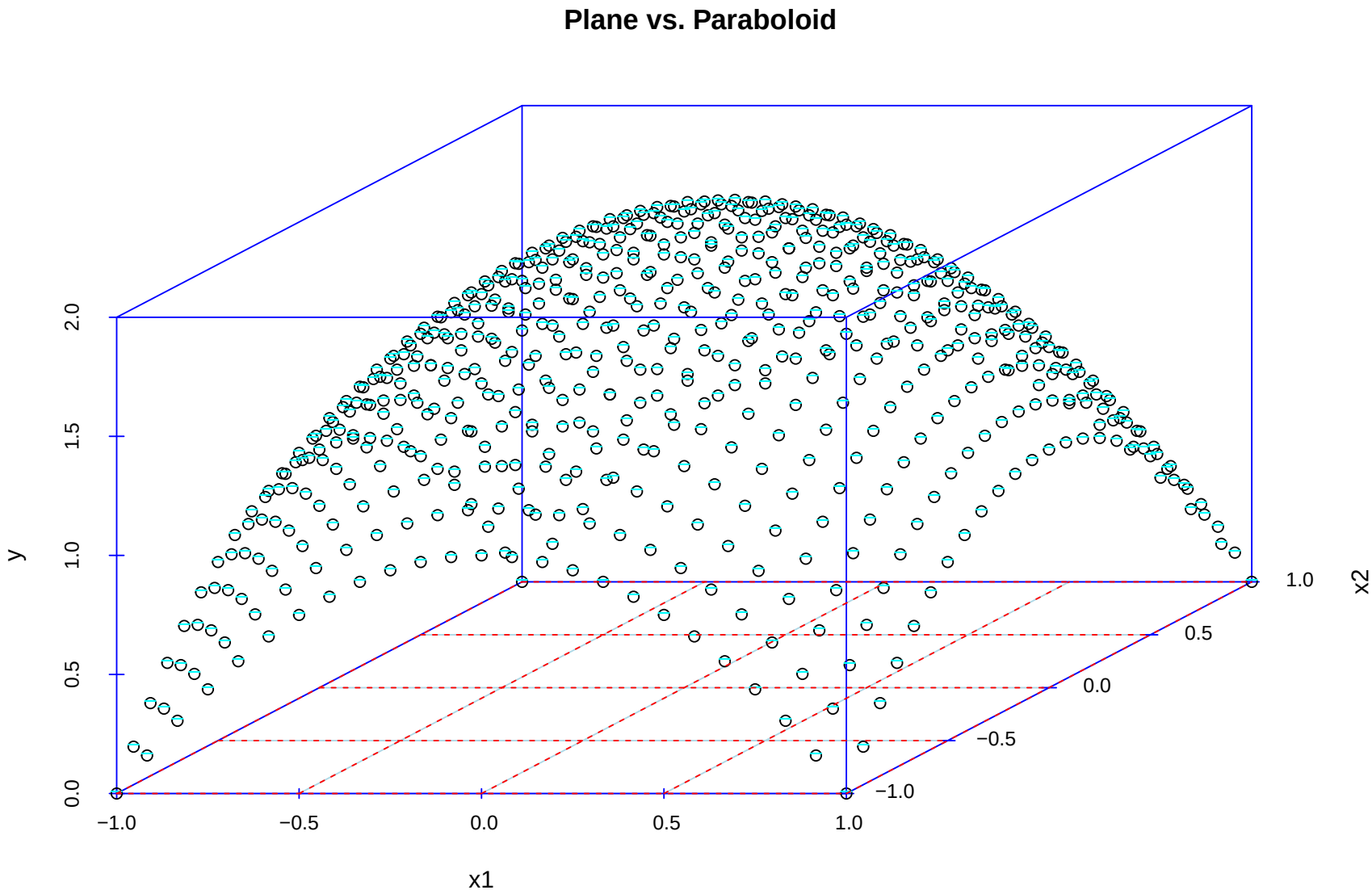


2D Example 3

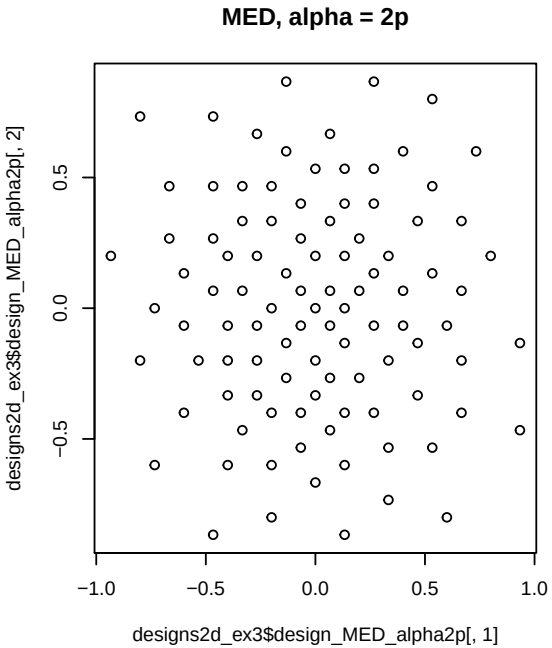
2D Example 3 Parameters

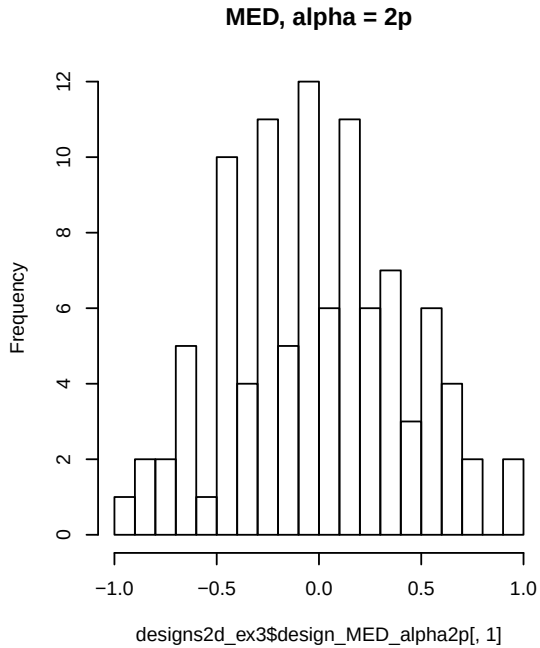
```
# Priors
sigmasq01 = 0.01
mu0 = c(0, 0, 0)
V0 = diag(rep(sigmasq01,length(mu0)))
mu1 = c(-0.6, 0, 0.6, 0, 0.6)
V1 = diag(rep(sigmasq01,length(mu1)))
sigmasq = 1
betaT = mu1
```

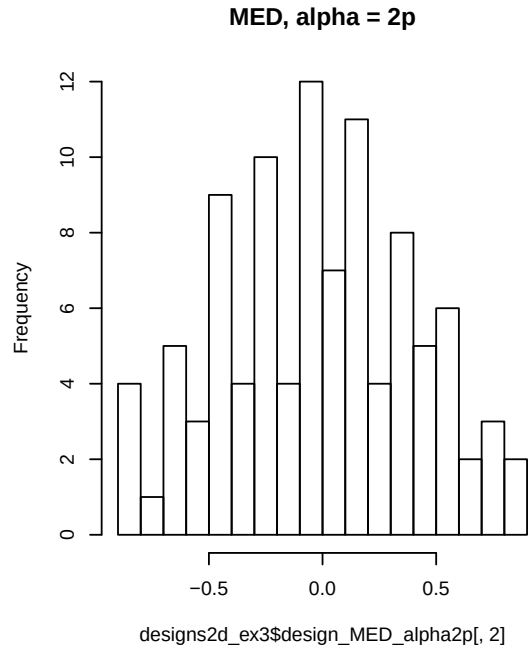
2D Example 3 Plots

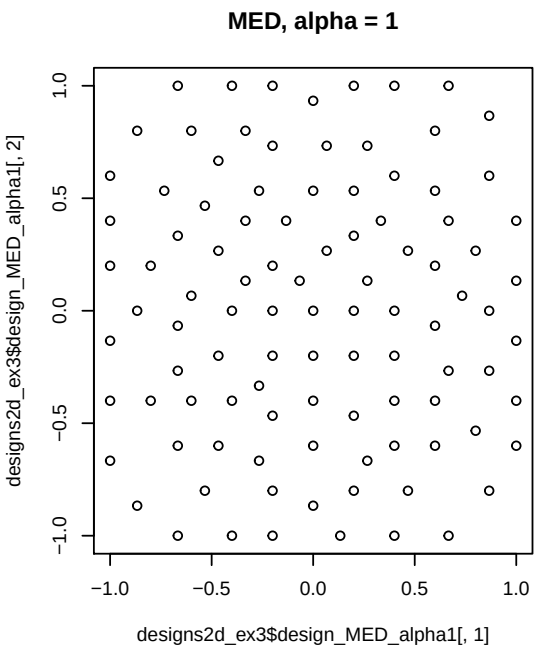


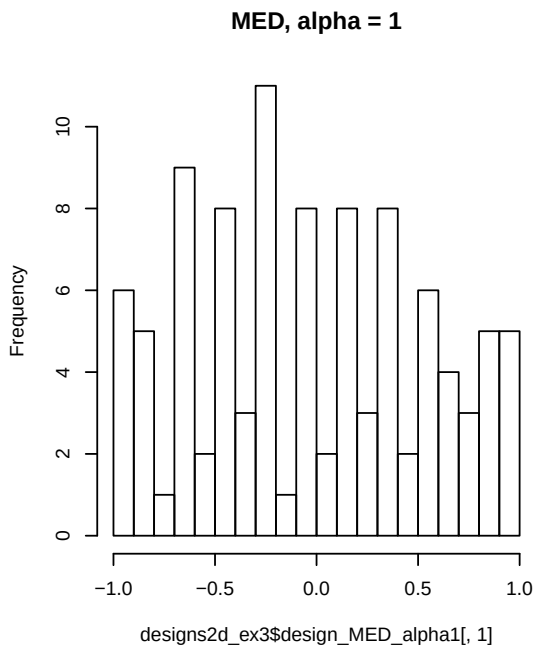
2D Example 3 MEDs

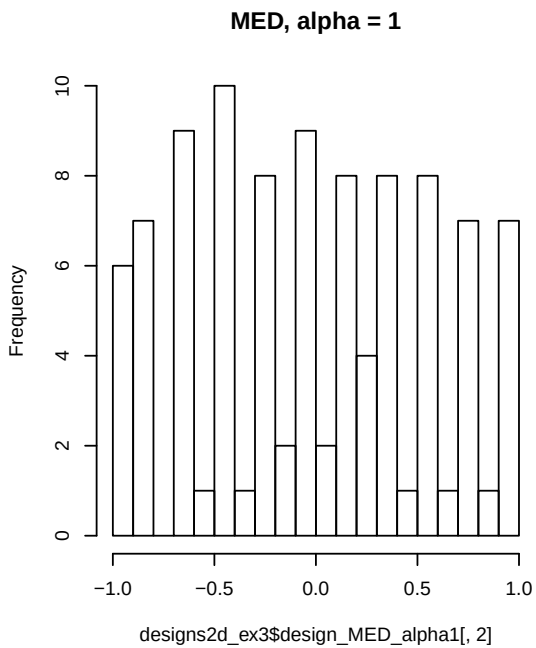




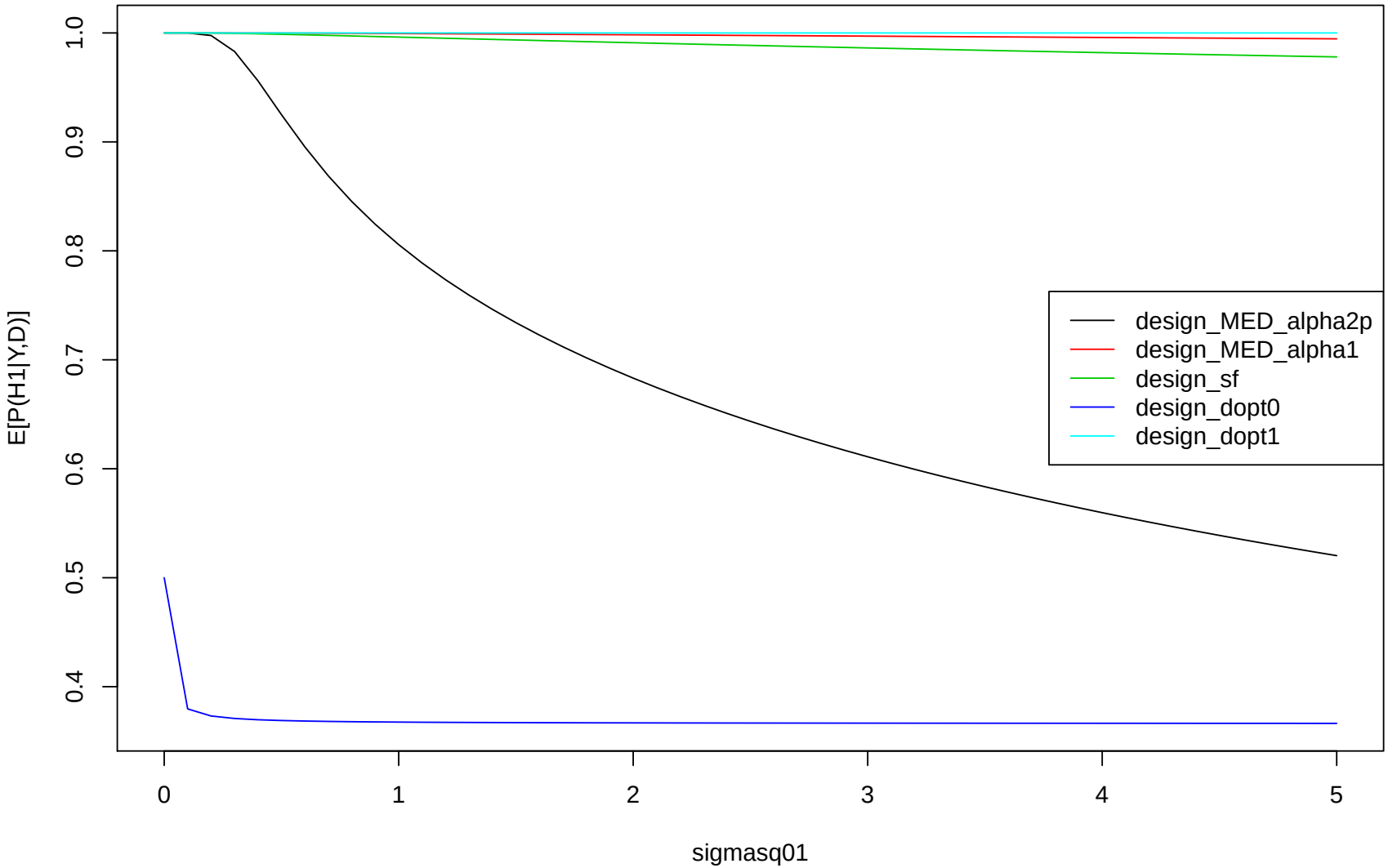




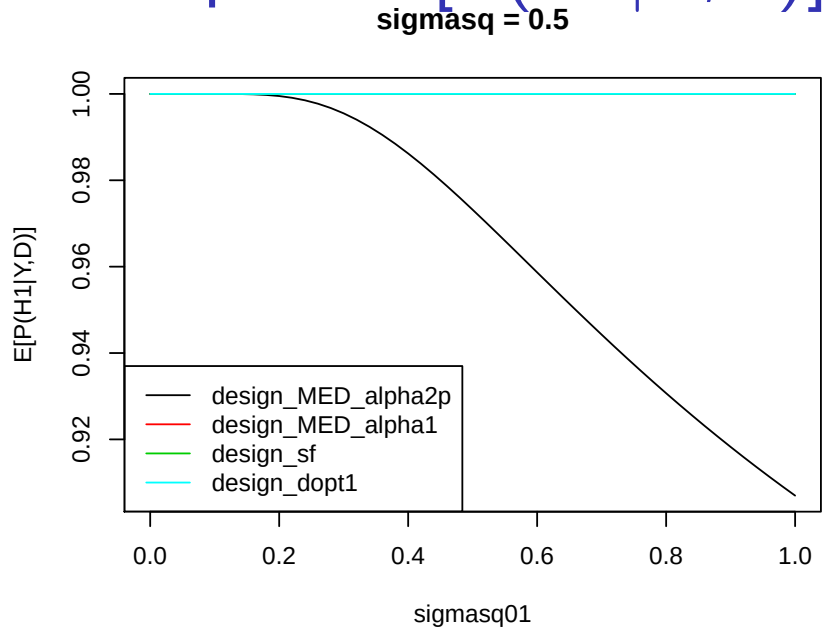


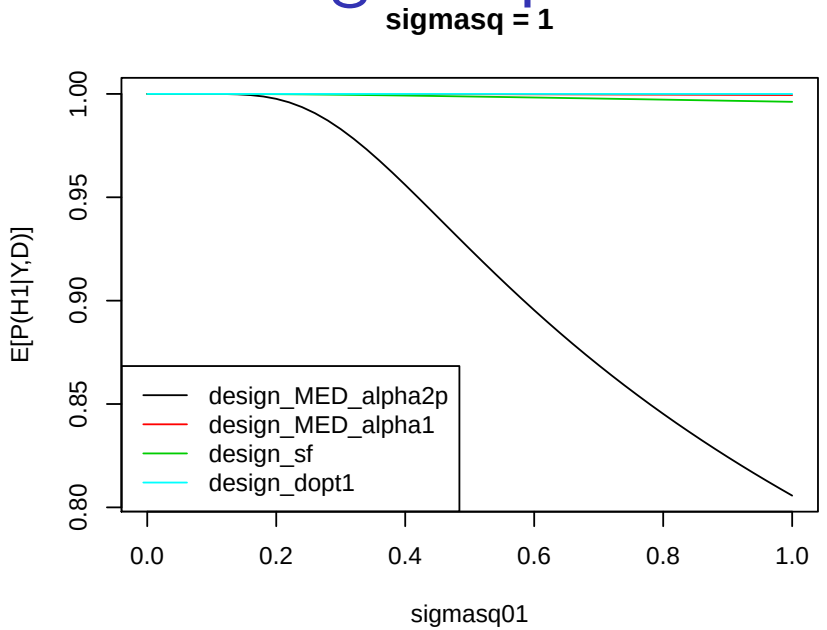


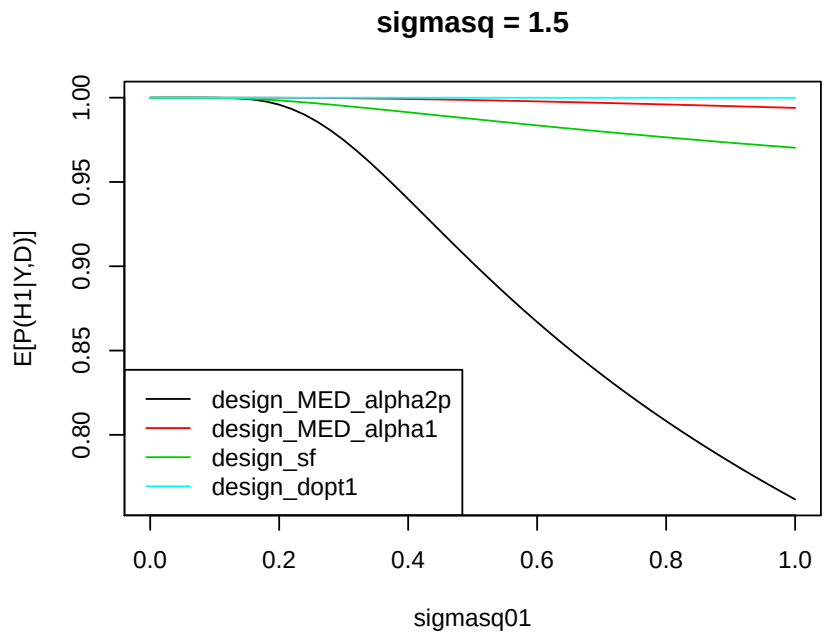
2D Example 3 E[P(H1|Y,D)]



2D Example 3 E[P(H1|Y,D)] Different sigmasq







MSE(Bn|Y,D,H1, BT)

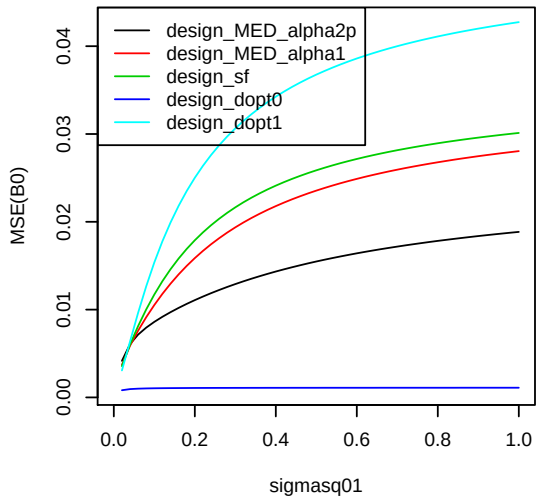
- `sigmasq01 = 0.5`

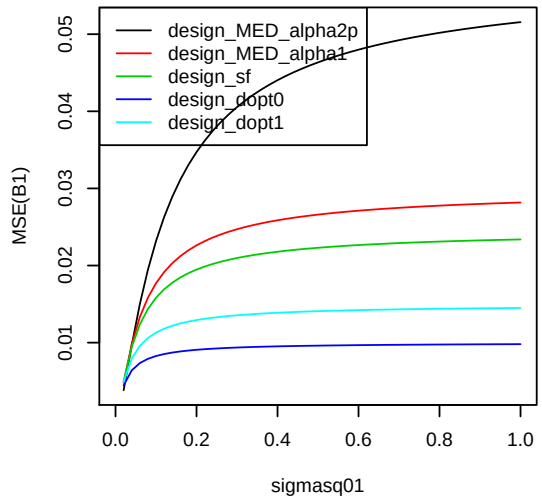
	design_MED_alpha2p	design_MED_alpha1	design_sf	design_dopt0	design_dopt1
MSE(B0)	0.01548	0.02355	0.02586	0.00110	0.03676
MSE(B1)	0.04634	0.02660	0.02230	0.00961	0.01408
MSE(B2)	0.10594	0.06508	0.05582	0.00110	0.03559
MSE(B3)	0.04595	0.02664	0.02230	0.00961	0.01408
MSE(B4)	0.10854	0.06364	0.05582	0.00110	0.03559

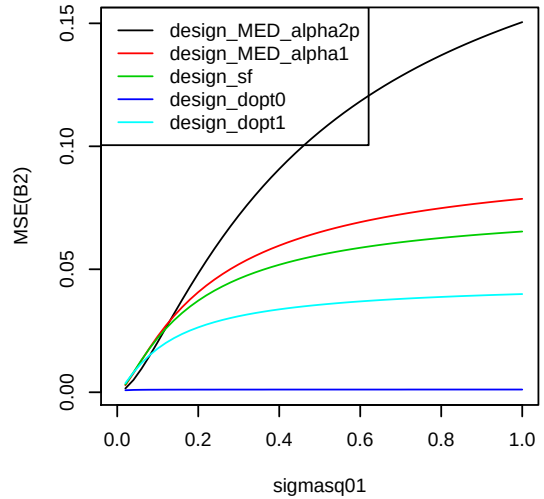
- `sigmasq01 = 1`

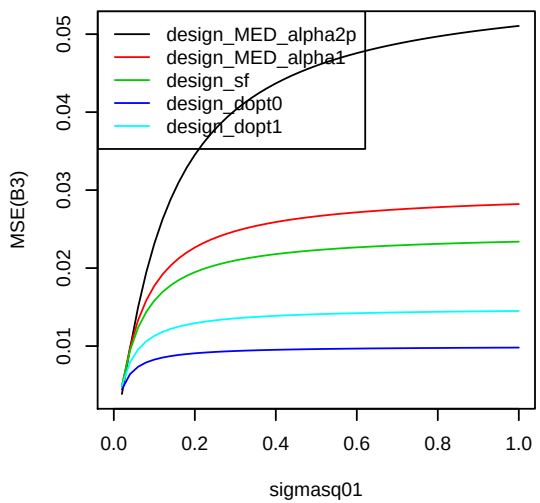
	design_MED_alpha2p	design_MED_alpha1	design_sf	design_dopt0	design_dopt1
MSE(B0)	0.01885	0.02805	0.03012	0.0011	0.04274
MSE(B1)	0.05156	0.02817	0.02338	0.0098	0.01450
MSE(B2)	0.15049	0.07865	0.06535	0.0011	0.03992
MSE(B3)	0.05105	0.02821	0.02338	0.0098	0.01450
MSE(B4)	0.15712	0.07649	0.06535	0.0011	0.03992

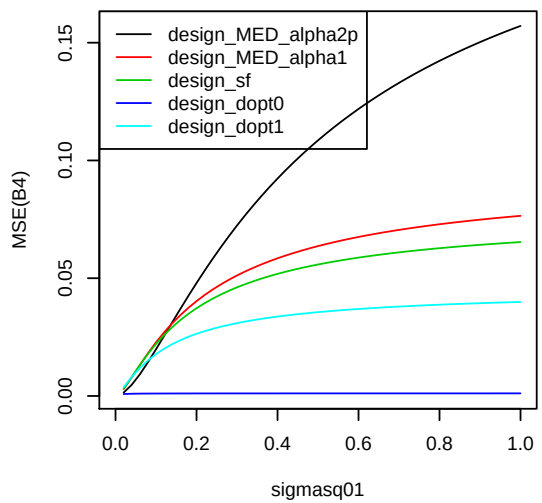
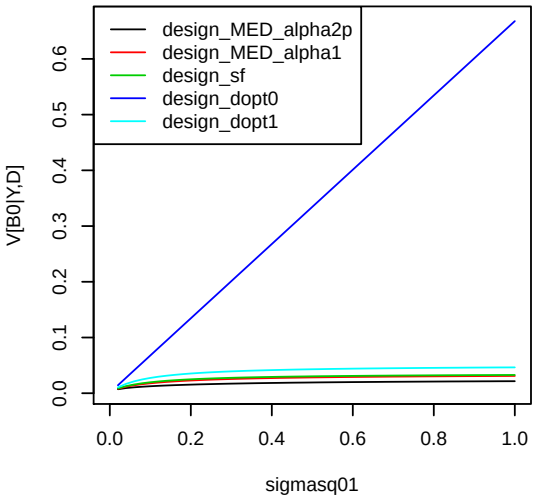
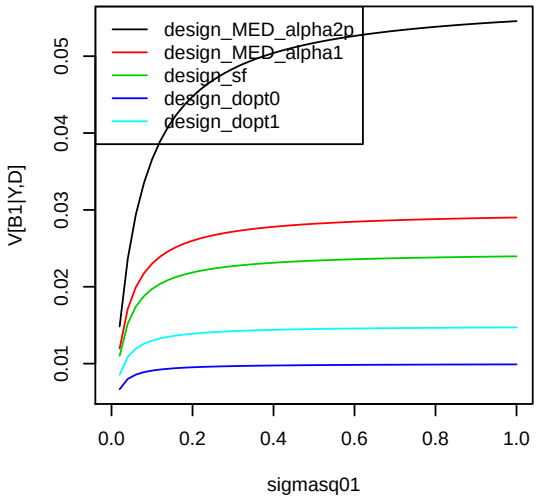
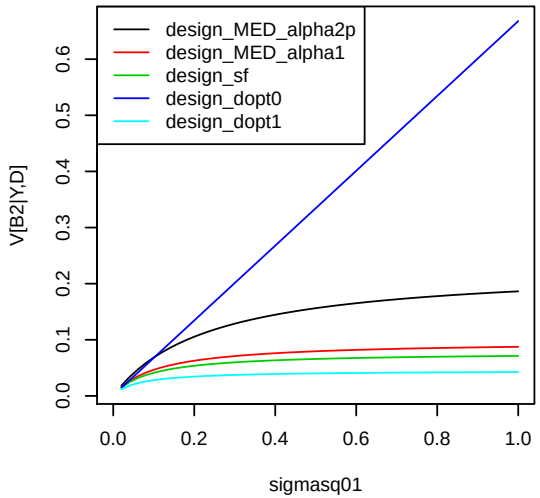
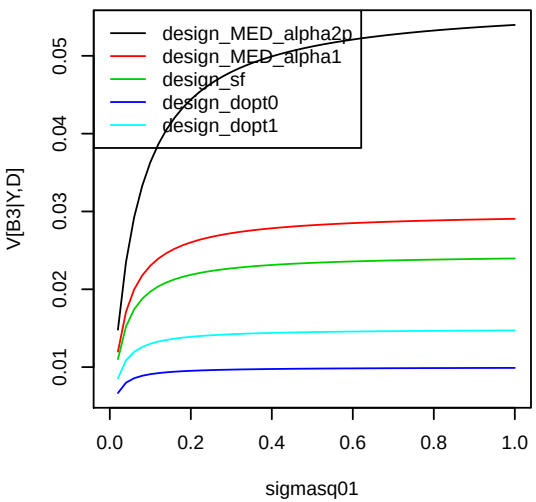
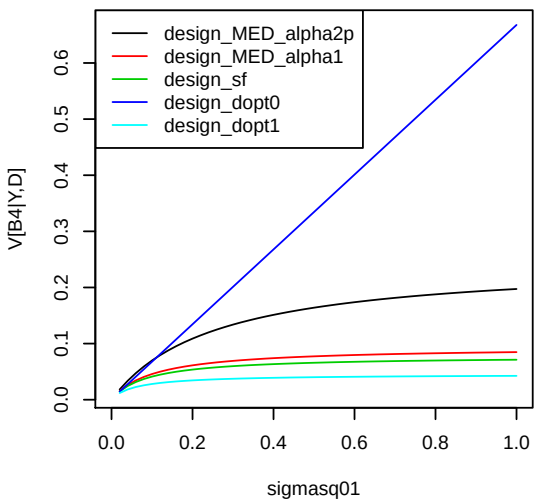
MSEs


design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B0)
0.00
0.01
0.02
0.03
0.04
0.0
0.2
0.4
0.6
0.8
1.0
sigmasq01


design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B1)
0.01
0.02
0.03
0.04
0.05
0.0
0.2
0.4
0.6
0.8
1.0
sigmasq01


design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B2)
0.00
0.05
0.10
0.15
0.0
0.2
0.4
0.6
0.8
1.0
sigmasq01


design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B3)
0.01
0.02
0.03
0.04
0.05
0.0
0.2
0.4
0.6
0.8
1.0
sigmasq01


design_MED_alpha2p
design_MED_alpha1
design_sf
design_dopt0
design_dopt1
MSE(B4)
0.00
0.05
0.10
0.15
0.0
0.2
0.4
0.6
0.8
1.0
sigmasq01

Posterior Variances | H1