

7. Post condition: $\text{product} = A \times B$.

Base case: At the start of 1st iteration, $y=B$, $x=A$, $\text{product}=0$
if B is even, $x=2A$, $y=\frac{B}{2}$.

$$xy + \text{product} = 2A \cdot \frac{B}{2} + \text{product} = A \times B + \text{product} = A \times B.$$

invariant holds in base case when B is even.

if B is odd, $\text{product} = \text{product} + x$, $y=B-1$.

$$xy + \text{product} = A(B-1) + \text{product} + A = A \times B$$

invariant holds at the end of base case when B is odd.

Induction hypothesis: At the start of i of the loop.

$$xy + \text{product} = A \times B \text{ holds.}$$

if y is even, $x=2x$, $y=\frac{y}{2}$.

$$xy + \text{product} = 2x \cdot \frac{y}{2} + \text{product} = A \times B.$$

invariant holds at the end of i -th iteration, At the start of $i+1$ of the loop if y is even.

if y is odd, $\text{product} = \text{product} + x$, $y=y-1$

$$xy + \text{product} = x(y-1) + \text{product} + x = A \times B$$

invariant holds at the end of i -th iterations, At the start of $i+1$ of the loop if y is odd.

Loop Termination: The loop ends After $\lceil \frac{B+1}{2} \rceil$ iterations.

- when it ends $y=0$

- Invariant $xy + \text{product} = A \times B$

$$0 + \text{product} = A \times B.$$

$$\text{product} = A \times B.$$

therefore, by invariant, $\text{product} = A \times B$ we end the loop.