Step2: $T(n) = \left(\frac{n(n+1)}{2}\right)^2$

1st: prove $T(n) \in O(n^4)$.

$$T(n) = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^4+2n^3+n^2}{4} \le \frac{n^4+2n^4+n^4}{4} = n^4.$$

$\therefore T(n) \in O(n^4)$ for $c_1 = 1$ $n_1 = 1$

2nd: prove $T(n) \in \Omega(n^4)$.

$$T(n) = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^4+2n^3+n^2}{4} \ge \frac{n^4}{4}$$

$\therefore T(n) \in \Omega(n^4)$ for $c_2 = \frac{1}{4}$, $n_2 = 1$.

Therefore $T(n) \in \Theta(n^4)$ for $n_0 = \max(n_1, n_2) = 1$, $c_1 = \frac{1}{4}$, $c_2 = 1$

$$\frac{1}{4}n^4 \le T(n) \le n^4$$

3. a). $E_1 = 0$

$E_2 = E_1 + 2 + 1 = 3$

$E_3 = E_2 + 3 + 1 = 3 + 4$

$E_4 = E_3 + 4 + 1 = 3 + 4 + 5$

$E_5 = E_4 + 5 + 1 = 3 + 4 + 5 + 6$.

$\vdots$

$E_k = E_{k-1} + k + 1 = 3 + 4 + \cdots + k + 1$

$$E_k = \frac{(k+1+3)(k-1)}{2} = \frac{k^2+3k-4}{2}$$

b) proof: base case, when $k=1$, $E_k = E_1 = 0$.

$$\frac{k^2+3k-4}{2} = 0 = E_1$$

Therefore $E_k = \frac{k^2+3k-4}{2}$ is true when $k=1$.

Inductive Step: Suppose $E_n = \frac{n^2+3n-4}{2}$ for $k=n$, $n \ge 1$.

Let $k=n+1$. $E_{n+1} = E_n + n + 1 + 1 = \frac{n^2+3n-4}{2} + n + 2$

$$= \frac{n^2+5n}{2}$$

$$= \frac{(n+1)^2 + 3(n+1) - 4}{2}.$$

$= RHS$.

Thus, $E_k = \frac{k^2+3k-4}{2}$ is true for all $k \ge 1$. ∎