Solution To Assignment 1

Answer to Q3

To determine the highest bandwidth available along a path connecting two switching centers, denoted as a and b, within a telephone network graph, we can utilize a modified variant of Dijkstra's algorithm. The following outlines the algorithm:

1. Initialize a priority queue q and a distance array d, which will store the maximum bandwidth values for each vertex.

2. Assign negative infinity to all distances in d, except for a, which is set to positive infinity.

3. Enqueue a into q with a priority of positive infinity.

4. While q is not empty, perform the following steps:

- Dequeue a vertex v from q.

- For each neighboring vertex n of v, execute the following actions:

  - Let bw represent the bandwidth of the edge connecting v and n.

  - Let max denote the maximum bandwidth value between v and n.

  - If max surpasses the recorded maximum bandwidth value for n, update the maximum bandwidth value of n in d, and enqueue n into q with a priority of max.

5. Return the maximum bandwidth value recorded for b in d.

Answer to Q4

The Floyd-Warshall algorithm provides a solution. Here is an algorithm to compute the transitive reduction:

1. Create a matrix R of size $n \times n$, where n represents the number of vertices in the graph. Initialize all elements of R to 0.

2. Iterate through each edge (u, v) in the graph and set $R[u][v]$ to 1.

3. For each vertex k from 1 to n, perform the following steps:

a. For every pair of vertices u and v, excluding the case when u is equal to k or v is equal to k, check if both $R[u][k]$ and $R[k][v]$ are equal to 1.

b. If $R[u][k]$ and $R[k][v]$ are both 1, it implies there is a path from u to v through k. In such cases, set $R[u][v]$ to 0.

4. The resulting matrix R represents the transitive reduction of the graph.

The time complexity of this algorithm is O(n^3), where n denotes the number of vertices in the graph. The algorithm examines all possible vertex pairs and checks for the existence of transitive edges through intermediate vertices. By efficiently computing the transitive closure of a graph, the Floyd-Warshall algorithm enables derivation of the transitive reduction.