For example, the symbolic notation $'a + \infty = +\infty'$ means that from the relations

$$\lim_{n\to\infty} a_n = a \qquad \textit{and} \qquad \lim_{n\to\infty} = +\infty \tag{1.21}$$

we always get

$$\lim_{n\to\infty}(a_n + b_n) = +\infty$$

as a consequence. Similarly, the notation $'a(+\infty) = +\infty$ for $a > 0'$ means that from (1.21) and $a > 0$, it follows that

$$\lim_{n\to\infty} a_n b_n = +\infty$$

Example 3 : $\lim_{n\to\infty} n^2 = \lim_{n\to\infty} n \lim_{n\to\infty} n = +\infty.$

Rational expressions: We set

$$a := \frac{\alpha_k n^k + \alpha_{k-1}n^{k-1} + \cdots + \alpha_0}{\beta_m n^m + \beta_{m-1}n^{m-1} + \cdots + \beta_0}, \qquad n = 1, 2, \ldots$$

for fixed $k, m = 0, 1, 2 \ldots$ and fixed real numbers α_r, β_s with $\alpha_k \neq 0$ and $\beta_m \neq 0$. Then we have:

$$\lim_{n\to\infty} a_n = \begin{cases} \frac{\alpha_k}{\beta_m} & for\ k = m, \\ 0 & for\ k < m, \\ +\infty & for\ k > m\ and\ \alpha_k/\beta_m > 0, \\ -\infty & for\ k > m\ and\ \alpha_k/\beta_m < 0. \end{cases}$$

Example 4 : $\lim_{n\to\infty} \frac{n^2+1}{n^3+1} = 0.$

Indeterminant expressions: In the case of

$$\boxed{+\infty - \infty,\ 0 \cdot (\pm\infty),\ \frac{0}{0},\ \frac{\infty}{\infty},\ 0^0,\ 0^\infty,\ \infty^0} \tag{1.22}$$

one must be extremely careful! There are no general rules for manipulating these expressions. In different cases one gets different results.
Example 5 $(+\infty - \infty)$:

$$\lim_{n\to\infty}(2n - n) = +\infty, \ \lim_{n\to\infty}(n - 2n) = -\infty, \ \lim_{n\to\infty}((n+1) - n) = 1.$$

Example 6 $(0 \cdot \infty)$:

$$\lim_{n\to\infty}\left(\frac{1}{n} \cdot n\right) = 1, \ \lim_{n\to\infty}\left(\frac{1}{n} \cdot n^2\right) = \lim_{n\to\infty} n = +\infty.$$

In certain case expressions as in (1.22) can be given meaning and calculated by means of l'Hospital' rule (cf. 1.3.1.3).

1.2.4 Criteria for convergence of sequences

Basic idea: *Example 1:* We consider the iteration procedure

$$a_{n+1} = \frac{a_n}{2} + \frac{1}{a_n}, \qquad n = 1, 2, \ldots \tag{1.23}$$