Association of Computing Activities

Algorithmic Graph Theory

Programming Assignment 1

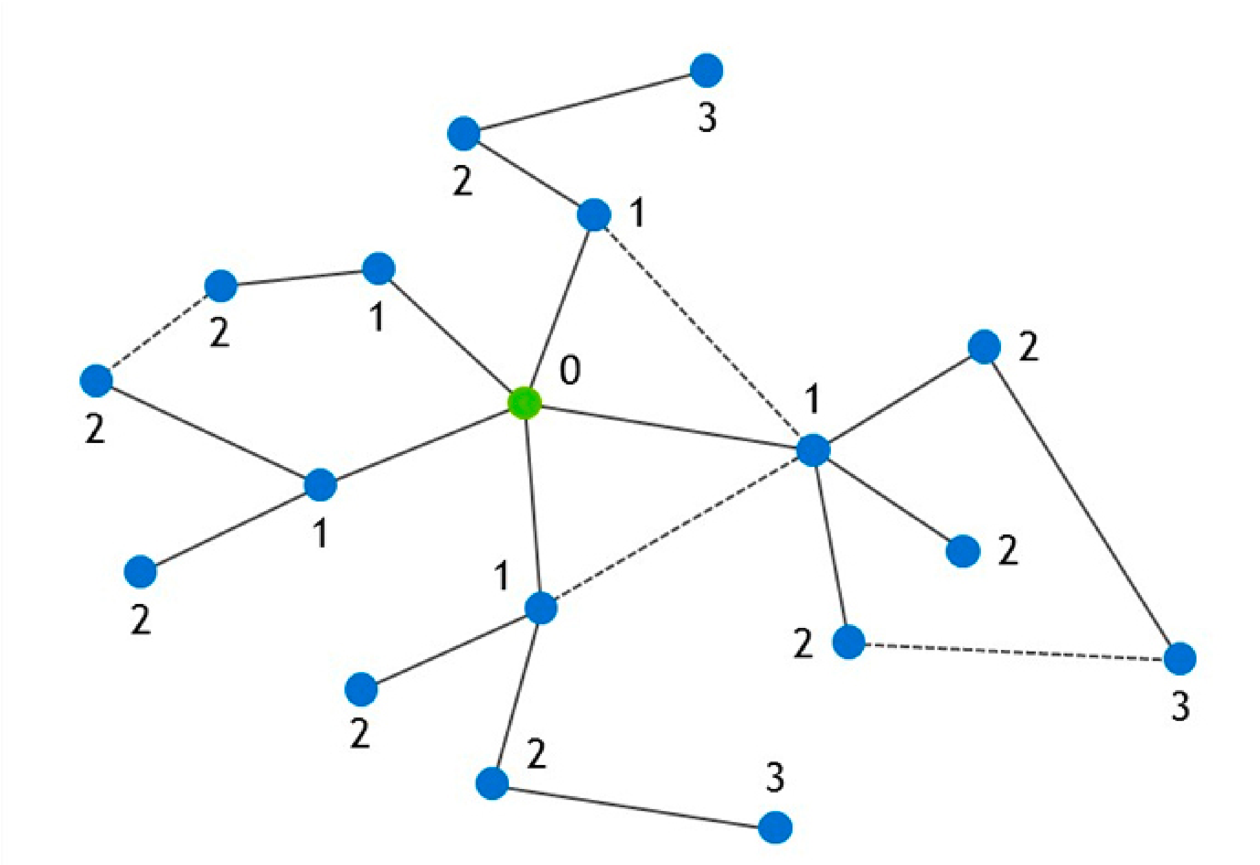
Topics : SSSP, APSP

2023

Homework N° 1

Brief Recap

The shortest path problem involves finding the shortest path between two vertices (or nodes) in a graph. Algorithms such as the *Floyd – Warshall algorithm* and different variations of *Dijkstra's algorithm* are used to find solutions to the shortest path problem. Applications of the shortest path problem include those in road networks, logistics, communications, electronic design, power grid contingency analysis, and community detection.



Shortest Routes I

There are n cities and m flight connections between them. Your task is to determine the length of the shortest route from Syrjälä to every city.

Input : The first input line has two integers n and m: the number of cities and flight connections. The cities are numbered $1, 2, \ldots, n$ and city 1 is Syrjälä.

After that, there are m lines describing the flight connections. Each line has three integers a, b and c: a flight begins at city a, ends at city b, and its length is c. Each flight is a one-way flight.

You can assume that it is possible to travel from Syrjälä to all other cities.

Output : Print n integers: the shortest route lengths from Syrjälä to cities $1, 2, \ldots, n$.

Solution: https://cses.fi/problemset/task/1671

Hint : A very straight-forward application of Dijkstra.

Shortest Routes II

There are n cities and m roads between them. Your task is to process q queries where you have to determine the length of the shortest route between two given cities.

Input : The first input line has three integers n, m and q: the number of cities, roads, and queries.

Then, there are m lines describing the roads. Each line has three integers a, b and c: there is a road between cities a and b whose length is c. All roads are two-way roads.

Finally, there are q lines describing the queries. Each line has two integers a and b: determine the length of the shortest route between cities a and b.

Output : Print the length of the shortest route for each query. If there is no route, print -1 instead.

Solution: https://cses.fi/problemset/task/1672

Hint : Simple application of APSP algorithm on an undirected graph.

Flight Discount

Your task is to find a minimum-price flight route from Syrjälä to Metsälä. You have one discount coupon, using which you can halve the price of any single flight during the route. However, you can only use the coupon once.

Input : The first input line has two integers n and m: the number of cities and flight connections. The cities are numbered $1, 2, \ldots, n$. City 1 is Syrjälä, and city n is Metsälä.

After this there are m lines describing the flights. Each line has three integers a, b, and c: a flight begins at city a, ends at city b, and its price is c. Each flight is unidirectional.

You can assume that it is always possible to get from Syrjälä to Metsälä.

Output : Print one integer: the price of the cheapest route from Syrjälä to Metsälä.

When you use the discount coupon for a flight whose price is x, its price becomes $\lfloor x/2 \rfloor$ (it is rounded down to an integer).

Solution: https://cses.fi/problemset/task/1195

Hint : In Dijkstra's algorithm for directed graphs, modify the distance array to track whether the discount has been used or not.

Flight Routes

Your task is to find the k shortest flight routes from Syrjälä to Metsälä. A route can visit the same city several times.

Note that there can be several routes with the same price and each of them should be considered (see the example).

Input : The first input line has three integers $n, m, andk$: the number of cities, the number of flights, and the parameter k. The cities are numbered $1, 2, \ldots, n$. City 1 is Syrjälä, and city n is Metsälä.

After this, the input has m lines describing the flights. Each line has three integers a, b, and c: a flight begins at city a, ends at city b, and its price is c. All flights are one-way flights.

You may assume that there are at least k distinct routes from Syrjälä to Metsälä.

Output : Print k integers: the prices of the k cheapest routes sorted according to their prices.

Solution: https://cses.fi/problemset/task/1196

Hint : Single source k shortest path problem. Read more about the same [https://en.wikipedia.org/wiki/K_shortest_path_routing]

Path sum: four ways

In the 5 by 5 matrix below, the minimal path sum from the top left to the bottom right, by moving left, right, up, and down, is indicated in bold red and is equal to 2297.

$$\begin{pmatrix} \color{red}{131} & 673 & \color{red}{234} & \color{red}{103} & \color{red}{18} \\ \color{red}{201} & \color{red}{96} & \color{red}{342} & 965 & \color{red}{150} \\ 630 & 803 & 746 & \color{red}{422} & \color{red}{111} \\ 537 & 699 & 497 & \color{red}{121} & 956 \\ 805 & 732 & 524 & \color{red}{37} & \color{red}{331} \end{pmatrix}$$

Find the minimal path sum from the top left to the bottom right by moving left, right, up, and down in https://projecteuler.net/resources/documents/0083_matrix.txt(right click and "Save Link/Target As..."), a 31K text file containing an 80 by 80 matrix.

Solution: https://projecteuler.net/problem=83

Hint : Try to represent the matrix in the form of a directed graph(each cell points to its neighbors). Then apply either Dijkstra or A* algorithm. Read more about A* algorithm here : [https://www.geeksforgeeks.org/a-search-algorithm/]

Risky Moon

A moon could be described by the sphere $C(r)$ with centre $(0,0,0)$ and radius r.

There are stations on the moon at the points on the surface of $C(r)$ with integer coordinates. The station at $(0,0,r)$ is called North Pole station, the station at $(0,0,-r)$ is called South Pole station.

All stations are connected with each other via the shortest road on the great arc through the stations. A journey between two stations is risky. If d is the length of the road between two stations, $\left(\frac{d}{\pi r}\right)^2$ is a measure for the risk of the journey (let us call it the risk of the road). If the journey includes more than two stations, the risk of the journey is the sum of risks of the used roads.

A direct journey from the North Pole station to the South Pole station has the length πr and risk 1. The journey from the North Pole station to the South Pole station via $(0,r,0)$ has the same length, but a smaller risk:

$$\left(\frac{\frac{1}{2}\pi r}{\pi r}\right)^2 + \left(\frac{\frac{1}{2}\pi r}{\pi r}\right)^2 = 0.5$$

The minimal risk of a journey from the North Pole station to the South Pole station on $C(r)$ is $M(r)$.

You are given that $M(7) = 0.1784943998$ rounded to 10 digits behind the decimal point.

Find $\sum_{n=1}^{15} M(2^n - 1)$.

Give your answer rounded to 10 digits behind the decimal point in the form a.bcdefghijk.

Solution: https://projecteuler.net/problem=353

Hint : Bonus question. Attempt this question only if you have nothing else to do in life.