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INTRODUCTION TO DATA SCIENCE

Lecture 5

Recommendation Techniques

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Where can one meet a Recommender System?

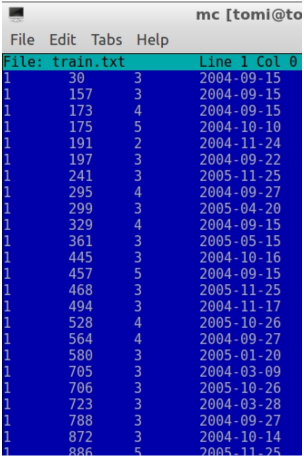
The Big Bang

- Contest begun on October 2, 2006
  - 100M ratings (1-5 stars) from 480K users on 18K movies
  - decrease RMSE of Cinematch (0.9525) at least with 10% ($\leq$0.8572)
- Grand Prize $1.000.000, Annual Progress Prizes $50.000

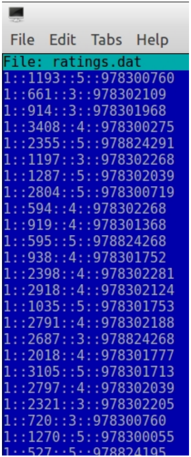
Rank	Team Name	Best Test Score	% Improvement	Best Submit Time
Grand Prize - RMSE = 0.8567 - Winning Team: BellKor's Pragmatic Chaos				
1	BellKor's Pragmatic Chaos	0.8567	10.06	2009-07-26 18:18:28
2	The Ensemble	0.8567	10.06	2009-07-26 18:38:22
3	Grand Prize Team	0.8582	9.90	2009-07-10 21:24:40
4	Opera Solutions and Vandelay United	0.8588	9.84	2009-07-10 01:12:31
5	Vandelay Industries !	0.8591	9.81	2009-07-10 00:32:20
6	PragmaticTheory	0.8594	9.77	2009-06-24 12:06:56
7	BellKor in BigChaos	0.8601	9.70	2009-05-13 08:14:09
8	Dace_	0.8612	9.59	2009-07-24 17:18:43
9	Feeds2	0.8622	9.48	2009-07-12 13:11:51
10	BigChaos	0.8623	9.47	2009-04-07 12:33:59
11	Opera Solutions	0.8623	9.47	2009-07-24 00:34:07
12	BellKor	0.8624	9.46	2009-07-26 17:19:11
Progress Prize 2008 - RMSE = 0.8627 - Winning Team: BellKor in BigChaos				

Netflix and Movielens data

Netflix


```
mc [tomi@to
File  Edit  Tabs  Help
File: train.txt              Line 1 Col 0
1        30     3        2004-09-15
1        157    3        2004-09-15
1        173    4        2004-09-15
1        175    5        2004-10-10
1        191    2        2004-11-24
1        197    3        2004-09-22
1        241    3        2005-11-25
1        295    4        2004-09-27
1        299    3        2005-04-20
1        329    4        2004-09-15
1        361    3        2005-05-15
1        445    3        2004-10-16
1        457    5        2004-09-15
1        468    3        2005-11-25
1        494    3        2004-11-17
1        528    4        2005-10-26
1        564    4        2004-09-27
1        580    3        2005-01-20
1        705    3        2004-03-09
1        706    3        2005-10-26
1        723    3        2004-03-28
1        788    3        2004-09-27
1        872    3        2004-10-14
1        886    5        2005-11-25
```

Movielens (100K, 1M)


```
File  Edit  Tabs  Help
File: ratings.dat
1::1193::5::978300760
1::661::3::978302109
1::914::3::978301968
1::3408::4::978300275
1::2355::5::978824291
1::1197::3::978302268
1::1287::5::978302039
1::2804::5::978300719
1::594::4::978302268
1::919::4::978301368
1::595::5::978824268
1::938::4::978301752
1::2398::4::978302281
1::2918::4::978302124
1::1035::5::978301753
1::2791::4::978302188
1::2687::3::978824268
1::2018::4::978301777
1::3105::5::978301713
1::2797::4::978302039
1::2321::3::978302205
1::720::3::978300760
1::1270::5::978300055
1::527::5::978824195
```

Closely related fields

Information Retrieval

- unstructured data, various topics (IR) vs. repositories focused on a single topic (RS)
- relevant content for the query (IR) vs. relevant content for the user (RS)

Data mining & Machine Learning

- hardly measurable, subjective evaluation criteria (RS) besides some classic, objective evaluation measures (ML)

Human-Computer Interaction

- RS should convince the user to try the recommended items
- clear, transparent and trustworthy system logic
- provide details about recommended items and opportunity to refine recommendations

Users, Items and their characteristics

Users

- set of users $\mathcal{U}$
- user attributes $\mathcal{A}^{user} \subset \mathbb{R}^k$
  - age, income, marital status, education, profession, nationality, …
  - preferred sport, hobbies, favourite movies, …
- user characteristics $\chi^{user} : \mathcal{U} \rightarrow \mathcal{A}^{user}$
  - *sensitive* information, hard to obtain

Items

- set of items $\mathcal{I}$
- item attributes $\mathcal{A}^{item} \subset \mathbb{R}^l$
  - movies: title, genre, year, director, actors, budget, nominations, …
- item characteristics $\chi^{item} : \mathcal{I} \rightarrow \mathcal{A}^{item}$
  - quite *costly* to obtain

User feedback

$\phi : \mathcal{D} \to \mathcal{F}$

- feedback values $\mathcal{F} \subset \mathbb{R}$ observed on $\mathcal{D} \subset \mathcal{U} \times \mathcal{I}$

Implicit feedback

- information obtained about users by watching their natural *interaction with the system*
  - view, listen, scroll, bookmark, save, purchase, link, copy&paste, ...
- no burden on the user

Explicit feedback

- *rating* items on a rating scale (Likert's scale)
- *scoring* items
- *ranking* a collection of items
- *pairwise ranking* of two presented items
- *provide* a list of preferred items

The recommendation task

Given

- $\mathcal{U}, \mathcal{I}$ and ϕ
- χ^{user}, χ^{item}
- some background knowledge κ

To learn

- **model** $\hat{\phi} : \mathcal{U} \times \mathcal{I} \rightarrow \mathbb{R}$ such that $acc(\hat{\phi}, \phi, \mathcal{T})$ is maximal
  - a set of "unseen" (or future) user-item pairs $\mathcal{T} \subseteq (\mathcal{U} \times \mathcal{I}) \setminus \mathcal{D}$
  - *acc* is the accuracy of $\hat{\phi}$ w.r.t. ϕ measured on $\mathcal{T}$

It looks as a simple prediction task, **however**

- χ^{user}, χ^{item} and κ are often unknown
- usually, $\mathcal{F} = \{1\}$ in case of implicit feedback

Two distinguished tasks

Rating prediction from explicit feedback

- How would Steve rate the movie Titanic more likely?

	Titanic	Pulp Fiction	Iron Man	Forrest Gump	The Mummy
Joe	1	4	5		3
Ann	5	1		5	2
Mary	4	1	2	5	
Steve	?	3	4		4

- $\hat{\phi}(u, i)$ – predicted rating of the user u for an item i

Item recommendation from implicit feedback

- Which movie(s) would does Steve see/buy more likely?

	Titanic	Pulp Fiction	Iron Man	Forrest Gump	The Mummy
Joe	1	1	1		1
Ann	1	1		1	1
Mary	1	1	1	1	
Steve	?	1	1	?	1

- $\hat{\phi}(u, i)$ – predicted likelihood of a “positive” implicit feedback (ranking score) of the user u for an item i

Types of RS

Knowledge-based

- recommendations are based on knowledge about users' needs and preferences
  - χ^{item}, κ, χ^{user}
- won't deal with it in this lecture

Content-based

- learn user's interests based on the features of items previously rated by the user, using supervised machine learning techniques
  - χ^{item}, ϕ

Collaborative-filtering

- recognize similarities between users according to their feedbacks and recommend objects preferred by the like-minded users
  - ϕ (also χ^{item} and/or χ^{user} can be utilized)

Hybrid

Neighborhood-based Collaborative Filtering

Recommendation $\hat{\phi}(u,i)$ for user u on item i using ϕ

- user-based
  - $\hat{\phi}(u,i)$ computed using feedback given by k **most similar users**

$$\mathcal{N}_i^{u,k} = \arg\max_{\mathcal{U}'} \sum_{\substack{v \in \mathcal{U}', v \neq u \\ \mathcal{U}' \subseteq \mathcal{U}_i, |\mathcal{U}'| = k}} sim(u,v)$$

    - $\mathcal{U}_i = \{v \in \mathcal{U} \mid \phi(v,i) \text{ is defined on } \mathcal{D}\}$

- item-based
  - $\hat{\phi}(u,i)$ computed using feedback given by k **most similar items**

$$\mathcal{N}_u^{i,k} = \arg\max_{\mathcal{I}'} \sum_{\substack{j \in \mathcal{I}', j \neq i \\ \mathcal{I}' \subseteq \mathcal{I}_u, |\mathcal{I}'| = k}} sim(i,j)$$

    - $\mathcal{I}_u = \{j \in I \mid \phi(u,j) \text{ is defined on } \mathcal{D}\}$

Item recommendation

What is the likelihood of an item i being liked by the user u?

- a simple **k-nearest-neighbor** approach[1]
  - user-based
    - an average similarity of most similar users which liked the item i

$$\hat{\phi}_{ui} = \frac{\sum_{v \in \mathcal{N}_i^{u,k}} sim(u,v)}{k}$$

  - item-based
    - an average similarity of most similar items liked by the user u

$$\hat{\phi}_{ui} = \frac{\sum_{j \in \mathcal{N}_u^{i,k}} sim(i,j)}{k}$$

assume that only (implicit) feedback ϕ is available

- users and items represented by **sparse vectors**
  - cosine-vector similarity sim_{cv}

[1] Simplified notation: $\phi(u,i) \rightsquigarrow \phi_{ui}$, $\mathcal{I}_u \cap \mathcal{I}_v \rightsquigarrow \mathcal{I}_{uv}$, $\mathcal{U}_i \cap \mathcal{U}_j \rightsquigarrow \mathcal{U}_{ij}$

Item recommendation – example

$sim_{cv}(i,j)$	Titanic	Pulp Fiction	Iron Man	Forrest Gump	The Mummy
Titanic	1.0	0.87	0.67	0.82	0.67
Pulp Fiction	–	1.0	0.87	0.71	0.87
Iron Man	–	–	1.0	0.41	0.67
Forrest Gump	–	–	–	1.0	0.41
The Mummy	–	–	–	–	1.0

$sim_{cv}(u,v)$	Joe	Ann	Mary	Steve
Joe	1.0	0.75	0.75	0.87
Ann	–	1.0	0.75	0.58
Mary	–	–	1.0	0.58
Steve	–	–	–	1.0

user-based[1]

- $\mathcal{N}^{Steve,2}_{Titanic} = \{Joe, Ann\}$, $\hat{\phi}_{ST} = \frac{s_{cv}(S,J)+s_{cv}(S,A)}{2} = \frac{0.87+0.58}{2} = 0.725$
- $\mathcal{N}^{Steve,2}_{ForrestGump} = \{Ann, Mary\}$, $\hat{\phi}_{ST} = \frac{s_{cv}(S,A)+s_{cv}(S,M)}{2} = \frac{0.58+0.58}{2} = 0.58$

item-based

- $\mathcal{N}^{Titanic,2}_{Steve} = \{PulpFiction, IronMan\}$, $\hat{\phi}_{ST} = \frac{s_{cv}(T,P)+s_{cv}(T,I)}{2} = \frac{0.87+0.67}{2} = 0.77$
- $\mathcal{N}^{ForrestGump,2}_{Steve} = \{PulpFiction, IronMan\}$, $\hat{\phi}_{ST} = \frac{s_{cv}(F,P)+s_{cv}(F,I)}{2} = \frac{0.71+0.41}{2} = 0.56$

[1] s_{cv} – cosine–vector similarity

Rating prediction

How would the user rate an item?

- user's/item's ratings are **biased**
    - optimistic, pessimistic users
    - items rated above or below average

mean-centered rating prediction

- user-based

$$\hat{\phi}_{ui} = \overline{\phi}_u + \frac{\sum_{v \in \mathcal{N}_i^{u,k}} sim(u,v) \cdot (\phi_{vi} - \overline{\phi}_v)}{\sum_{v \in \mathcal{N}_i^{u,k}} |sim(u,v)|}$$

    - $\overline{\phi}_u = \frac{\sum_{i \in \mathcal{I}_u} \phi(u,i)}{|\mathcal{I}_u|}$
- item-based

$$\hat{\phi}_{ui} = \overline{\phi}_i + \frac{\sum_{j \in \mathcal{N}_u^{i,k}} sim(i,j) \cdot (\phi_{uj} - \overline{\phi}_j)}{\sum_{v \in \mathcal{N}_u^{i,k}} |sim(i,j)|}$$

    - $\overline{\phi}_i = \frac{\sum_{u \in \mathcal{U}_i} \phi(u,i)}{|\mathcal{U}_i|}$

Pearson-correlation similarity

What similarity measure to use?

- sim_{cv} doesn't take into account the mean and variances of ratings

pearson-correlation similarity

$$sim_{pc}(u,v) = \frac{\sum_{i \in \mathcal{I}_{uv}} (\phi_{ui} - \overline{\phi}_u)(\phi_{vi} - \overline{\phi}_v)}{\sqrt{\sum_{i \in \mathcal{I}_{uv}} (\phi_{ui} - \overline{\phi}_u)^2 \ \sum_{i \in \mathcal{I}_{uv}} (\phi_{vi} - \overline{\phi}_v)^2}}$$

$$sim_{pc}(i,j) = \frac{\sum_{u \in \mathcal{U}_{ij}} (\phi_{ui} - \overline{\phi}_i)(\phi_{uj} - \overline{\phi}_j)}{\sqrt{\sum_{u \in \mathcal{U}_{ij}} (\phi_{ui} - \overline{\phi}_i)^2 \ \sum_{i \in \mathcal{U}_{ij}} (\phi_{uj} - \overline{\phi}_j)^2}}$$

Rating prediction – example

$sim_{pc}(i,j)$	Titanic	Pulp Fiction	Iron Man	Forrest Gump	The Mummy
Titanic	1.0	-0.956	-0.815	NaN	-0.581
Pulp Fiction	–	1.0	0.948	NaN	0.621
Iron Man	–	–	1.0	NaN	0.243
Forrest Gump	–	–	–	1.0	NaN
The Mummy	–	–	–	–	1.0

NaN values are usually converted to zero (rare in case of enough data)

$sim_{pc}(u,v)$	Joe	Ann	Mary	Steve
Joe	1.0	-0.716	-0.762	-0.005
Ann	–	1.0	0.972	0.565
Mary	–	–	1.0	0.6
Steve	–	–	–	1.0

user-based

- $\mathcal{U}_{Titanic} = \{Joe, Ann, Mary\}$, $\mathcal{N}_{Titanic}^{Steve,2} = \{Mary, Ann\}$
- $\overline{\phi}_{Steve} = \frac{11}{3} = 3.67$, $\overline{\phi}_{Mary} = \frac{12}{4} = 3$, $\overline{\phi}_{Ann} = \frac{13}{4} = 3.25$
- $\hat{\phi}_{ST} = \overline{\phi}_S + \frac{s_{pc}(S,M)\cdot(\phi_{MT}-\overline{\phi}_M)+s_{pc}(S,A)\cdot(\phi_{AT}-\overline{\phi}_A)}{|s_{pc}(S,M)|+|s_{pc}(S,A)|} = 3.67 + \frac{0.6\cdot(4-3)+0.565\cdot(5-3.25)}{0.6+0.565} = 1.36$

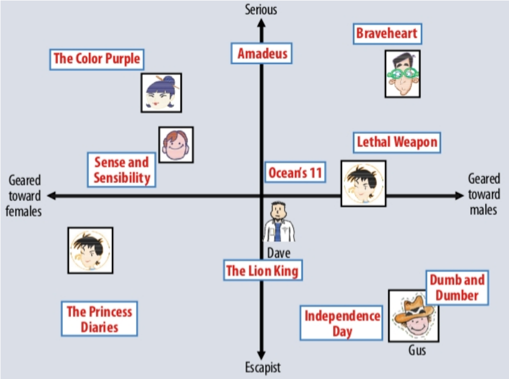
item-based

- $\mathcal{I}_{Steve} = \{\underline{P}ulp\ Fiction, \underline{I}ron\ Man, The\ \underline{M}ummy\}$, $\mathcal{N}_{Steve}^{Titanic,2} = \{\underline{I}ron\ Man, The\ \underline{M}ummy\}$
- $\overline{\phi}_T = \frac{10}{3} = 3.34$, $\overline{\phi}_I = \frac{11}{3} = 3.67$, $\overline{\phi}_M = \frac{9}{3} = 3$
- $\hat{\phi}_{ST} = \overline{\phi}_T + \frac{s_{pc}(T,I)\cdot(\phi_{SI}-\overline{\phi}_I)+s_{pc}(T,M)\cdot(\phi_{SM}-\overline{\phi}_M)}{|s_{pc}(T,I)|+|s_{pc}(T,M)|} = 3.34 + \frac{-.815\cdot(4-3.67)-.581\cdot(4-3)}{0.815+0.581} = 2.73$

A latent space representation

Map users and items to a common latent space

- where dimensions or **factors** represent
  - items' **implicit properties**
  - users' **interest** in items' hidden properties



[1]

[1] The picture is taken from *Y. Koren et al. (2009). Matrix Factorization Techniques for Recommender Systems. Computer 42 (8).*

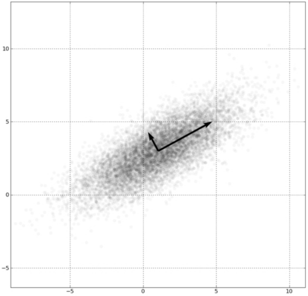
Known factorization models (1/2)

ϕ represented as a user-item matrix $\Phi^{n \times m}$

- n users, m items

Principal Component Analysis (PCA)

- transform data to a new coordinate system
    - variances by any projection of the data lies on coordinates in decreasing order



[2]

[2] The picture is taken from wikipedia.

Known factorization models (2/2)

Singular Value Decomposition (SVD)

$$\Phi = W^{n \times k} \Sigma^{k \times k} H^{n \times k^T}$$

- $W^T W = I$, $H^T H = I$
- column vectors of W are orthonormal eigenvectors of $\Phi\Phi^T$
- column vectors of H are orthonormal eigenvectors of $\Phi^T\Phi$
- Σ contains eigenvallues of W in descending order

PCA, SVD computed algebraically

- Φ is a **big** and **sparse** matrix
  - approximations of PCA[1], SVD[2]

[1] T.Raiko et al. (2007). Principal Component Analysis for Sparse High-Dimensional Data. Neural Information Processing, LNCS. 4984.

[2] A.K. Menon and Ch. Elkan (2011). Fast Algorithms for Approximating the Singular Value Decomposition. ACM Trans. Knowl. Discov. Data 5 (2).

MF – rating prediction (1/2)

recommendation task

- to find $\hat{\phi} : \mathcal{U} \times \mathcal{I} \to \mathbb{R}$ such that $acc(\hat{\phi}, \phi, \mathcal{T})$ is maximal
  - acc is the **expected** accuracy on $\mathcal{T}$
  - training $\hat{\phi}$ on $\mathcal{D}$ such that the **empirical** loss $err(\hat{\phi}, \phi, \mathcal{D})$ is minimal

a simple, **approximative** MF **model**

- only $W^{n \times k}$ and $H^{m \times k}$
- k – the number of factors

$$\Phi^{n \times m} \approx \hat{\Phi}^{n \times m} = WH^T$$

- predicted rating $\hat{\phi}_{ui}$ of the user u for the item i

$$\hat{\phi}_{ui} = w_u h_i^T$$

MF – rating prediction (2/2)

the **loss** function $err(\hat{\phi}, \phi, \mathcal{D})$

- squared loss

$$err(\hat{\phi}, \phi, \mathcal{D}) = \sum_{(u,i)\in\mathcal{D}} e_{ui}^2 = \sum_{(u,i)\in\mathcal{D}} (\phi_{ui} - \hat{\phi}_{ui})^2 = \sum_{(u,i)\in\mathcal{D}} (\phi_{ui} - w_u h_i^T)^2$$

the **objective function**

- **regularization** term $\lambda \geq 0$ to prevent overfitting
  - penalizing the magnitudes of parameters

$$f(\hat{\phi}, \phi, \mathcal{D}) = \sum_{(u,i)\in\mathcal{D}} (\phi_{ui} - w_u h_i^T)^2 + \lambda(\|W\|^2 + \|H\|^2)$$

The task is to find parameters W and H such that, given λ, the objective function $f(\hat{\phi}, \phi, \mathcal{D})$ is minimal.

Gradient descent

How to find a minimum of an "objective" function $f(\Theta)$?

- in case of MF, $\Theta = W \cup H$, and
- $f(\Theta)$ refers to the error of approximation of Φ by WH^T

Gradient descent

input: f, α, Σ^2, *stopping criteria*
initialize $\Theta \sim \mathcal{N}(0, \Sigma^2)$
repeat
$\quad \Theta \leftarrow \Theta - \alpha \frac{\partial f}{\partial \Theta}(\Theta)$
until approximate minimum is reached
return Θ

stopping criteria

- $|\Theta^{old} - \Theta| < \epsilon$
- maximum number of iterations reached
- a combination of both

Stochastic gradient descent

if f can be written as

$$f(\Theta) = \sum_{i=1}^{n} f_i(\Theta)$$

Stochastic gradient descent (SGD)

input: f_i, α, Σ^2, *stopping criteria*
initialize $\Theta \sim \mathcal{N}(0, \Sigma^2)$
repeat
  for all i in random order **do**
    $\Theta \leftarrow \Theta - \alpha \frac{\partial f_i}{\partial \Theta}(\Theta)$
until approximate minimum is reached
return Θ

MF with SGD

updating parameters **iteratively** for each data point ϕ_{ui} in the opposite direction of the **gradient** of the objective function at the given point until a **convergence** criterion is fulfilled.

- updating the vectors w_u and h_i for the data point $(u, i) \in D$

$$\frac{\partial f}{\partial w_u}(u, i) = -2(e_{ui} h_i - \lambda w_u)$$
$$\frac{\partial f}{\partial h_i}(u, i) = -2(e_{ui} w_u - 2\lambda h_i)$$

$$w_u(u, i) \leftarrow w_u - \alpha \frac{\partial f}{\partial w_u}(u, i) = w_u + \alpha(e_{ui} h_i - \lambda w_u)$$
$$h_i(u, i) \leftarrow h_i - \alpha \frac{\partial f}{\partial h_i}(u, i) = h_i + \alpha(e_{ui} w_u - \lambda h_i)$$

where $\alpha > 0$ is a **learning rate**.

MF with SGD – Algorithm

Hyper-parameters: $k, iters$ *(the max number of iteration)*, $\alpha, \lambda, \Sigma^2$

$W \leftarrow \mathcal{N}(0, \Sigma^2)$

$H \leftarrow \mathcal{N}(0, \Sigma^2)$

for $iter \leftarrow 1, \ldots, iters \cdot |\mathcal{D}|$ **do**

    draw randomly (u, i) from $\mathcal{D}$

    $\hat{\phi}_{ui} \leftarrow 0$

    for $j \leftarrow 1, \ldots, k$ **do**

        $\hat{\phi}_{ui} \leftarrow \hat{\phi}_{ui} + W[u][j] \cdot H[i][j]$

    $e_{ui} = \phi_{ui} - \hat{\phi}_{ui}$

    for $j \leftarrow 1, \ldots, k$ **do**

        $temp_w \leftarrow W[u][j]$

        $temp_h \leftarrow H[i][j]$

        $W[u][j] \leftarrow W[u][j] + \alpha * (e_{ui} * temp_h - \lambda * temp_w)$

        $H[i][j] \leftarrow H[i][j] + \alpha * (e_{ui} * temp_w - \lambda * temp_h)$

return $\{W, H\}$

MF with SGD – Example[2]

Let's have the following hyper-parameters:
$K = 2,\ \alpha = 0.1,\ \lambda = 0.15,\ iter = 150,\ \sigma^2 = 0.01$

$\Phi =$

1	4	5		3
5	1		5	2
4	1	2	5	
	3	4		4

Results are:

$W =$

1.1995242	1.1637173
1.8714619	-0.02266505
2.3267753	0.27602595
2.033842	0.539499

$H^T =$

1.6261001	1.1259034	2.131041	2.2285593	1.6074764
-0.40649664	0.7055319	1.0405376	0.39400166	0.49699315

Results[1] are:

$\hat{\Phi} =$

1.477499	2.171588	3.767126	3.131717	2.506566
3.052397	2.091094	3.964578	4.161733	2.997066
3.671365	2.814469	5.245668	5.294111	3.877419
3.087926	2.670543	4.895569	4.745101	3.537480

[1] Note, that these hyper-parameters are just picked up in an ad-hoc manner. One should search for the "best" hyper-parameter combinations using e.g. grid-search (a brute-force approach).

[2] Thanks to my colleague Thai-Nghe Nguyen for computing an example.

Biased MF

baseline estimate

- user-item bias
$$b_{ui} = \mu + b'_u + b''_i$$
  - μ – average rating across the whole $\mathcal{D}$
  - b', b'' – vectors of user and item biases, respectively

prediction
$$\hat{\phi}_{ui} = \mu + b'_u + b''_i + w_u h_i$$

objective function to minimize

$$f(\phi, \hat{\phi}, \mathcal{D}) = \sum_{(u,i)\in\mathcal{D}} (\phi_{ui} - \mu - b'_u - b''_i - w_u h_i)^2 + \lambda(\|W\|^2 + \|H\|^2 + b'^2 + b''^2)$$

Biased MF with SGD

similar to unbiased MF

- initialize average and biases

$$\mu = \frac{\sum_{(u,i)\in\mathcal{D}}}{|\mathcal{D}|}$$

$$b^{'} \leftarrow (\overline{\phi}_{u_1}, \ldots, \overline{\phi}_{u_n})$$

$$b^{''} \leftarrow (\overline{\phi}_{i_1}, \ldots, \overline{\phi}_{i_m})$$

- update average and biases

$$\mu \leftarrow \mu - \frac{\partial f}{\partial \mu}(u,i) = \mu + \alpha e_{ui}$$

$$b^{'} \leftarrow b^{'} - \frac{\partial f}{\partial b^{'}}(u,i) = b^{'} + \alpha(e_{ui} - \lambda b^{'})$$

$$b^{''} \leftarrow b^{''} - \frac{\partial f}{\partial b^{''}}(u,i) = b^{''} + \alpha(e_{ui} - \lambda b^{''})$$

MF – item recommendation

to predict a personalized **ranking score**[1] $\hat{\phi}_{ui}$

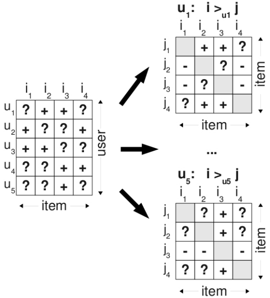
- how the item i is preferred to other items for the user u
- to find W and H such that $\hat{\Phi} = WH^T$

$$\hat{\phi}_{ui} = w_u h_i^T$$

problem: positive feedback only

- **pairwise ranking** data

$$\mathcal{D}_p = \{(u, i, j) \in \mathcal{D} | i \in \mathcal{I}_u \wedge j \in \mathcal{I} \setminus \mathcal{I}_u\}$$



[1] S. Rendle et al. (2009). BPR: Bayesian Personalized Ranking from Implicit Feedback. 25th Conference on Uncertainty in Artificial Intelligence.

MF – Bayesian Personalized Ranking (1/3)

Bayesian formulation of the problem

- $\succ$ – the unknown preference structure (ordering)
  - we use the derived pairwise ranking data $\mathcal{D}_p$
- Θ – parameters of an arbitrary prediction model
  - in case of MF, $\Theta = W \cup H$

$$p(\Theta| \succ) \propto p(\succ |\Theta)p(\Theta)$$

prior probability

- assume independence of parameters
- assume, $\Theta \sim N(0, \frac{1}{\lambda} I)$

$$p(\Theta) = \prod_{\theta \in \Theta} \sqrt{\frac{\lambda}{2\pi}}\, e^{-\frac{1}{2}\lambda\theta^2}$$

MF – Bayesian Personalized Ranking (2/3)

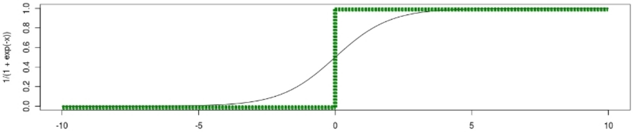
likelihood

- assume users' feedbacks are independent
- assume, ordering of each pair is independent

$$p(\succ|\Theta) = \prod_{u\in\mathcal{U}} p(\succ_u|\Theta) = \prod_{(u,i,j)\in\mathcal{D}_p} p(i \succ_u j|\Theta)$$

- using the ranking scores $\hat{\phi}$

$$p(i \succ_u j|\Theta) = p(\hat{\phi}_{ui} - \hat{\phi}_{uj} > 0) = \sigma(\hat{\phi}_{ui} - \hat{\phi}_{uj}) = \frac{1}{1+e^{-(\hat{\phi}_{ui}-\hat{\phi}_{uj})}}$$



MF – Bayesian Personalized Ranking (3/3)

maximum **a posteriori estimation** of Θ

$$\arg\max_{\Theta} p(\Theta, \succ) =$$

$$\arg\max_{\Theta} p(\succ|\Theta)p(\Theta) =$$

$$\arg\max_{\Theta} ln\, p(\succ|\Theta)p(\Theta) =$$

$$\arg\max_{\Theta} ln \prod_{(u,i,j)\in\mathcal{D}_p} \sigma(\hat{\phi}_{ui} - \hat{\phi}_{uj}) \sqrt{\frac{\lambda}{2\pi}}\, e^{-\frac{1}{2}\lambda\theta^2}$$

$$\arg\max_{\Theta} \underbrace{\sum_{(u,i,j)\in\mathcal{D}_p} ln\, \sigma(\hat{\phi}_{ui} - \hat{\phi}_{uj}) \; - \lambda\|\Theta\|^2}_{BPR\text{–}OPT}$$

Finding parameters for BPR-OPT

Stochastic gradient ascent

$$\frac{\partial BPR-OPT}{\partial \Theta} \propto \sum_{(u,i,j)\in \mathcal{D}_p} \frac{e^{-(\hat{\phi}_{ui}-\hat{\phi}_{uj})}}{1+e^{-(\hat{\phi}_{ui}-\hat{\phi}_{uj})}} \frac{\partial}{\partial \Theta}(\hat{\phi}_{ui}-\hat{\phi}_{uj}) - \lambda\Theta$$

$$\frac{\partial}{\partial \theta}(\hat{\phi}_{ui}-\hat{\phi}_{uj}) = \begin{cases} (h_i - h_j) & if\ \theta = w_u \\ w_u & if\ \theta = h_i \\ -w_u & if\ \theta = h_j \\ 0 & else \end{cases}$$

<u>LearnBPR</u>

input: $f_i, \alpha, \Sigma^2, stopping\ criteria$
initialize $\Theta \sim \mathcal{N}(0, \Sigma^2)$
repeat
  draw $(u, i, j) \in \mathcal{D}_p$ randomly
  $\Theta \leftarrow \Theta + \alpha \frac{\partial BPR-OPT}{\partial \Theta}(\Theta)$
until approximate maximum is reached
return Θ

BPR-OPT vs AUC

Area under the ROC curve (AUC)

- probability that the ranking of a randomly drawn pair is correct

$$AUC = \sum_{u \in \mathcal{U}} AUC(u) = \frac{1}{|\mathcal{U}|} \frac{1}{|\mathcal{I}_u|\ |\mathcal{I} \setminus \mathcal{I}_u|} \sum_{(u,i,j) \in \mathcal{D}_p} \delta(\hat{\phi}_{ui} \succ \hat{\phi}_{uj})$$

- $\delta(\hat{\phi}_{ui} \succ \hat{\phi}_{uj}) = 1$ if $\hat{\phi}_{ui} \succ \hat{\phi}_{uj}$, and 0, else

Smoothed AUC objective function with regularization of parameters

$$AUC - OPT = \sum_{(u,i,j) \in \mathcal{D}_p} \sigma(\hat{\phi}_{ui} - \hat{\phi}_{uj}) - \lambda \|\Theta\|^2$$

$$BPR - OPT = \sum_{(u,i,j) \in \mathcal{D}_p} ln\, \sigma(\hat{\phi}_{ui} - \hat{\phi}_{uj}) - \lambda \|\Theta\|^2$$

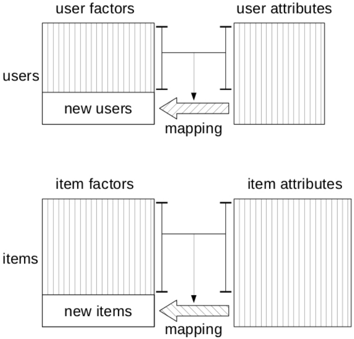
The cold-start problem

arises when not enough collaborative information is available

- new user or new item

possible solutions

- recommend popular items, “predict” global average, ...
- utilize item attributes[1]

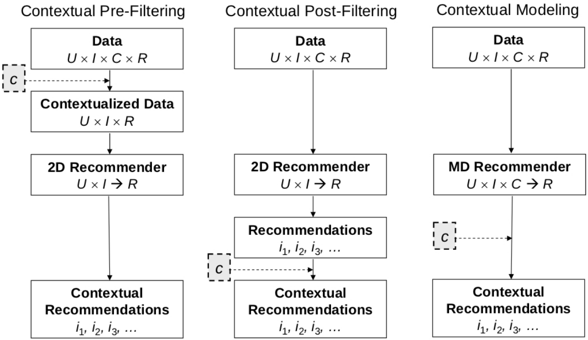


[1] Z. Gantner et al. (2010). Learning Attribute-to-Feature Mappings for Cold-Start Recommendations. 10th IEEE International Conference on Data Mining.

Context-aware recommendation

Context is any additional information, besides $\chi^{user}, \chi^{item}, \phi$ and κ, that is relevant for the recommendation[1]

- time, location, companion (when, where and with whom the user wants to watch some movie)



[1] Picture from *G. Adomavicius and A. Tuzhilin: Context-Aware Recommender Systems. Tutorial on the 2nd ACM International Conference on Recommender Systems, 2008.*
`http://ids.csom.umn.edu/faculty/gedas/talks/RecSys2008-tutorial.pdf`

Evaluating RS (1/3)

experiments

- **offline**
  - no interaction with real users, need to simulate user behaviour
  - low cost, short time
  - answers only a few questions, e.g. the predictive power of techniques
- **user studies**
  - observing test subjects' behaviour in the system
  - questionnaries
  - expensive, small scale,
- **online evaluation**
  - redirect a small part of the traffic to an alternative recommendation engine
  - risky – we can loose some customers
  - good to do after an offline testing of an recommendation engine showes good results

Evaluating RS (2/3)

properties of a recommender system

- user preference
  - Which one from different RS users prefer more?
- prediction accuracy
  - How precise recommendations does a RS provide?
- coverage
  - What proportion of all items can a RS ever recommend? To what proportion of users can a system recommend? How rich a user profile should be for making recommendation?
  - cold-start as a subproblem ("coldness" of an item)
- confidence
  - How confident the system is with its recommendation? (e.g. depends on amount of data in CF...)
- novelty
  - Does the system recommends items the user did not know about?
- trust
  - What is the users' trust in recommendation?

Evaluating RS (3/3)

- serendipity
  - How surprising the recommendations are? (e.g. a new movie with the user's favourite actor can be novel but not surprising)
- diversity
  - How "colorful" the recommendations are?
- utility
  - How useful a RS is for the provider/user? (e.g. generated revenue)
- robustness
  - How stable a RS is in presence of fake information?
- privacy
  - How users' privacy is retained in a RS ?
- adaptivity
  - How does a RS adapt to changes in the item collection?
- scalability
  - How scalable a RS is?

Pros and Cons of various types of RS

Knowledge-based

- pros: no cold-start, deterministic
- cons: knowledge-engineering needed, static

Content-based

- pros: no collaborative information needed
- cons: content is needed, cold-start for new users, no serendipity

Collaborative-filtering

- pros: no user nor item attributes needed, serendipity
- cons: cold-start for new users and items


That's all Folks!

References

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- Andreas W. Neumann (2009). Recommender Systems for Information Providers: Designing Customer Centric Paths to Information. Physica-Verlag Heidelberg.
- Dietmar Jannach, Markus Zanker, Alexander Felfernig and Gerhard Friedrich (2010). Recommender Systems: An Introduction. Cambridge University Press.
- Francesco Ricci, Lior Rokach, Bracha Shapira and Paul B. Kantor (2010). Recommender Systems Handbook. Springer-Verlag.
- Charu C. Aggarwal (2016). Recommender Systems: The Textbook. Springer International Publishing.

Homework

1. Download a MovieLens dataset
   - `https://grouplens.org/datasets/movielens/`
2. Implement a simple and a biased matrix factorization technique using stochastic gradient descent
3. Make experiments with these two techniques for various hyper-parameter settings
   - Don't forget, it's a prediction task, so take care of the quality of the model (if it's not overfitting, etc.)

Questions?

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