Evaluations for Examples

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9/29/2019

1 Dimension

Example 1

Example 2

Example 2.1 (Better than Example 2)

Example 3

Example 4

Example 4.1

Example 5

Summary

- In these slides, I look at different pairs of sharp hypotheses for generating MED, and how they evaluate based on
  - $E[P(H_1|Y, D)|\beta_T]$
  - $MSE(\beta_n|Y, D, H_1, \beta_T)$
- And we try to find an example where the quadratic D-Optimal design isn't best for the $E[P(H_1|Y, D)|\beta_T]$ evaluations. (Example 4.1, but also 1 & 5)
- This time, $\beta_T \sim N(\mu_1, V_1)$ (whereas last time, $\beta_T = \mu_1$)
- Things to consider:
  - For posterior evaluations, when $\beta_T = \mu_T$, it might not make sense to compare the D-optimal designs
  - We want to pay attention to cases of $\beta_T \neq \mu_T$ where D-optimal designs are expected to do worse than MEDs
  - And find cases where space-filling design does worse than MEDs

Observations from Last Time (Example 1: Estimating Linear Term's Coefficient)

In last slides, we saw what happens when $\beta_T = \mu_T$.

- In Example 1, where the MED concentrates at -1 and 1, we saw that:
  - for the linear term's coefficient, $MSE(\beta_{n,1}|Y, D, H_1, \beta_T)$ was smaller (i.e. better) for the MED than for space-filling design, but
  - for the intercept and quadratic term's coefficients, $MSE(\beta_{n,0}|Y, D, H_1, \beta_T)$ and $MSE(\beta_{n,2}|Y, D, H_1, \beta_T)$ were larger (i.e. worse) for MED compared to the space-filling design except at smaller values of the prior variance on β.
  - This makes me think that, like the linear D-optimal design suggests, concentrating at -1 and 1 is best for estimating the linear term - however, it is not best for estimating the intercept and quadratic term.

Observations from Last Time (Example 2: Estimating Intercept and Quadratic Term's Coefficient)

- In Example 2, where MED concentrates at -1, 1, and 0 (but mostly at 0), we saw that:
  - For $E[P(H_1|Y, D)|\beta_T]$, MED seems to be a compromise between the quadratic D-optimal design and the space-filling design (yay!)
  - For $MSE(\beta_n|Y, D, H_1, \beta_T)$,
    - $MSE(\beta_{n,0}|Y, D, H_1, \beta_T)$: MED is better than space-filling design
    - $MSE(\beta_{n,1}|Y, D, H_1, \beta_T)$: MED, surprisingly, is worse than space-filling design (perhaps it focuses too much in the middle than at the edges)
    - $MSE(\beta_{n,2}|Y, D, H_1, \beta_T)$: MED and space-filling are comparable... MED with $\alpha = 1$ does better than space-filling design, but MED with $\alpha = 2p$ does worse than space-filling design

Observations from Last Time (Example 3: Estimating Intercept well)

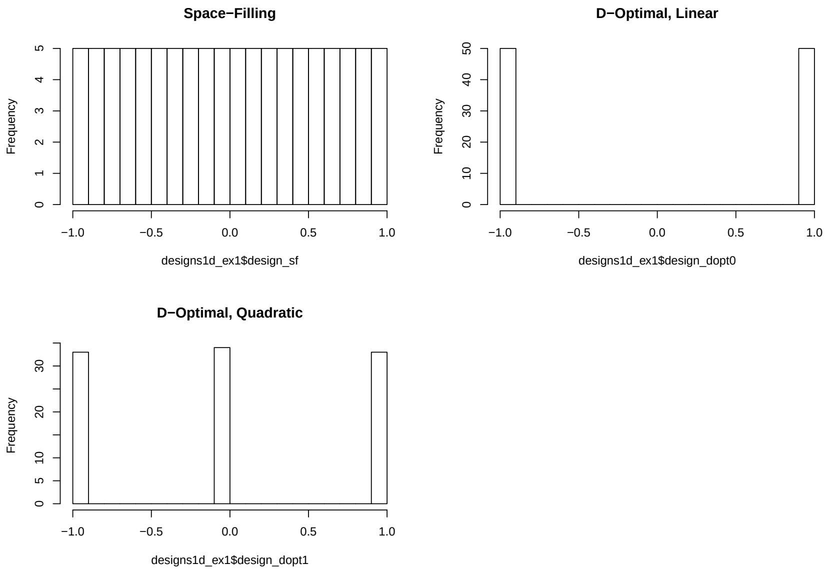
- In Example 3, MEDs concentrate at 0
  - For $E[P(H_1|Y, D)|\beta_T]$, the space-filling design does worse than the MED with $\alpha = 1$, but a lot better than the MED $\alpha = 2p$, which was disturbing.
  - For $MSE(\beta_n|Y, D, H_1, \beta_T)$,
    - $MSE(\beta_{n,0}|Y, D, H_1, \beta_T)$: As expected, given what we saw in Example 2, MEDs did better than the space-filling design, since they concentrate more points at 0
    - For $MSE(\beta_{n,1}|Y, D, H_1, \beta_T)$ and $MSE(\beta_{n,2}|Y, D, H_1, \beta_T)$, though, MED did worse than the space-filling design.

Why look at different BetaT

- $MSE(\beta_n|Y, D, H_1, \beta_T)$ evaluations: we want to show examples where the D-optimal design doesn't do well. Since D-optimal designs specify a general model, it seems that it would require specific models to show that it's not "optimal" in certain senses.

1 Dimension

Non-MED Designs



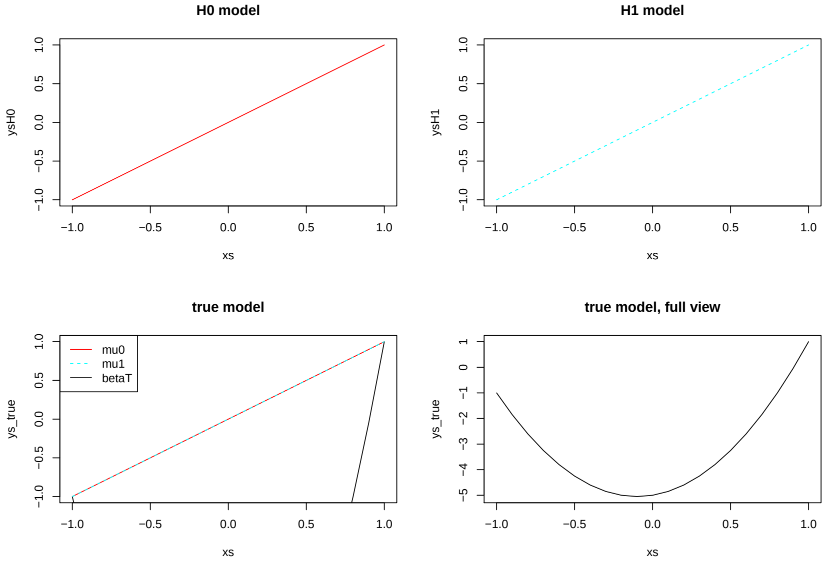
Example 1

Example 1 Parameters

- The idea here is a model with no assumptions about the value for the quadratic term. The result is a more spread-out version of the linear D-optimal design.
- Idea: Should we instead apply this to model comparison with no assumptions about the value for the linear term? i.e. $\mu_0 = (0)$ and $\mu_1 = (0, 0)$?
- In any case...

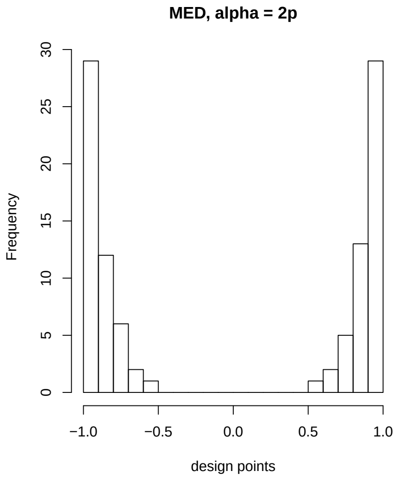
```
# Priors
sigmasq01 = 0.01
mu0 = c(0,1)
V0 = diag(rep(sigmasq01,length(mu0)))
mu1 = c(0,1,0)
V1 = diag(rep(sigmasq01,length(mu1)))
sigmasq = 1
betaT = c(-5, 1, 5)
```

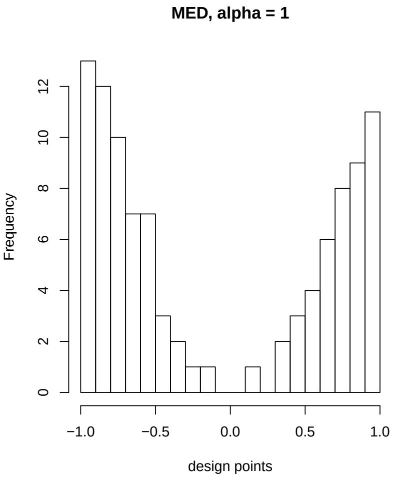
Example 1 Plots



Example 1 MEDs

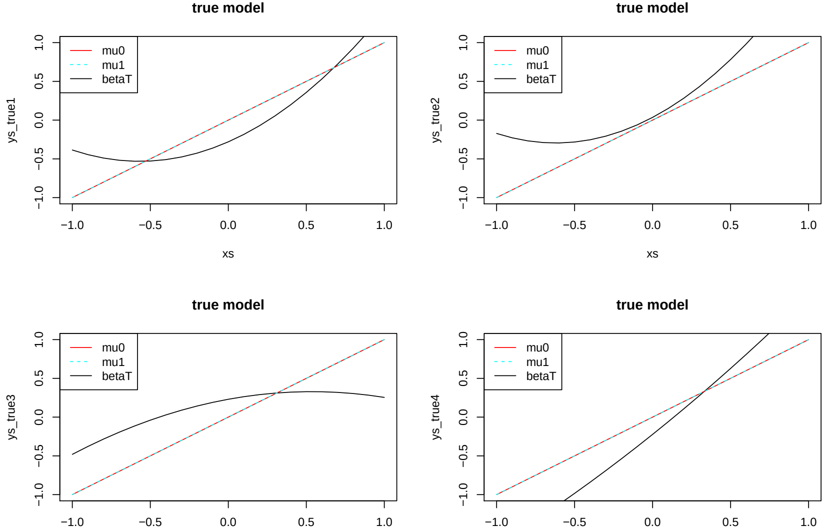
- The original MED (with $\alpha = 1$) has a more space-filling tendency than the adjusted MED (with no power on $f_D(\cdot,\cdot) = \text{Wasserstein}(\cdot,\cdot)$, i.e. $\alpha = 2p$).



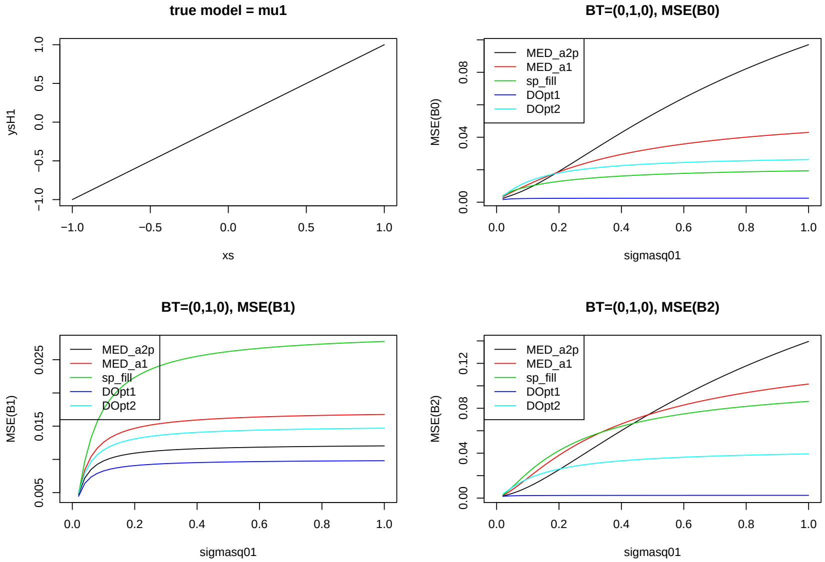


Example 1, Different BetaT

Now we generate different values of $\beta_T \sim N((0,1,0)^T, V_1)$ using `sigmasq01 = 0.25`. (should `sigmasq01` be larger for more variety?)

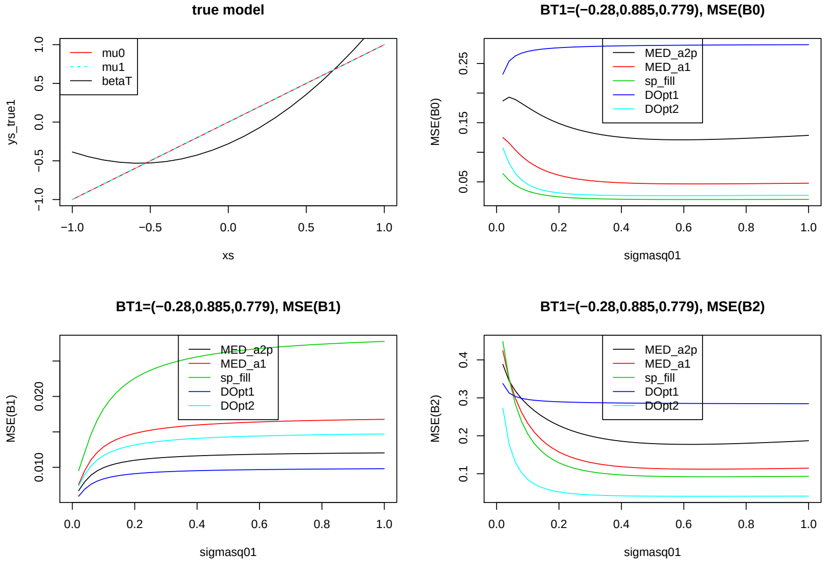


MSEs, BT = mu1


true model = mu1
ysH1
xs
BT=(0,1,0), MSE(B0)
MSE(B0)
sigmasq01
MED_a2p
MED_a1
sp_fill
DOpt1
DOpt2
BT=(0,1,0), MSE(B1)
MSE(B1)
sigmasq01
MED_a2p
MED_a1
sp_fill
DOpt1
DOpt2
BT=(0,1,0), MSE(B2)
MSE(B2)
sigmasq01
MED_a2p
MED_a1
sp_fill
DOpt1
DOpt2

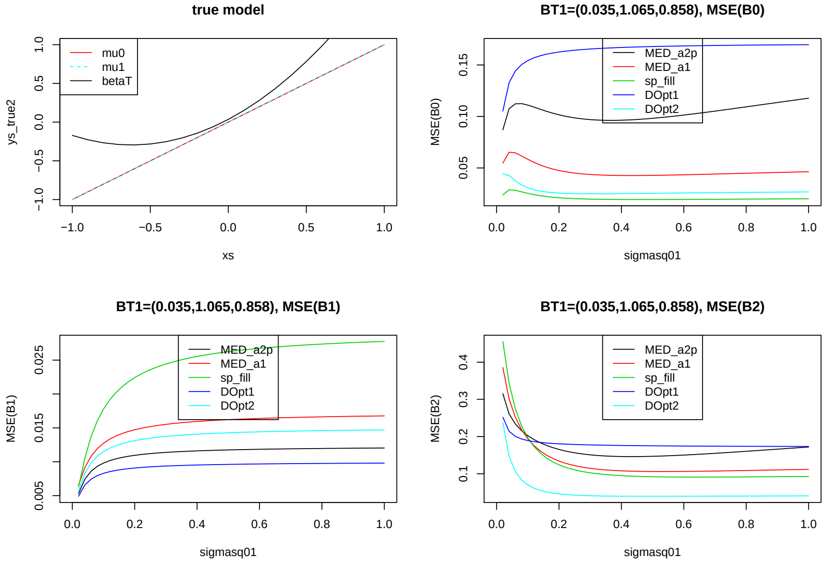
MSEs, BT(1)

`betaTs[ ,1]`



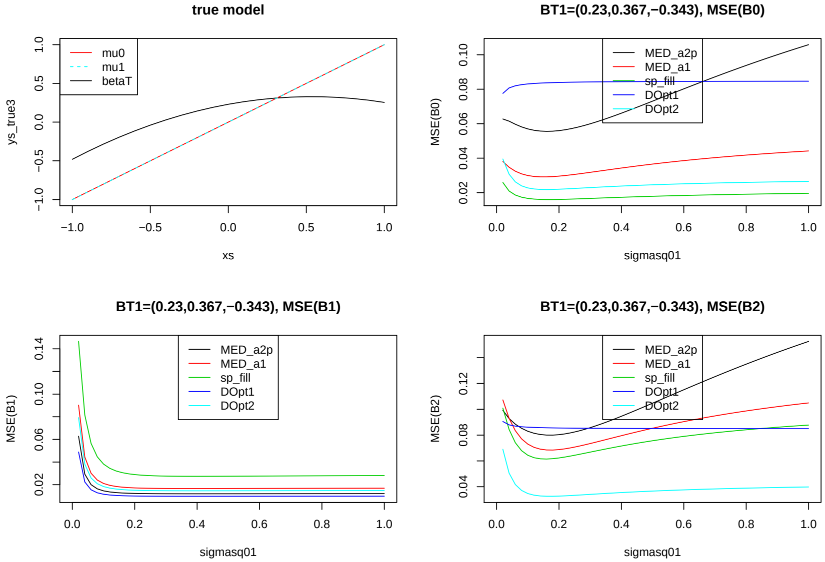
MSEs, BT(2)

`betaTs[ ,2]`



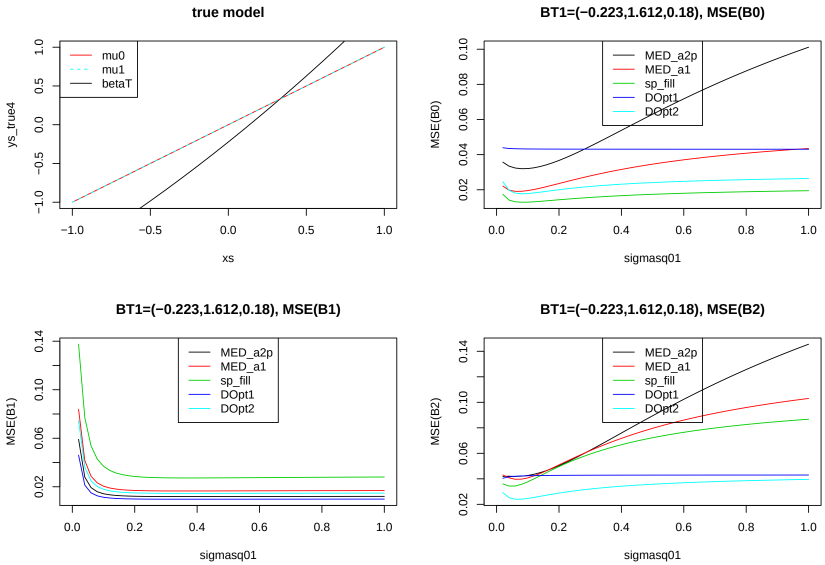
MSEs, BT(3)

`betaTs[ ,3]`

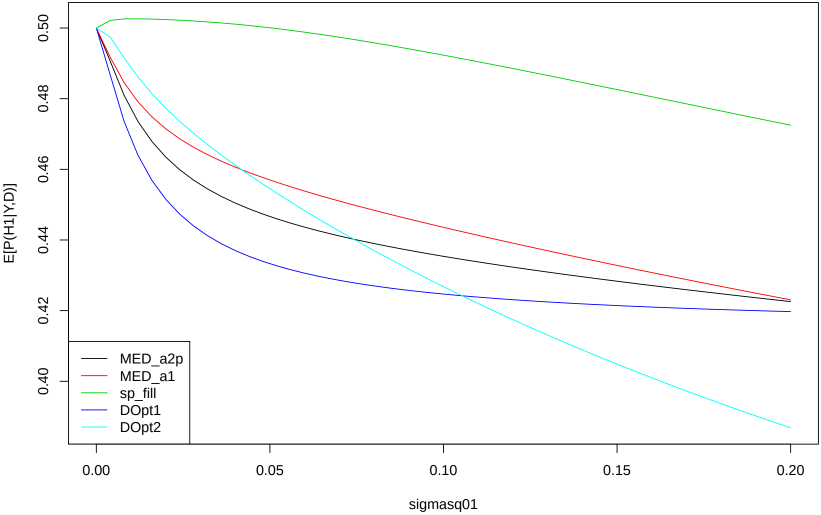


MSEs, BT(4)

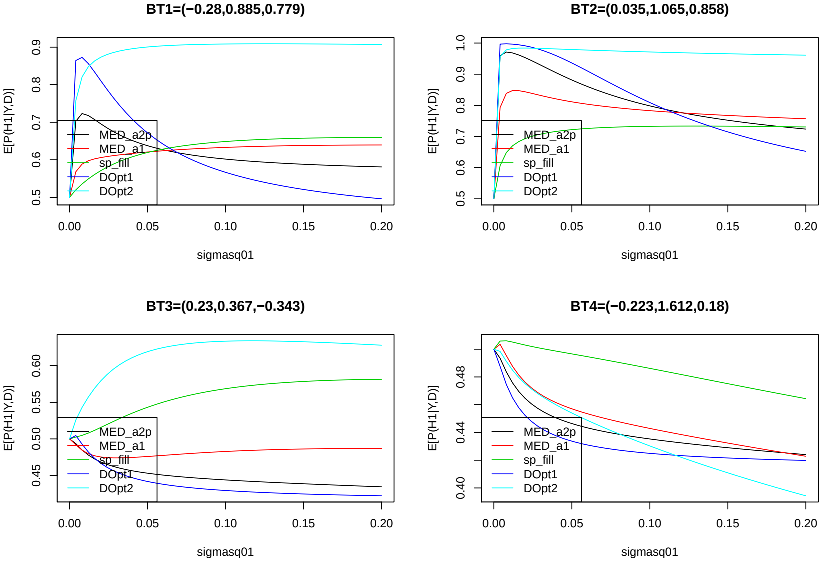
`betaTs[ ,4]`



Example 1 E[P(H1|Y,D, betaT=mu1)]



Example 1 E[P(H1|Y,D, betaT)]

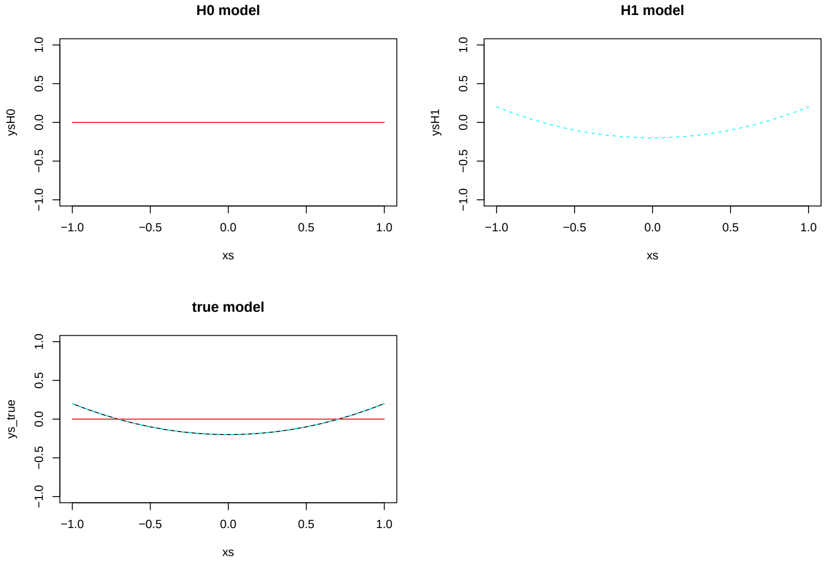


Example 2

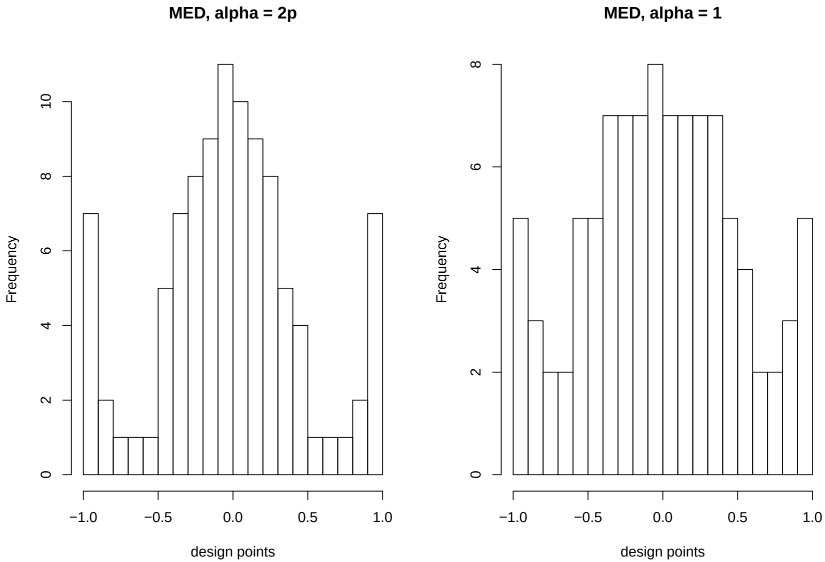
Example 2 Parameters

```r
# Priors
sigmasq01 = 0.01
mu0 = c(0, 0)
V0 = diag(rep(sigmasq01,length(mu0)))
mu1 = c(-0.2, 0, 0.4)
V1 = diag(rep(sigmasq01,length(mu1)))
sigmasq = 1
betaT = mu1
```

Example 2 Plots

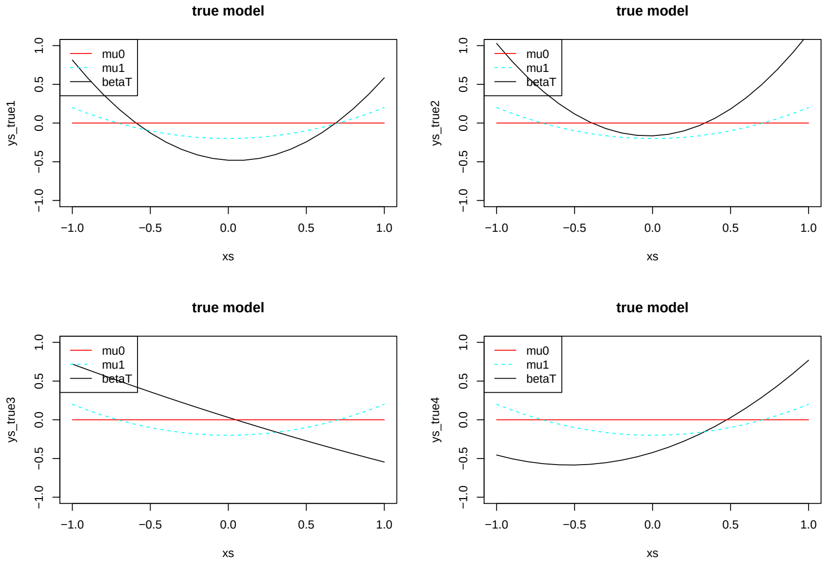


Example 2 MEDs

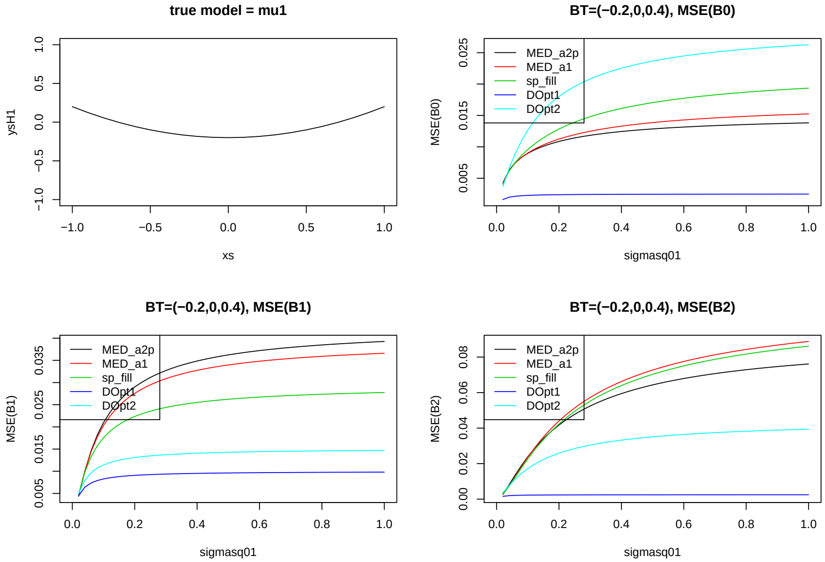


Example 2, Different BetaT

Now we generate different values of $\beta_T \sim N((-0.2, 0, 0.4)^T, V_1)$ using `sigmasq01 = 0.25`

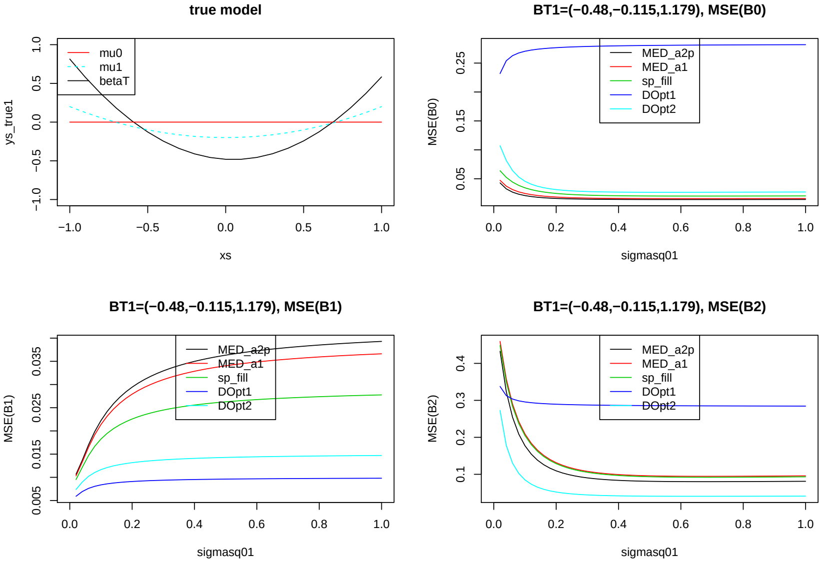


MSEs, BT = mu1


true model = mu1
ysH1
xs
BT=(−0.2,0,0.4), MSE(B0)
MSE(B0)
sigmasq01
MED_a2p
MED_a1
sp_fill
DOpt1
DOpt2
BT=(−0.2,0,0.4), MSE(B1)
MSE(B1)
BT=(−0.2,0,0.4), MSE(B2)
MSE(B2)

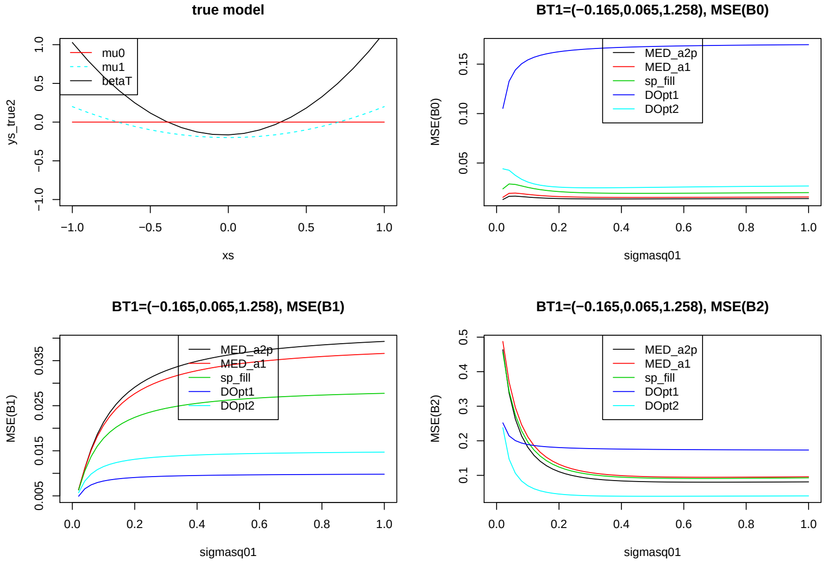
MSEs, BT(1)

betaTs[,1]



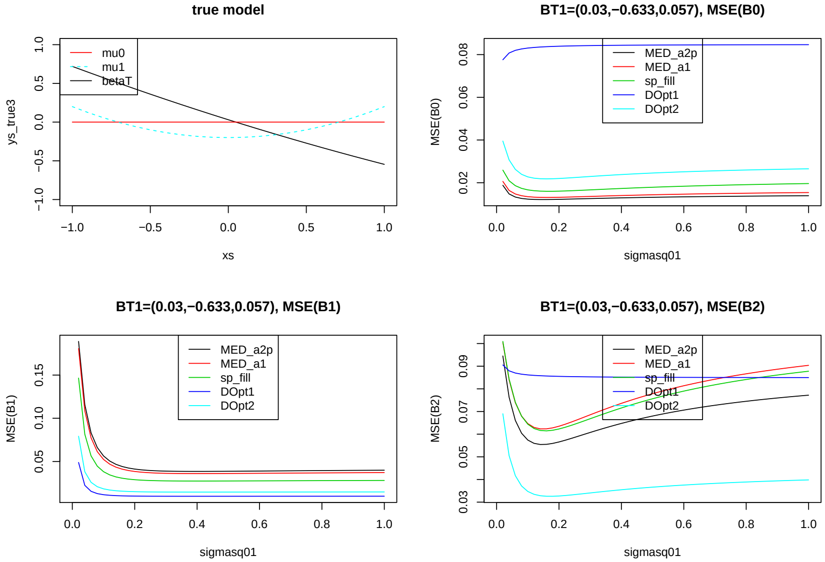
MSEs, BT(2)

`betaTs[ ,2]`



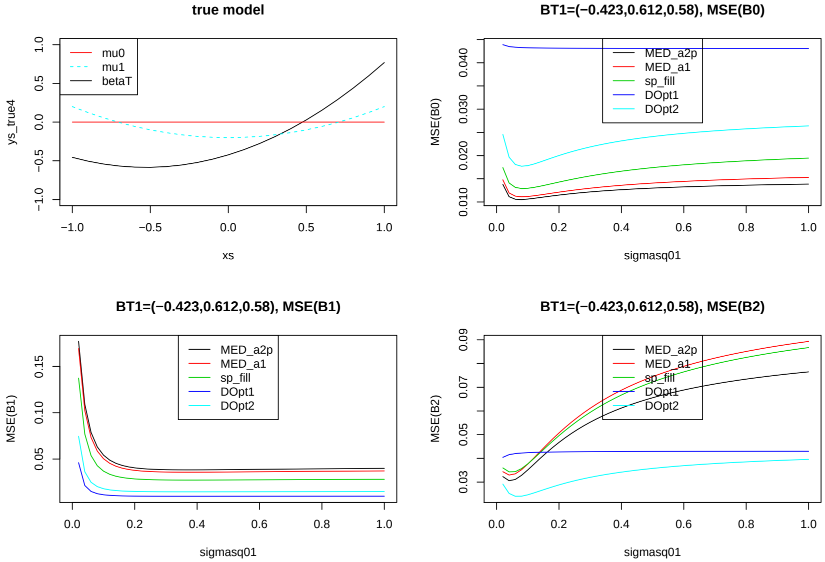
MSEs, BT(3)

`betaTs[ ,3]`

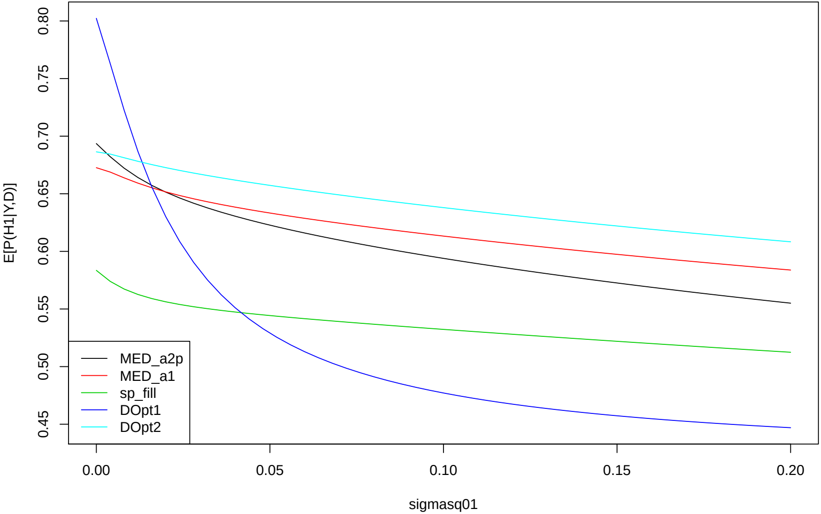


MSEs, BT(4)

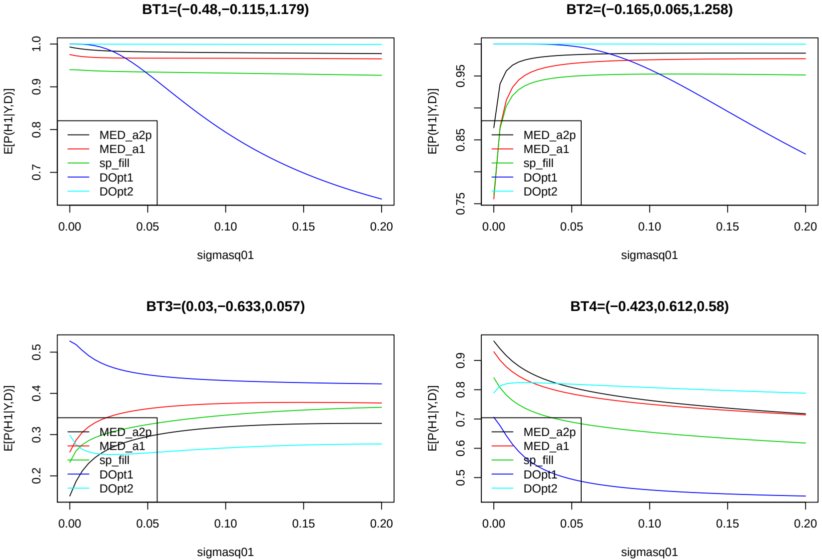
`betaTs[ ,4]`



Example 2 E[P(H1|Y,D, betaT=mu1)]



Example 2 E[P(H1|Y,D)]

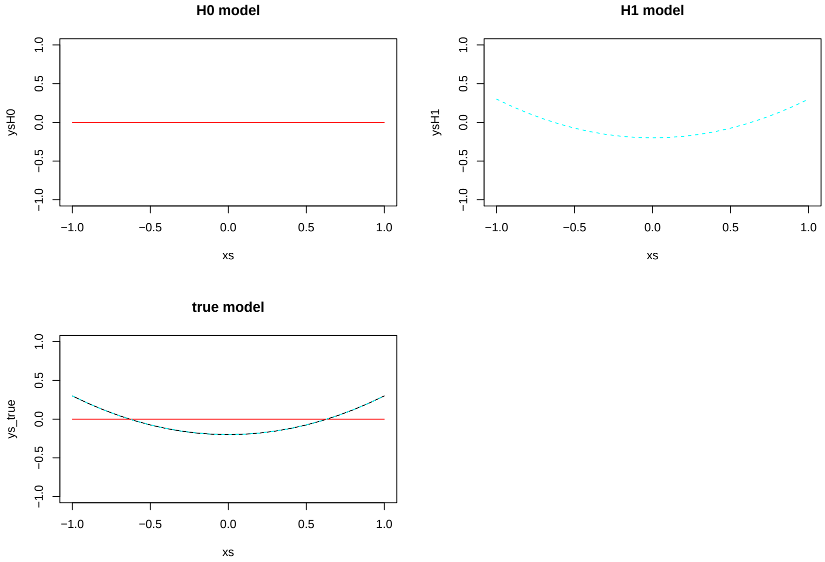


Example 2.1 (Better than Example 2)

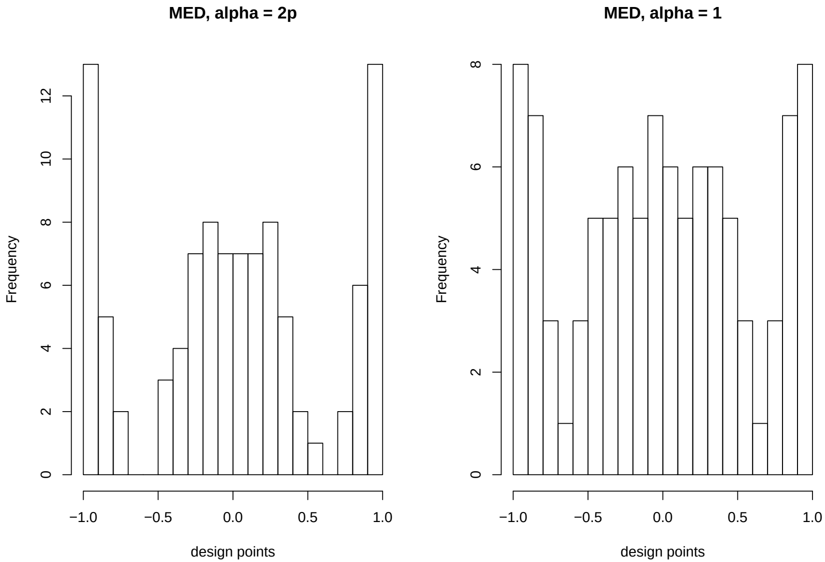
Example 2.1 Parameters

```r
# Priors
sigmasq01 = 0.01
mu0 = c(0, 0)
V0 = diag(rep(sigmasq01,length(mu0)))
mu1 = c(-0.2, 0, 0.5)
V1 = diag(rep(sigmasq01,length(mu1)))
sigmasq = 1
betaT = mu1
```

Example 2.1 Plots

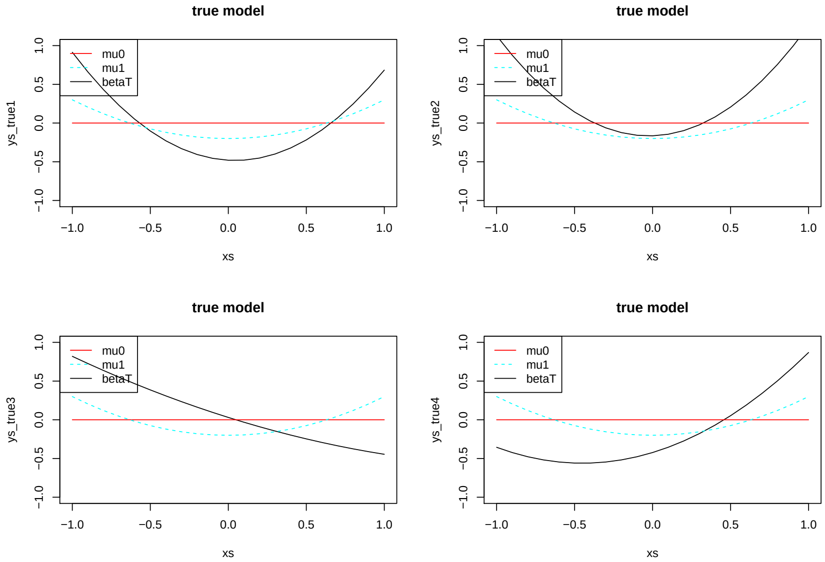


Example 2.1 MEDs

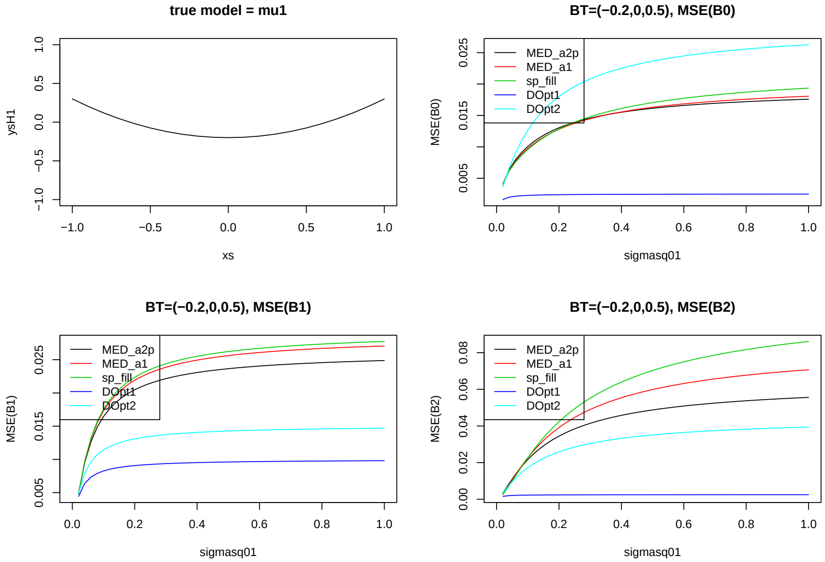


Example 2.1, Different BetaT

Now we generate different values of $\beta_T \sim N((-0.2, 0, 0.5)^T, V_1)$ using `sigmasq01 = 0.25`

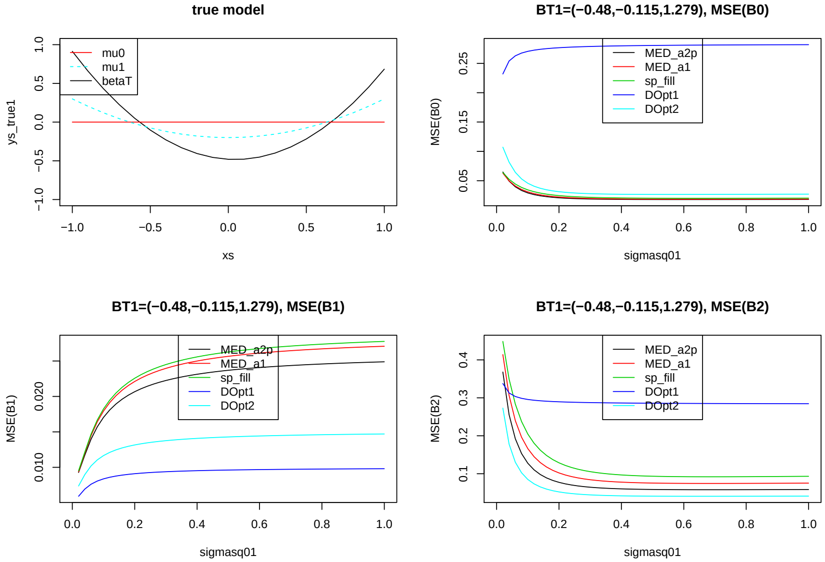


MSEs, BT = mu1


true model = mu1
ysH1
xs
BT=(−0.2,0,0.5), MSE(B0)
MSE(B0)
sigmasq01
BT=(−0.2,0,0.5), MSE(B1)
MSE(B1)
sigmasq01
BT=(−0.2,0,0.5), MSE(B2)
MSE(B2)
sigmasq01
MED_a2p
MED_a1
sp_fill
DOpt1
DOpt2

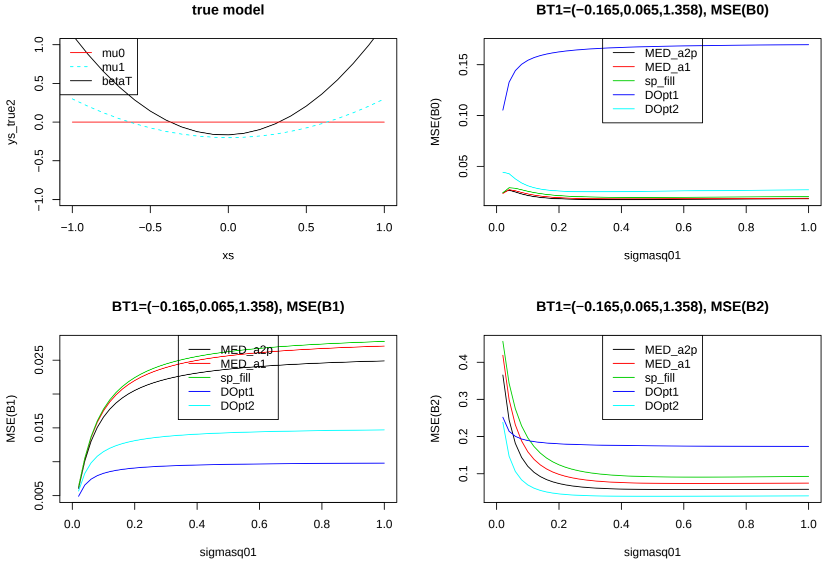
MSEs, BT(1)

`betaTs[ ,1]`



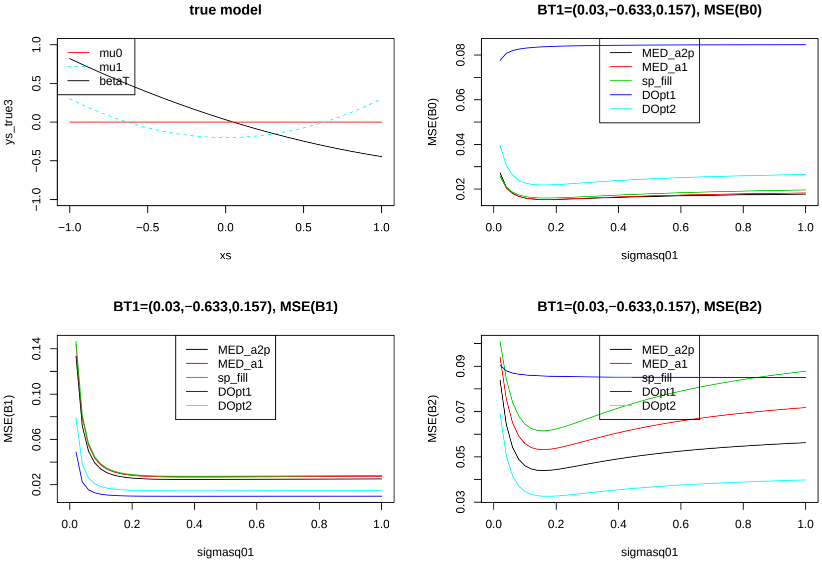
MSEs, BT(2)

`betaTs[ ,2]`



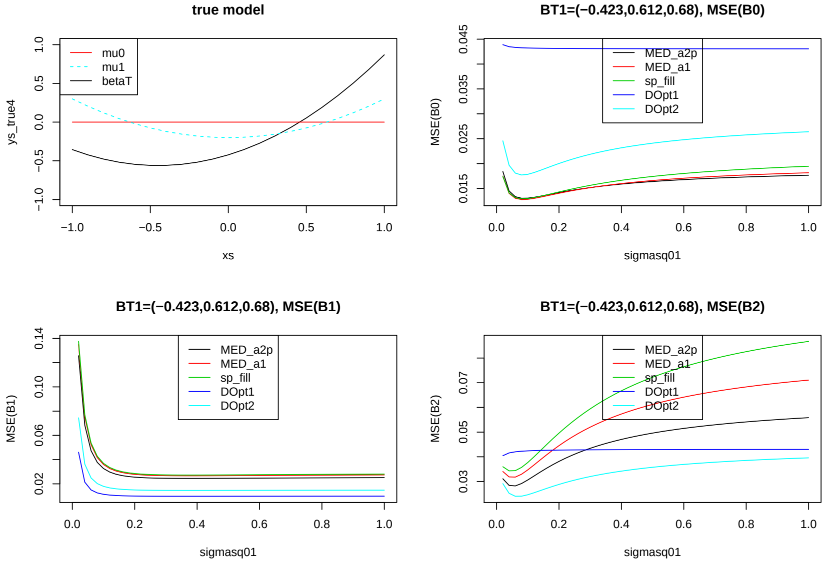
MSEs, BT(3)

`betaTs[ ,3]`

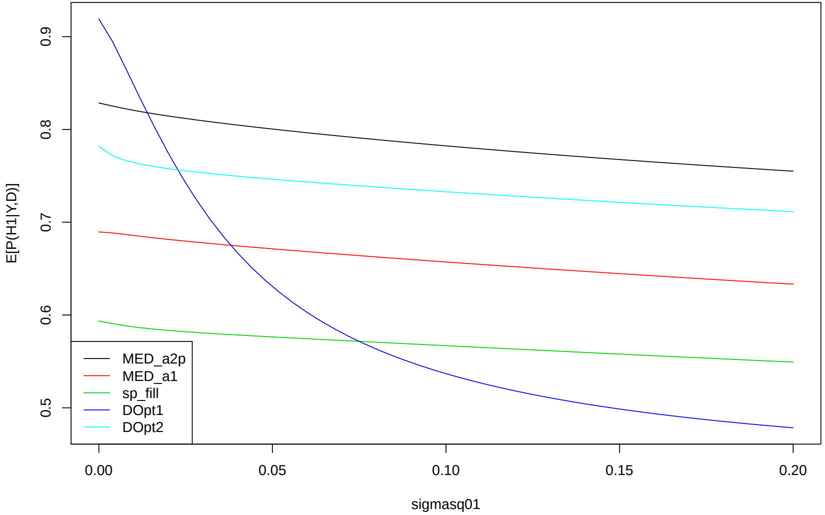


MSEs, BT(4)

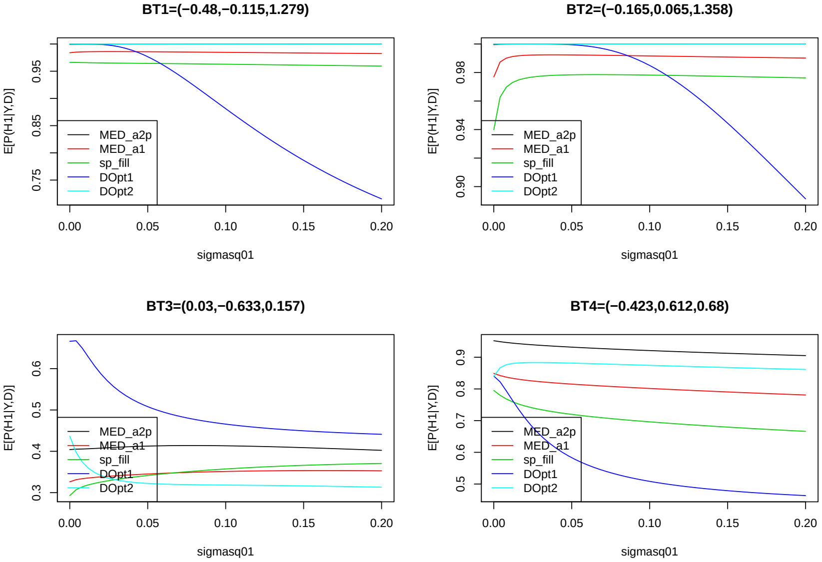
`betaTs[ ,4]`



Example 2.1 E[P(H1|Y,D, betaT=mu1)]



Example 2.1 E[P(H1|Y,D)]

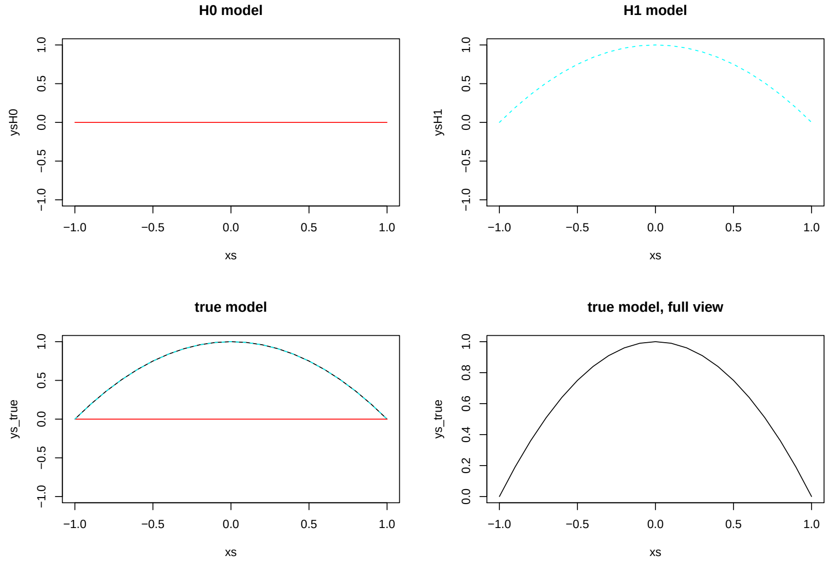


Example 3

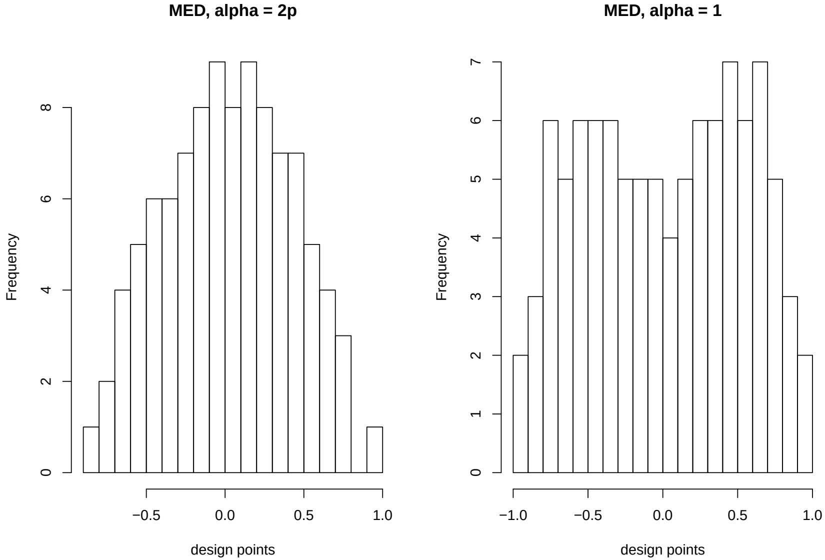
Example 3 Parameters

```r
# Priors
sigmasq01 = 0.01
mu0 = c(0, 0, 0)
V0 = diag(rep(sigmasq01,length(mu0)))
mu1 = c(2, 0, -1, 0, -1)
V1 = diag(rep(sigmasq01,length(mu1)))
sigmasq = 1
betaT = mu1
```

Example 3 Plots

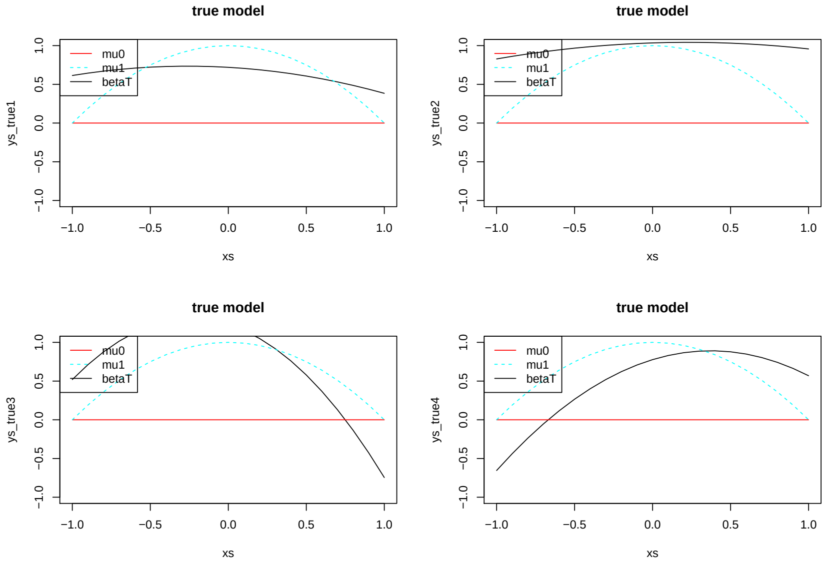


Example 3 MEDs

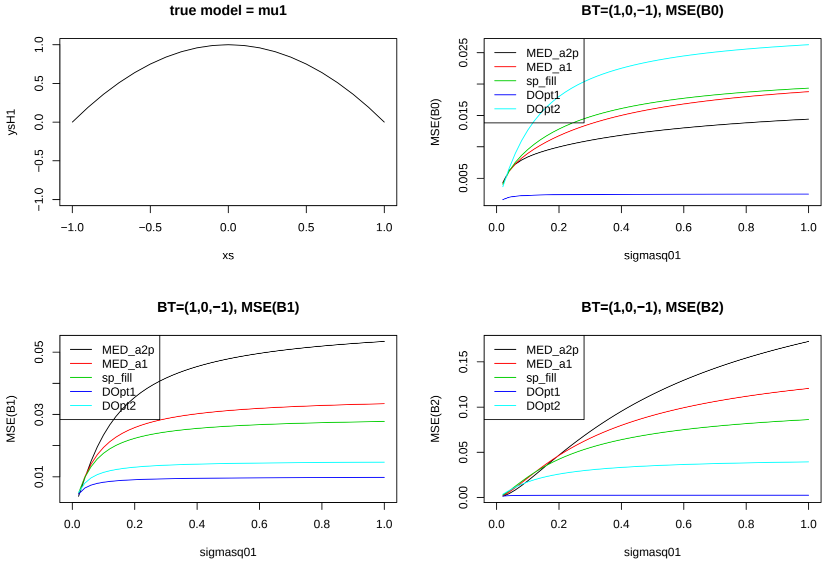


Example 3, Different BetaT

Now we generate different values of $\beta_T \sim N((0,1,0)^T, V_1)$ using `sigmasq01 = 0.25`



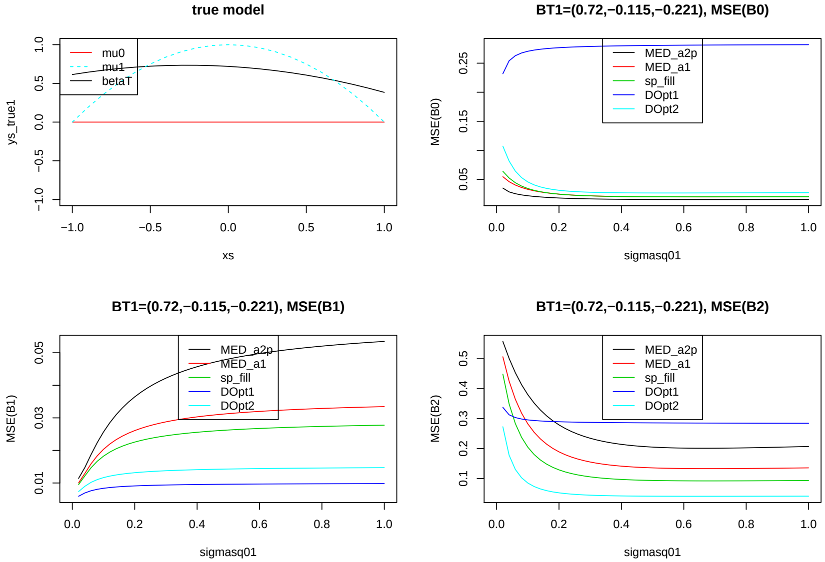
MSEs, BT = mu1



true model = mu1
ysH1
xs
BT=(1,0,−1), MSE(B0)
MSE(B0)
sigmasq01
BT=(1,0,−1), MSE(B1)
MSE(B1)
sigmasq01
BT=(1,0,−1), MSE(B2)
MSE(B2)
sigmasq01
MED_a2p
MED_a1
sp_fill
DOpt1
DOpt2

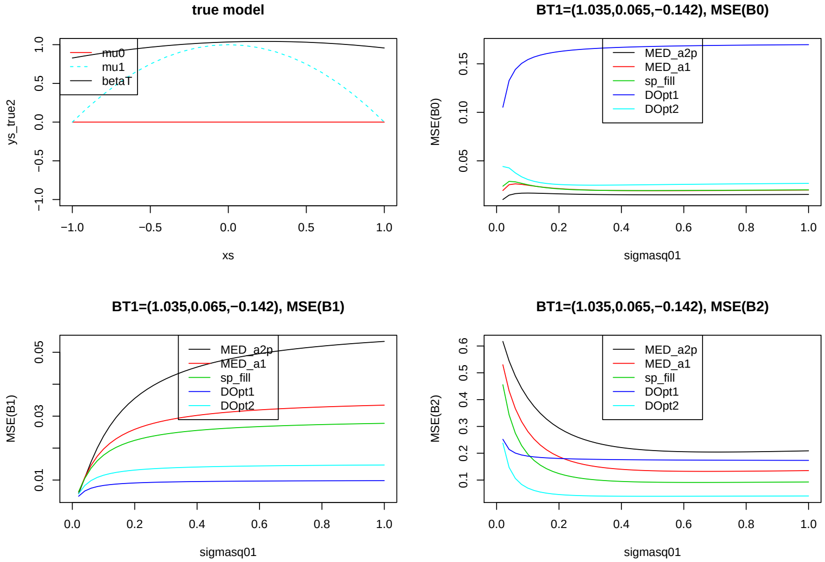
MSEs, BT(1)

`betaTs[ ,1]`



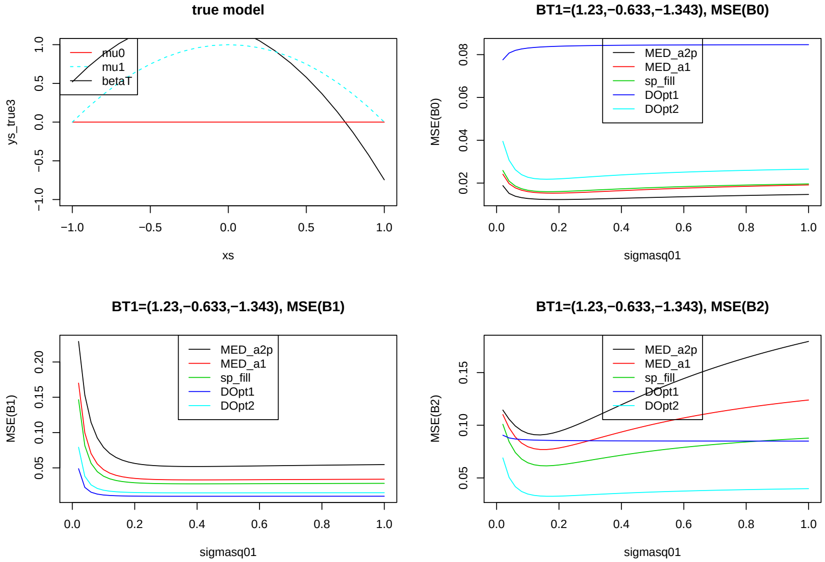
MSEs, BT(2)

`betaTs[ ,2]`



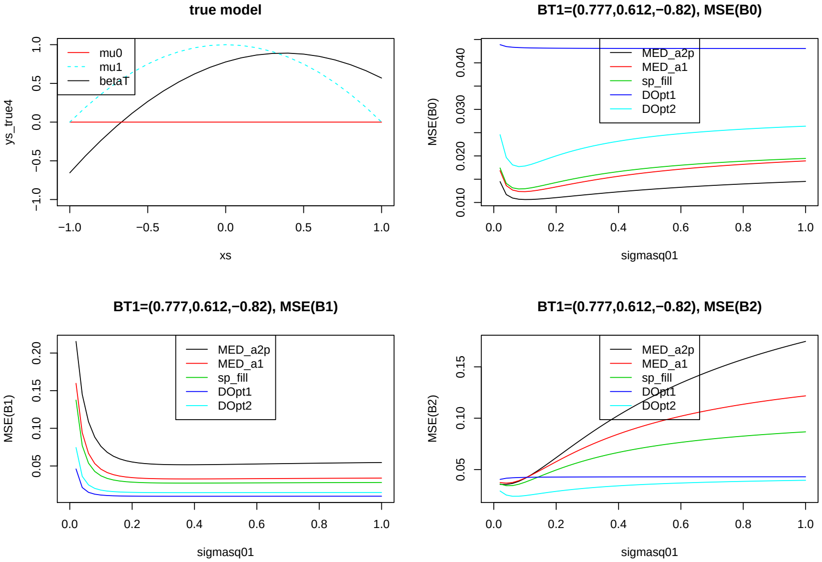
MSEs, BT(3)

`betaTs[ ,3]`

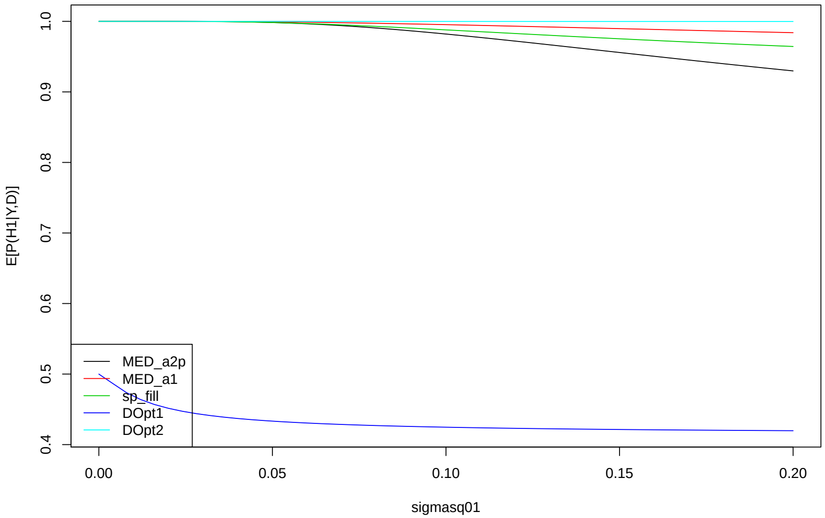


MSEs, BT(4)

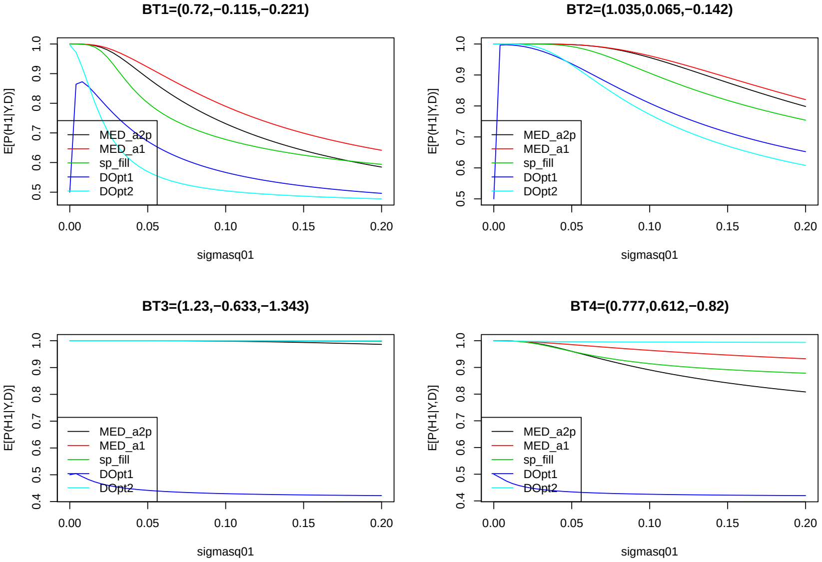
`betaTs[ ,4]`



Example 3 E[P(H1|Y,D, betaT=mu1)]



Example 3 E[P(H1|Y,D)]

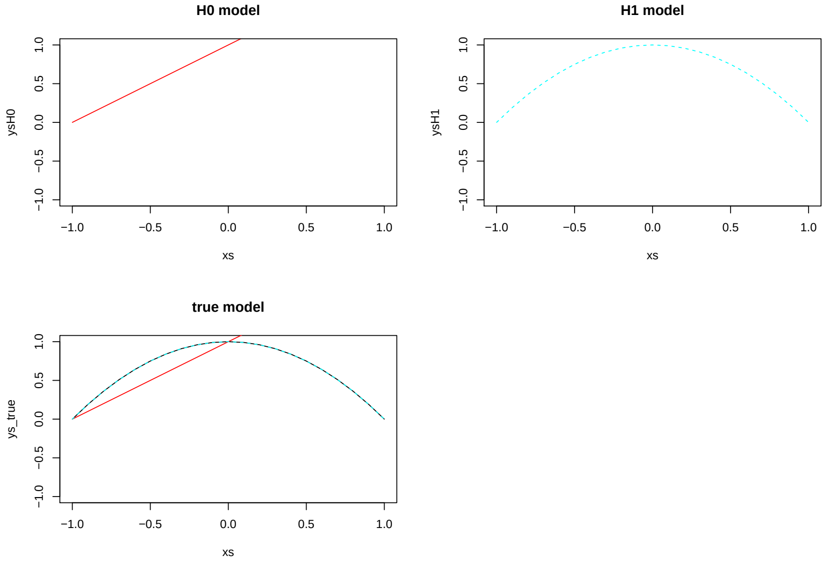


Example 4

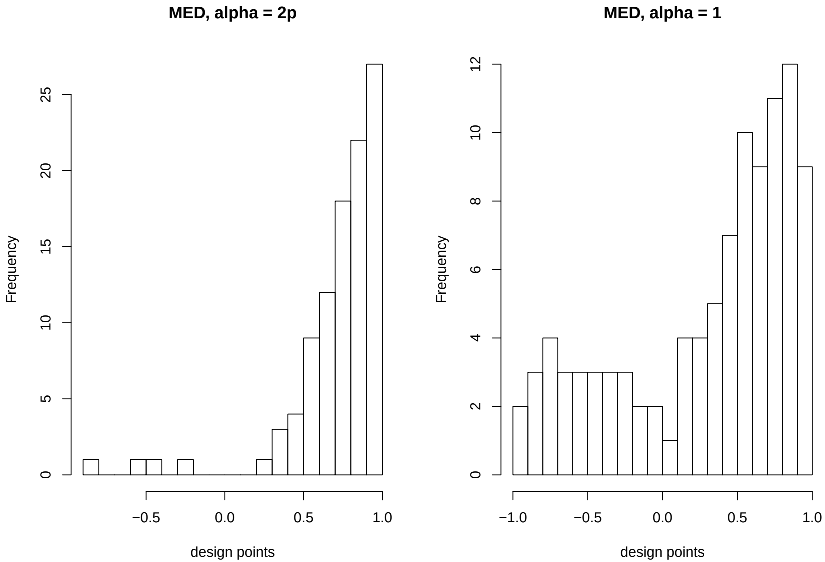
Example 4 Parameters

```
# Priors
sigmasq01 = 0.01
mu0 = c(1, 1)
V0 = diag(rep(sigmasq01,length(mu0)))
mu1 = c(1, 0, -1)
V1 = diag(rep(sigmasq01,length(mu1)))
sigmasq = 1
betaT = mu1
```

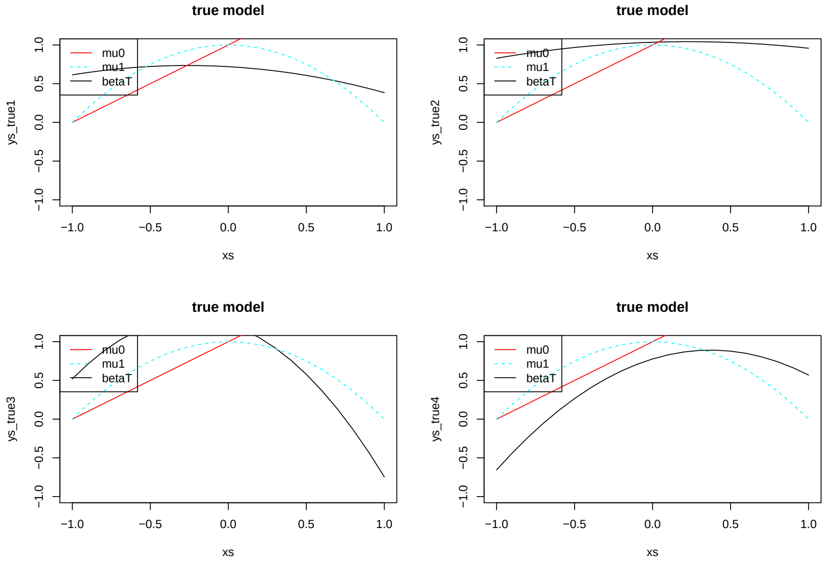
Example 4 Plots



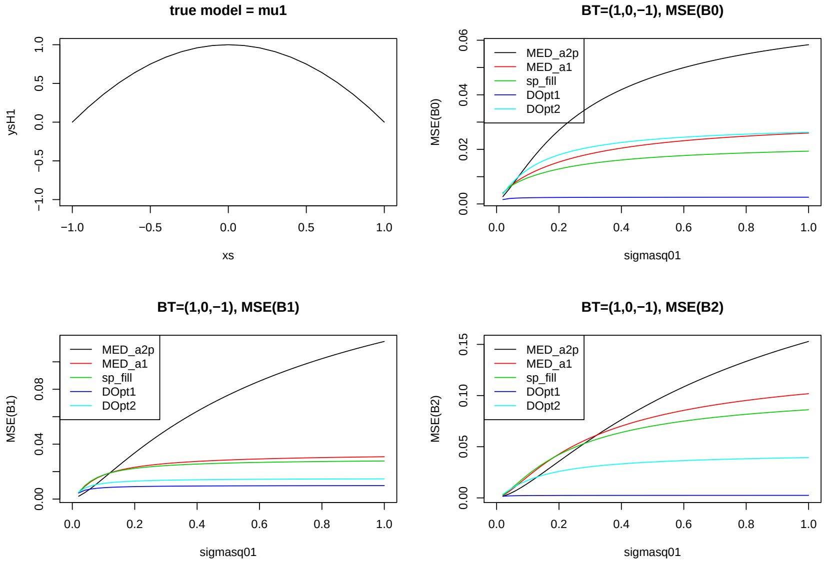
Example 4 MEDs



Example 4, Different BetaT

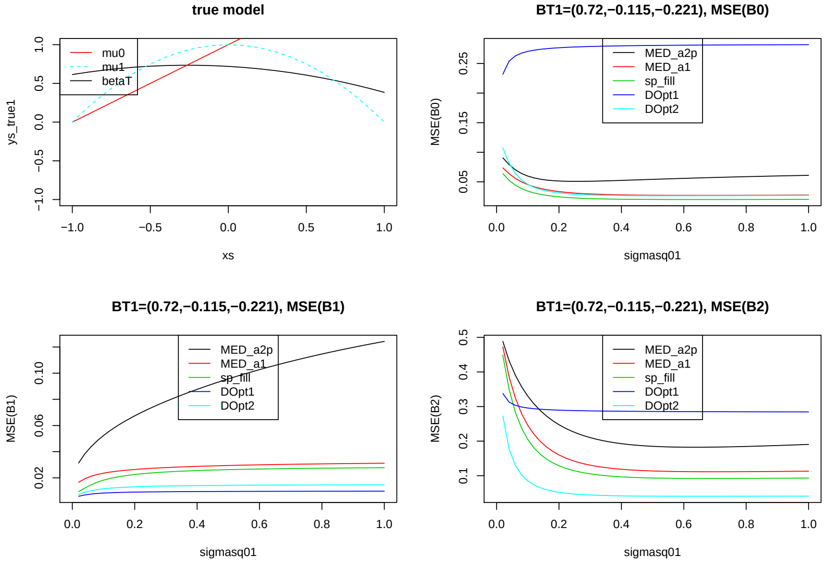


MSEs, BT $= \text{mu1}$



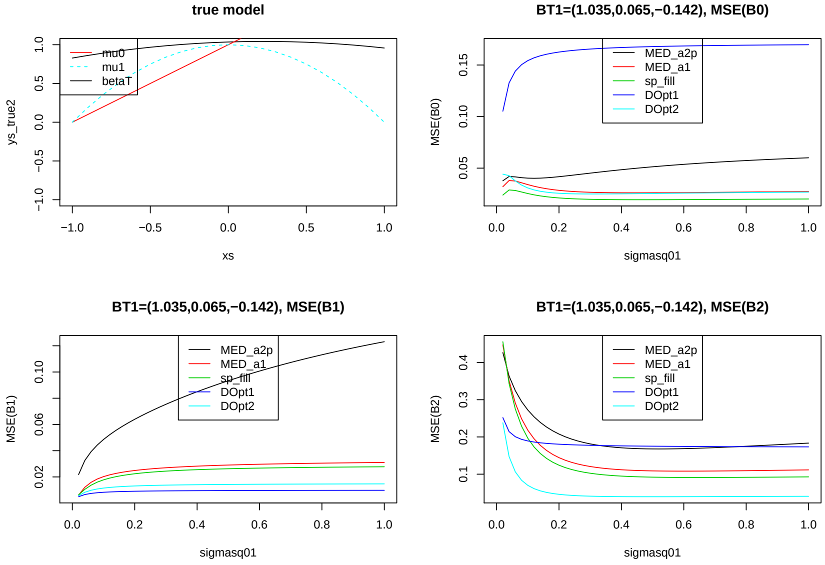
MSEs, BT(1)

`betaTs[ ,1]`



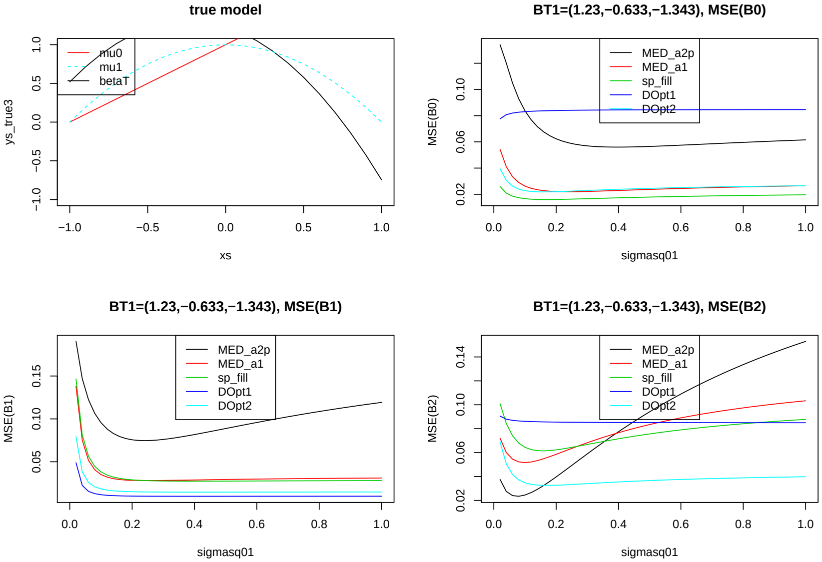
MSEs, BT(2)

`betaTs[ ,2]`



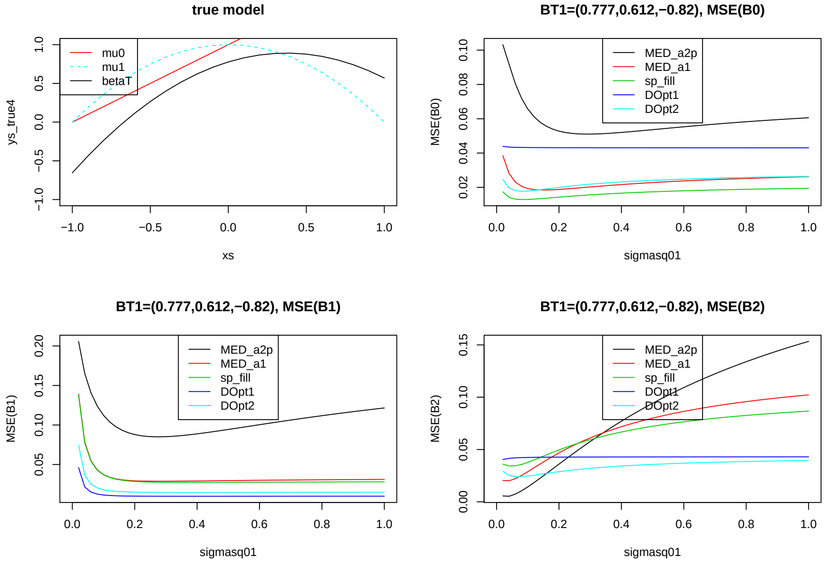
MSEs, BT(3)

`betaTs[ ,3]`

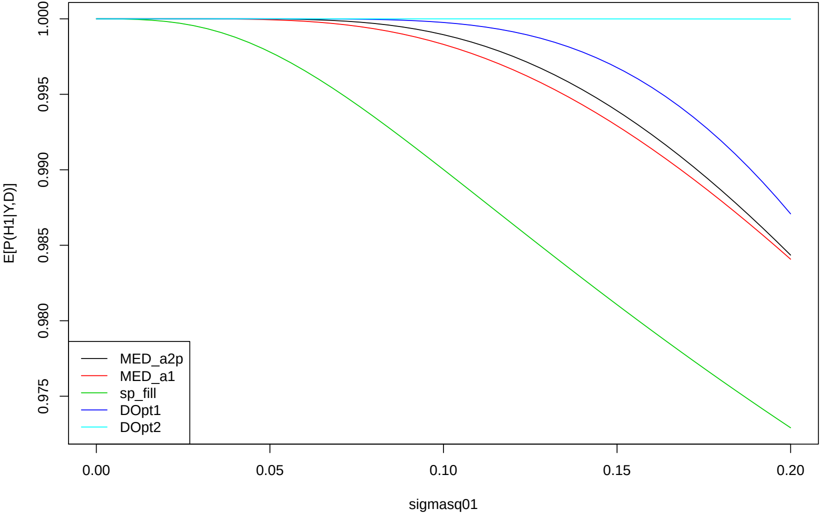


MSEs, BT(4)

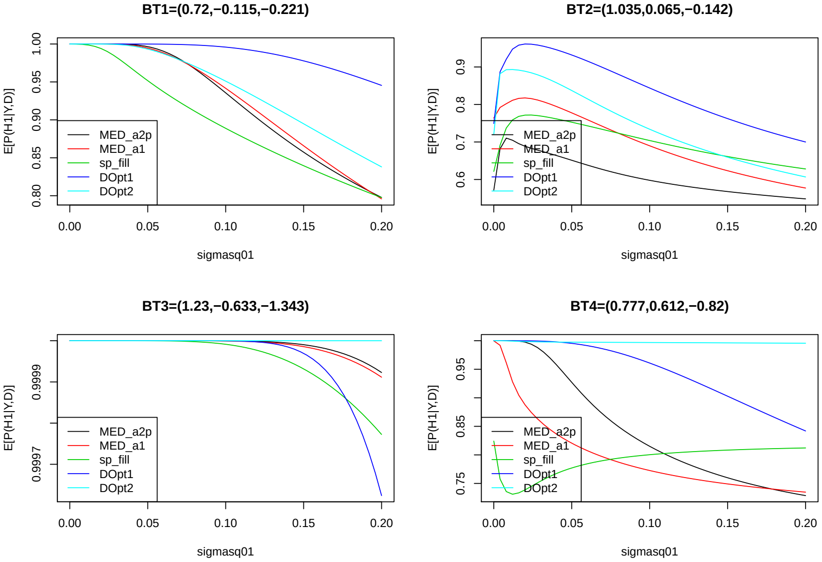
`betaTs[ ,4]`



Example 4 E[P(H1|Y,D, betaT=mu1)]



Example 4 E[P(H1|Y,D)]

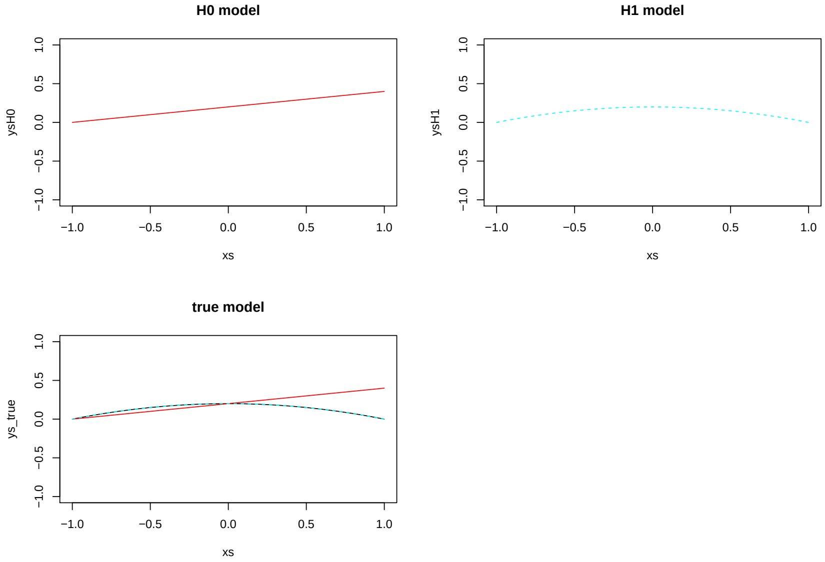


Example 4.1

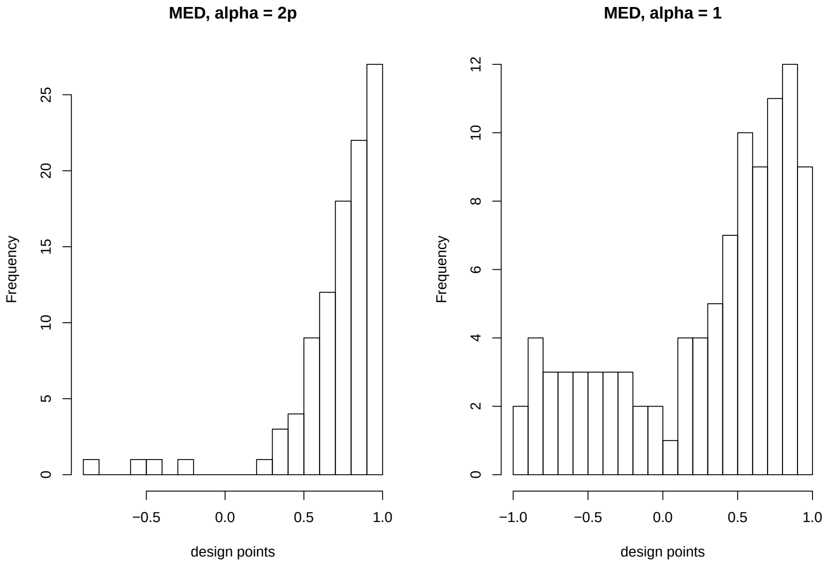
Example 4.1 Parameters

```r
# Priors
sigmasq01 = 0.01
mu0 = c(0.2, 0.2)
V0 = diag(rep(sigmasq01,length(mu0)))
mu1 = c(0.2, 0, -0.2)
V1 = diag(rep(sigmasq01,length(mu1)))
sigmasq = 1
betaT = mu1
```

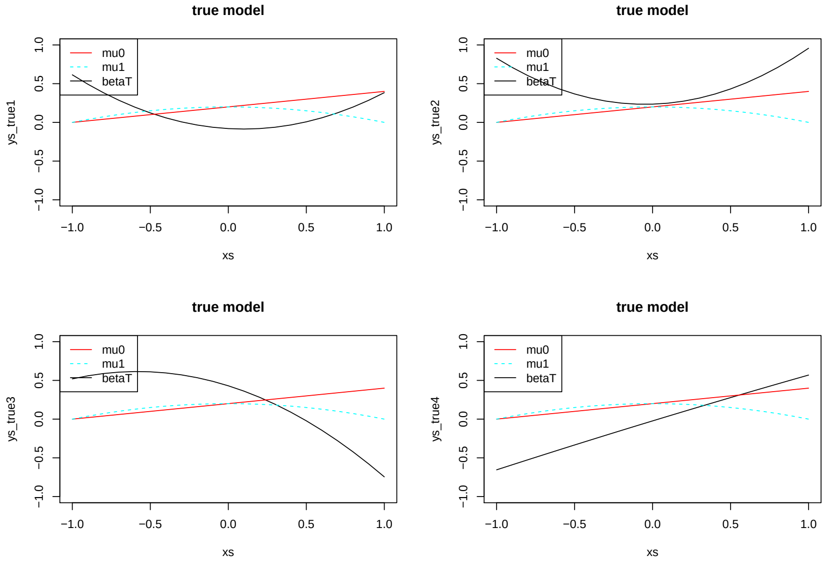
Example 4.1 Plots



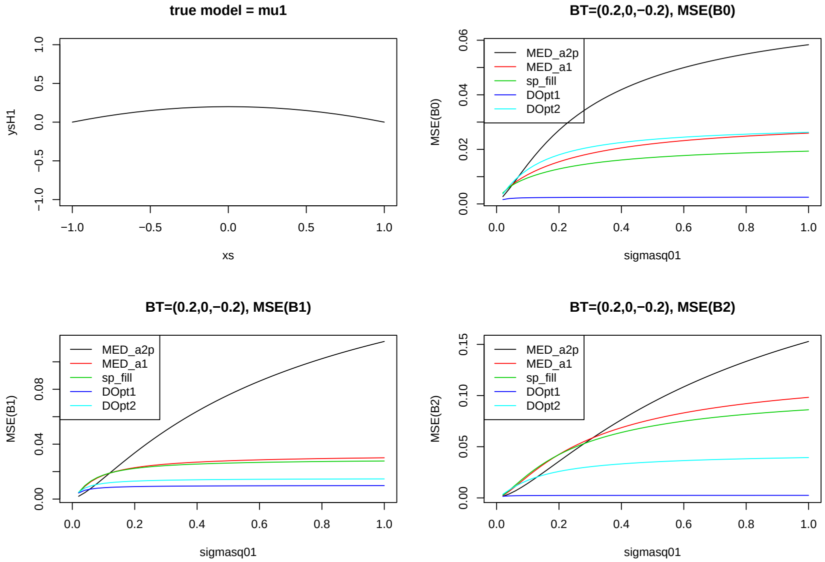
Example 4.1 MEDs



Example 4.1, Different BetaT



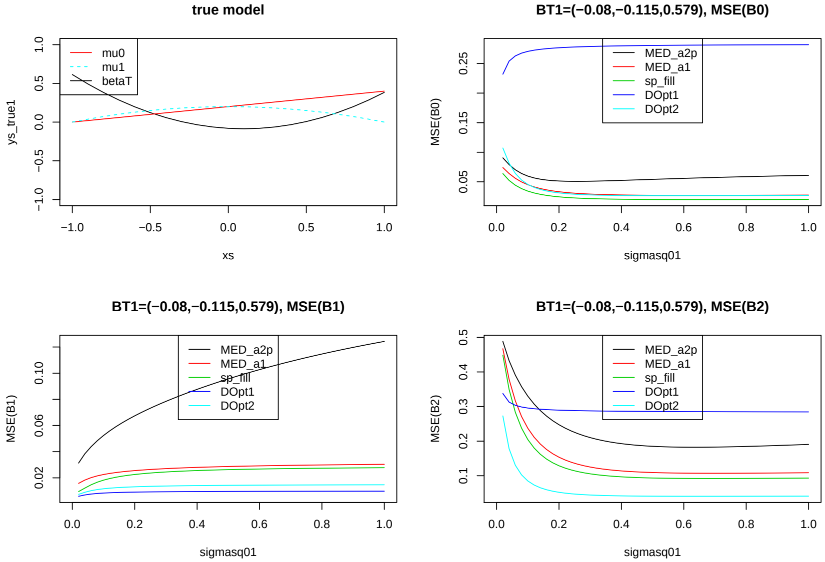
MSEs, BT = mu1



true model = mu1
ysH1
xs
BT=(0.2,0,−0.2), MSE(B0)
MSE(B0)
sigmasq01
BT=(0.2,0,−0.2), MSE(B1)
MSE(B1)
sigmasq01
BT=(0.2,0,−0.2), MSE(B2)
MSE(B2)
sigmasq01
MED_a2p
MED_a1
sp_fill
DOpt1
DOpt2

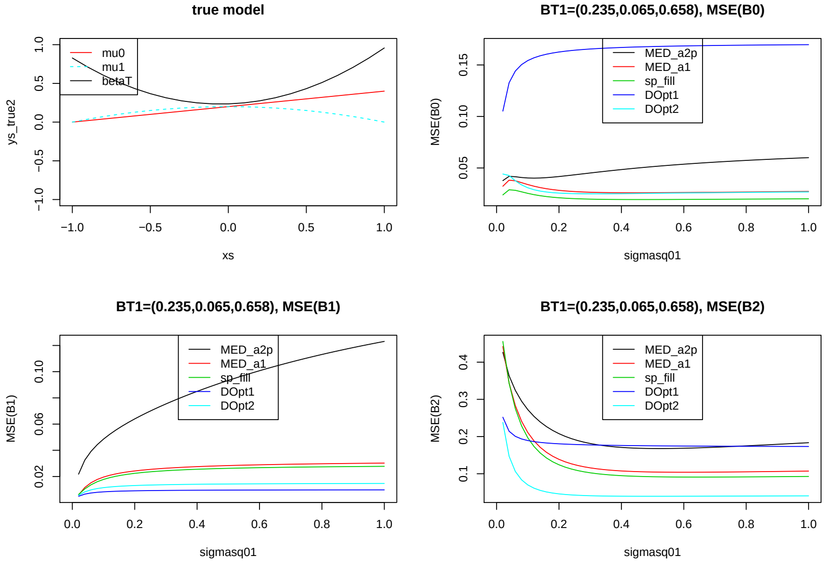
MSEs, BT(1)

`betaTs[ ,1]`



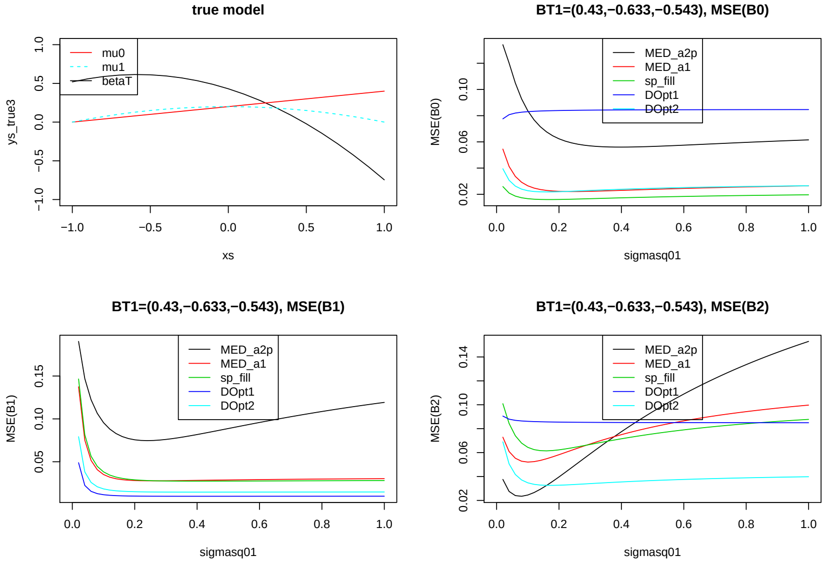
MSEs, BT(2)

`betaTs[ ,2]`



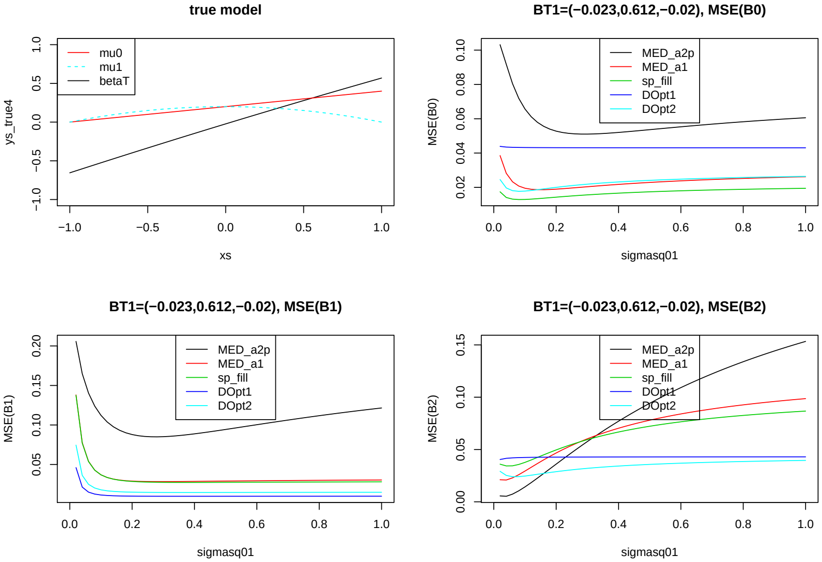
MSEs, BT(3)

`betaTs[ ,3]`

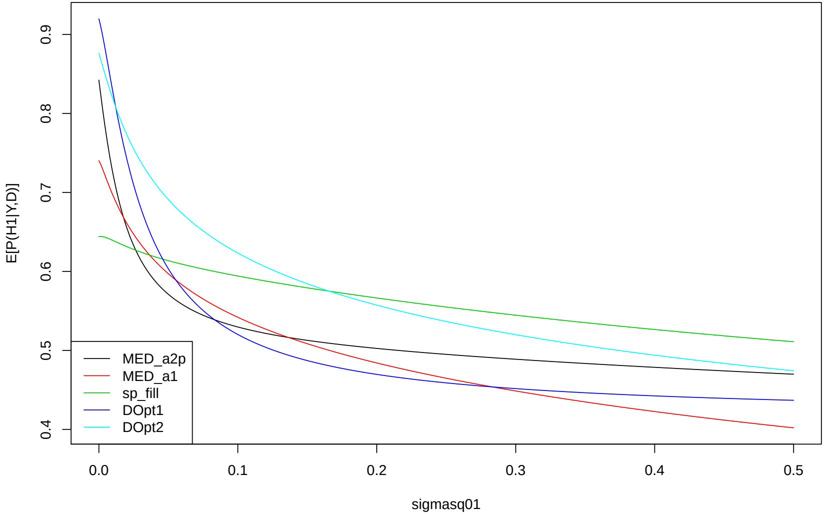


MSEs, BT(4)

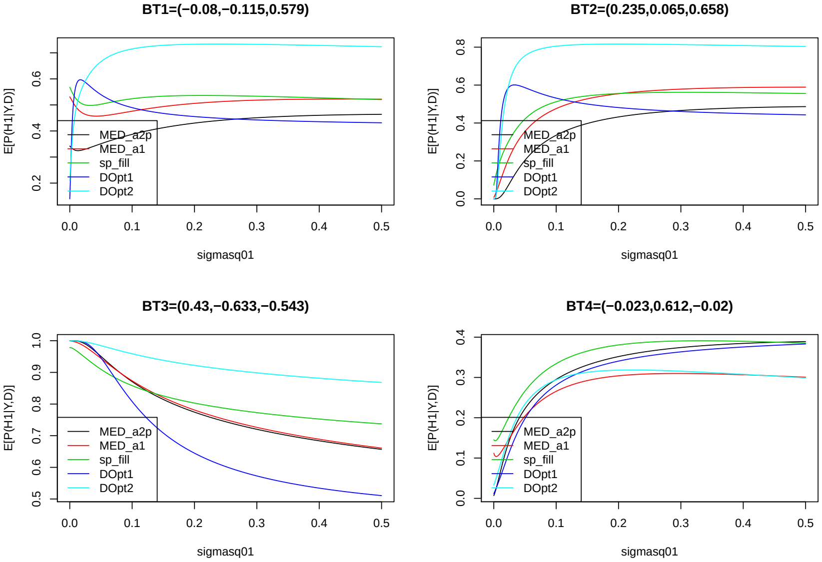
`betaTs[ ,4]`



Example 4.1 E[P(H1|Y,D, betaT=mu1)]



Example 4.1 E[P(H1|Y,D)]

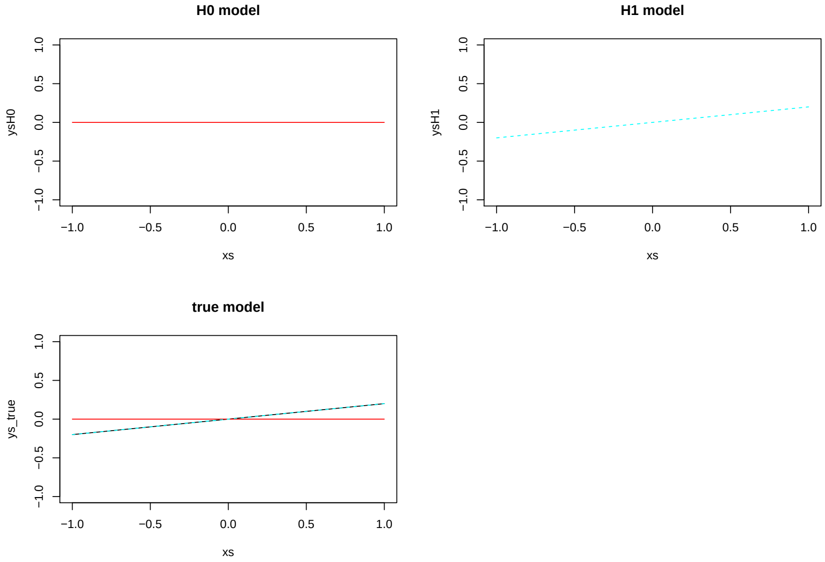


Example 5

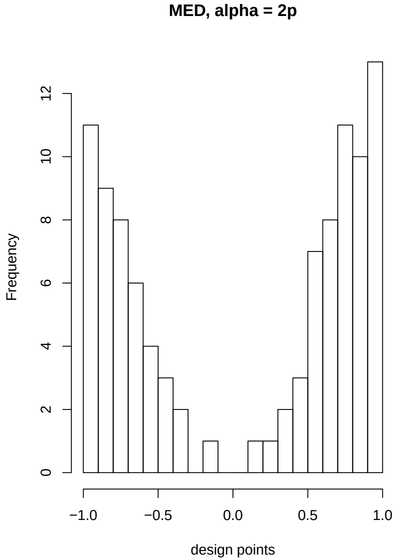
Example 5 Parameters

```
# Priors
sigmasq01 = 0.01
mu0 = c(0, 0)
V0 = diag(rep(sigmasq01,length(mu0)))
mu1 = c(0, 0.2)
V1 = diag(rep(sigmasq01,length(mu1)))
sigmasq = 1
betaT = mu1
```

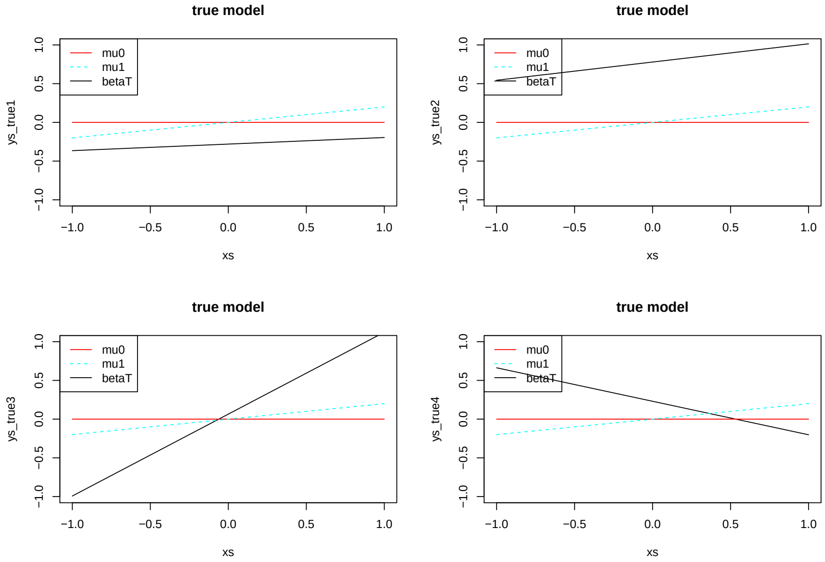
Example 5 Plots



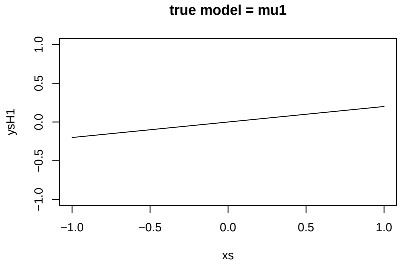
Example 5 MEDs



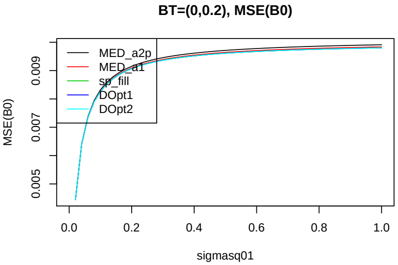
Example 5, Different BetaT



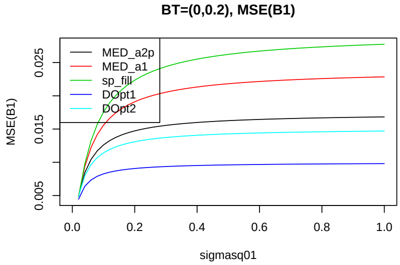
MSEs, BT $=$ mu1



true model = mu1
ysH1
−1.0
−0.5
0.0
0.5
1.0
xs



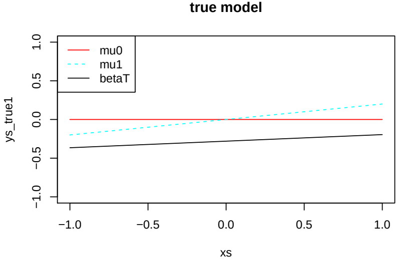
BT=(0,0.2), MSE(B0)
MED_a2p
MED_a1
sp_fill
DOpt1
DOpt2
MSE(B0)
0.005
0.007
0.009
0.0
0.2
0.4
0.6
0.8
1.0
sigmasq01

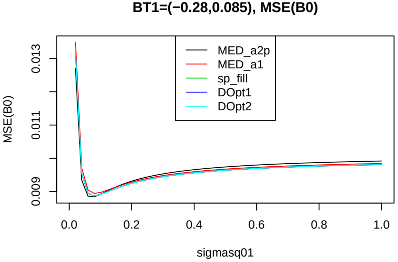


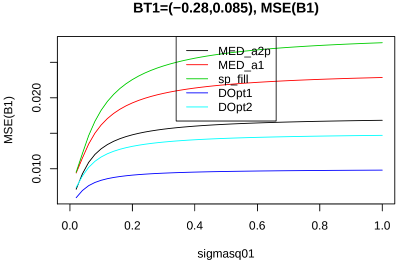
BT=(0,0.2), MSE(B1)
MED_a2p
MED_a1
sp_fill
DOpt1
DOpt2
MSE(B1)
0.005
0.015
0.025
0.0
0.2
0.4
0.6
0.8
1.0
sigmasq01

MSEs, BT(1)

`betaTs[ ,1]`

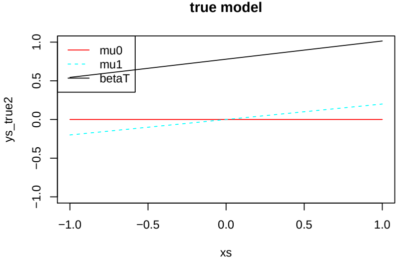


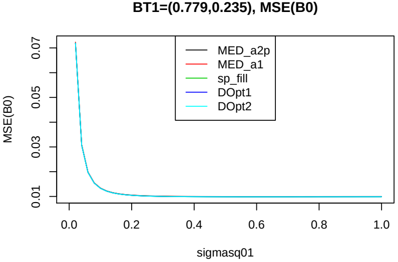


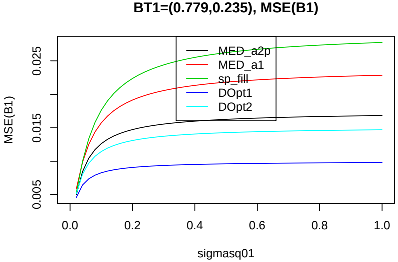


MSEs, BT(2)

`betaTs[ ,2]`

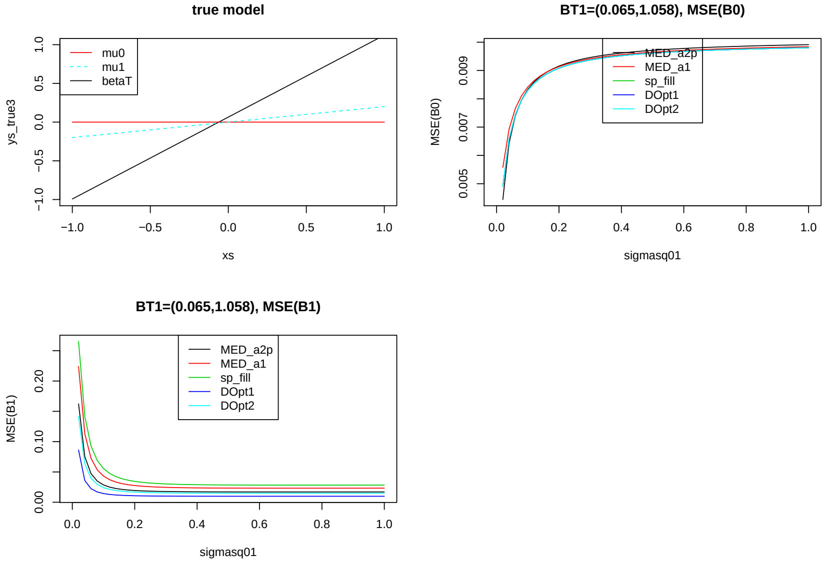






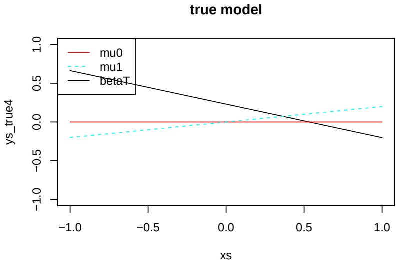
MSEs, BT(3)

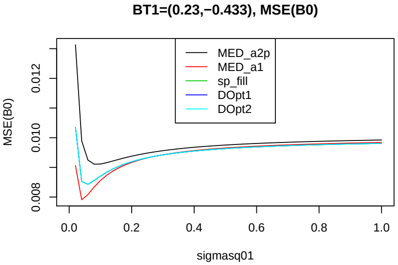
`betaTs[ ,3]`

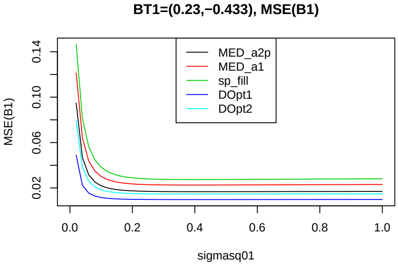


MSEs, BT(4)

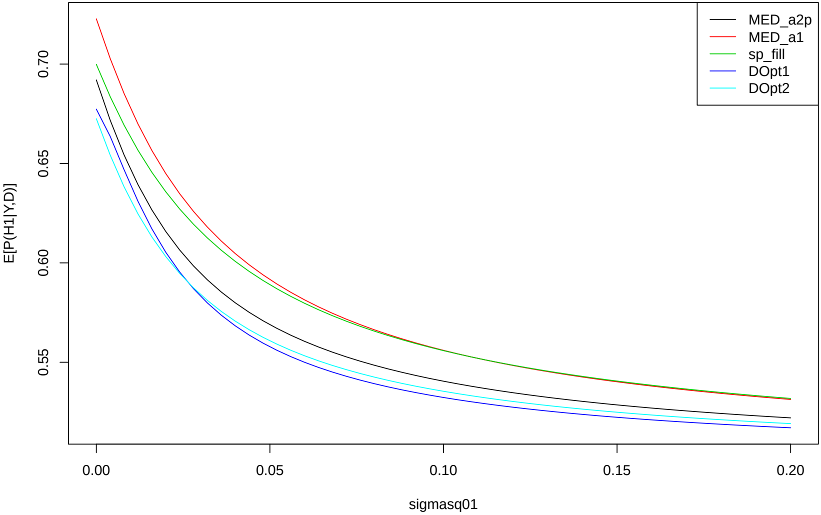
`betaTs[ ,4]`







Example 5 E[P(H1|Y,D, betaT=mu1)]



Example 5 E[P(H1|Y,D)]