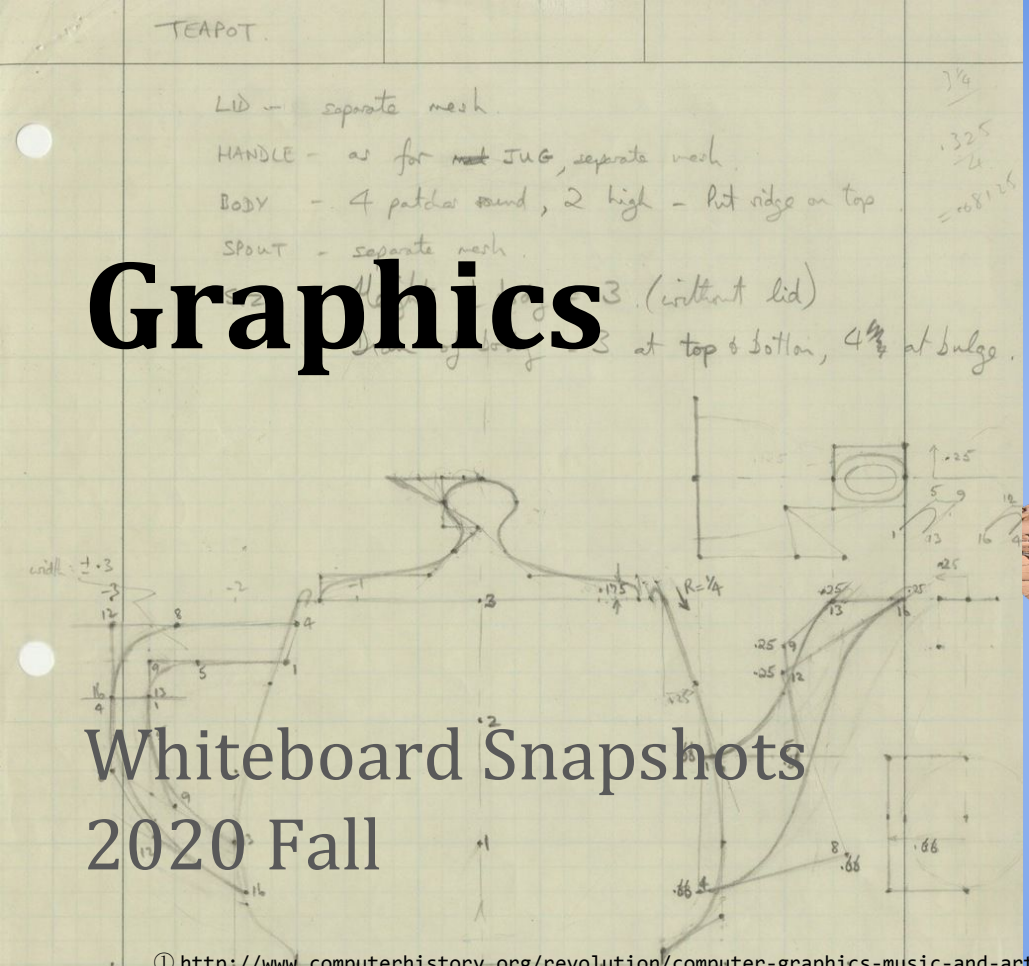
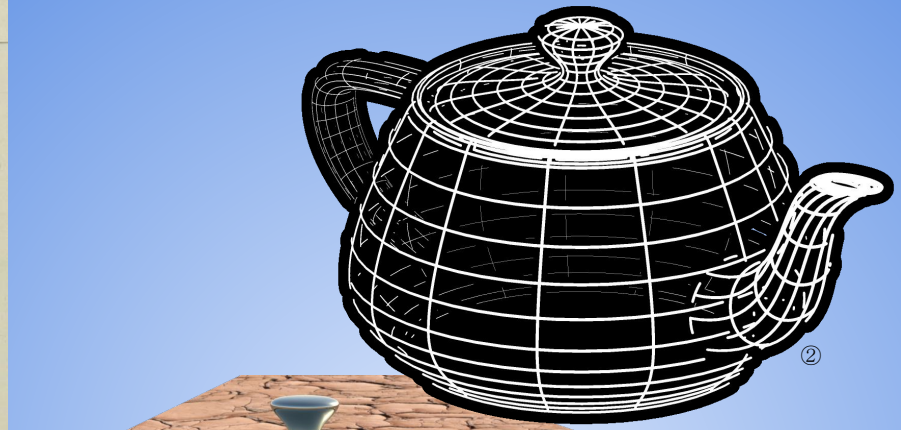
Graphics

Whiteboard Snapshots
2020 Fall




②


③


④

① http://www.computerhistory.org/revolution/computer-graphics-music-and-art/15/206/556
② http://www.cs.technion.ac.il/~gershon/site/img/gallery/gallery-pic-cat3-depth-cueing-2-big.jpg
③ http://www.omnigraphica.com/gallery/maingallery/original/Utah_teapot_1.png
④ http://unfold.be/assets/images/000/113/719/large-utanalog3.jpg

– NOTE – NOTE – NOTE – NOTE – NOTE – NOTE –

These pages are just snapshots of ***some*** of the stuff that's scribbled on the "whiteboard" (i.e., those blank pages in the Canvas conference). The ***intent*** is to make copies of whatever goes by.

However …

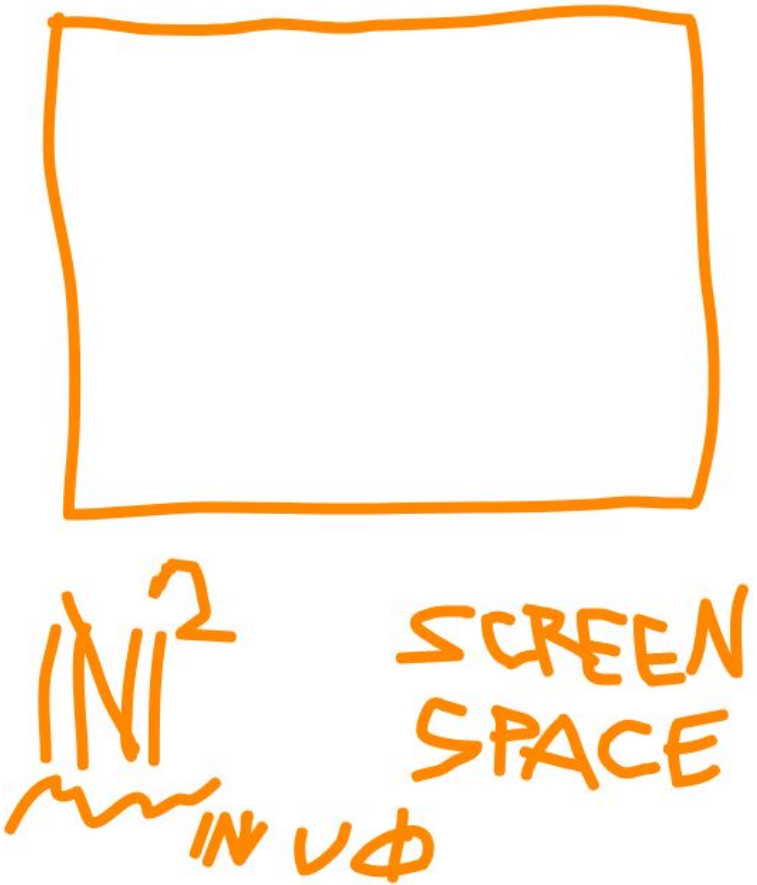
① There's ***no*** guarantee that there's a copy of everything. Sometimes the discussion is so fast that some pages will slip by unsnapped. ***Take your own notes in class***; let me know if something is missing or should be added.

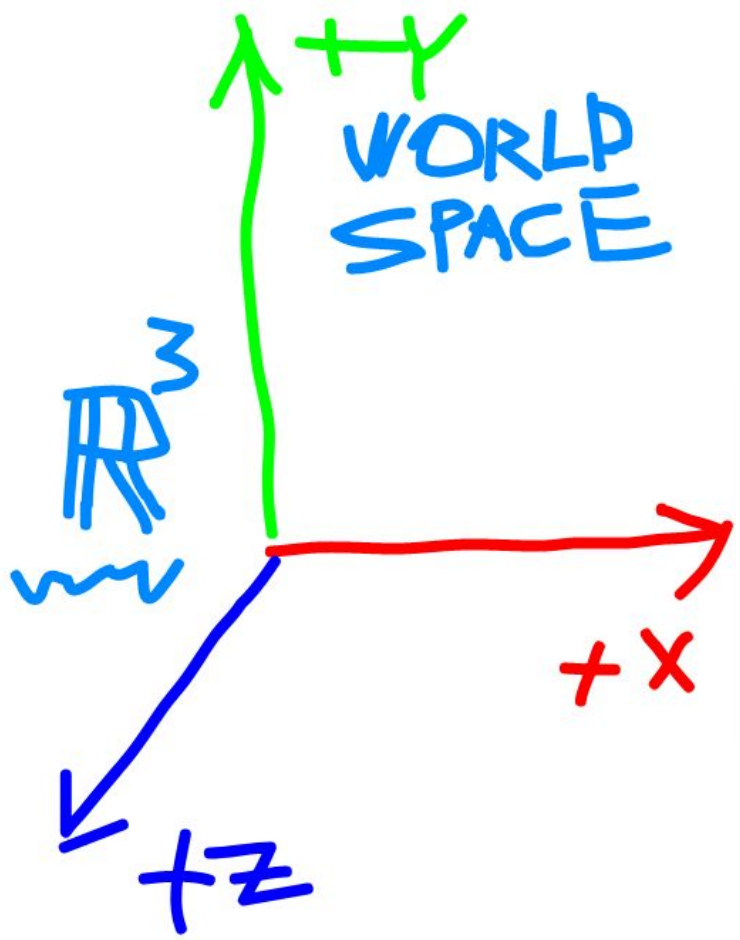
② I sometimes clean up the snapshots (that's why you'll see typeset pages intermixed with scribbles), there's no guarantee that any particular page will get typeset.

③ There are ***certainly*** tpyos (and even outright *mystekas* — *gasp!*) here. I fix those as I see them, but errors no doubt exist.

Be mindful of all of this as you review these pages. Tell me if anything is wrong. Thanks!

2D, Discrete, Finite





3D, Continuous, Infinite

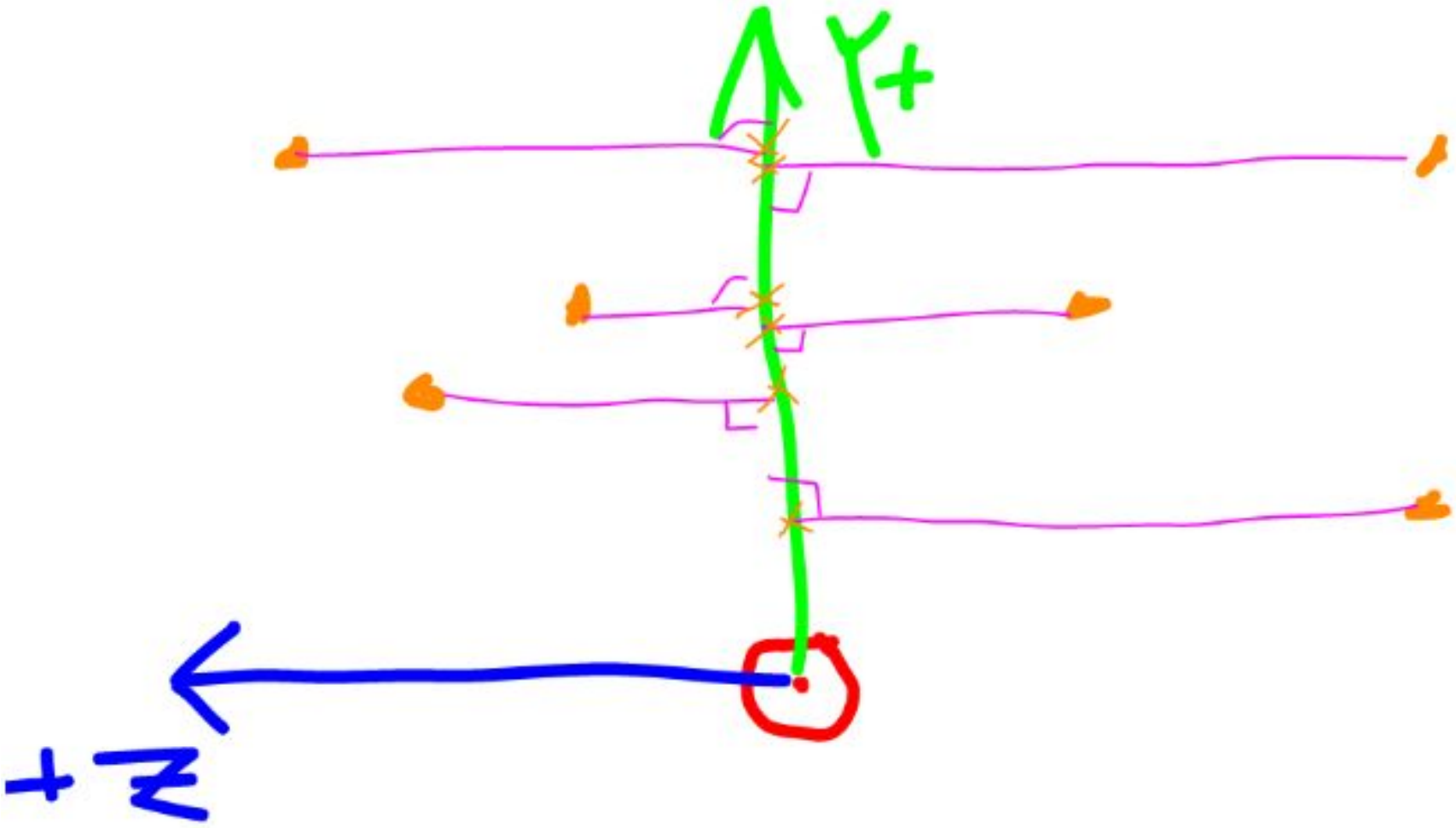
Projection

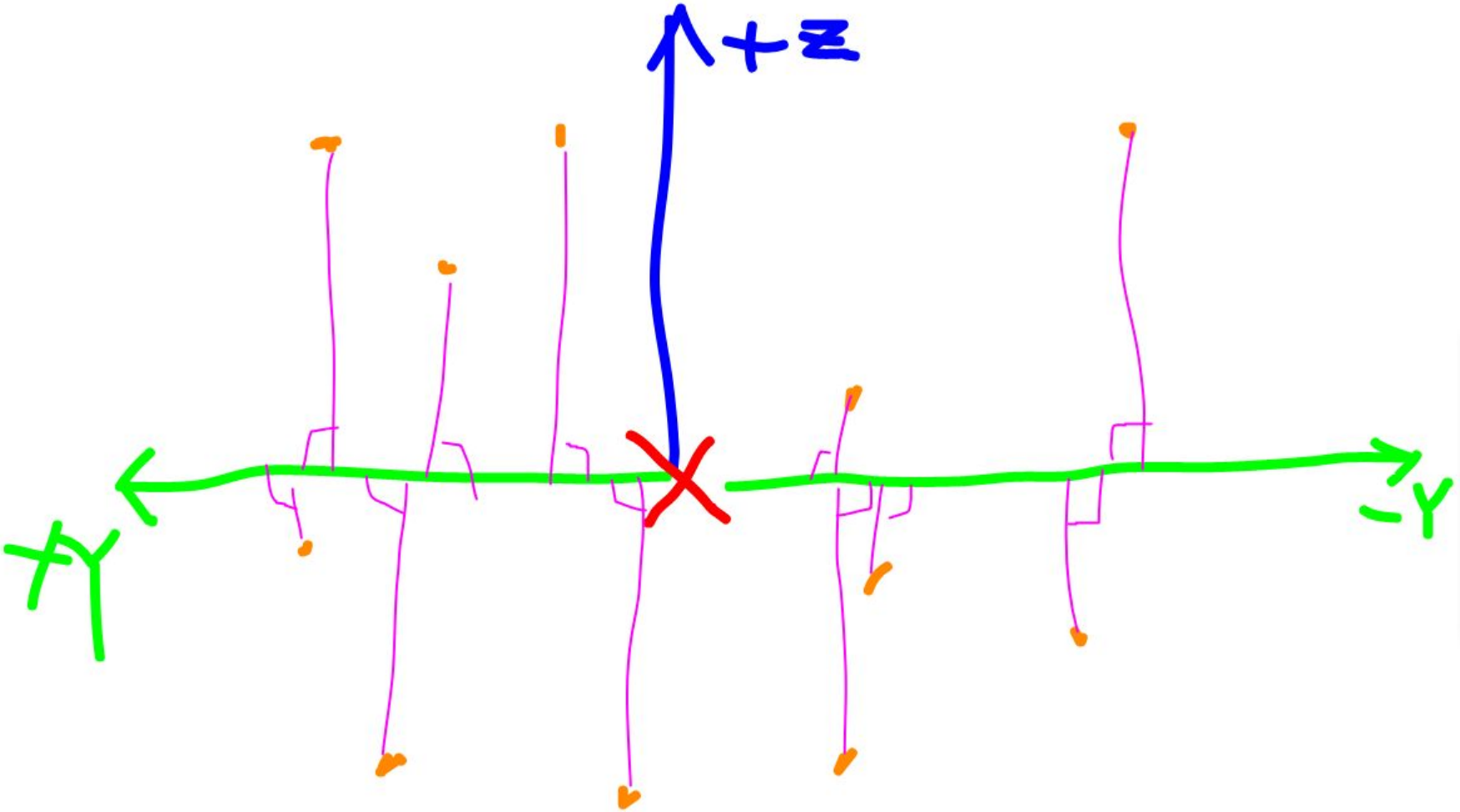
3D → 2D
WS SS

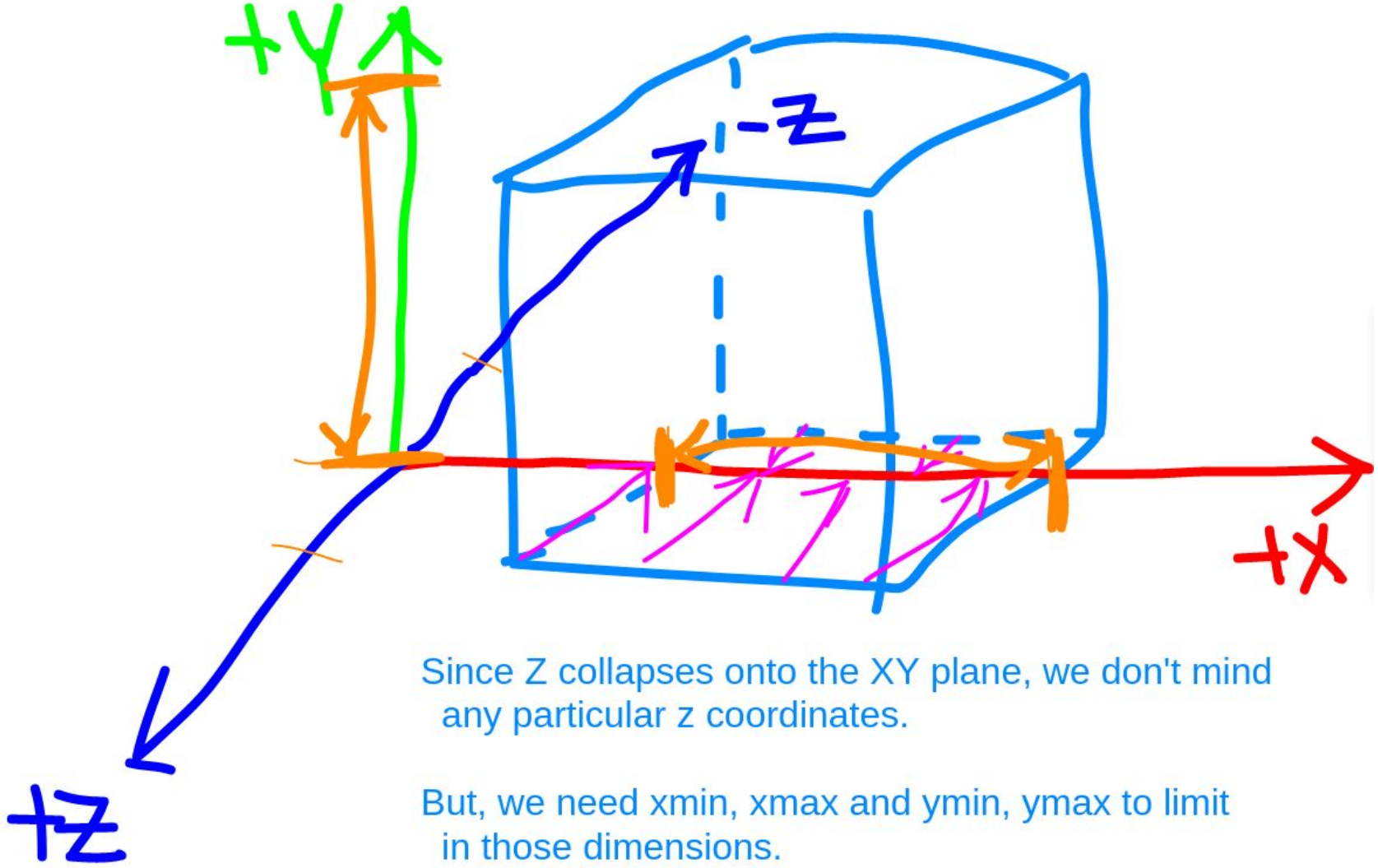
Parallel Projection (aka Orthogonal Projection)

One kind of Parallel Projection is simply to "project" the vertices onto the X-Y plane.

In other words, ignore the Z coordinate (or equivalently just set Z to 0).


Y+
+ N

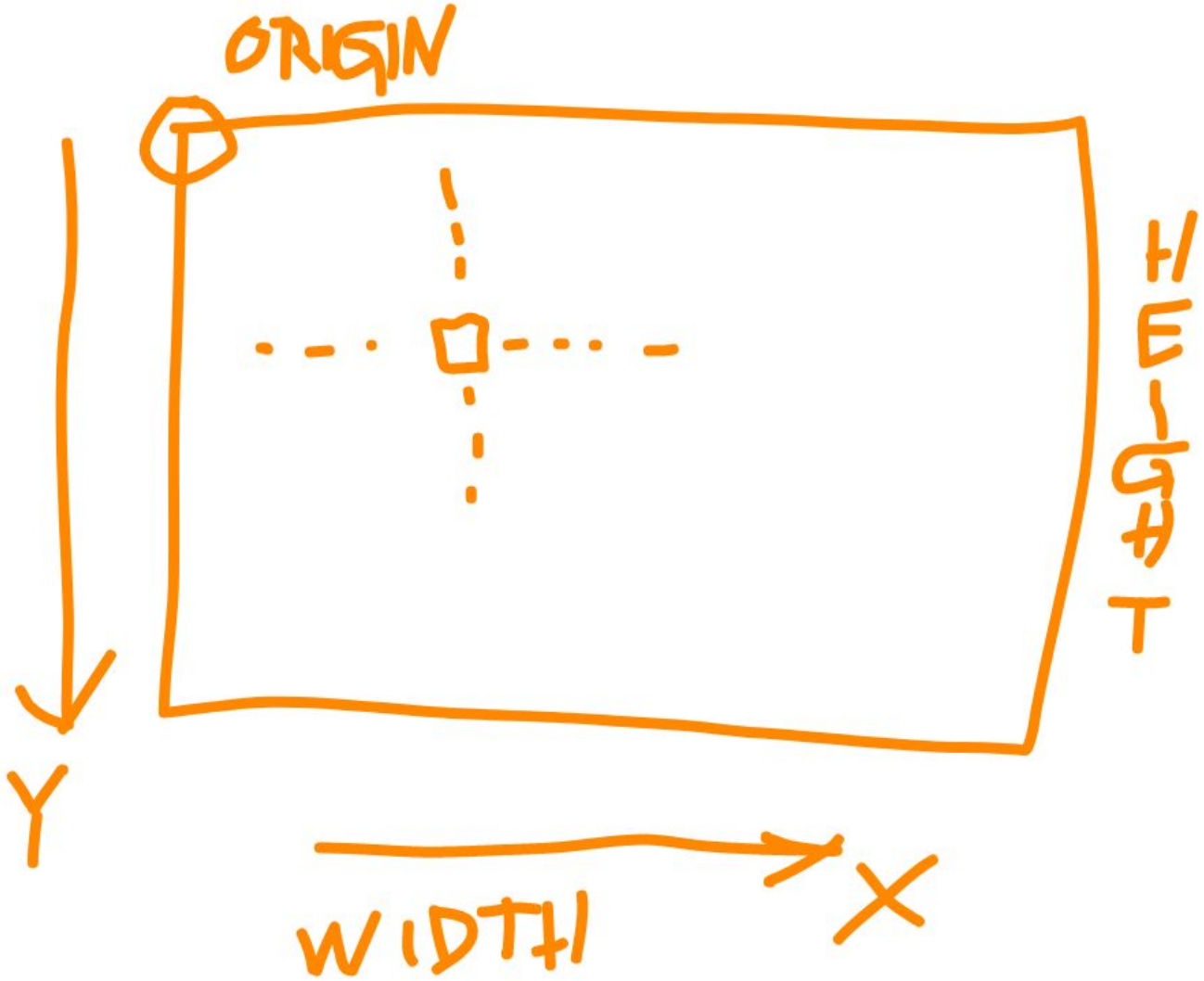

+Z
+Y
-Y

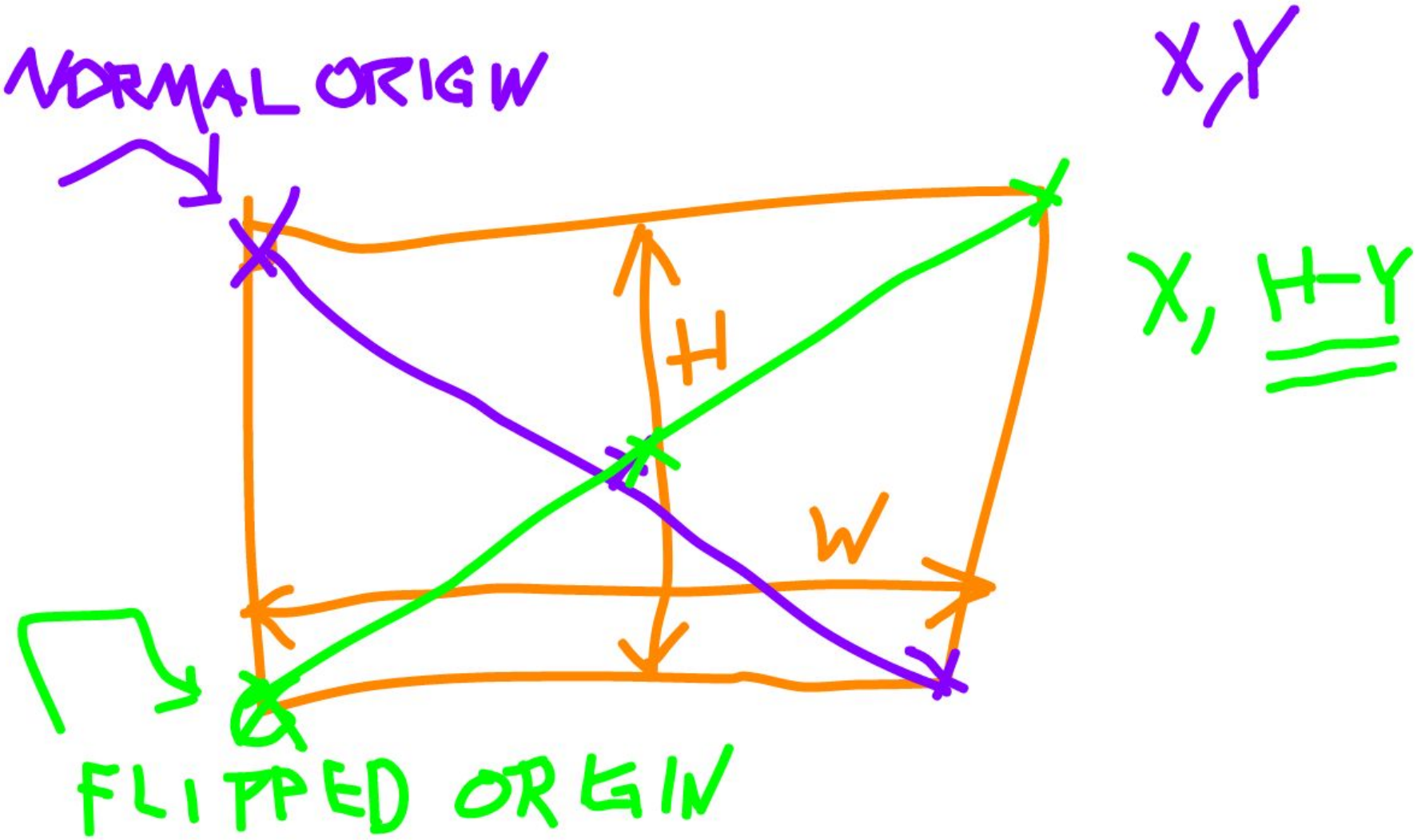


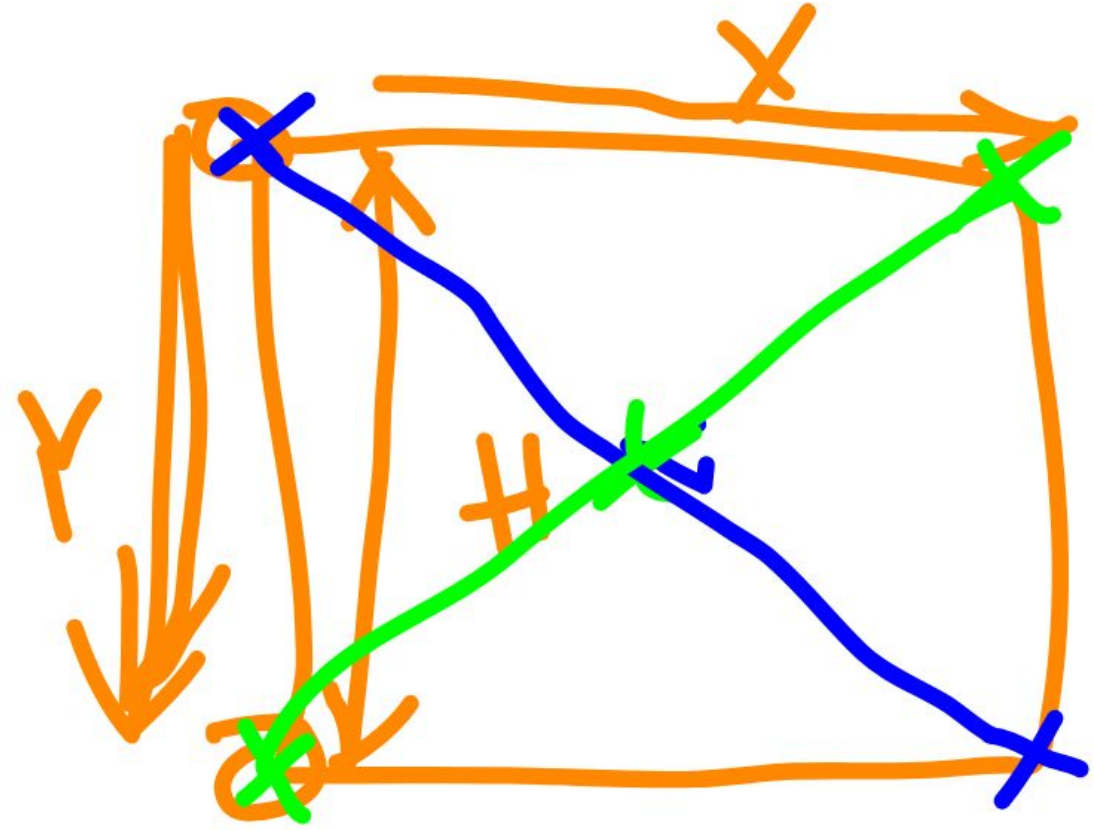
Since Z collapses onto the XY plane, we don't mind any particular z coordinates.

But, we need xmin, xmax and ymin, ymax to limit in those dimensions.

Screen Space

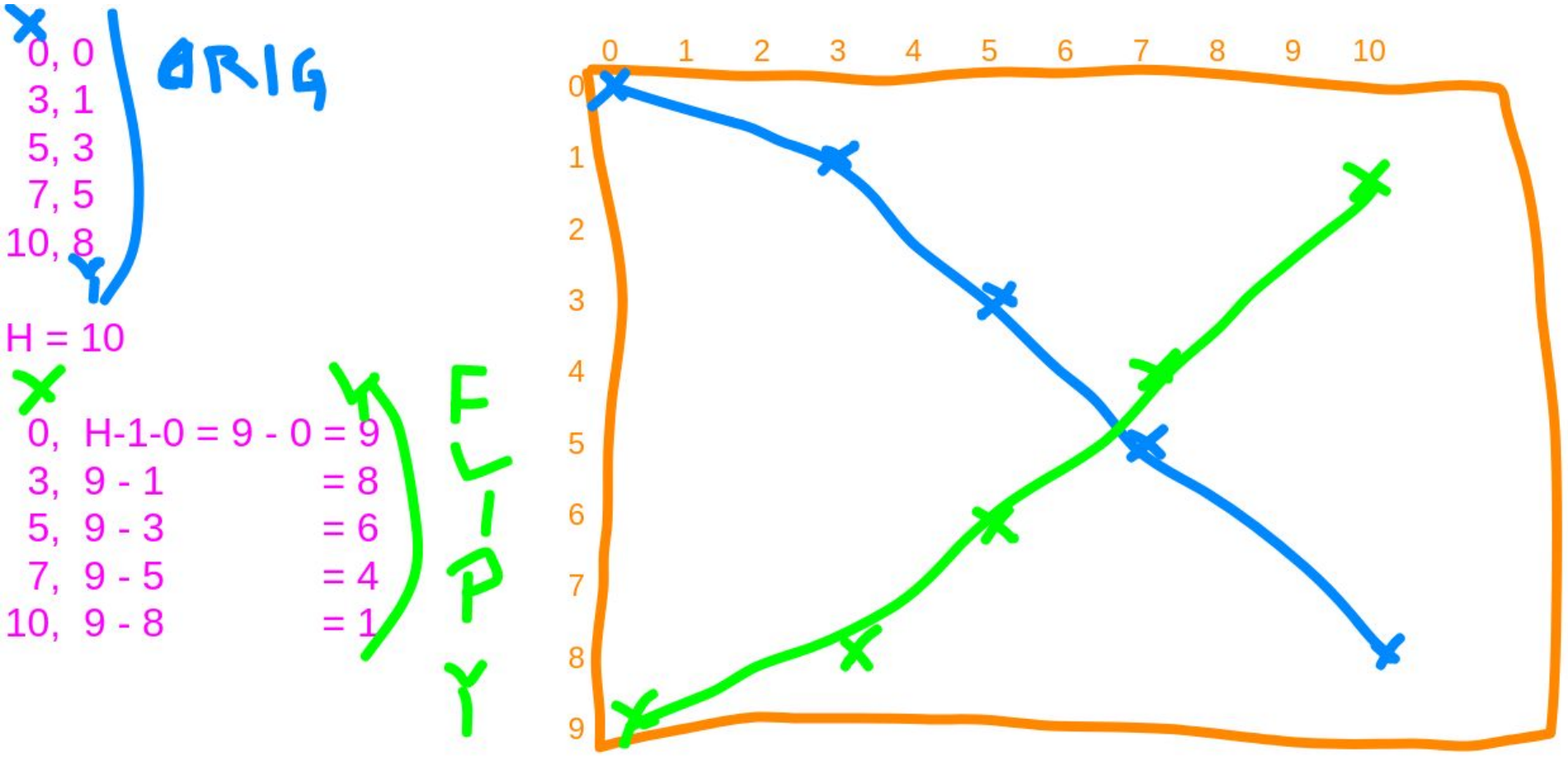


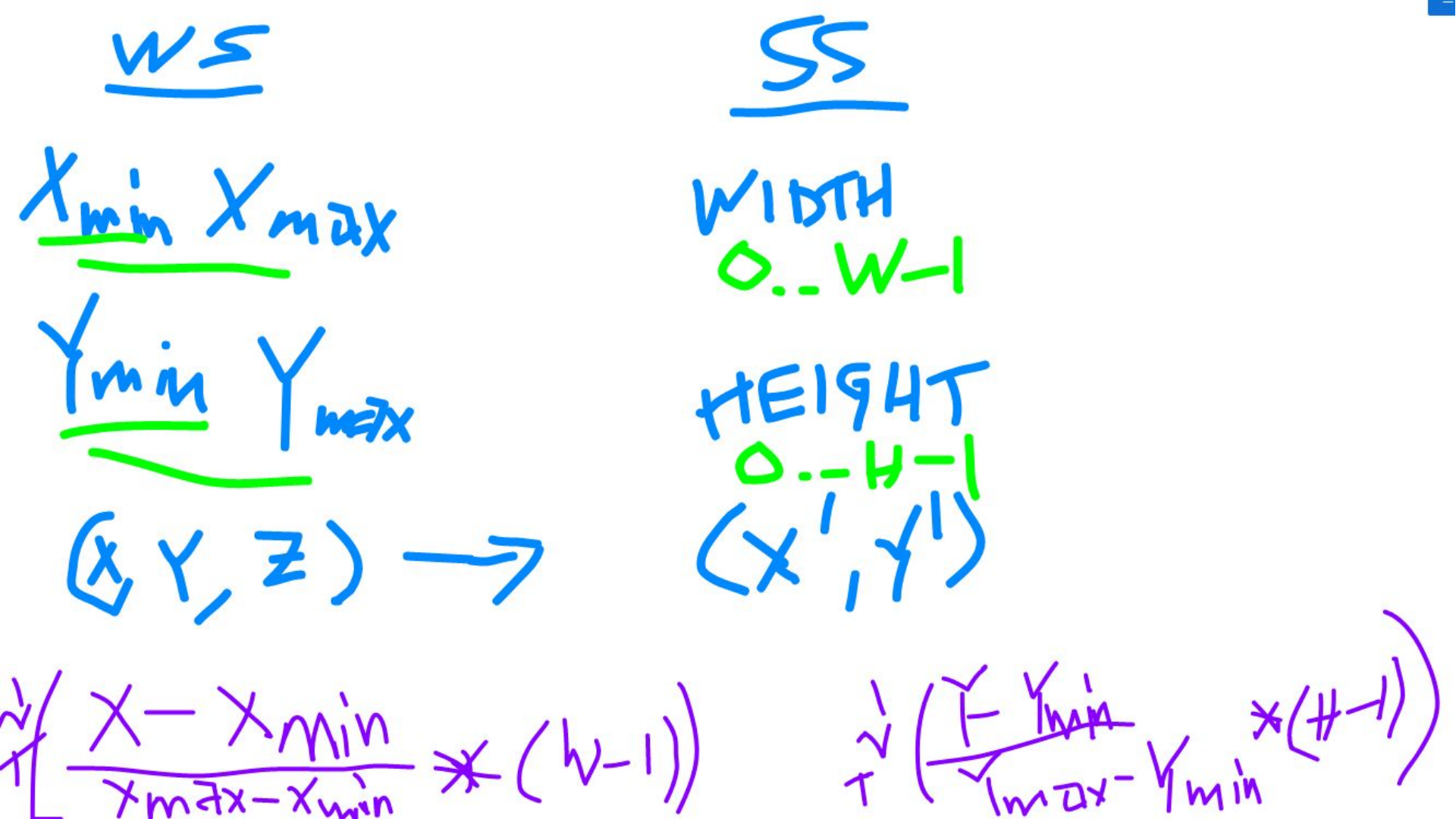

NORMAL ORIGIN
X, Y
X, H-Y
H
W
FLIPPED ORIGIN



X, Y

X, H-Y


X
0, 0
3, 1
5, 3
7, 5
10, 8
Y
ORIG
H = 10
X
0, H-1-0 = 9 - 0 = 9
3, 9 - 1 = 8
5, 9 - 3 = 6
7, 9 - 5 = 4
10, 9 - 8 = 1
Y
FLIP Y
0 1 2 3 4 5 6 7 8 9 10
0 1 2 3 4 5 6 7 8 9


WS
SS
X_{min} X_{max}
Y_{min} Y_{max}
(X, Y, Z) →
WIDTH
0..W-1
HEIGHT
0..H-1
(X', Y')
$\frac{X - X_{min}}{X_{max} - X_{min}} * (W-1)$
$\frac{Y - Y_{min}}{Y_{max} - Y_{min}} * (H-1)$

$$\frac{1}{2}\left[\frac{x-x_{min}}{x_{max}-x_{min}}\cdot(w-1)\right]$$

$$\left[x-x_{min}\right]\cdot\frac{w-1}{x_{max}-x_{min}}+1$$

xlate

scaling

$$t_x = -x_{min}$$

$$t_y = -y_{min}$$

$$s_x = \frac{w-1}{x_{max} - x_{min}}$$

$$s_y = \frac{H-1}{y_{max} - y_{min}}$$

$$P_s = STP_w$$

(See notes on next slide.)

$$\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & s_x t_x \\ 0 & s_y & s_y t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ 1 \end{bmatrix}$$

$$t_x = -xmin$$

$$t_y = -ymin$$

$$s_x = \frac{w - 1}{xmax - xmin}$$

$$s_y = \frac{h - 1}{ymax - ymin}$$

When forming the matrices to do the Parallel case from world space to screen space, use the 4x4 matrices shown on a later slide.

$$\begin{bmatrix} s_x & 0 & s_x t_x \\ 0 & s_y & s_y t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ 1 \end{bmatrix} = \begin{bmatrix} s_x x_w + s_x t_x \\ s_y y_w + s_y t_y \\ 1 \end{bmatrix} = P_s$$

$$x_s = s_x x_w + s_x t_x$$

$$y_s = s_y y_w + s_y t_y$$

constants

so 2 mults
2 adds
per point

Each FRAME is an instant of time -- *during* a frame, nothing moves. By "nothing", we mean NOTHING. The camera, Point of View, the Field of View, the orientation and position of all objects, the monitor width and height, etc., etc.

NOTHING changes *during* a frame. In-between frames, ANYTHING can change: the POV, the FOV, the window (monitor) width and height, the positions and orientations of all objects, etc., etc.

Because of this, while computing a frame, xmin, xmax, ymin, ymax, width, height, positions of points, orientations of models, etc., etc., are all FIXED and can be considered CONSTANTS.

Thus, when we are transforming from World Space to Screen Space, we can construct a single transformation matrix using x, y min/max and width and height and use it everywhere.

Perspective

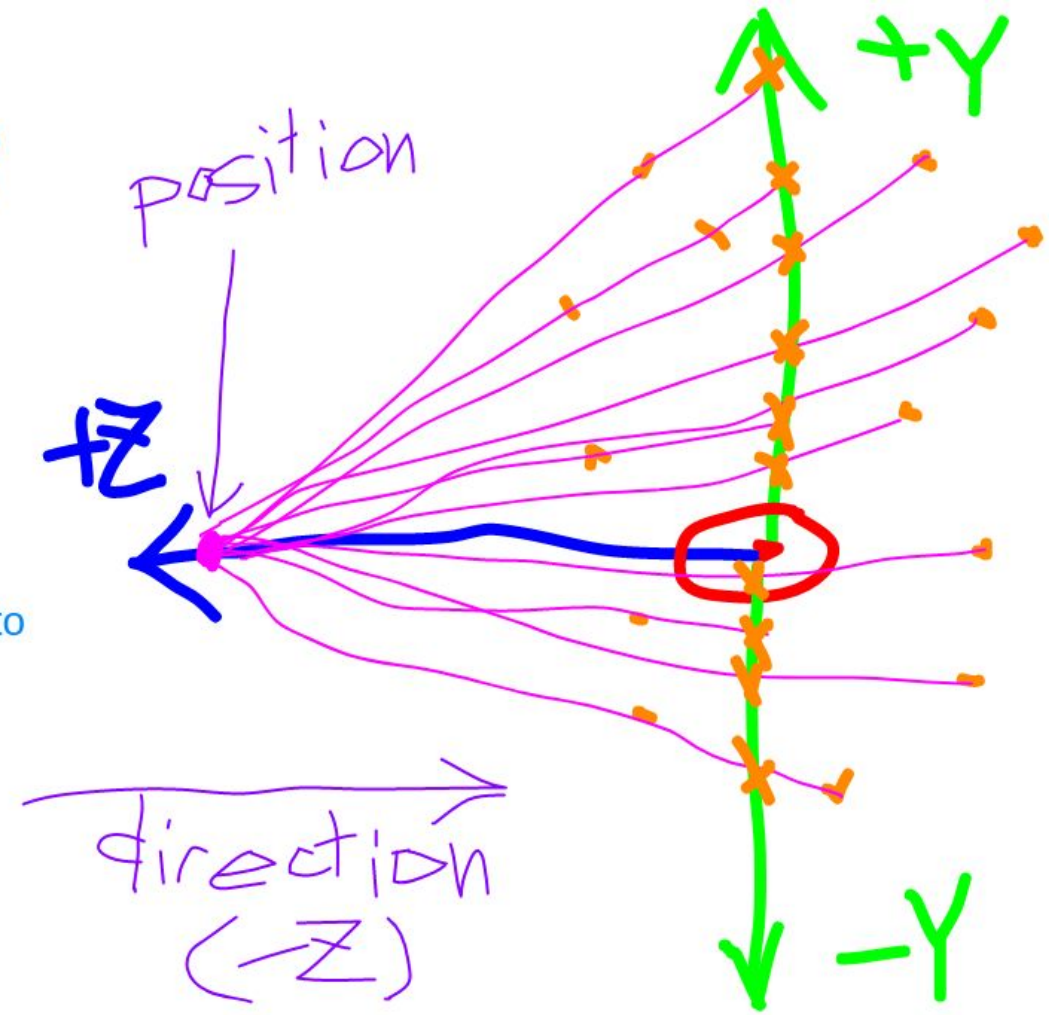
PERSPECTIVE projection uses a
- view position
- view direction
- view orientation

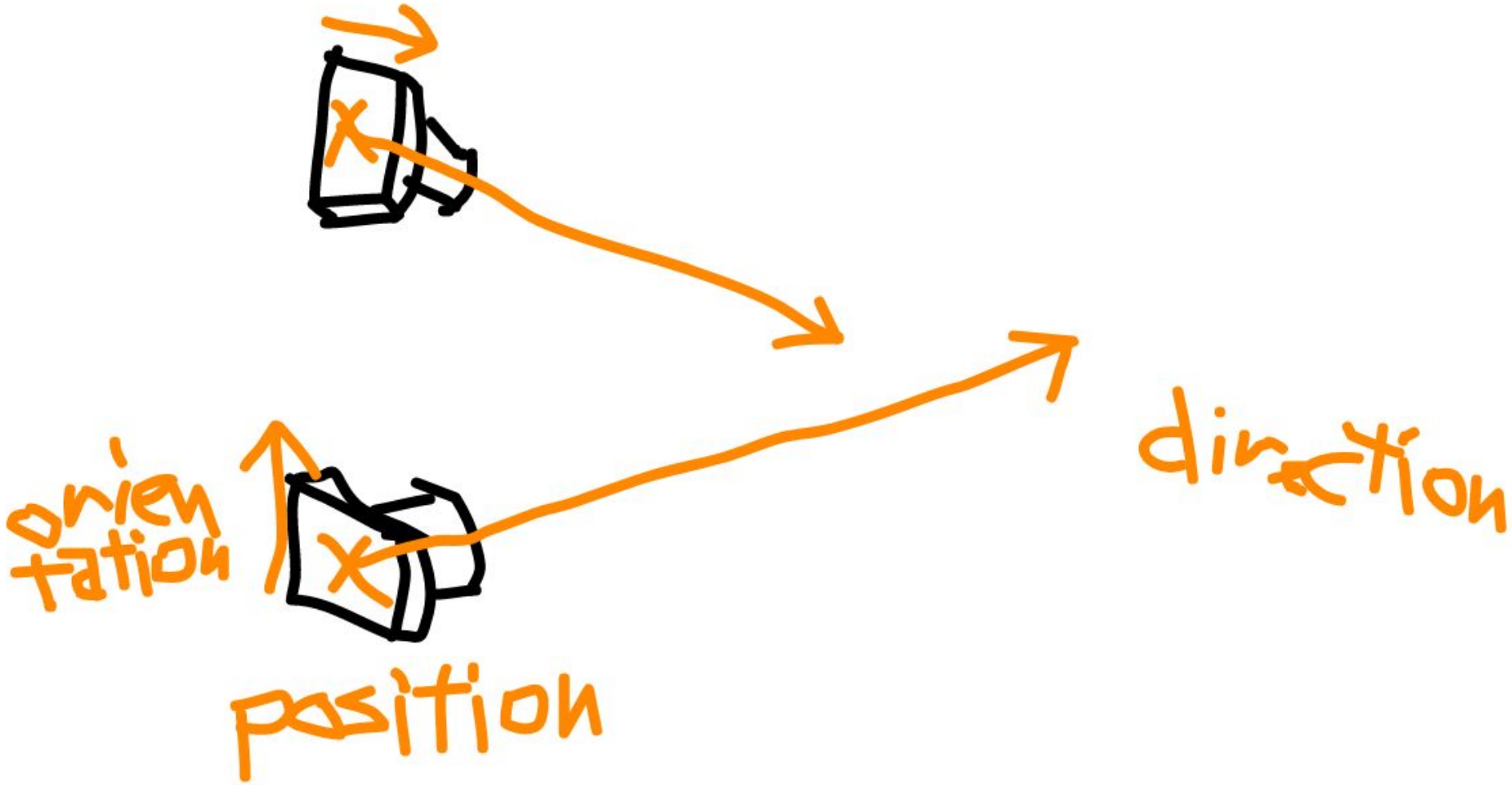
Like the PARALLEL projection, the world space points are projected onto the XY plane.

view position is ALWAYS (0,0,d), where d > 0.

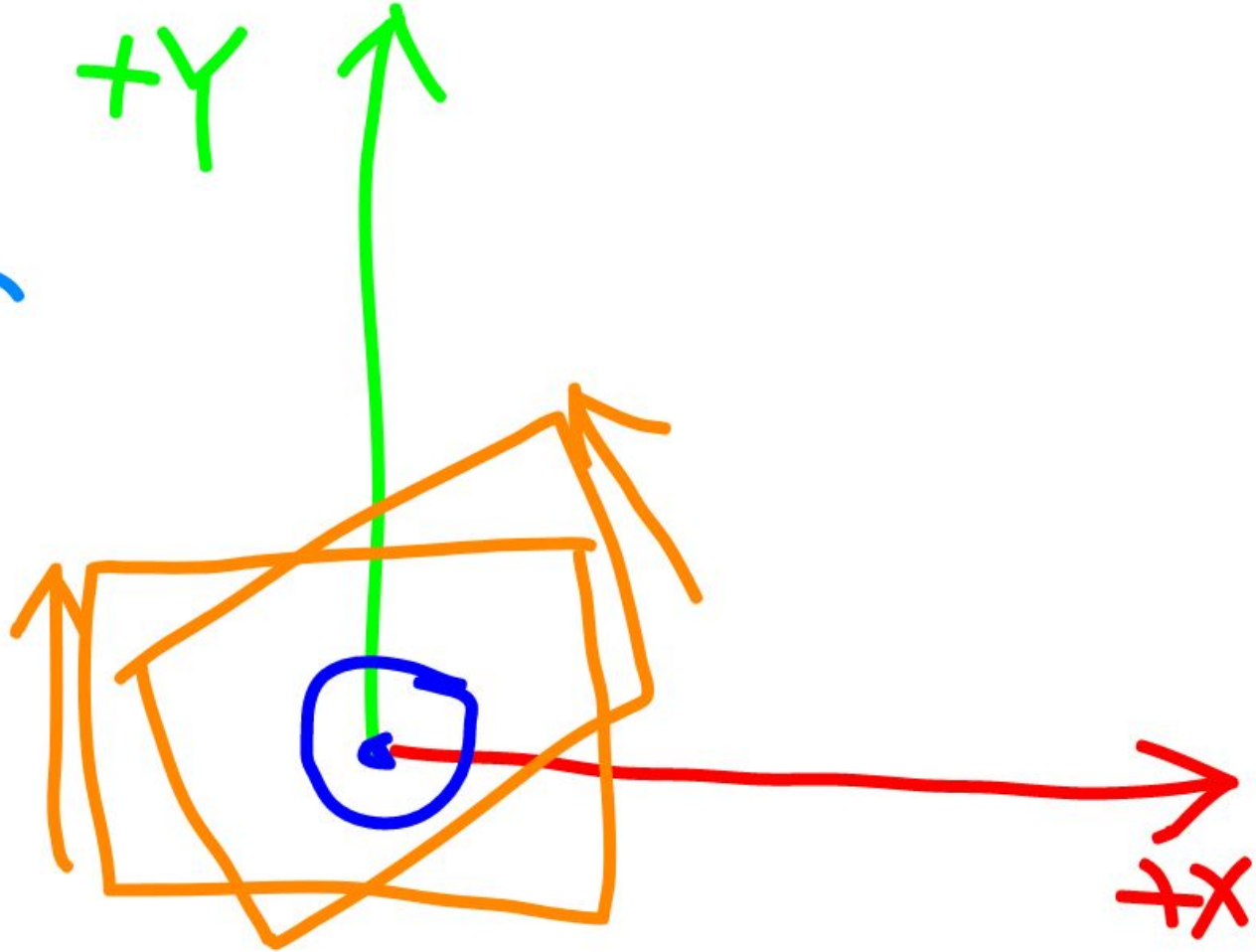
view direction is ALWAYS -Z.

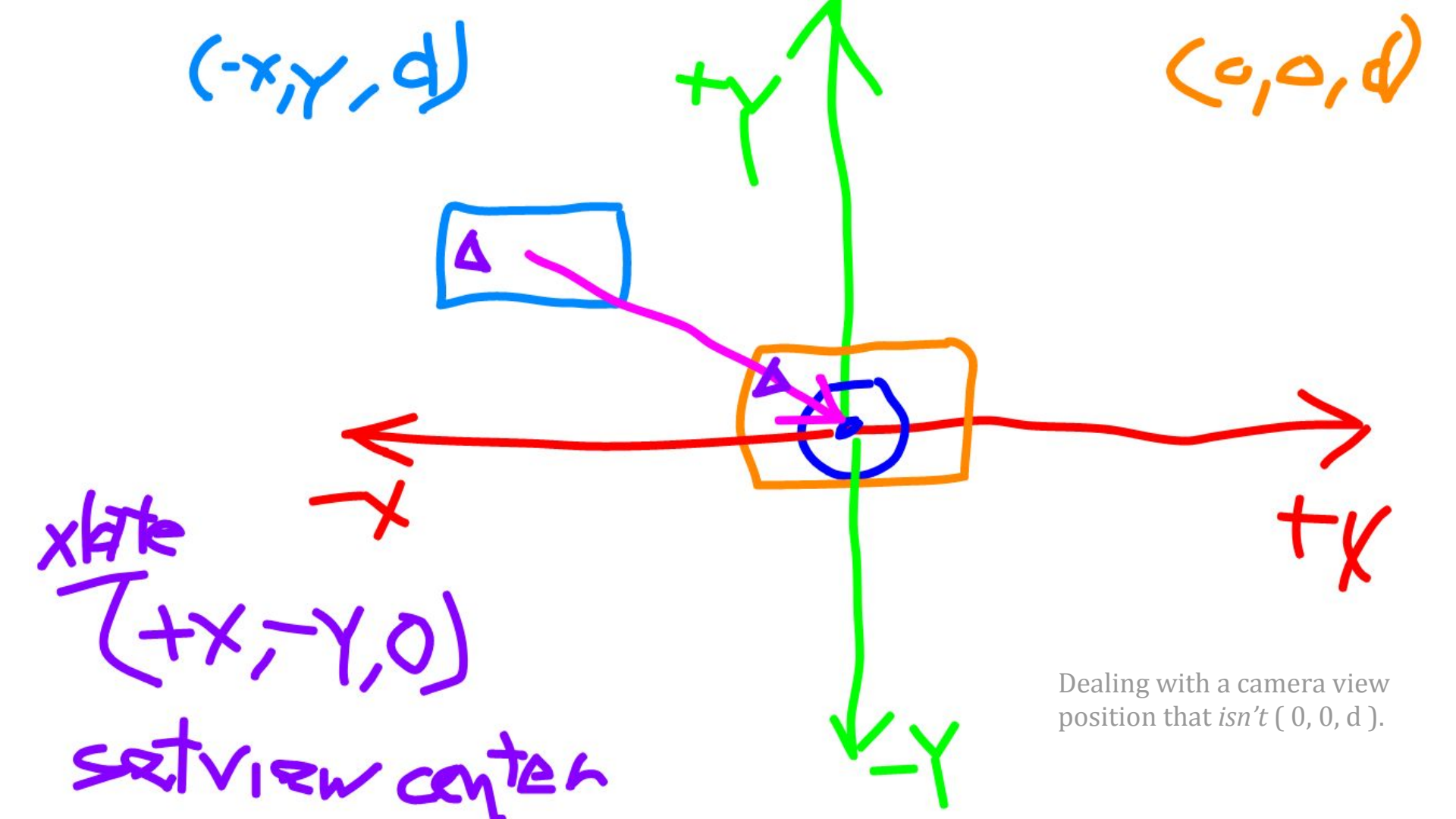
view orientation is ALWAYS Y-UP.



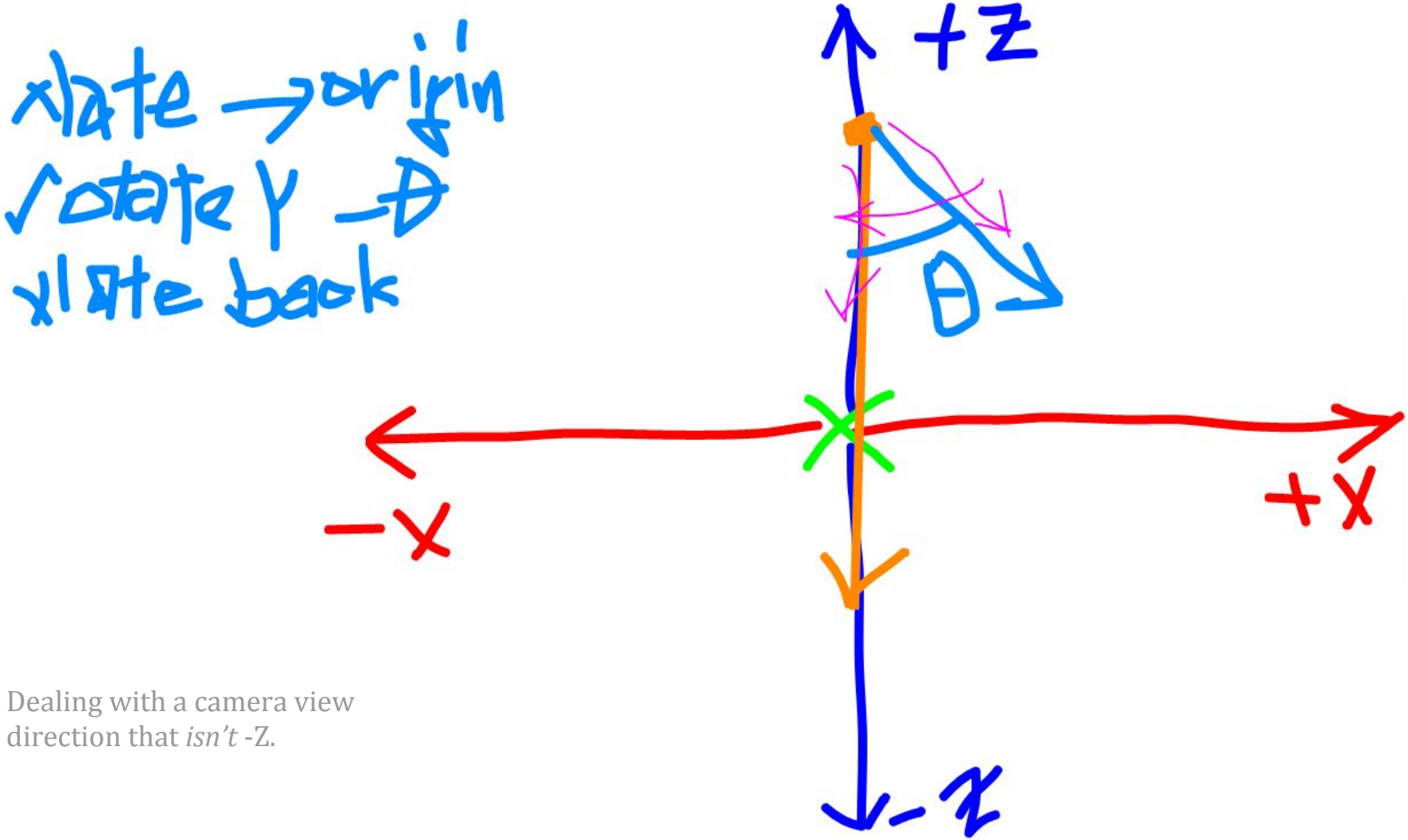

orien
tation
direction
position

VRw orientation

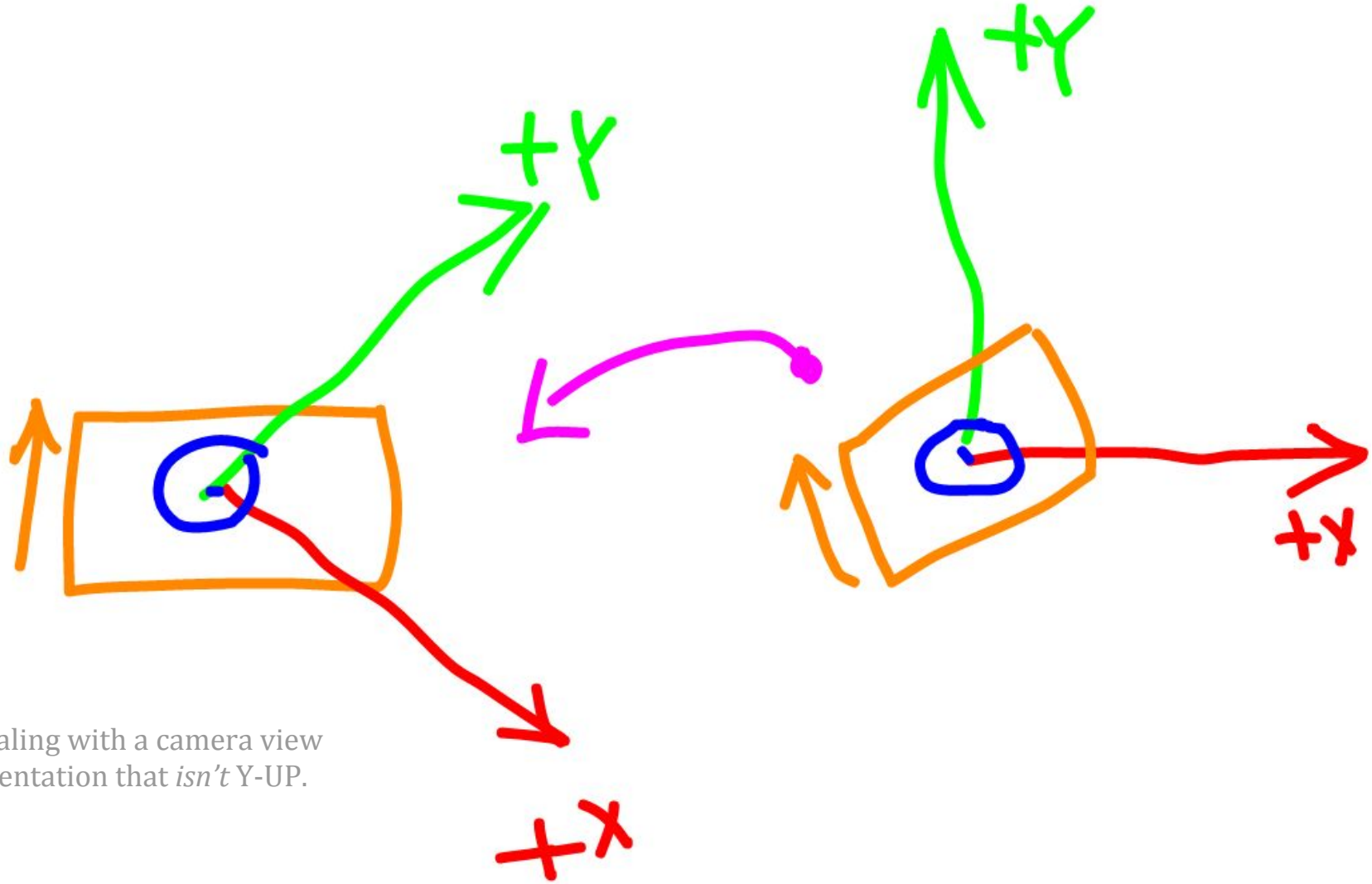




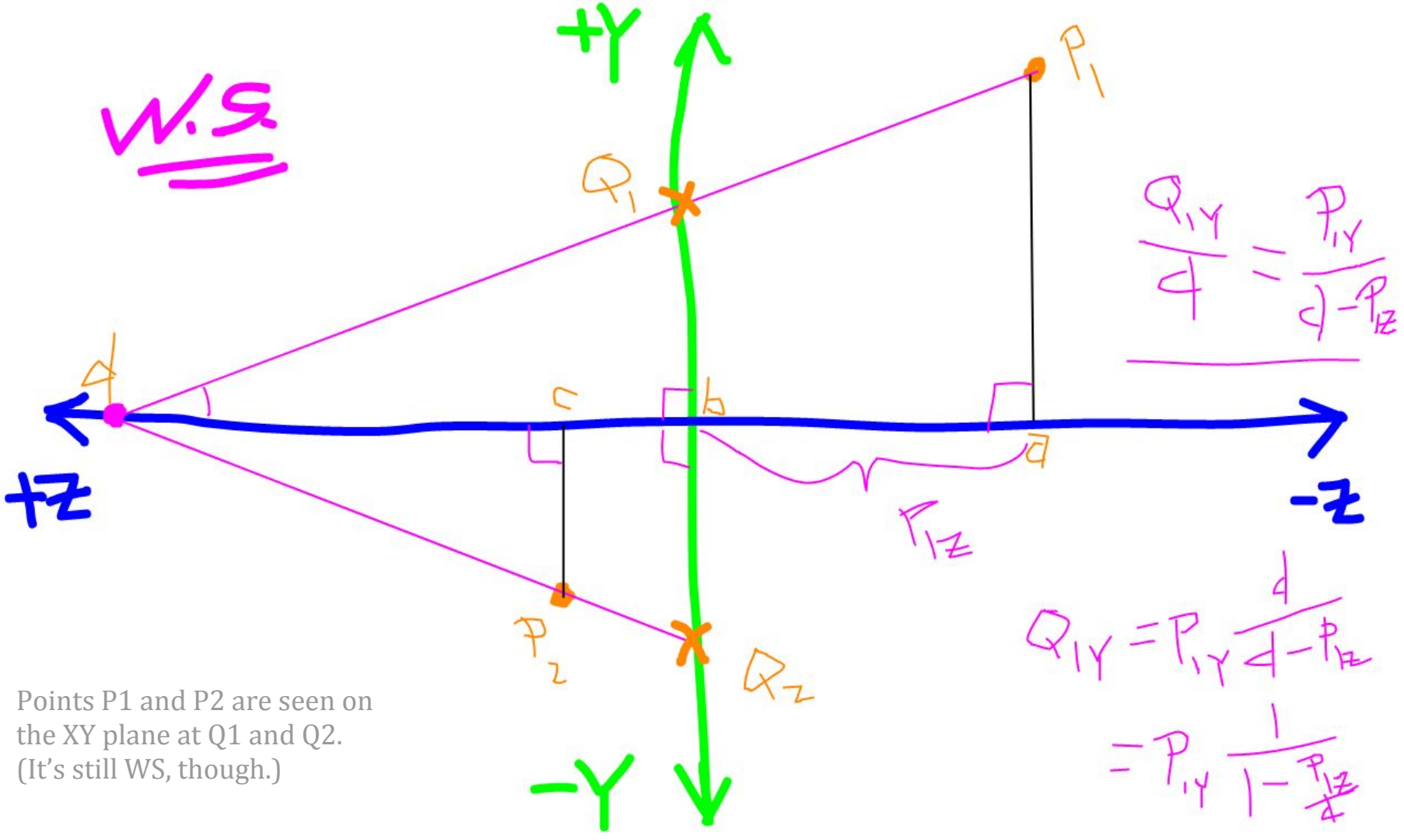
Dealing with a camera view position that *isn't* (0, 0, d).



Dealing with a camera view direction that *isn't* -Z.



Dealing with a camera view orientation that *isn't* Y-UP.

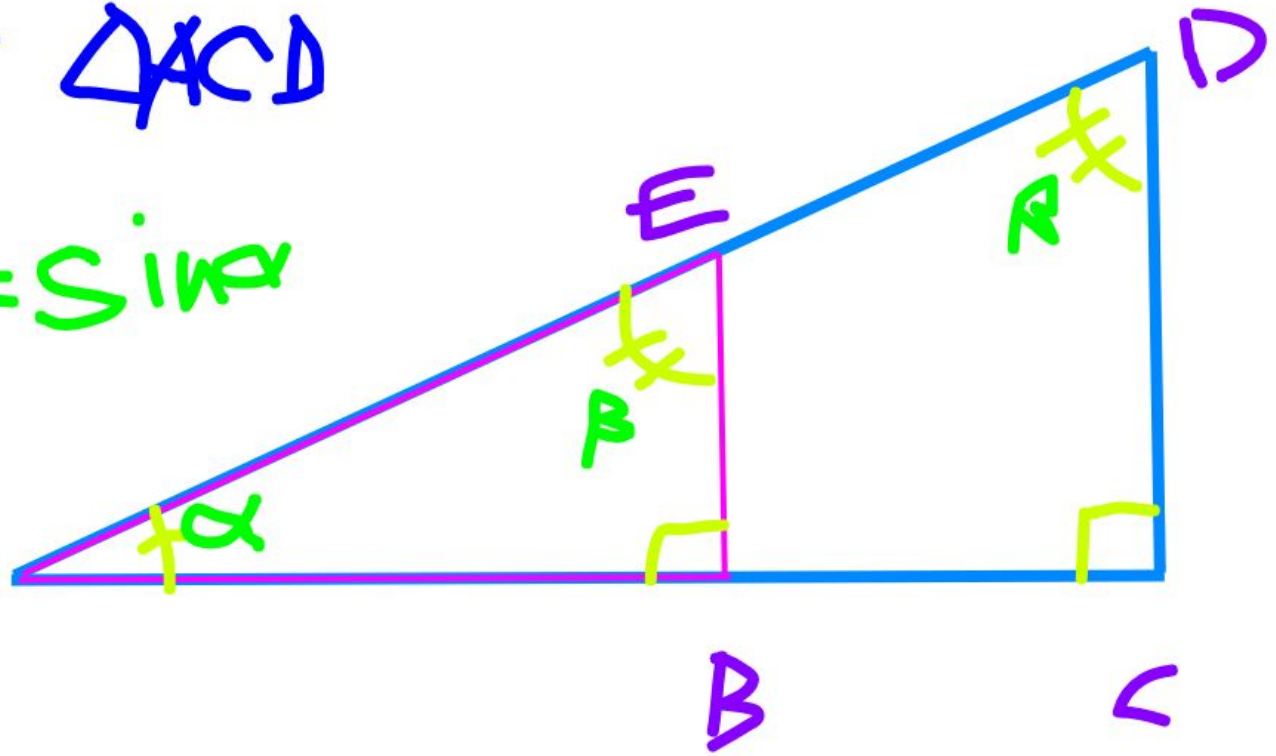


Points P1 and P2 are seen on the XY plane at Q1 and Q2. (It's still WS, though.)

$\Delta ABE \sim \Delta ACD$

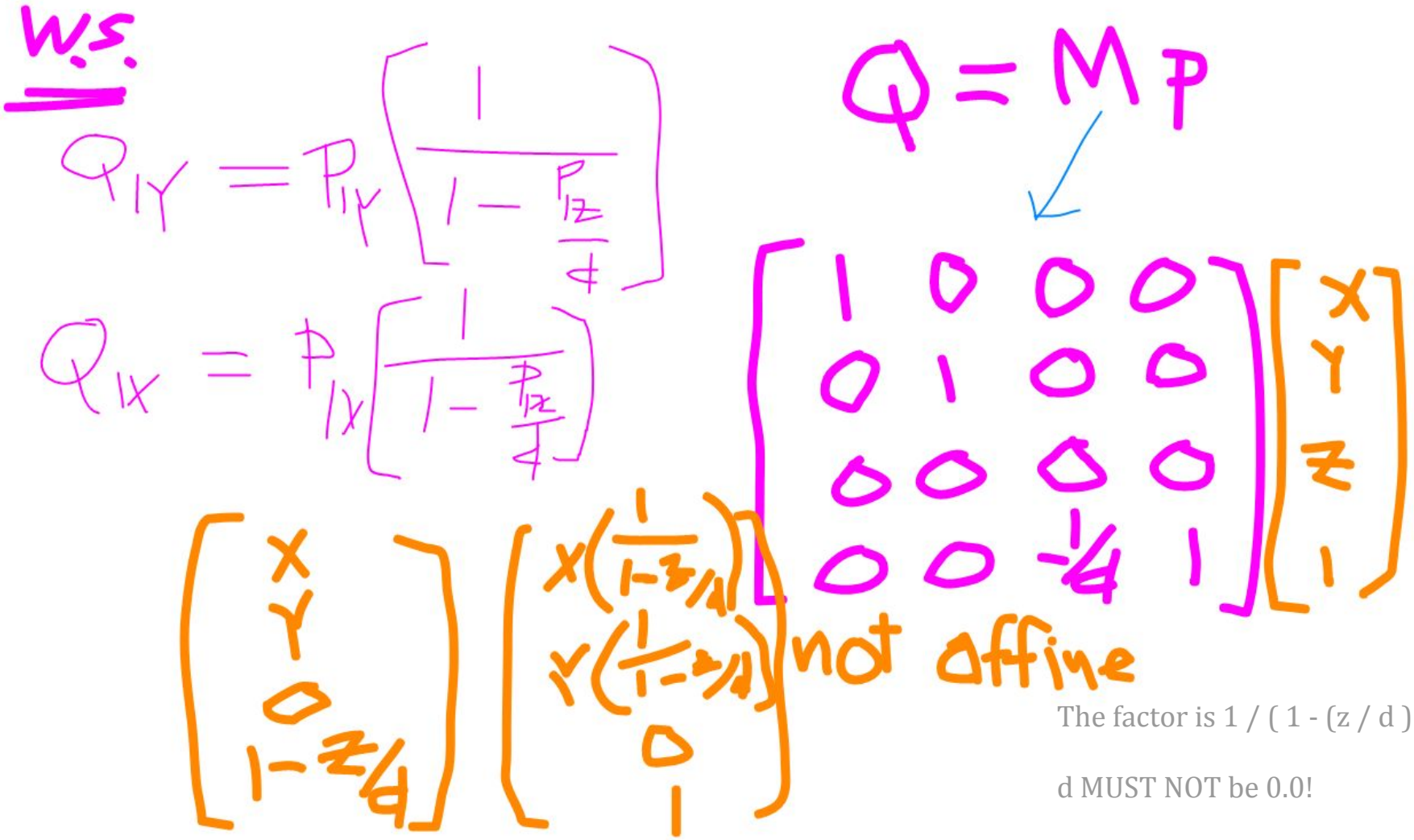
$\frac{EB}{AE} = \sin\alpha$

$\tan\alpha = \frac{ED}{AB} = \frac{DC}{AC}$



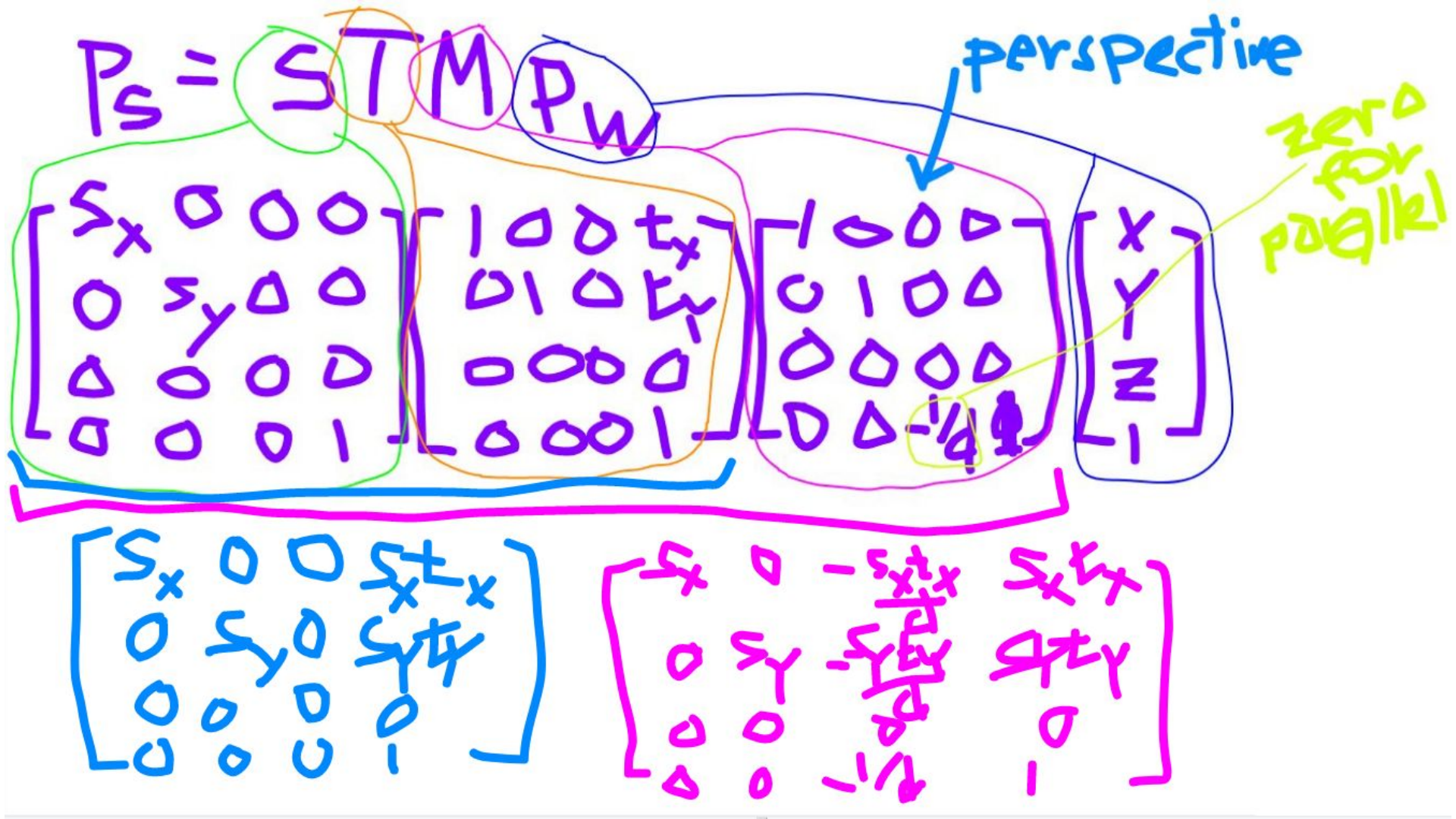
$\sin\alpha = \frac{DC}{AD} = \frac{EB}{AE}$

$\cos\alpha = \frac{AB}{AE} = \frac{AC}{AD}$



The factor is 1 / (1 - (z / d)).

d MUST NOT be 0.0!


$P_s = S\,T\,M\,P_w$
perspective
zero for parallel
$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
$\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/4 & 1 \end{bmatrix}$
$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$
$\begin{bmatrix} s_x & 0 & 0 & s_x t_x \\ 0 & s_y & 0 & s_y t_y \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
$\begin{bmatrix} s_x & 0 & -s_x t_x & s_x t_x \\ 0 & s_y & -s_y t_y & s_y t_y \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/4 & 1 \end{bmatrix}$

#- S ----------

$$\begin{bmatrix} sx & 0 & 0 & 0 \\ 0 & sy & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#- T ----------

$$\begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#-- S·T ----------

$$\begin{bmatrix} sx & 0 & 0 & sx \cdot tx \\ 0 & sy & 0 & sy \cdot ty \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#- M ----------

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-1}{d} & 1 \end{bmatrix}$$

#- S·T·M ----------

$$\begin{bmatrix} sx & 0 & \frac{-sx \cdot tx}{d} & sx \cdot tx \\ 0 & sy & \frac{-sy \cdot ty}{d} & sy \cdot ty \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-1}{d} & 1 \end{bmatrix}$$

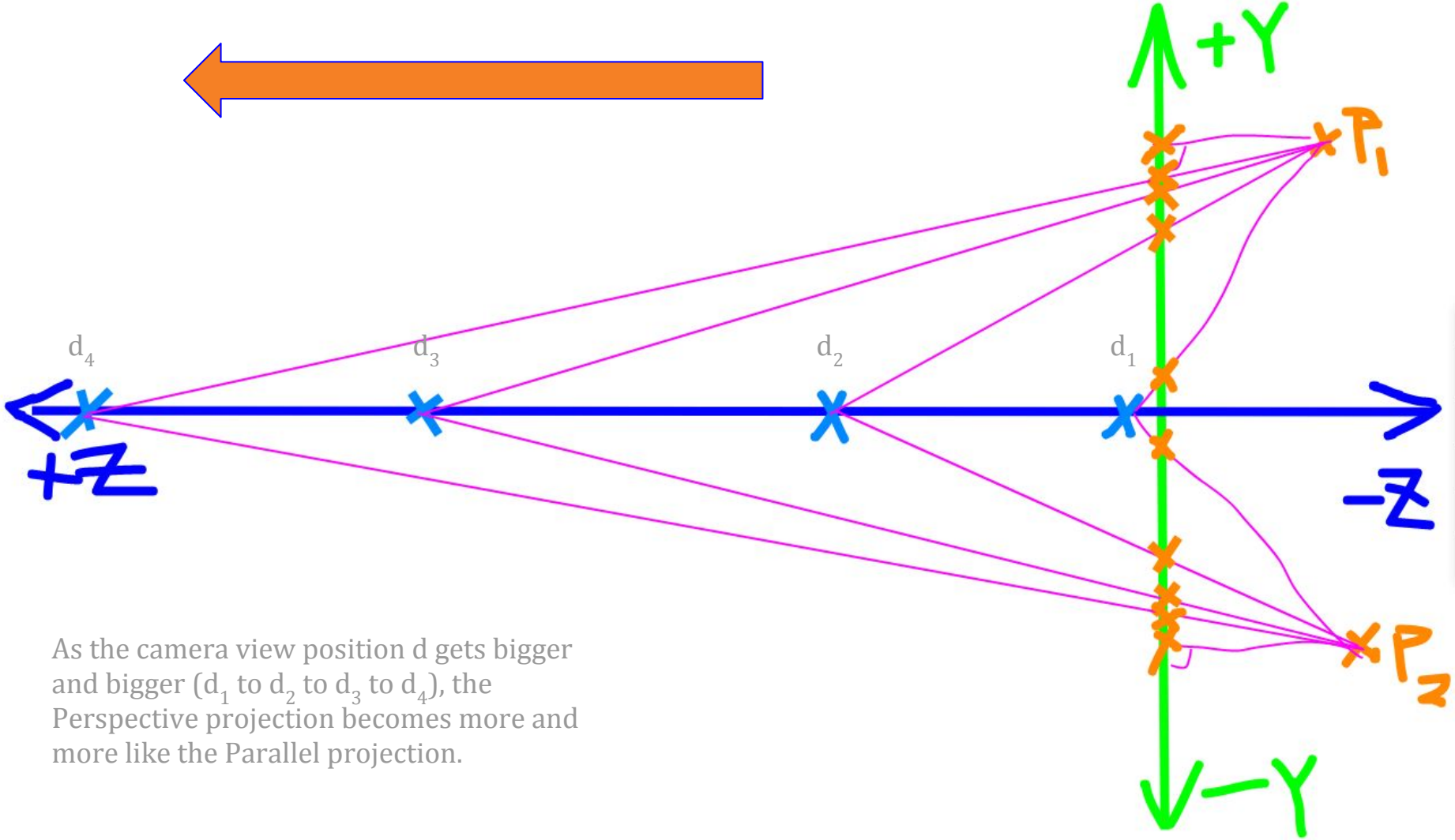
#----------

The definitions for t_x, t_y, s_x, and s_y are given on a previous slide.

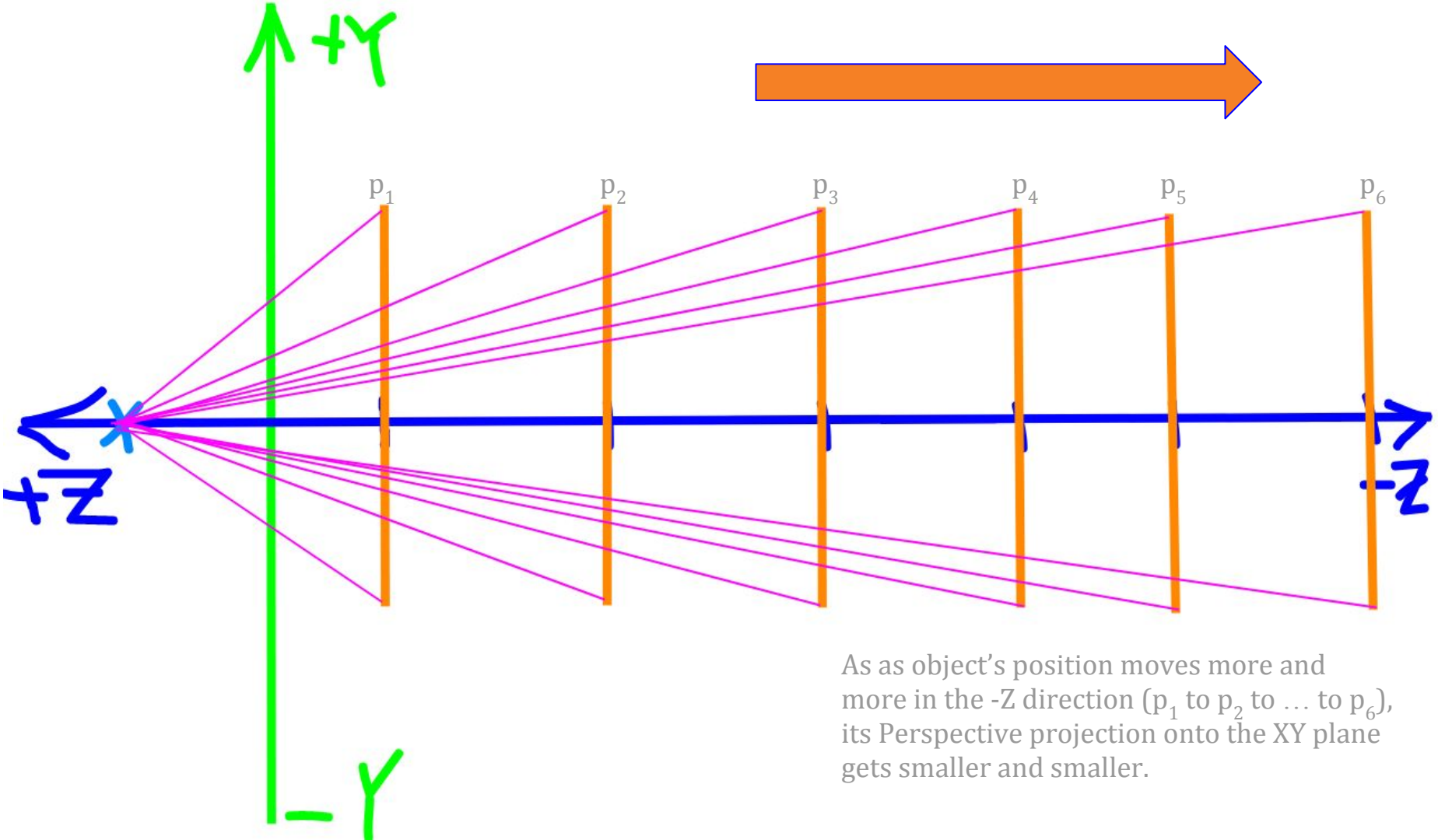
Be careful about forbidden cases! (Usually happens when one tries to divide by zero.)

When forming M for the Parallel case, -1/d is replaced by 0.

In the Perspective case, d must not be 0. (Obviously, right?)



As the camera view position d gets bigger and bigger (d_1 to d_2 to d_3 to d_4), the Perspective projection becomes more and more like the Parallel projection.



As as object's position moves more and more in the -Z direction (p_1 to p_2 to … to p_6), its Perspective projection onto the XY plane gets smaller and smaller.

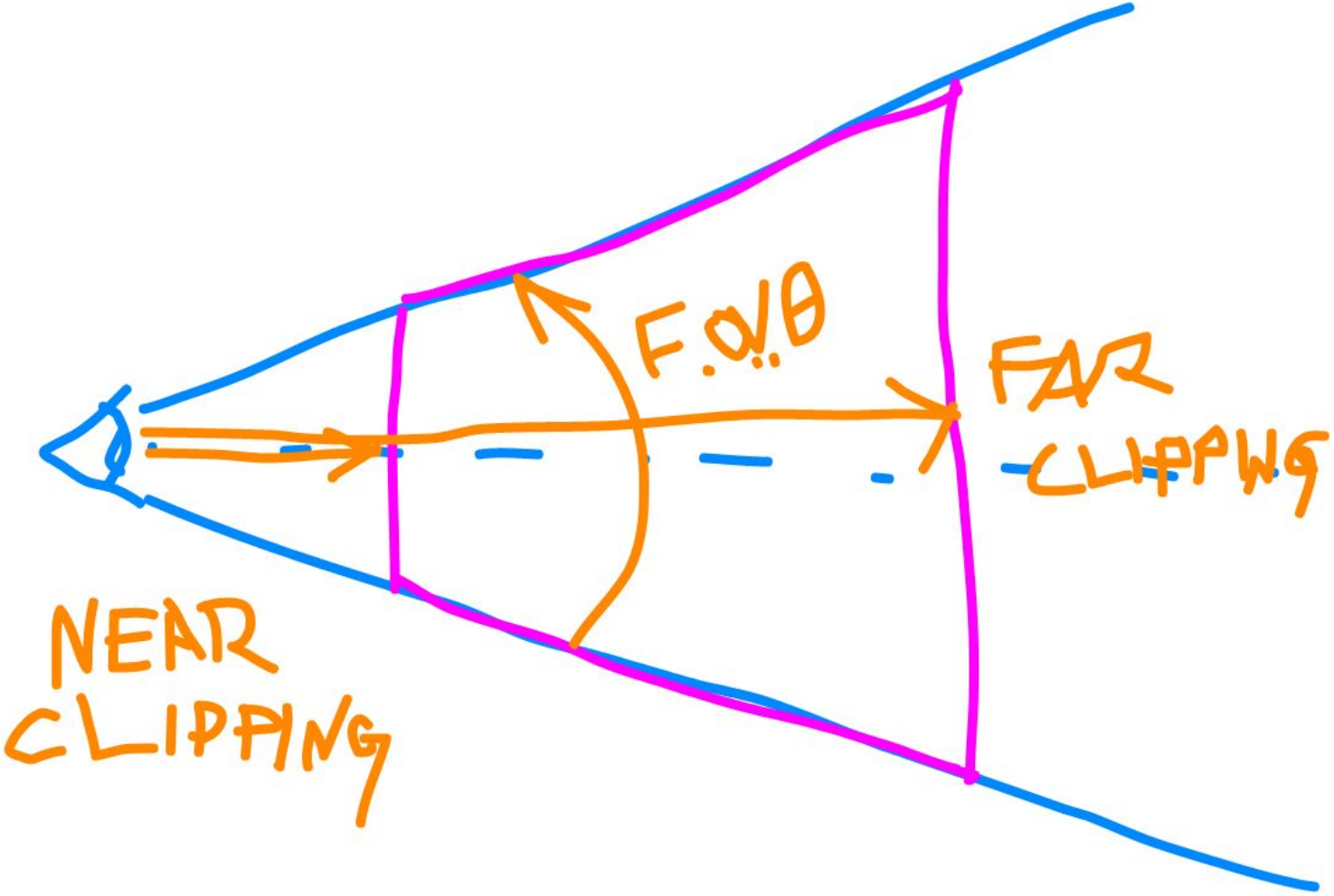
$$Y' = Y\left[\frac{1}{1-\frac{z}{d}}\right]$$

$$Y' = Y\left[\frac{1}{1-0}\right] = Y$$

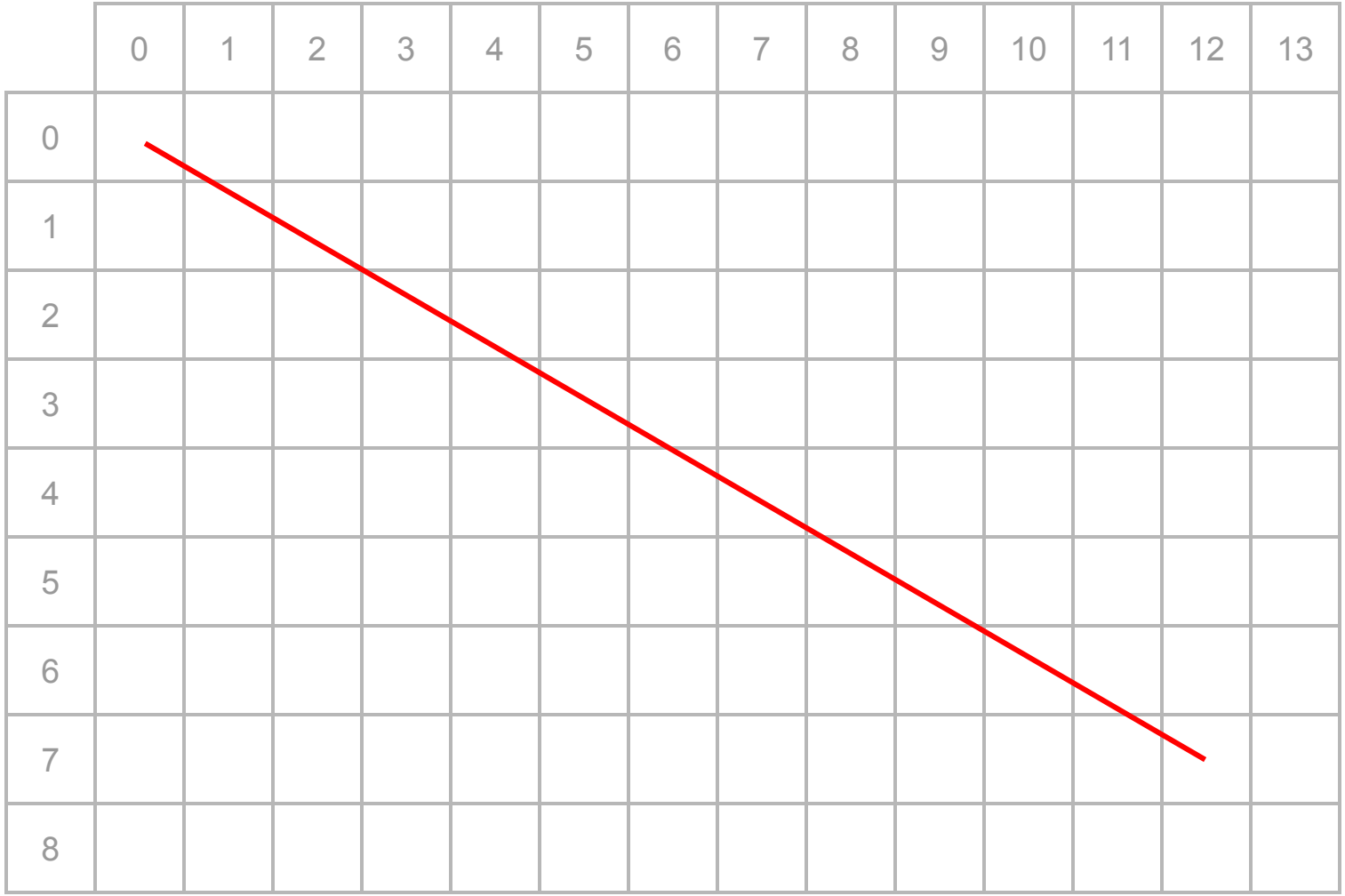
$$d \to \infty, \; z/d \to 0$$

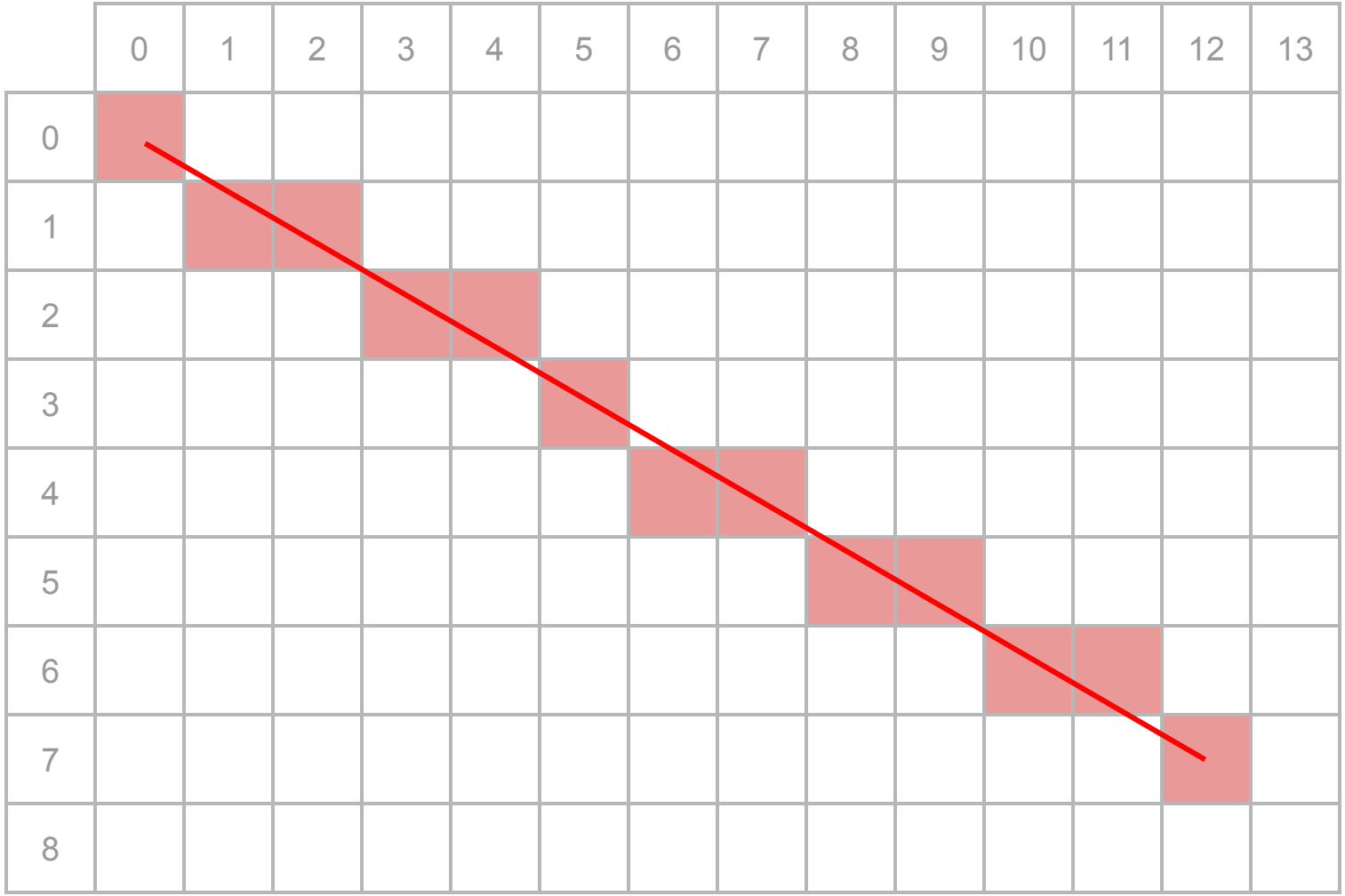
$$Y' = Y\left[\frac{1}{\infty}\right] = 0$$

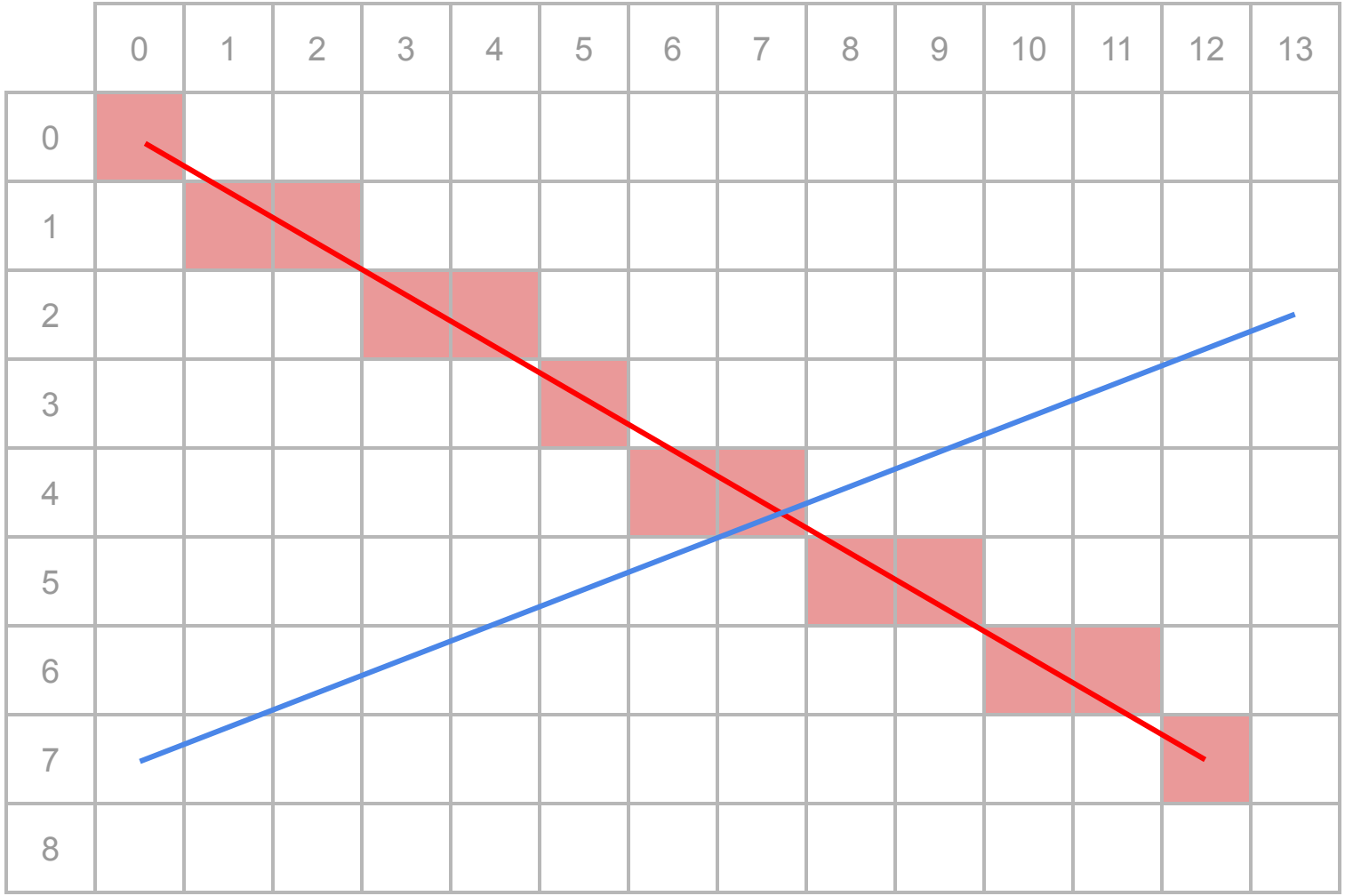
$$z \to -\infty, \; z/d \to \infty, \; 1 - \frac{z}{d} \to \infty$$

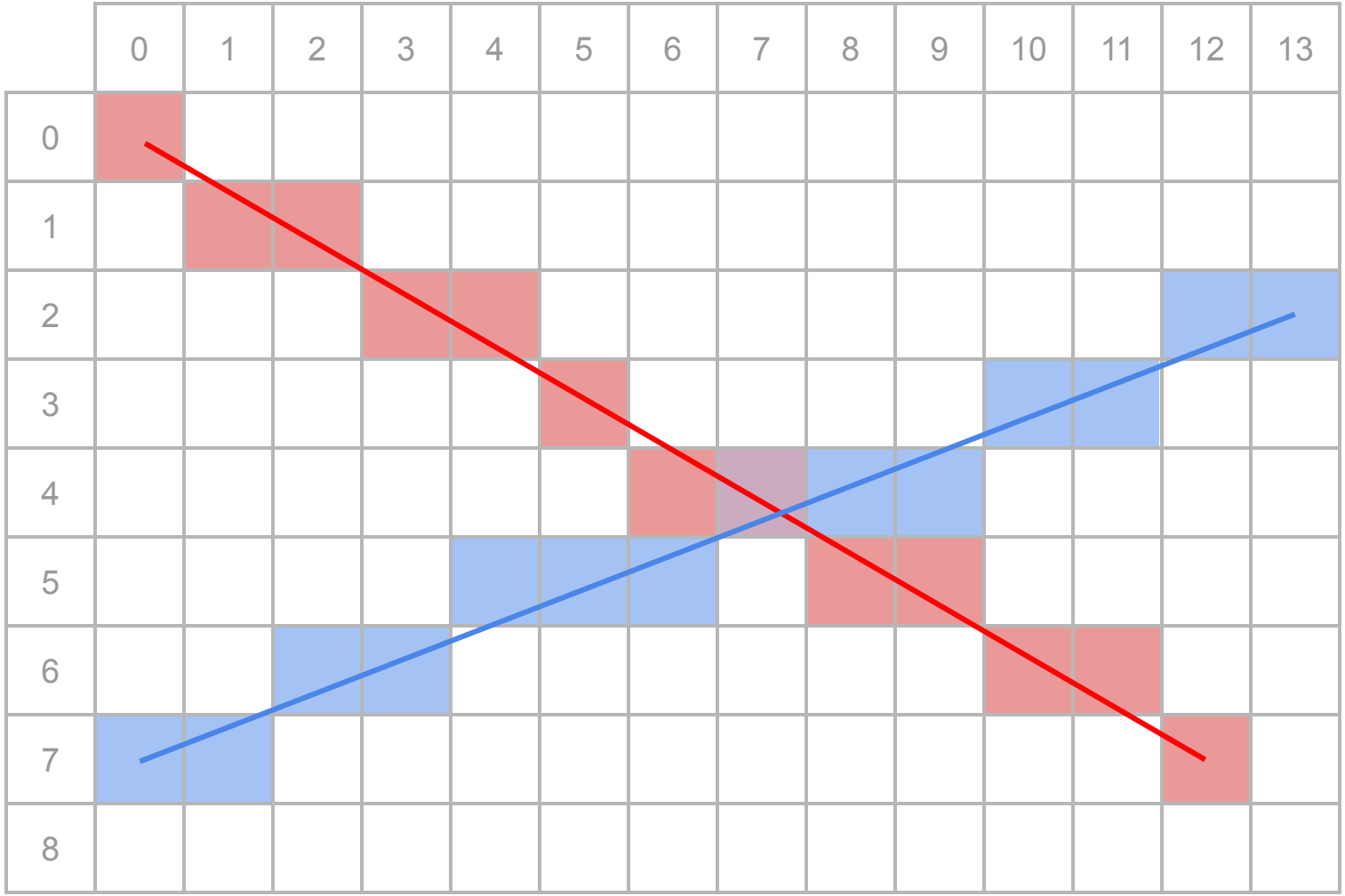

F.O.V θ
FAR
CLIPPING
NEAR
CLIPPING

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0														
1														
2														
3														
4														
5														
6														
7														
8														


0 1 2 3 4 5 6 7 8 9 10 11 12 13
0
1
2
3
4
5
6
7
8


0 1 2 3 4 5 6 7 8 9 10 11 12 13
0
1
2
3
4
5
6
7
8

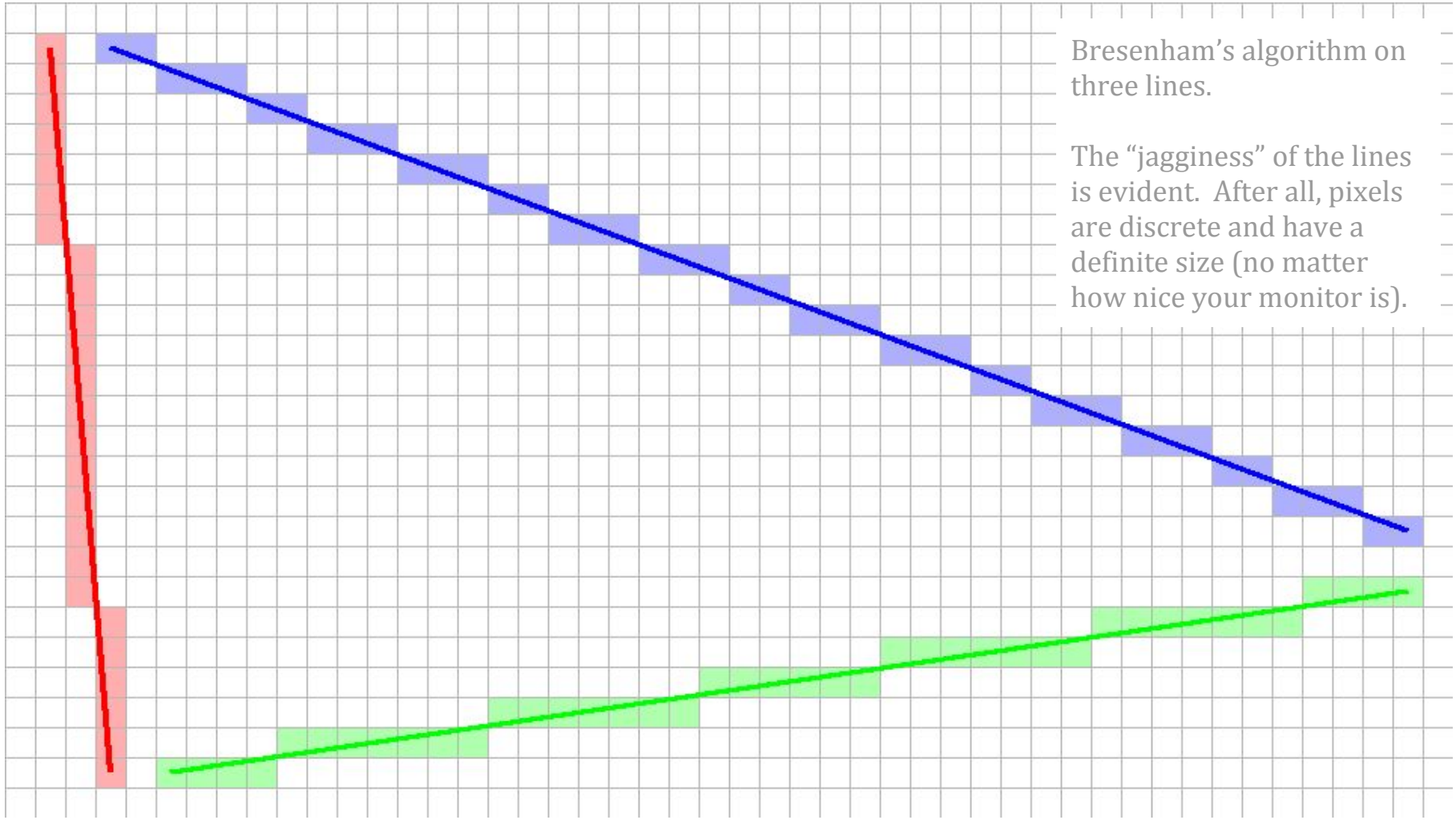

0 1 2 3 4 5 6 7 8 9 10 11 12 13
0 1 2 3 4 5 6 7 8


0 1 2 3 4 5 6 7 8 9 10 11 12 13
0 1 2 3 4 5 6 7 8

```
( 0,  0) -> (12,  7)
  ( 0,  0)
  ( 1,  1)
  ( 2,  1)
  ( 3,  2)
  ( 4,  2)
  ( 5,  3)
  ( 6,  4)
  ( 7,  4)
  ( 8,  5)
  ( 9,  5)
  (10,  6)
  (11,  6)
  (12,  7)
```

```
( 0,  7) -> (13,  2)
  ( 0,  7)
  ( 1,  7)
  ( 2,  6)
  ( 3,  6)
  ( 4,  5)
  ( 5,  5)
  ( 6,  5)
  ( 7,  4)
  ( 8,  4)
  ( 9,  4)
  (10,  3)
  (11,  3)
  (12,  2)
  (13,  2)
```

```python
def bresenham( x1, y1, x2, y2 ) :
    dx = x2 - x1
    dy = y2 - y1
    yincr = int( math.copysign( 1, dy ) )

    derr = abs( dy / dx )
    err  = 0.0
    y    = y1

    print( f'({x1:2d}, {y1:2d}) -> ({x2:2d}, {y2:2d})' )

    for x in range( x1, x2+1 ) :
        print( f'  ({x:2d}, {y:2d})' )

        err += derr

        if err >= 0.5 :
            y += yincr
            err -= 1.0

```


Bresenham's algorithm on three lines.
The "jagginess" of the lines is evident. After all, pixels are discrete and have a definite size (no matter how nice your monitor is).

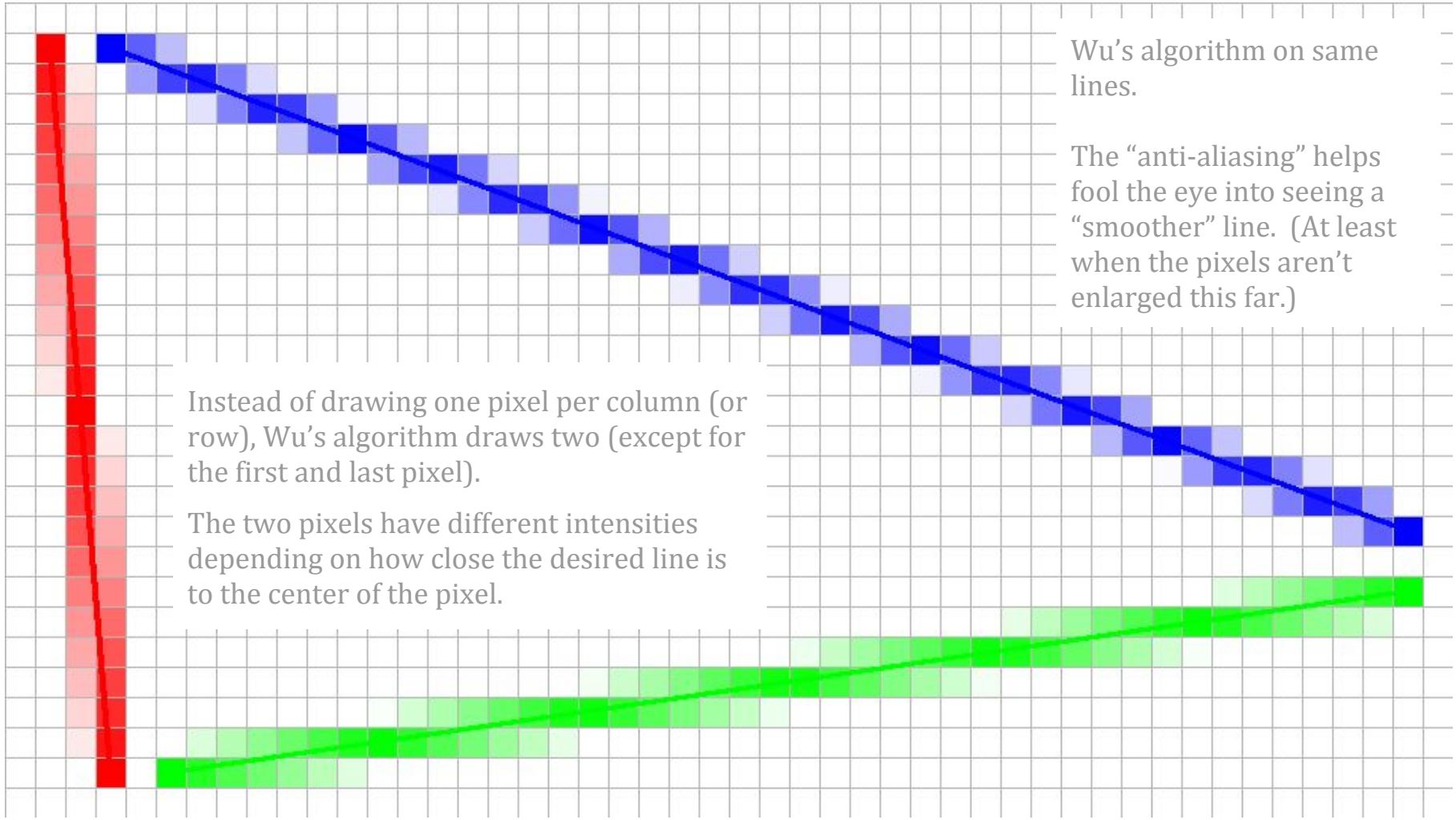
```
//-- Bresenham -------------------------
// (1,1) -> (3,25)
// dx 2, dy 24, xincr 1
// derr 0.0833333
// (  1,  1 ) +0
// (  1,  2 ) +0.0833333
// (  1,  3 ) +0.166667
// (  1,  4 ) +0.25
// (  1,  5 ) +0.333333
// (  1,  6 ) +0.416667
// (  1,  7 ) +0.5
// (  2,  8 ) -0.416667
// (  2,  9 ) -0.333333
// (  2, 10 ) -0.25
// (  2, 11 ) -0.166667
// (  2, 12 ) -0.0833333
// (  2, 13 ) -1.38778e-16
// (  2, 14 ) +0.0833333
// (  2, 15 ) +0.166667
// (  2, 16 ) +0.25
// (  2, 17 ) +0.333333
// (  2, 18 ) +0.416667
// (  2, 19 ) +0.5
// (  3, 20 ) -0.416667
// (  3, 21 ) -0.333333
// (  3, 22 ) -0.25
// (  3, 23 ) -0.166667
// (  3, 24 ) -0.0833333
// (  3, 25 ) -2.498e-16
```

```
//-- Bresenham -------------------------
// (5,25) -> (46,19)
// dx 41, dy -6, yincr -1
// derr 0.146341
// (  5, 25 ) +0
// (  6, 25 ) +0.146341
// (  7, 25 ) +0.292683
// (  8, 25 ) +0.439024
// (  9, 24 ) -0.414634
// ( 10, 24 ) -0.268293
// ( 11, 24 ) -0.121951
// ( 12, 24 ) +0.0243902
// ( 13, 24 ) +0.170732
// ( 14, 24 ) +0.317073
// ( 15, 24 ) +0.463415
// ( 16, 23 ) -0.390244
// ( 17, 23 ) -0.243902
// ( 18, 23 ) -0.097561
// ( 19, 23 ) +0.0487805
// ( 20, 23 ) +0.195122
// ( 21, 23 ) +0.341463
// ( 22, 23 ) +0.487805
// ( 23, 22 ) -0.365854
// ( 24, 22 ) -0.219512
// ( 25, 22 ) -0.0731707
// ( 26, 22 ) +0.0731707
// ( 27, 22 ) +0.219512
// ( 28, 22 ) +0.365854
// ( 29, 21 ) -0.487805
// ( 30, 21 ) -0.341463
// ( 31, 21 ) -0.195122
// ( 32, 21 ) -0.0487805
// ( 33, 21 ) +0.097561
// ( 34, 21 ) +0.243902
// ( 35, 21 ) +0.390244
// ( 36, 20 ) -0.463415
// ( 37, 20 ) -0.317073
// ( 38, 20 ) -0.170732
// ( 39, 20 ) -0.0243902
// ( 40, 20 ) +0.121951
// ( 41, 20 ) +0.268293
// ( 42, 20 ) +0.414634
// ( 43, 19 ) -0.439024
// ( 44, 19 ) -0.292683
// ( 45, 19 ) -0.146341
// ( 46, 19 ) +5.55112e-17
```

The raw info for the three lines using Bresenham's method. For each point, the data printed is,

```
// ( x, y ) current_error
```

```
//-- Bresenham -------------------------
// (3,1) -> (46,17)
// dx -43, dy 16, yincr 1
// derr 0.372093
// (  3,  1 ) +0
// (  4,  1 ) +0.372093
// (  5,  2 ) -0.255814
// (  6,  2 ) +0.116279
// (  7,  2 ) +0.488372
// (  8,  3 ) -0.139535
// (  9,  3 ) +0.232558
// ( 10,  4 ) -0.395349
// ( 11,  4 ) -0.0232558
// ( 12,  4 ) +0.348837
// ( 13,  5 ) -0.27907
// ( 14,  5 ) +0.0930233
// ( 15,  5 ) +0.465116
// ( 16,  6 ) -0.162791
// ( 17,  6 ) +0.209302
// ( 18,  7 ) -0.418605
// ( 19,  7 ) -0.0465116
// ( 20,  7 ) +0.325581
// ( 21,  8 ) -0.302326
// ( 22,  8 ) +0.0697674
// ( 23,  8 ) +0.44186
// ( 24,  9 ) -0.186047
// ( 25,  9 ) +0.186047
// ( 26, 10 ) -0.44186
// ( 27, 10 ) -0.0697674
// ( 28, 10 ) +0.302326
// ( 29, 11 ) -0.325581
// ( 30, 11 ) +0.0465116
// ( 31, 11 ) +0.418605
// ( 32, 12 ) -0.209302
// ( 33, 12 ) +0.162791
// ( 34, 13 ) -0.465116
// ( 35, 13 ) -0.0930233
// ( 36, 13 ) +0.27907
// ( 37, 14 ) -0.348837
// ( 38, 14 ) +0.0232558
// ( 39, 14 ) +0.395349
// ( 40, 15 ) -0.232558
// ( 41, 15 ) +0.139535
// ( 42, 16 ) -0.488372
// ( 43, 16 ) -0.116279
// ( 44, 16 ) +0.255814
// ( 45, 17 ) -0.372093
// ( 46, 17 ) -2.22045e-16
```


Wu's algorithm on same lines.
The "anti-aliasing" helps fool the eye into seeing a "smoother" line. (At least when the pixels aren't enlarged this far.)
Instead of drawing one pixel per column (or row), Wu's algorithm draws two (except for the first and last pixel).
The two pixels have different intensities depending on how close the desired line is to the center of the pixel.

```
//-- Wu --------------------------------
// (  1/ 2,  2 ) 0x15, 0xEA
// (  1/ 2,  3 ) 0x2A, 0xD5
// (  1/ 2,  4 ) 0x3F, 0xC0
// (  1/ 2,  5 ) 0x55, 0xAA
// (  1/ 2,  6 ) 0x6A, 0x95
// (  1/ 2,  7 ) 0x7F, 0x80
// (  1/ 2,  8 ) 0x95, 0x6A
// (  1/ 2,  9 ) 0xAA, 0x55
// (  1/ 2, 10 ) 0xBF, 0x40
// (  1/ 2, 11 ) 0xD5, 0x2A
// (  1/ 2, 12 ) 0xEA, 0x15
// (  1/ 2, 13 ) 0xFF, 0x00
// (  2/ 3, 14 ) 0x15, 0xEA
// (  2/ 3, 15 ) 0x2A, 0xD5
// (  2/ 3, 16 ) 0x3F, 0xC0
// (  2/ 3, 17 ) 0x55, 0xAA
// (  2/ 3, 18 ) 0x6A, 0x95
// (  2/ 3, 19 ) 0x7F, 0x80
// (  2/ 3, 20 ) 0x95, 0x6A
// (  2/ 3, 21 ) 0xAA, 0x55
// (  2/ 3, 22 ) 0xBF, 0x40
// (  2/ 3, 23 ) 0xD5, 0x2A
// (  2/ 3, 24 ) 0xEA, 0x15
```

```
//-- Wu --------------------------------
// ( 45, 19/20 ) 0x25, 0xDA  // (  8, 24/25 ) 0x8F, 0x70
// ( 44, 19/20 ) 0x4A, 0xB5  // (  7, 24/25 ) 0xB4, 0x4B
// ( 43, 19/20 ) 0x70, 0x8F  // (  6, 24/25 ) 0xDA, 0x25
// ( 42, 19/20 ) 0x95, 0x6A
// ( 41, 19/20 ) 0xBB, 0x44
// ( 40, 19/20 ) 0xE0, 0x1F
// ( 39, 20/21 ) 0x06, 0xF9
// ( 38, 20/21 ) 0x2B, 0xD4
// ( 37, 20/21 ) 0x51, 0xAE
// ( 36, 20/21 ) 0x76, 0x89
// ( 35, 20/21 ) 0x9C, 0x63
// ( 34, 20/21 ) 0xC1, 0x3E
// ( 33, 20/21 ) 0xE6, 0x19
// ( 32, 21/22 ) 0x0C, 0xF3
// ( 31, 21/22 ) 0x31, 0xCE
// ( 30, 21/22 ) 0x57, 0xA8
// ( 29, 21/22 ) 0x7C, 0x83
// ( 28, 21/22 ) 0xA2, 0x5D
// ( 27, 21/22 ) 0xC7, 0x38
// ( 26, 21/22 ) 0xED, 0x12
// ( 25, 22/23 ) 0x12, 0xED
// ( 24, 22/23 ) 0x38, 0xC7
// ( 23, 22/23 ) 0x5D, 0xA2
// ( 22, 22/23 ) 0x83, 0x7C
// ( 21, 22/23 ) 0xA8, 0x57
// ( 20, 22/23 ) 0xCD, 0x32
// ( 19, 22/23 ) 0xF3, 0x0C
// ( 18, 23/24 ) 0x18, 0xE7
// ( 17, 23/24 ) 0x3E, 0xC1
// ( 16, 23/24 ) 0x63, 0x9C
// ( 15, 23/24 ) 0x89, 0x76
// ( 14, 23/24 ) 0xAE, 0x51
// ( 13, 23/24 ) 0xD4, 0x2B
// ( 12, 23/24 ) 0xF9, 0x06
// ( 11, 24/25 ) 0x1F, 0xE0
// ( 10, 24/25 ) 0x44, 0xBB
// (  9, 24/25 ) 0x6A, 0x95
```

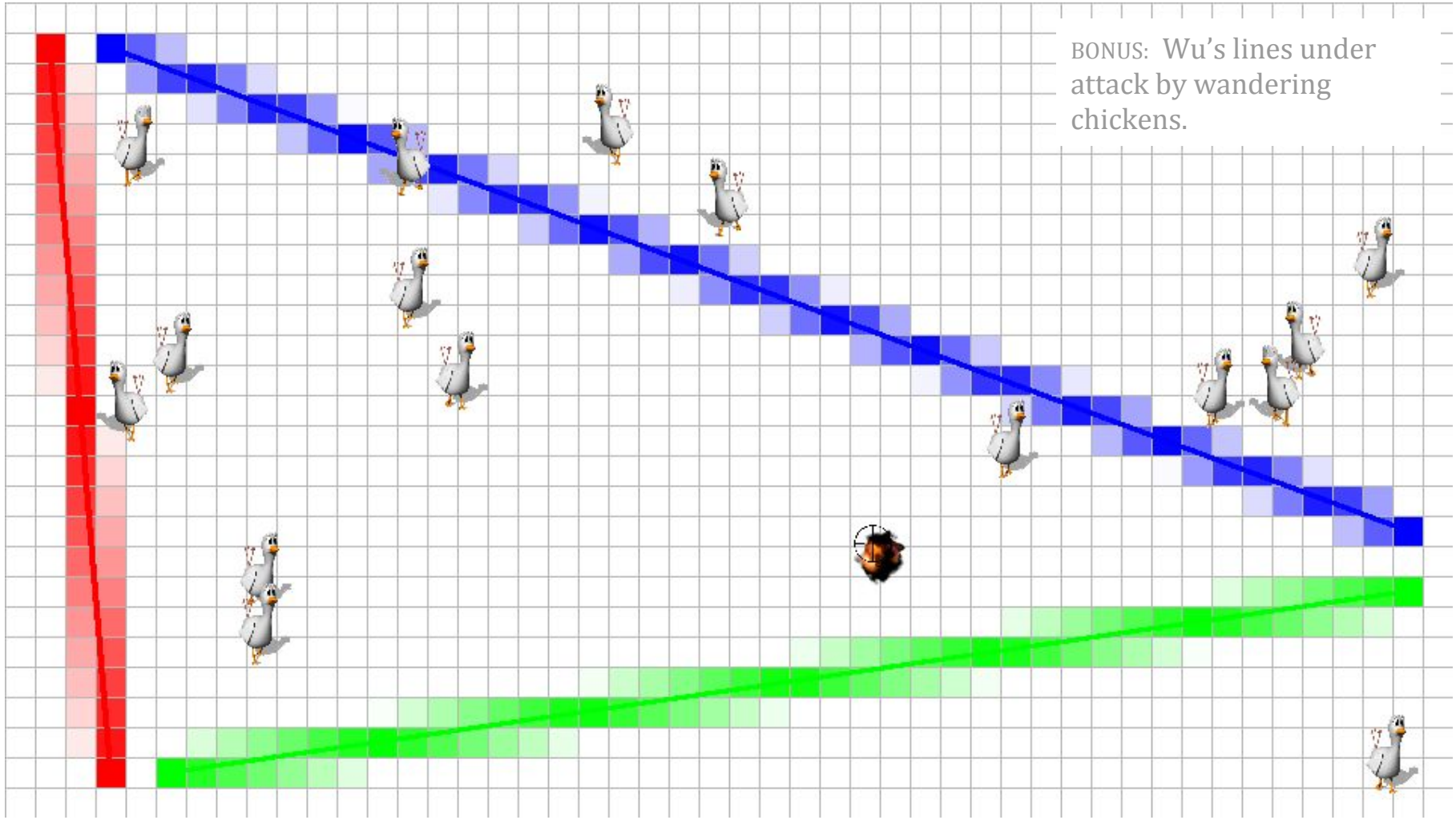
The raw info for the three lines using Wu's method. For each point, the data printed is the coordinates of the two pixels drawn. For "vertical" lines, two x coords are given. For "horizontal" lines, two y coords are given.

Also given is their intensities (on a scale of 0x00 to 0xFF).

(The endpoints of the lines are not shown in these lists.)

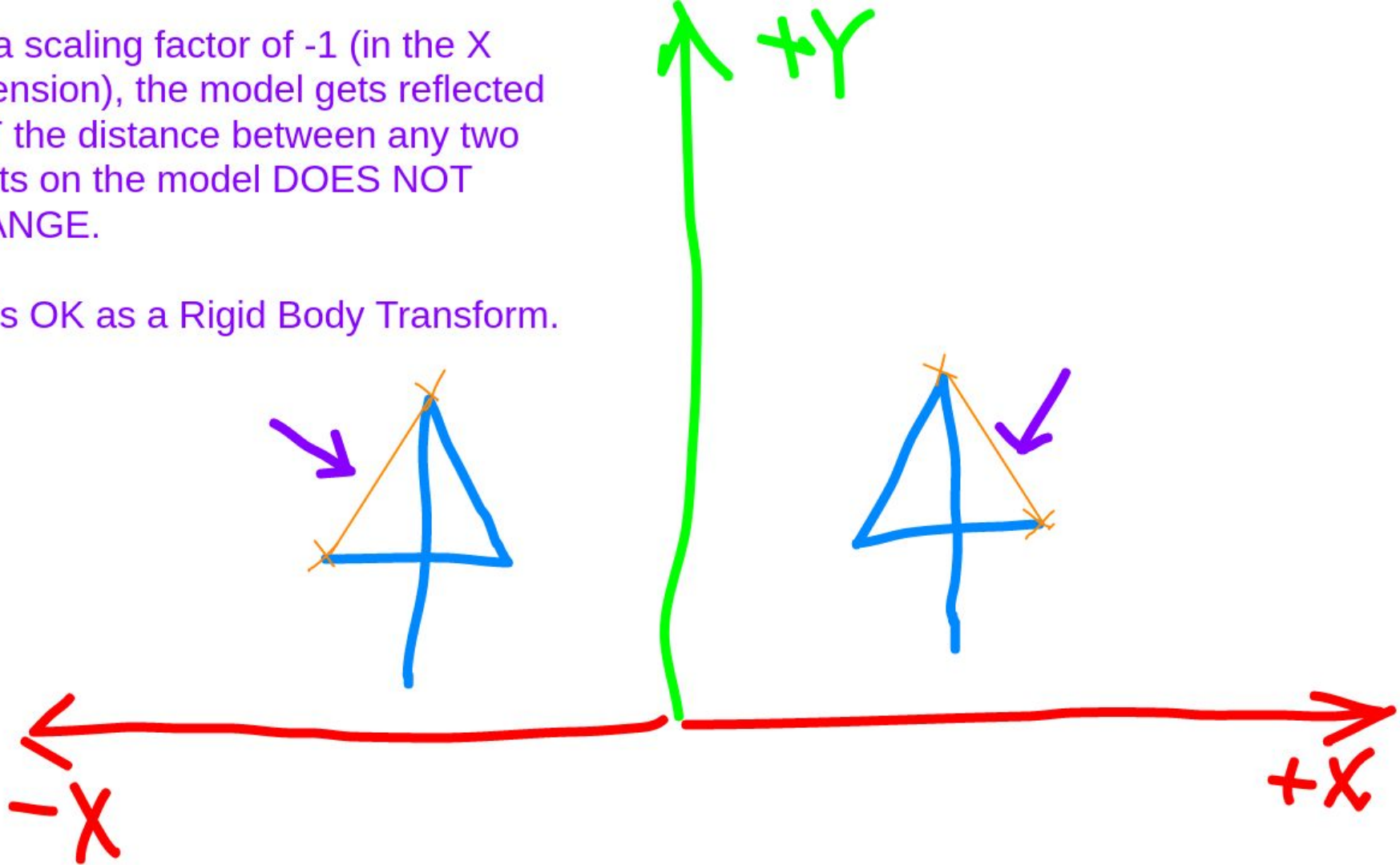
```
// ( 41, 15/16 ) 0x23, 0xDC
// ( 42, 15/16 ) 0x82, 0x7D
// ( 43, 15/16 ) 0xE2, 0x1D
// ( 44, 16/17 ) 0x41, 0xBE
// ( 45, 16/17 ) 0xA0, 0x5F
```

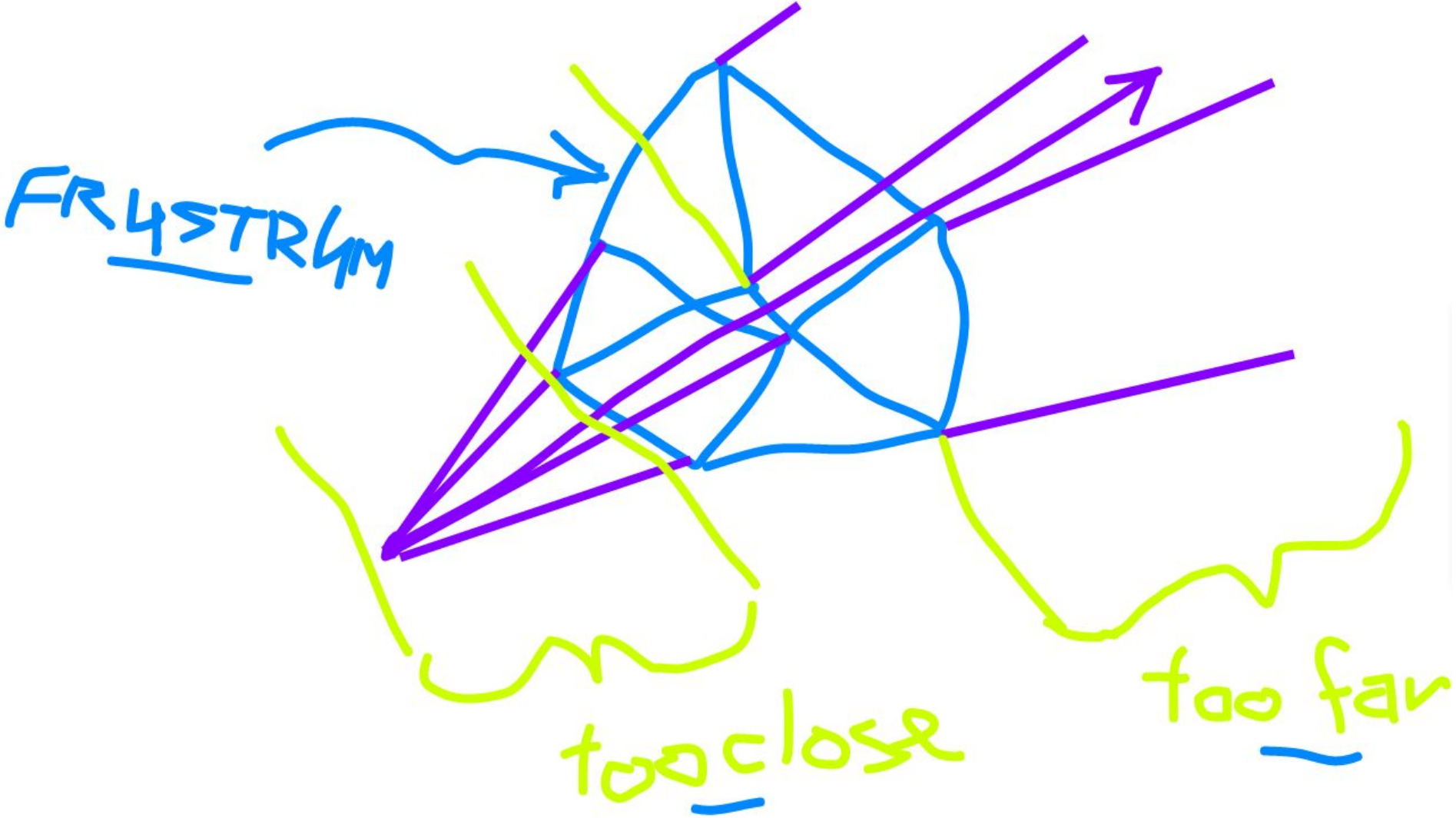
```
//-- Wu --------------------------------
// (  4,  1/ 2 ) 0x5F, 0xA0
// (  5,  1/ 2 ) 0xBE, 0x41
// (  6,  2/ 3 ) 0x1D, 0xE2
// (  7,  2/ 3 ) 0x7D, 0x82
// (  8,  2/ 3 ) 0xDC, 0x23
// (  9,  3/ 4 ) 0x3B, 0xC4
// ( 10,  3/ 4 ) 0x9A, 0x65
// ( 11,  3/ 4 ) 0xFA, 0x05
// ( 12,  4/ 5 ) 0x59, 0xA6
// ( 13,  4/ 5 ) 0xB8, 0x47
// ( 14,  5/ 6 ) 0x17, 0xE8
// ( 15,  5/ 6 ) 0x77, 0x88
// ( 16,  5/ 6 ) 0xD6, 0x29
// ( 17,  6/ 7 ) 0x35, 0xCA
// ( 18,  6/ 7 ) 0x94, 0x6B
// ( 19,  6/ 7 ) 0xF4, 0x0B
// ( 20,  7/ 8 ) 0x53, 0xAC
// ( 21,  7/ 8 ) 0xB2, 0x4D
// ( 22,  8/ 9 ) 0x11, 0xEE
// ( 23,  8/ 9 ) 0x71, 0x8E
// ( 24,  8/ 9 ) 0xD0, 0x2F
// ( 25,  9/10 ) 0x2F, 0xD0
// ( 26,  9/10 ) 0x8E, 0x71
// ( 27,  9/10 ) 0xEE, 0x11
// ( 28, 10/11 ) 0x4D, 0xB2
// ( 29, 10/11 ) 0xAC, 0x53
// ( 30, 11/12 ) 0x0B, 0xF4
// ( 31, 11/12 ) 0x6B, 0x94
// ( 32, 11/12 ) 0xCA, 0x35
// ( 33, 12/13 ) 0x29, 0xD6
// ( 34, 12/13 ) 0x88, 0x77
// ( 35, 12/13 ) 0xE8, 0x17
// ( 36, 13/14 ) 0x47, 0xB8
// ( 37, 13/14 ) 0xA6, 0x59
// ( 38, 14/15 ) 0x05, 0xFA
// ( 39, 14/15 ) 0x65, 0x9A
// ( 40, 14/15 ) 0xC4, 0x3B
```


BONUS: Wu's lines under attack by wandering chickens.

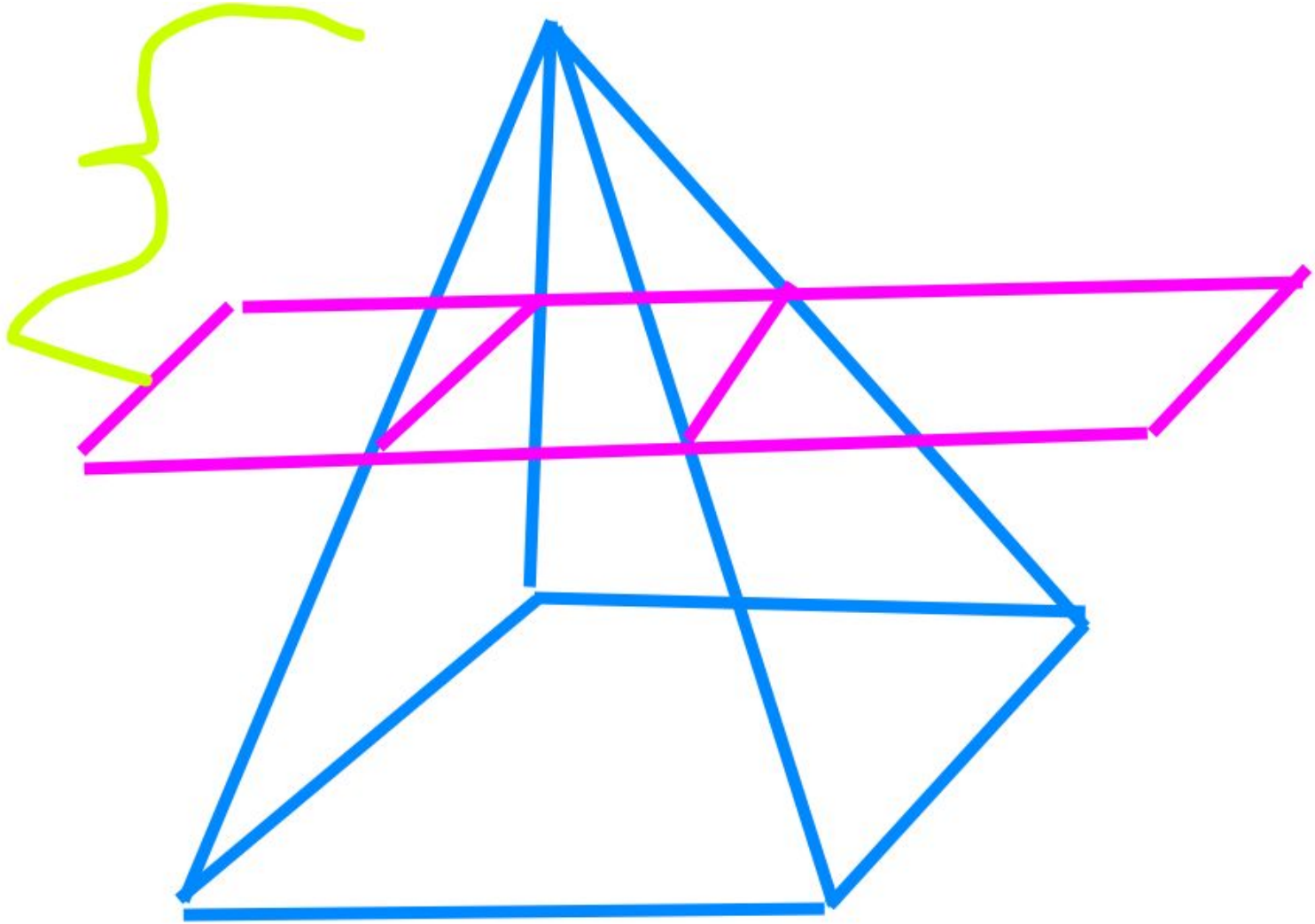
With a scaling factor of -1 (in the X dimension), the model gets reflected BUT the distance between any two points on the model DOES NOT CHANGE.

So, it's OK as a Rigid Body Transform.



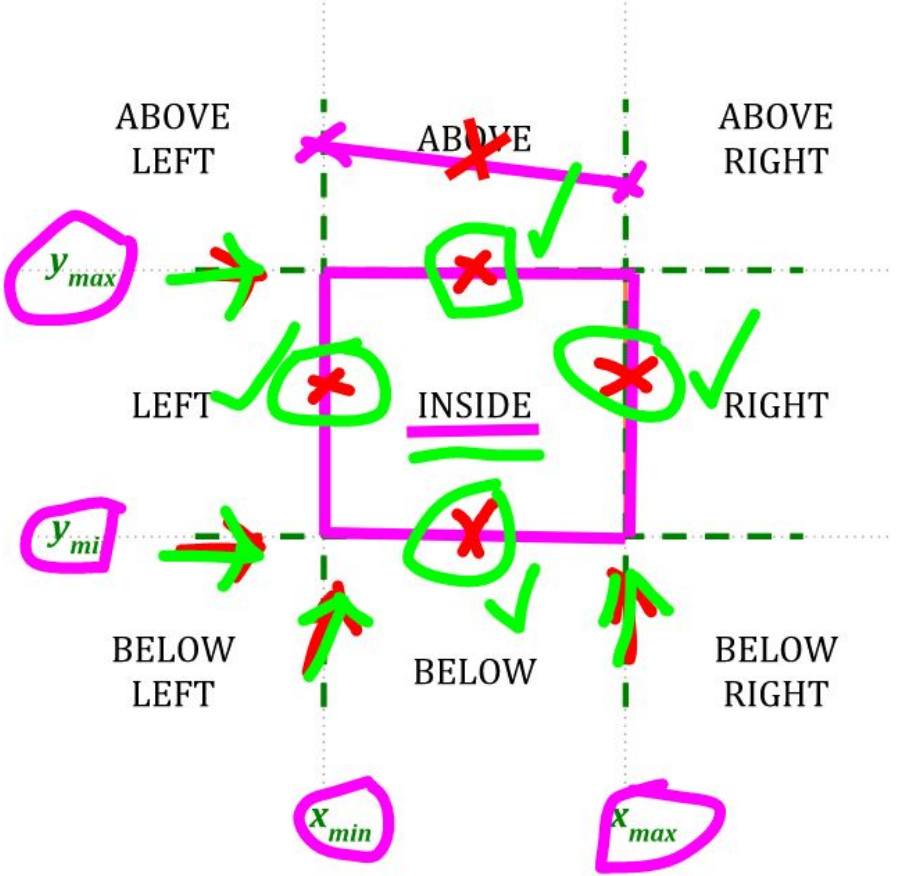

FRUSTRUM
too close
too far

too close



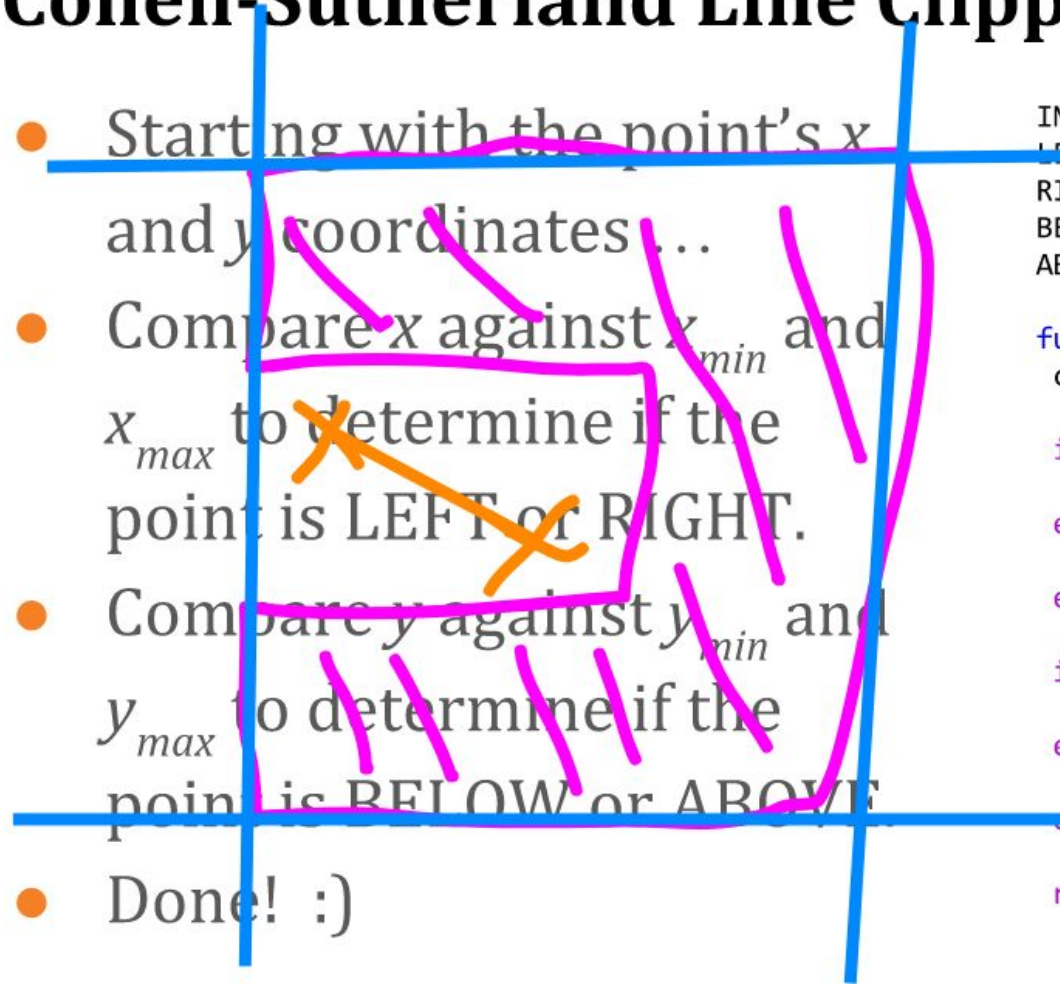
Cohen-Sutherland Line Clipping

- Divides the viewport space into nine areas.
- The central area is the *inside* space that the user sees.
- All other areas are *outside* and are not seen.
- INSIDE is bounded by the lines $x_{min}, x_{max}, y_{min}, y_{max}$.



Cohen-Sutherland Line Clipping

- Starting with the point's x and y coordinates…
- Compare x against x_{min} and x_{max} to determine if the point is LEFT or RIGHT.
- Compare y against y_{min} and y_{max} to determine if the point is BELOW or ABOVE.
- Done! :)



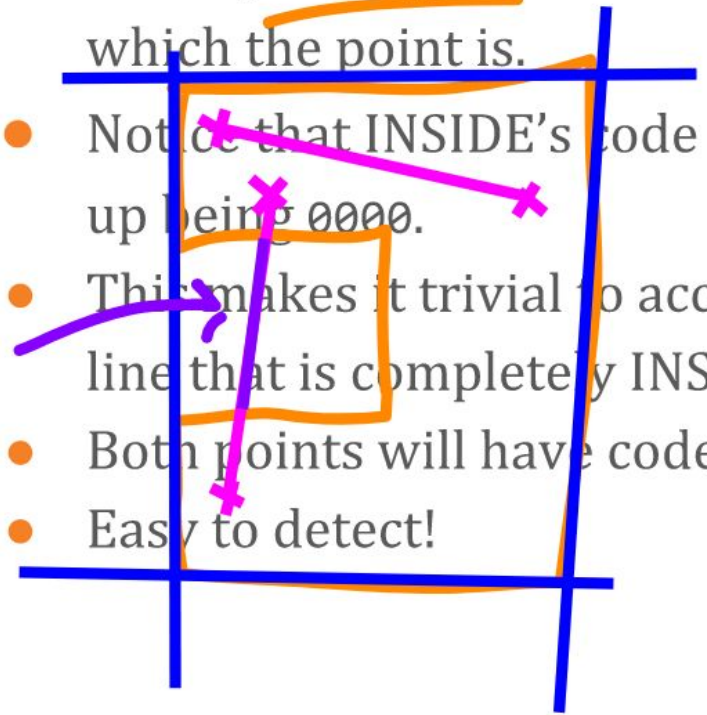
```
INSIDE = 0
LEFT   = 1
RIGHT  = 2
BELOW  = 4
ABOVE  = 8
```

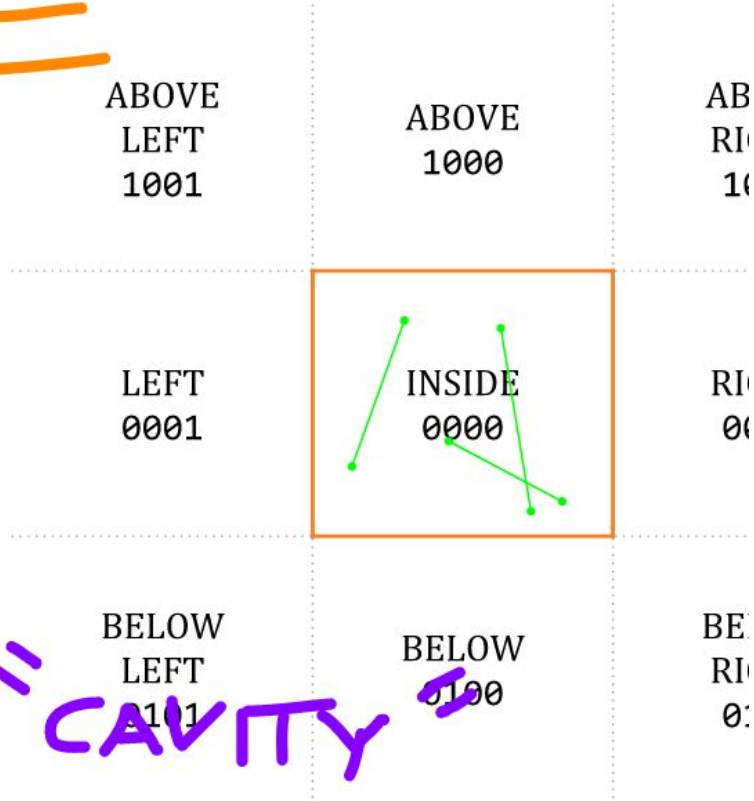
These are mutually exclusive powers of 2, so each is a unique bit.

```
func outcode( x, y, xMin, yMin, xMax, yMax )
 code = INSIDE

 if x < xMin then
   code = code | LEFT
 elif x > xMax then
   code = code | RIGHT
 endif

 if y < yMin then
   code = code | BELOW
 elif y > yMax then
   code = code | ABOVE
 endif

 return code
```

Cohen-Sutherland Line Clipping

- The result will be a 4-bit code corresponding to the area in which the point is.
- Notice that INSIDE's code ends up being `0000`.
- This makes it trivial to accept a line that is completely INSIDE.
- Both points will have code `0000`.
- Easy to detect!



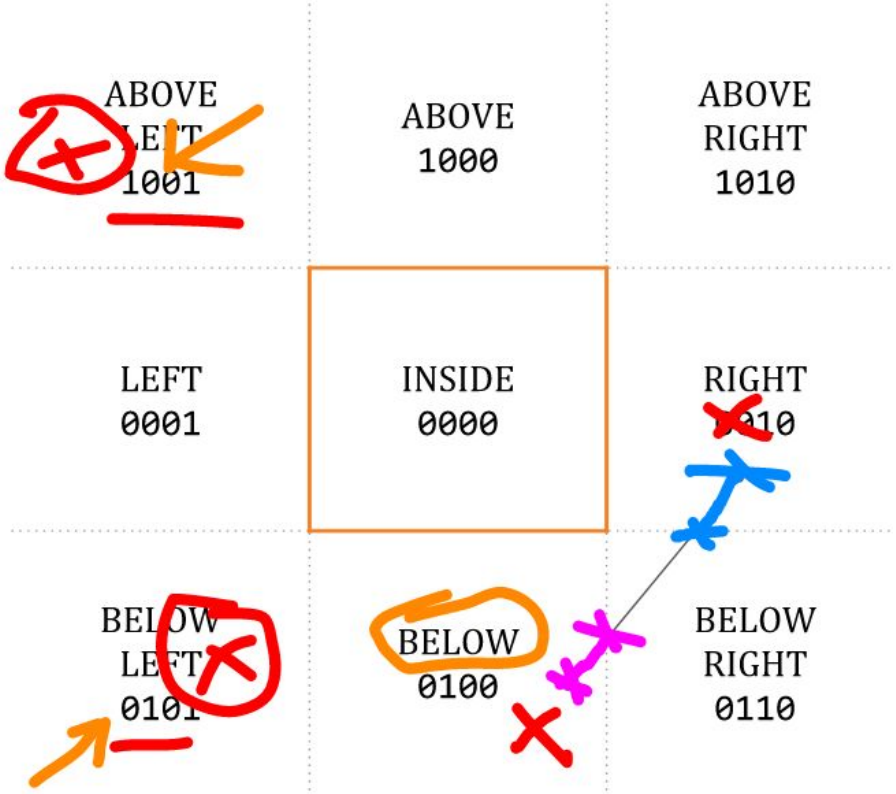


Cohen-Sutherland Line Clipping

- For example, both points of this line are outside the INSIDE region.

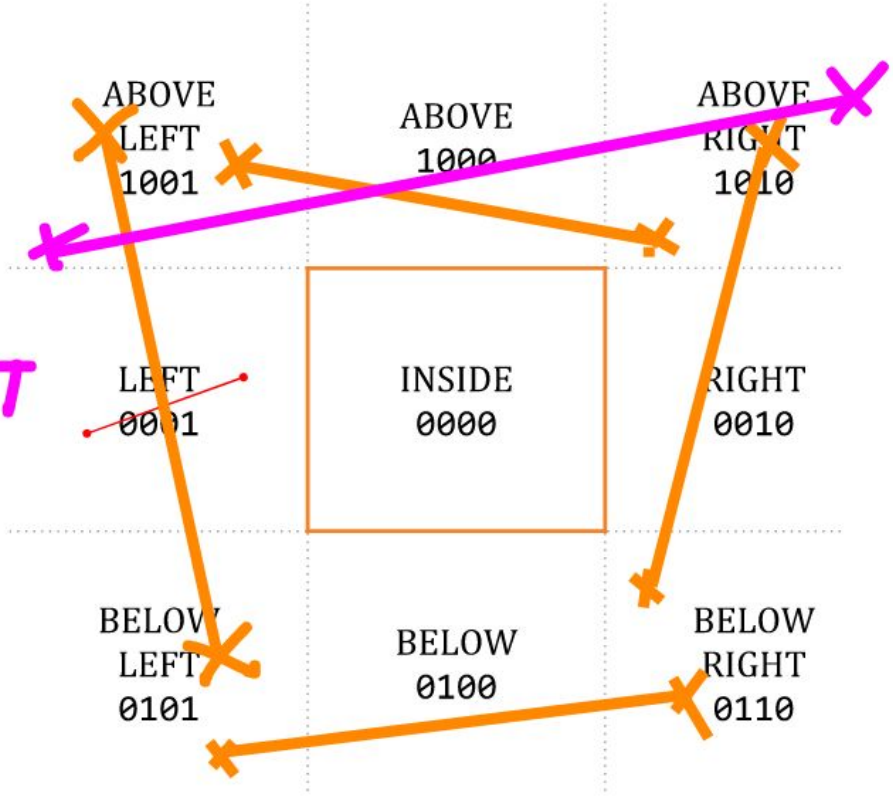
BOTH CODE = 0
TR. ACC.

BITWISE AND ≠ 0
TR. REJ.



Cohen-Sutherland Line Clipping

- What about trivial rejects?
- Lines whose points are both in the same region that is *not* INSIDE can be rejected.
  - Line is entirely in unseen region.
- Lines whose points are on the same side of INSIDE can be rejected.



REJECT

$c_1 \quad c_2$

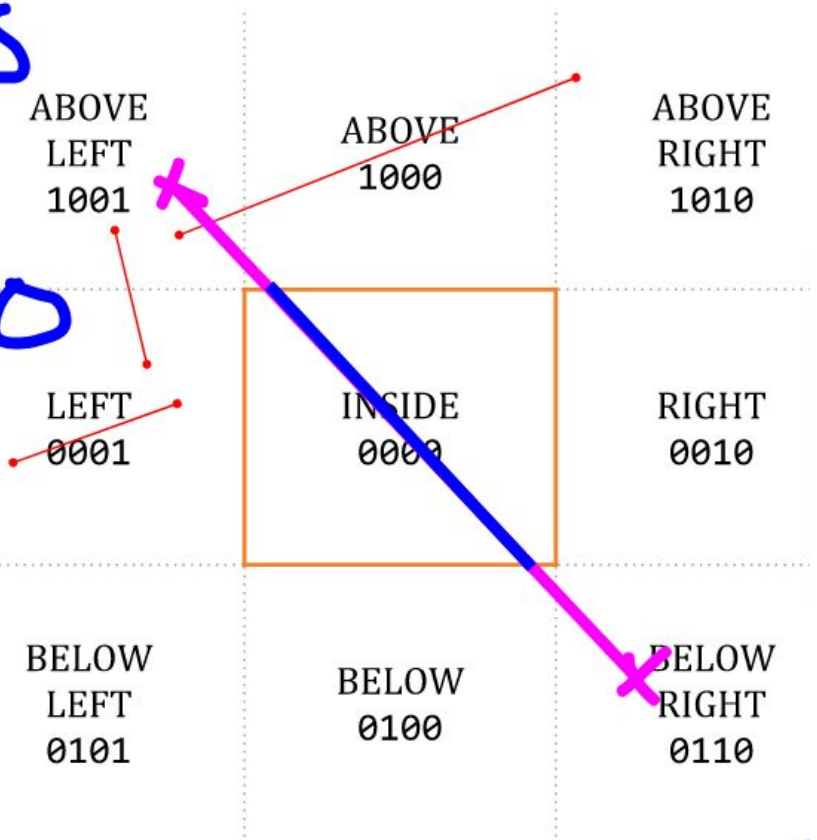
$(c_1 \& c_2) != \emptyset$

Cohen-Sutherland Line Clipping

- What about trivial rejects?
- Lines whose points are both in the same region that is *not* INSIDE can be rejected.
  - Line is entirely in unseen region
- Lines whose points are on the same side of INSIDE can be rejected.

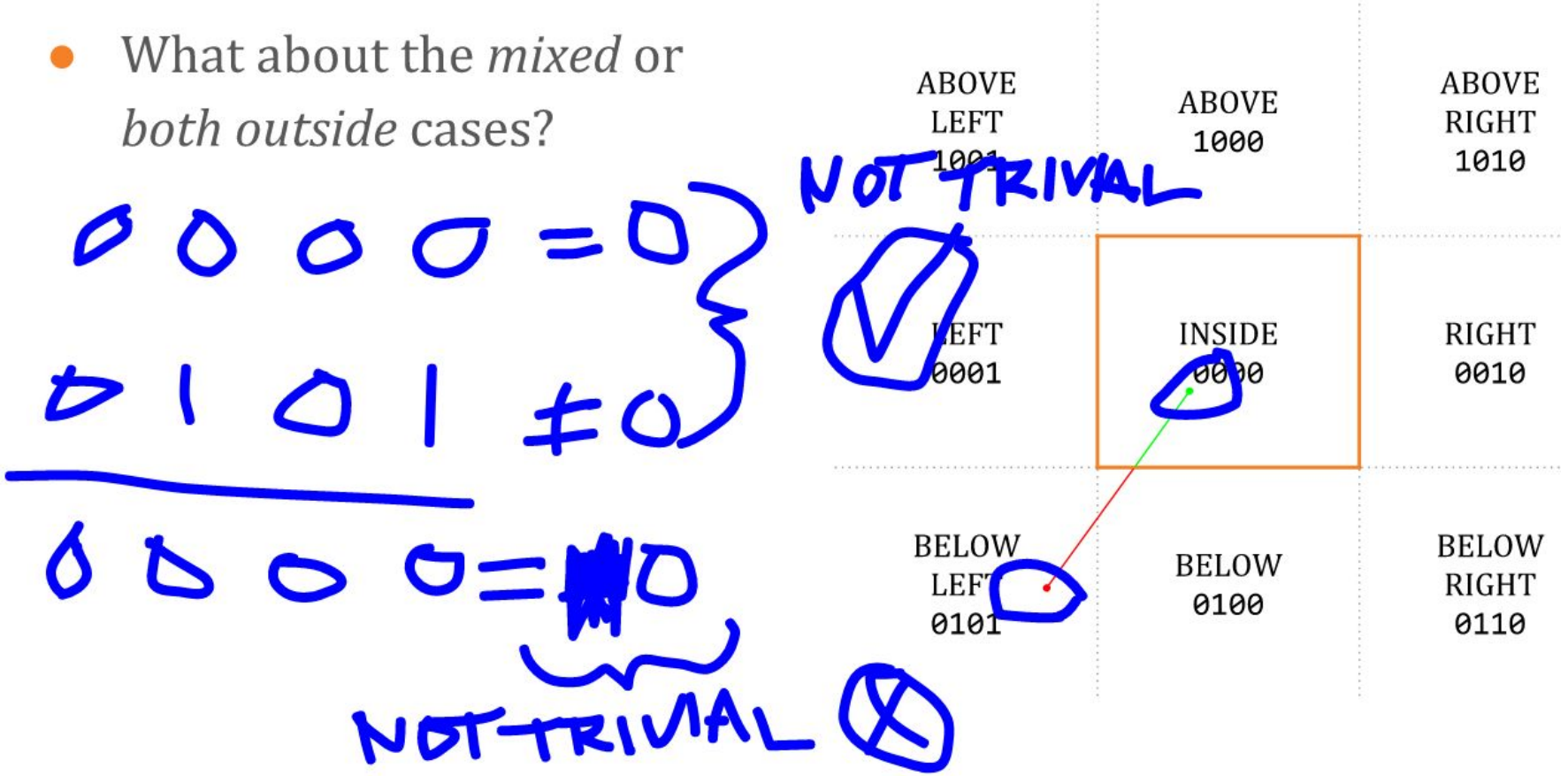
Cohen-Sutherland Line Clipping

- What about trivial rejects?
- Lines whose points are both in the same region that is *not* INSIDE can be rejected.
  - Line is entirely in unseen region.
- Lines whose points are on the same side of INSIDE can be rejected.
  - Line cannot intersect INSIDE region so nothing to draw.
- How to compute these relationships?
  - Bitwise AND of codes will be nonzero.



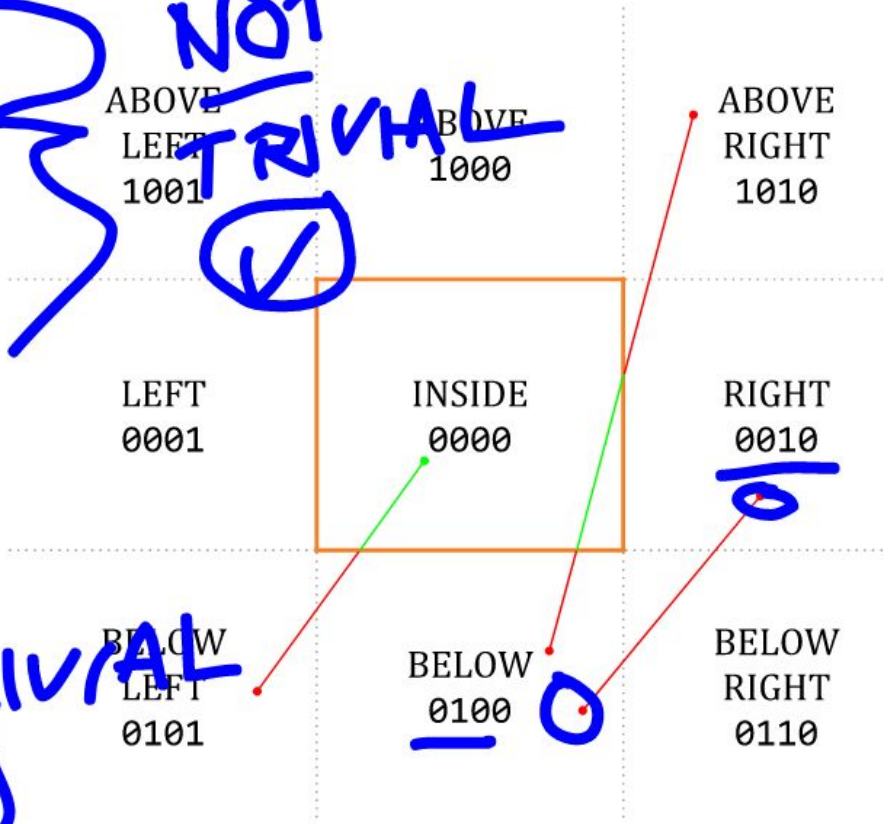
Cohen-Sutherland Line Clipping

- What about the *mixed* or *both outside* cases?



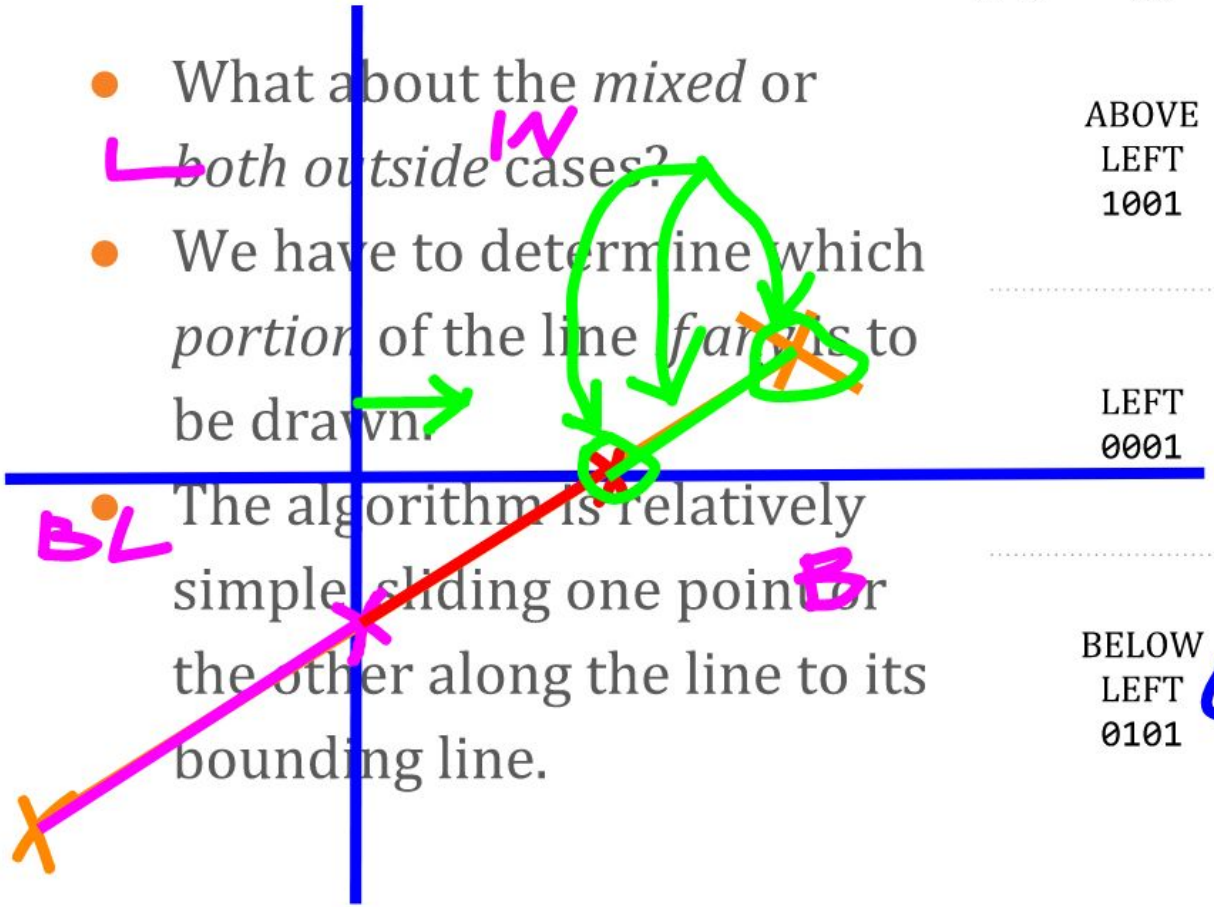
Cohen-Sutherland Line Clipping

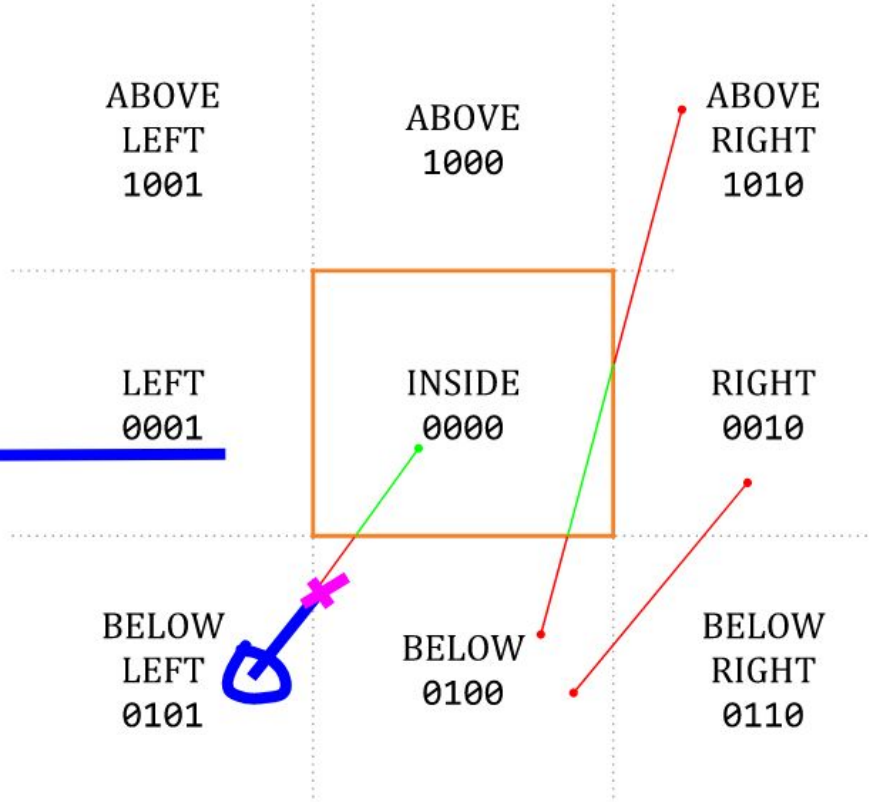
- What about the *mixed* or *both outside* cases?
- We have to determine which *portion* of the line *if any* is to be drawn

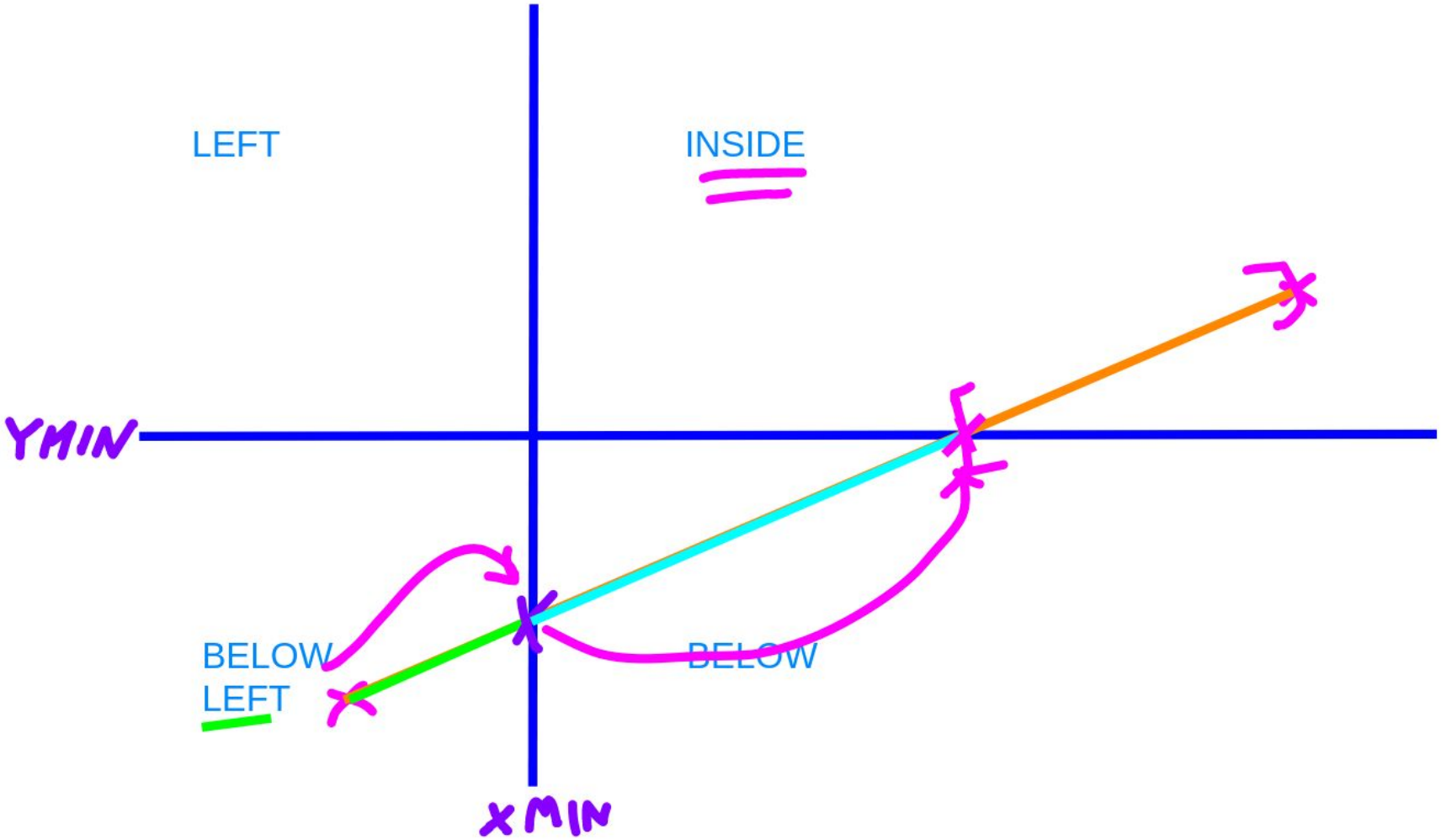


Cohen-Sutherland Line Clipping

- What about the *mixed* or *both outside* cases?
- We have to determine which *portion* of the line *far* is to be drawn.
- The algorithm is relatively simple, sliding one point or the other along the line to its bounding line.

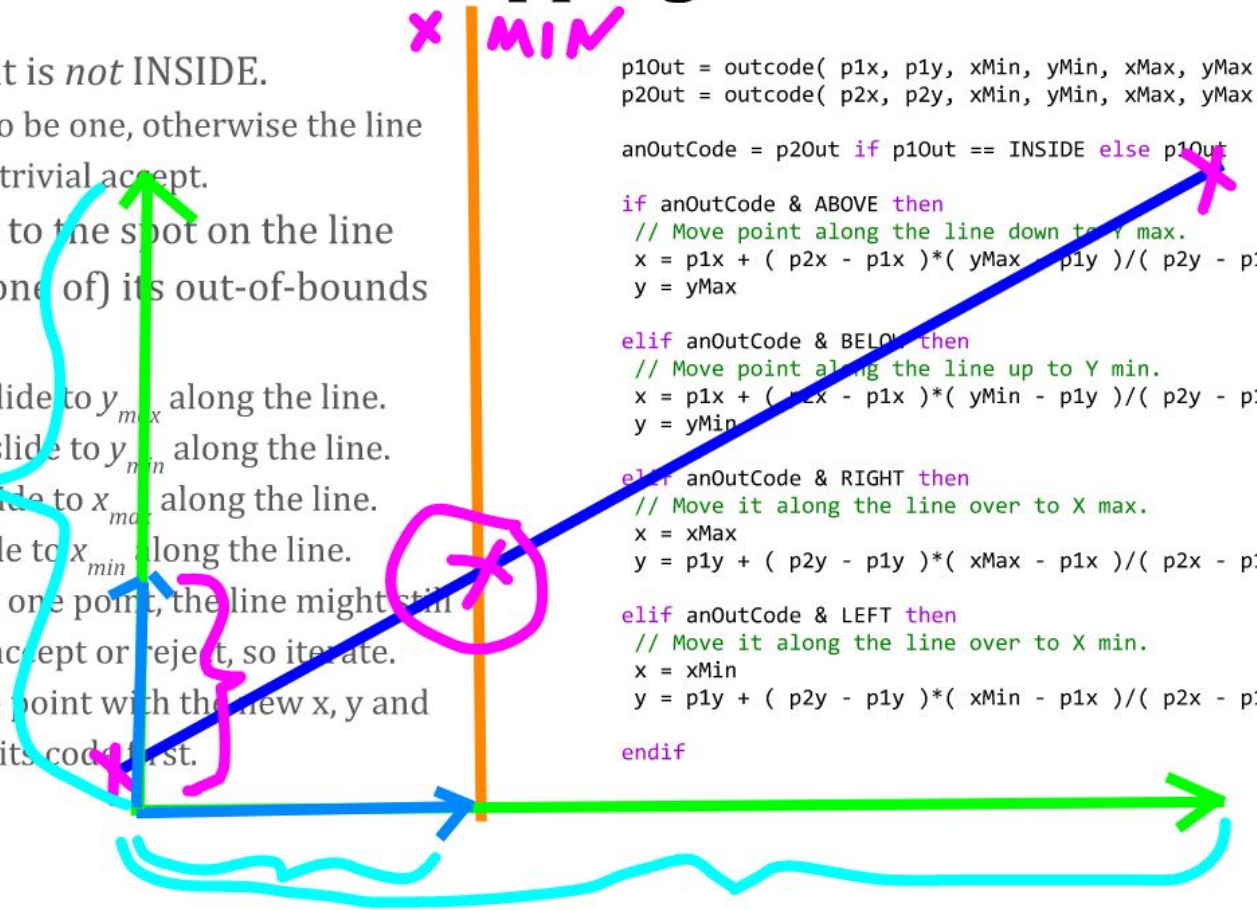




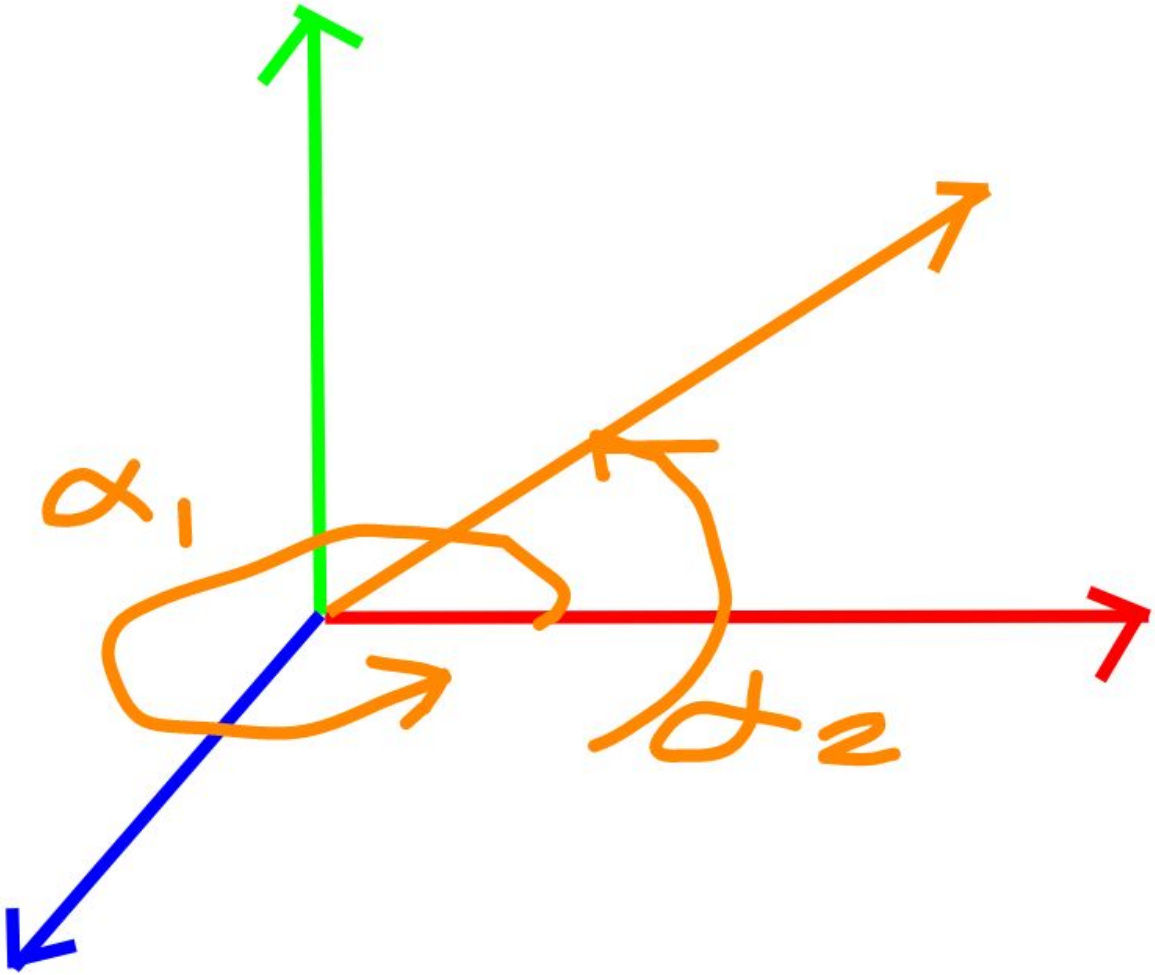

LEFT
INSIDE
YMIN
BELOW
LEFT
BELOW
XMIN

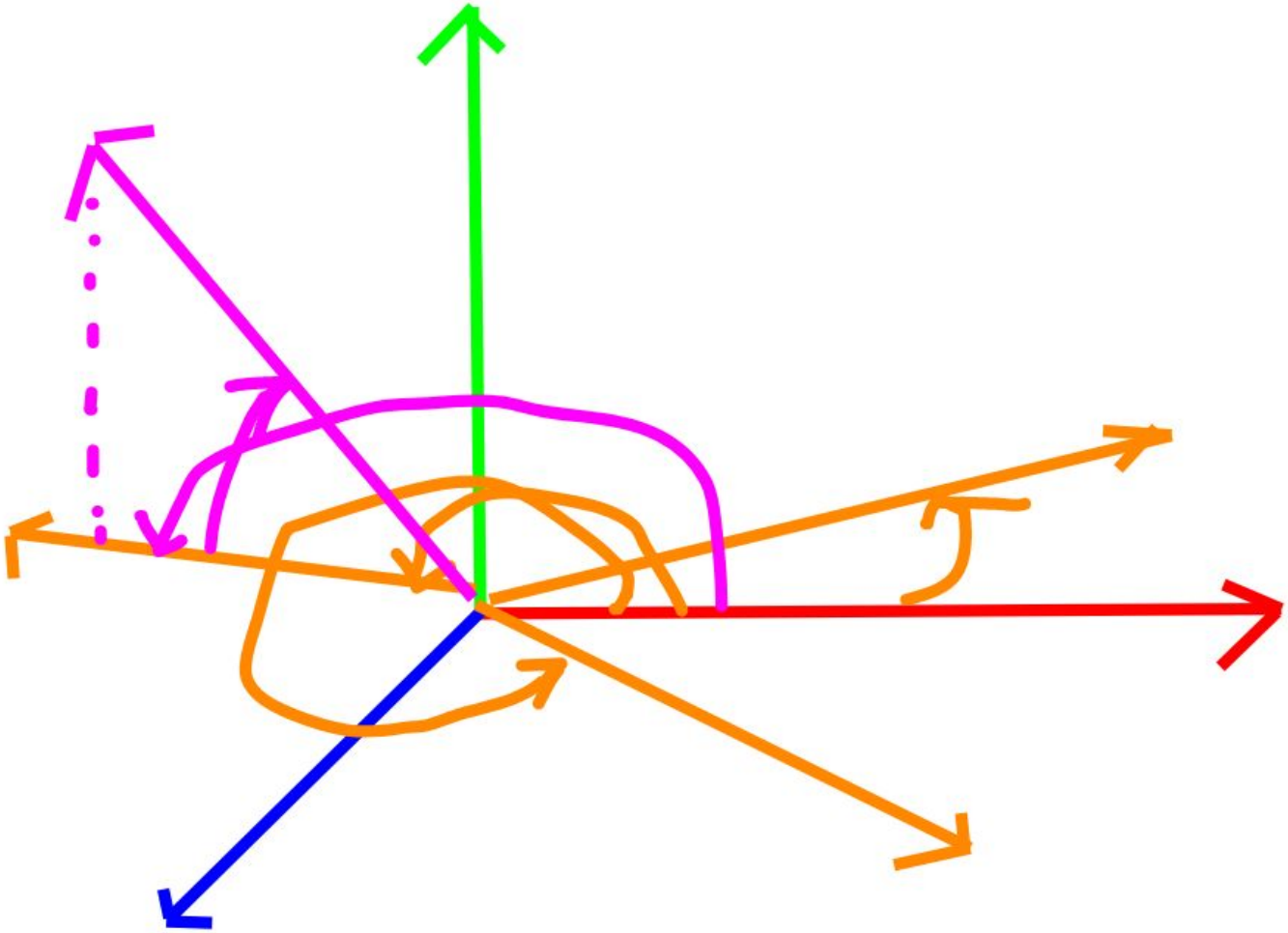
Cohen-Sutherland Line Clipping

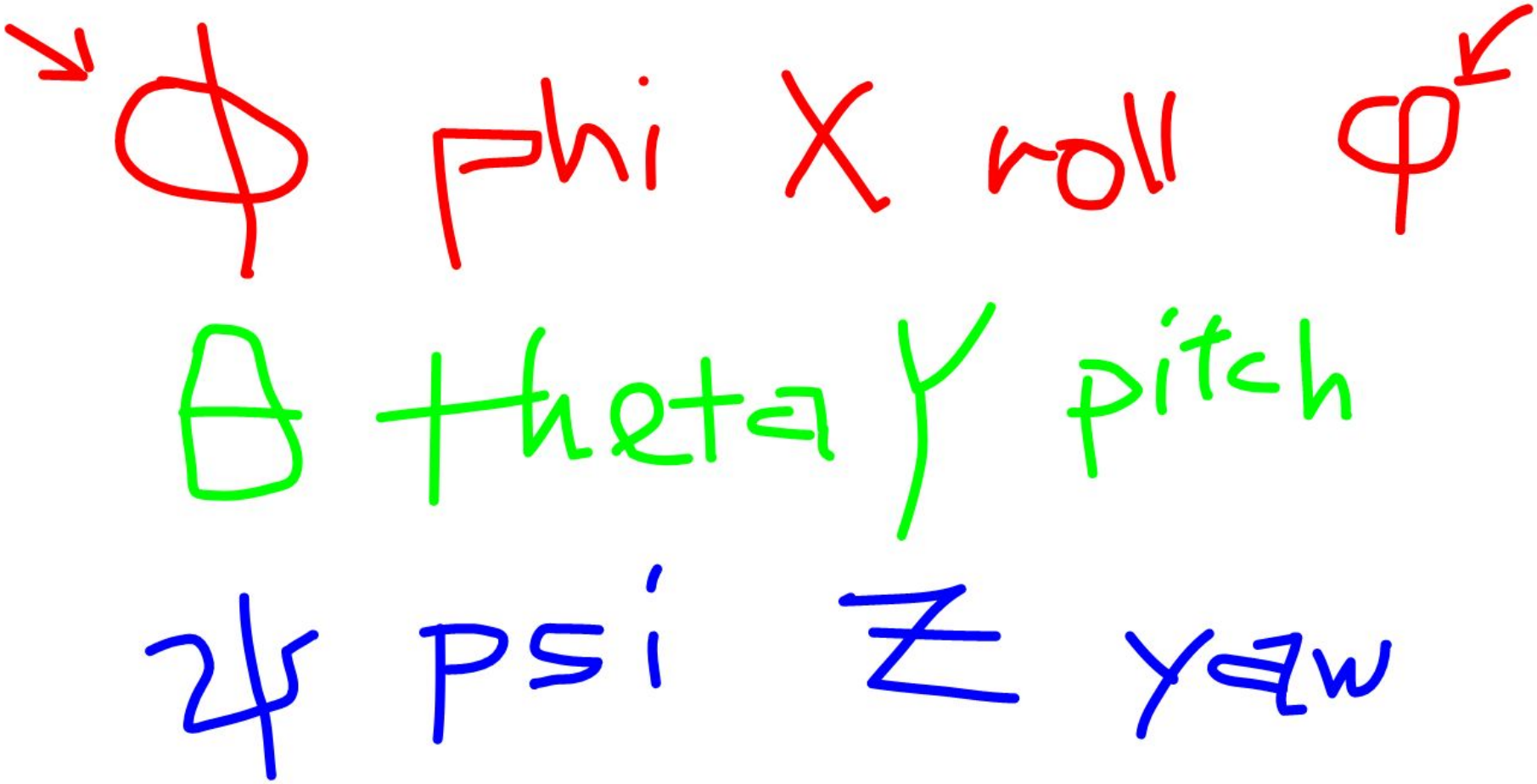
- Pick a point that is *not* INSIDE.
  - There has to be one, otherwise the line would be a trivial accept.
- Slide that point to the spot on the line that removes (one of) its out-of-bounds problems.
  - If ABOVE, slide to y_{max} along the line.
  - If BELOW, slide to y_{min} along the line.
  - If RIGHT, slide to x_{max} along the line.
  - If LEFT, slide to x_{min} along the line.
- Even after sliding one point, the line might still be non-trivial to accept or reject, so iterate.
  - Replace the point with the new x, y and recompute its code first.

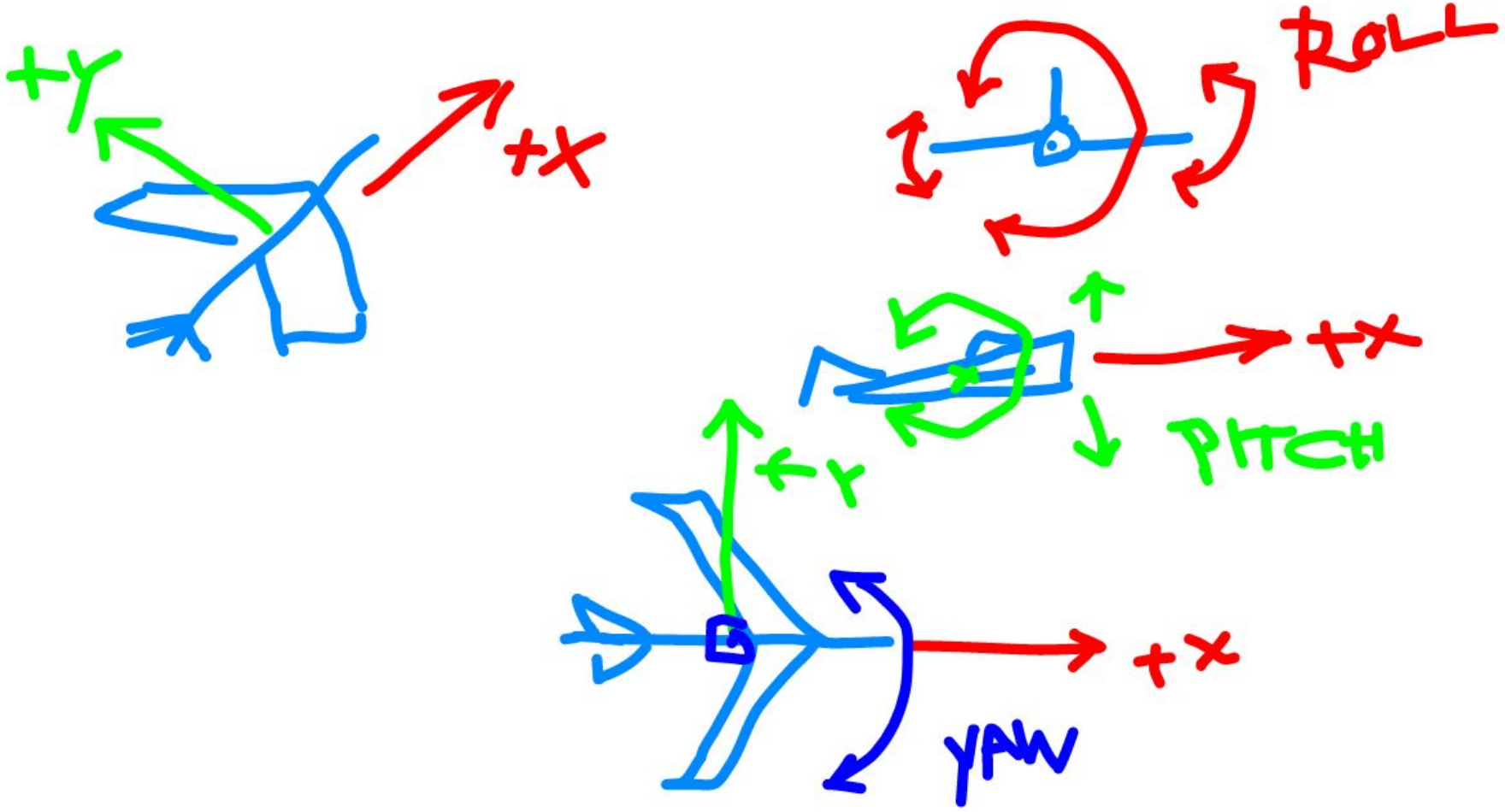


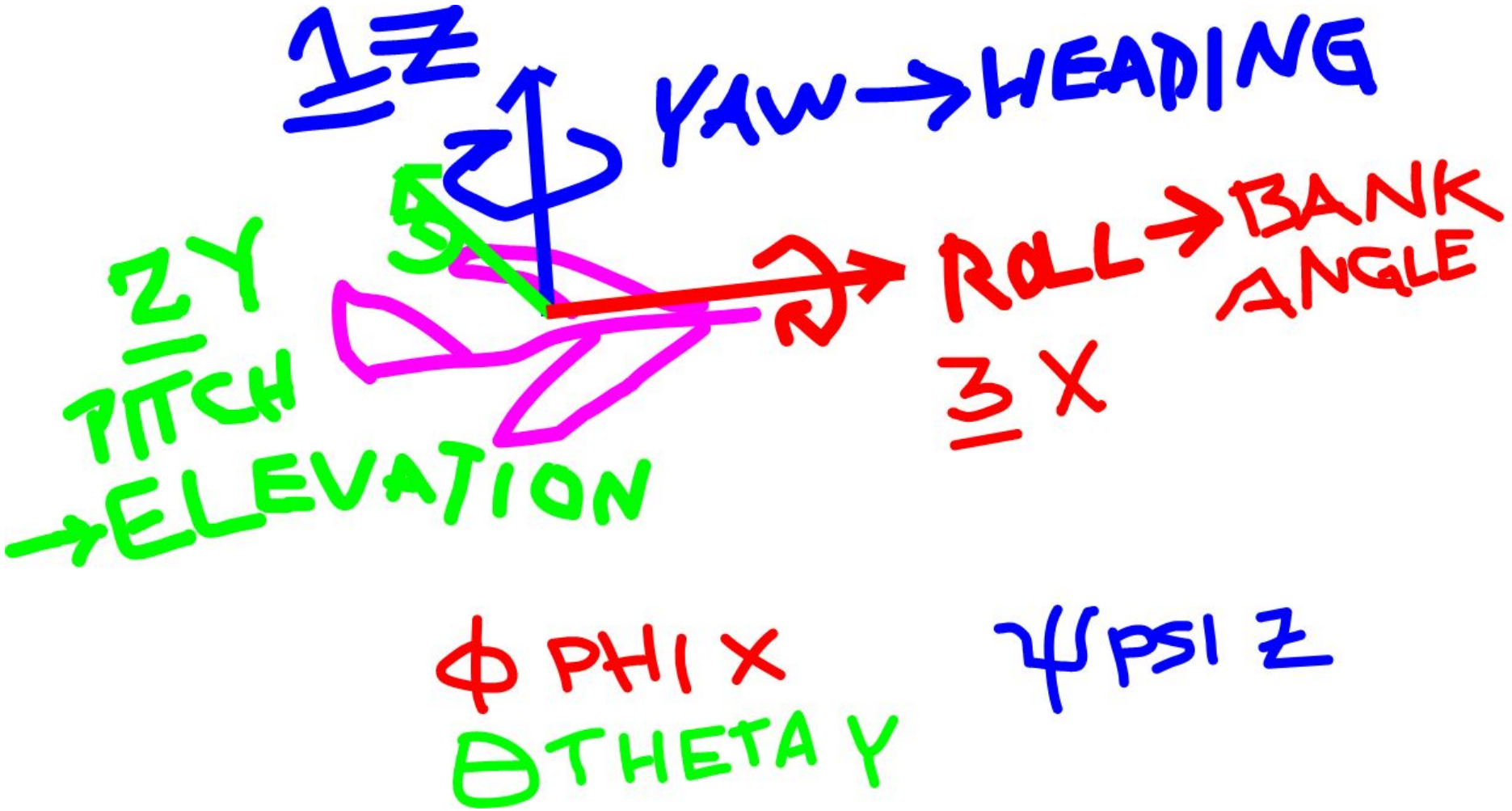

```
p1Out = outcode( p1x, p1y, xMin, yMin, xMax, yMax )
p2Out = outcode( p2x, p2y, xMin, yMin, xMax, yMax )

anOutCode = p2Out if p1Out == INSIDE else p1Out

if anOutCode & ABOVE then
 // Move point along the line down to Y max.
 x = p1x + ( p2x - p1x )*( yMax - p1y )/( p2y - p1y )
 y = yMax

elif anOutCode & BELOW then
 // Move point along the line up to Y min.
 x = p1x + ( p2x - p1x )*( yMin - p1y )/( p2y - p1y )
 y = yMin

elif anOutCode & RIGHT then
 // Move it along the line over to X max.
 x = xMax
 y = p1y + ( p2y - p1y )*( xMax - p1x )/( p2x - p1x )

elif anOutCode & LEFT then
 // Move it along the line over to X min.
 x = xMin
 y = p1y + ( p2y - p1y )*( xMin - p1x )/( p2x - p1x )

endif
```


α_1
α_2



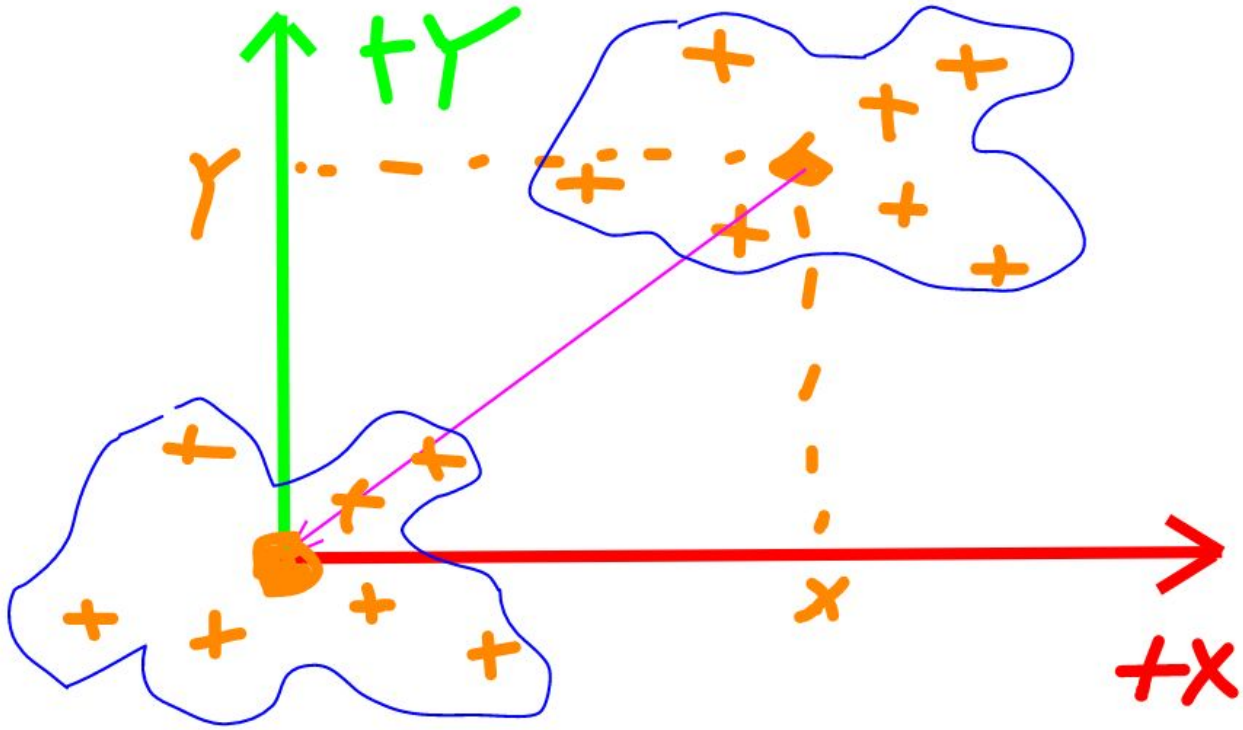

ϕ phi X roll φ
θ theta Y pitch
ψ psi Z yaw

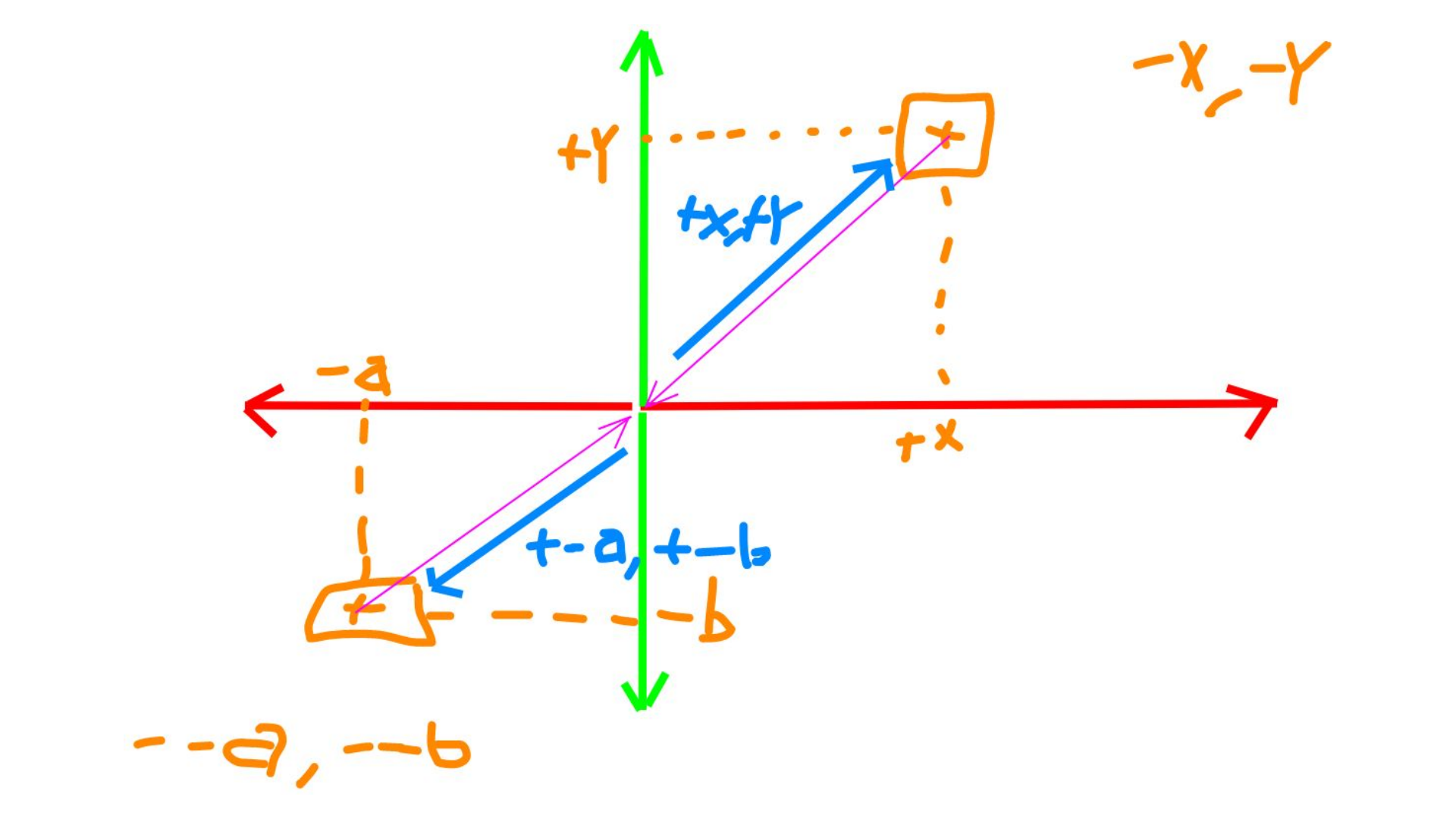

+Y
+X
ROLL
+X
PITCH
+Y
+X
YAW


1 Z
YAW → HEADING
2 Y
PITCH
→ ELEVATION
ROLL → BANK ANGLE
3 X
ϕ PHI X
ΘTHETA Y
ψ PSI Z

$t_x = -x$

$t_x = -y$

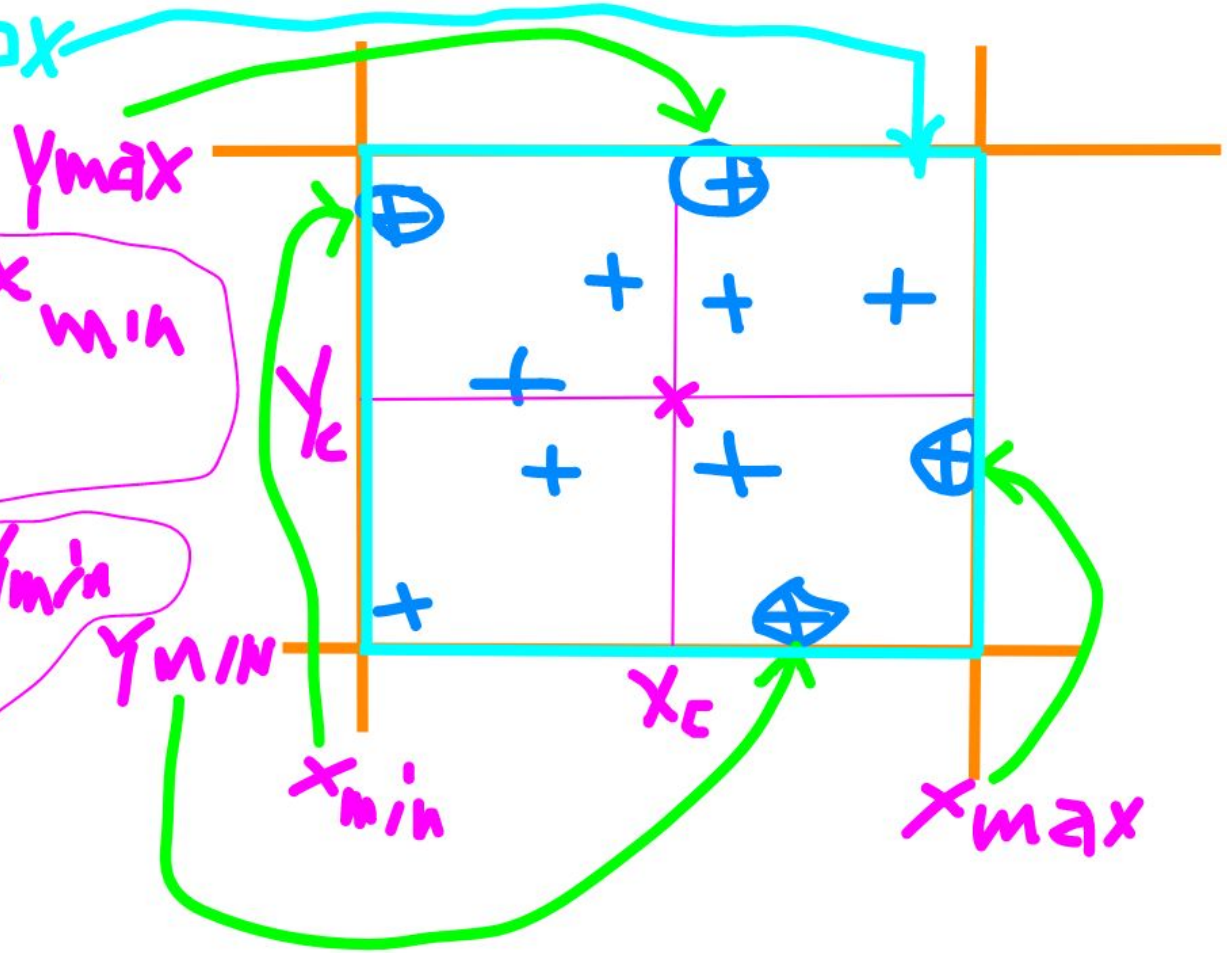



-x, -y
+y
+x, +y
-a
+x
+- a, +- b
- - - - -b
- -a, - -b

BOUNDING BOX

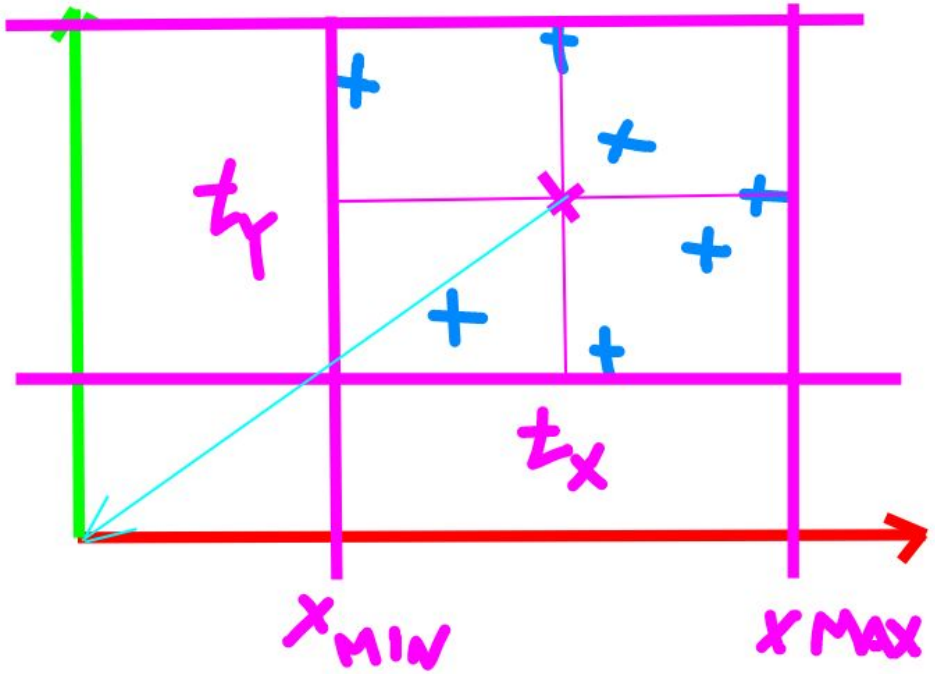
$$x_c = \frac{x_{max} + x_{min}}{2}$$

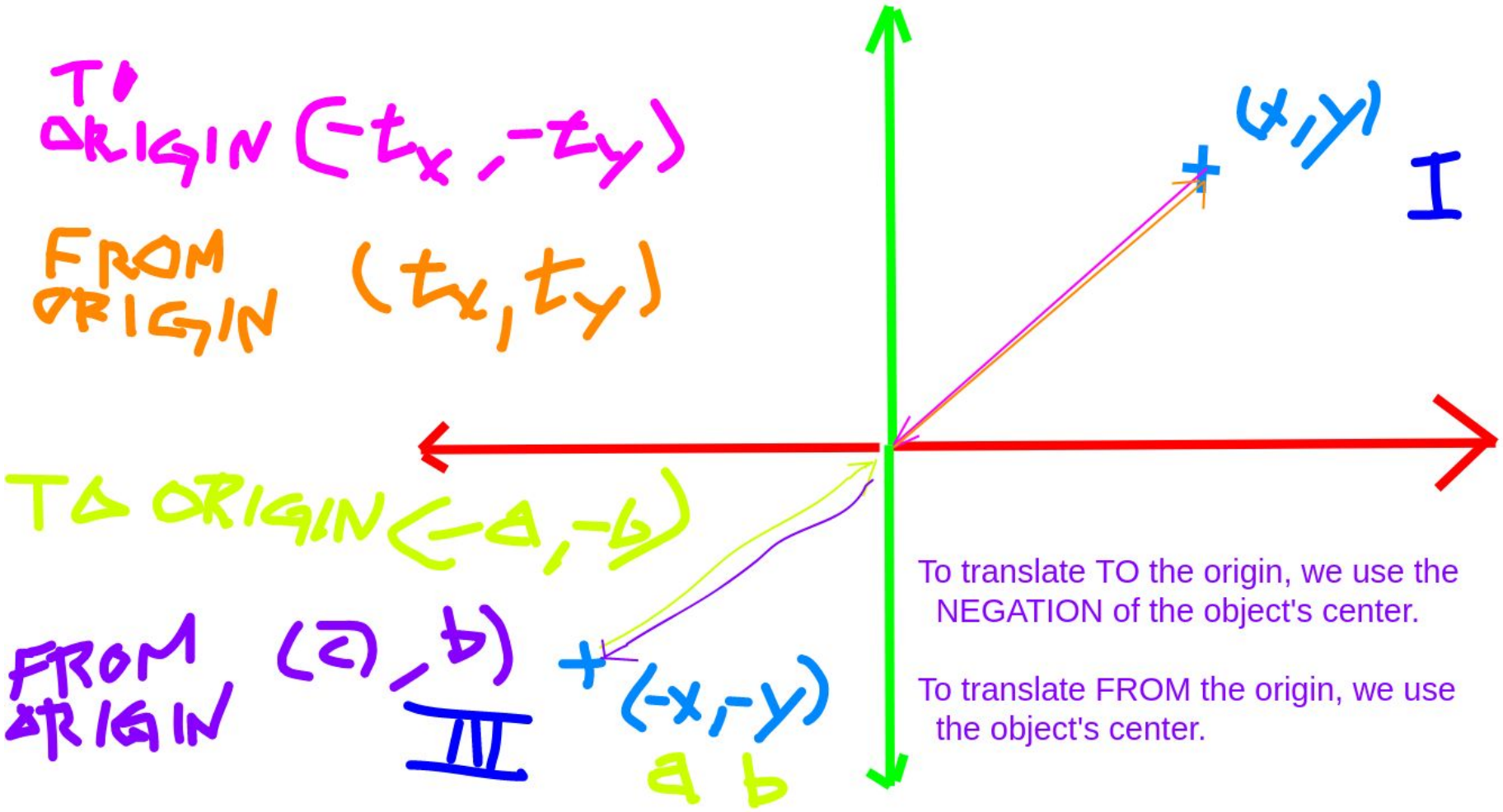
$$y_c = \frac{y_{max} + y_{min}}{2}$$



$$t_x = \frac{x_{max} + x_{min}}{2}$$

$$t_y = \frac{y_{max} + y_{min}}{2}$$

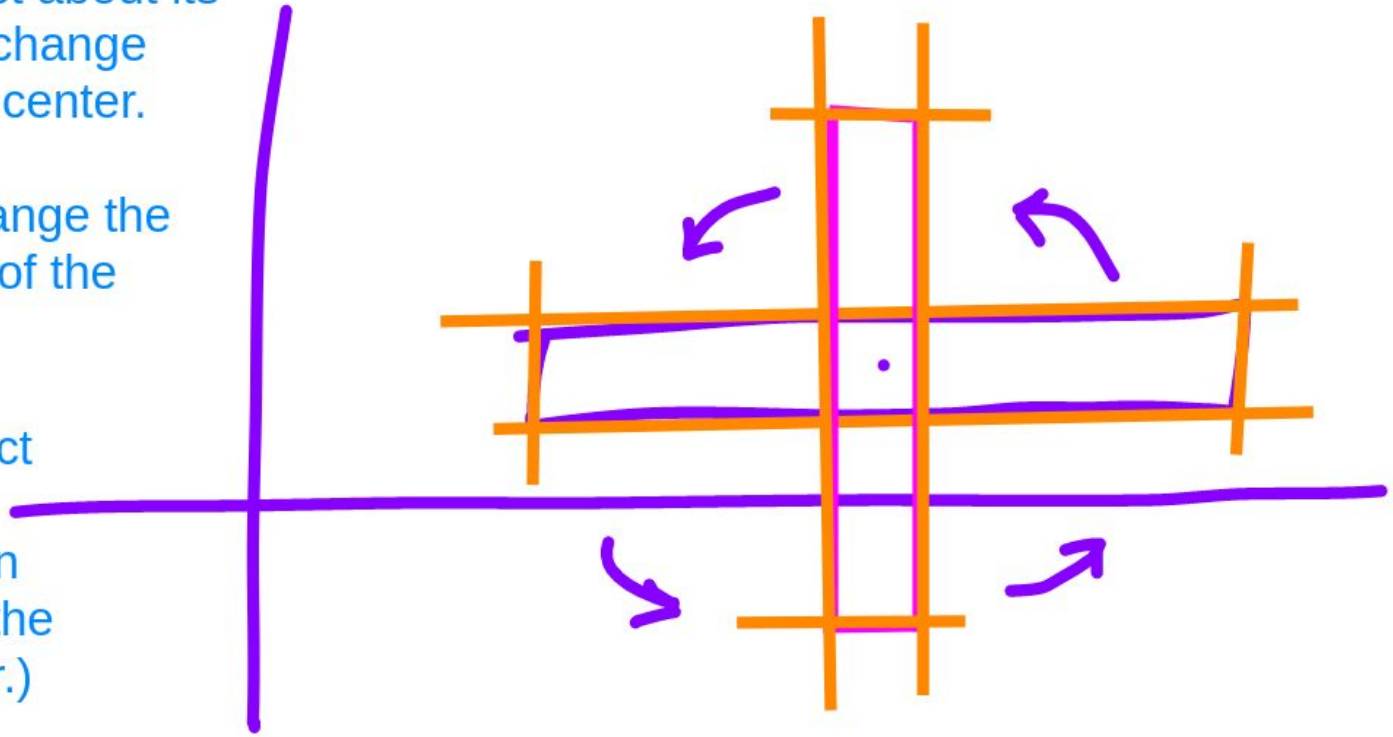



TO ORIGIN $(-t_x, -t_y)$
FROM ORIGIN (t_x, t_y)
TO ORIGIN $(-a, -b)$
FROM ORIGIN (a, b)
III
$(-x, -y)$
a b
(x, y)
I
To translate TO the origin, we use the NEGATION of the object's center.
To translate FROM the origin, we use the object's center.

Rotation of an object about its center does NOT change the location of the center.

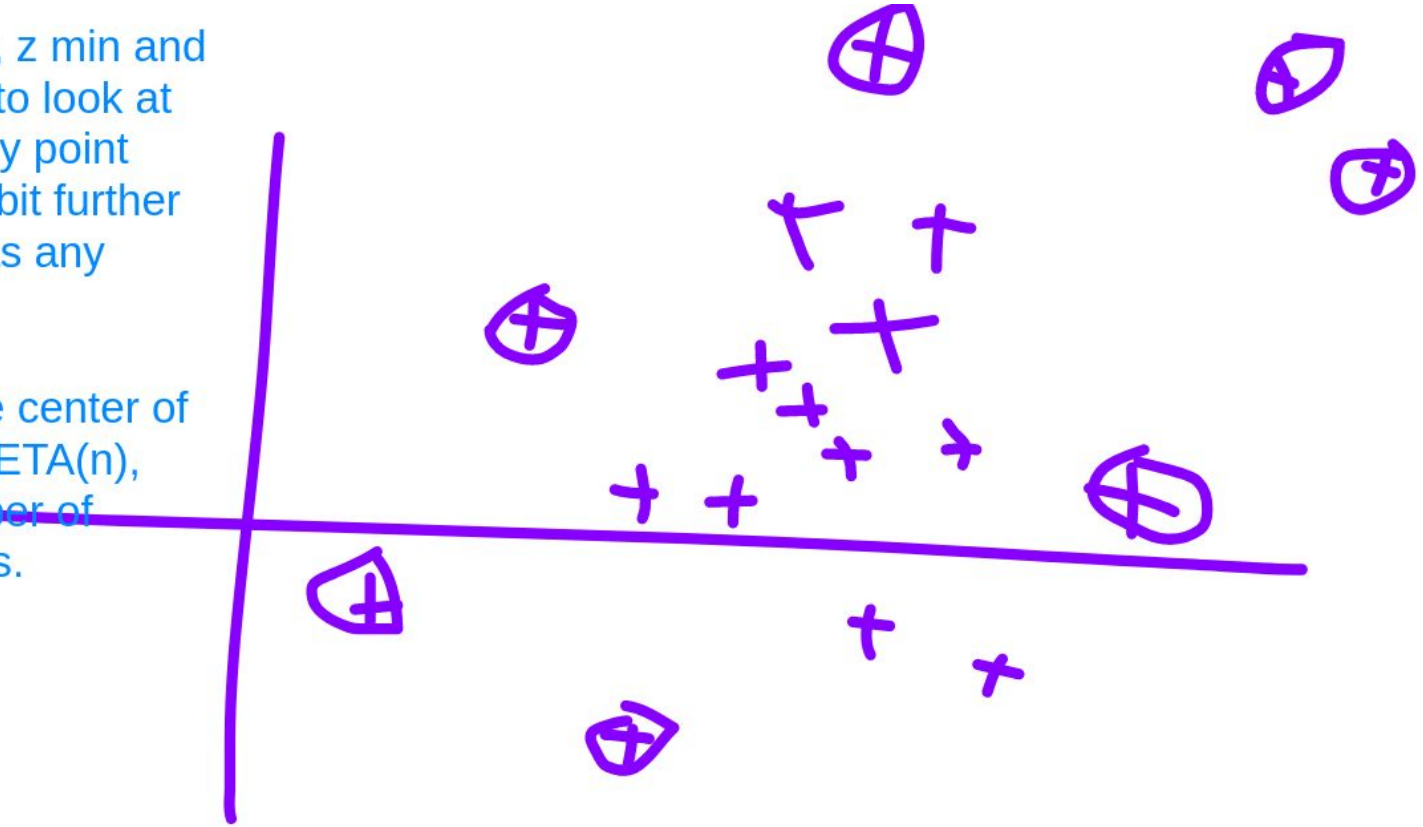
It can, however, change the BOUNDING BOX of the object.

(Suppose the object is a sphere. Any amount of rotation changes neither the BB nor the center.)



When computing x, y, z min and max, it's necessary to look at EVERY point (as any point could be just a little bit further out along any axis as any previous point).

Therefore, getting the center of an object is BIG-THETA(n), where n is the number of points the object has.

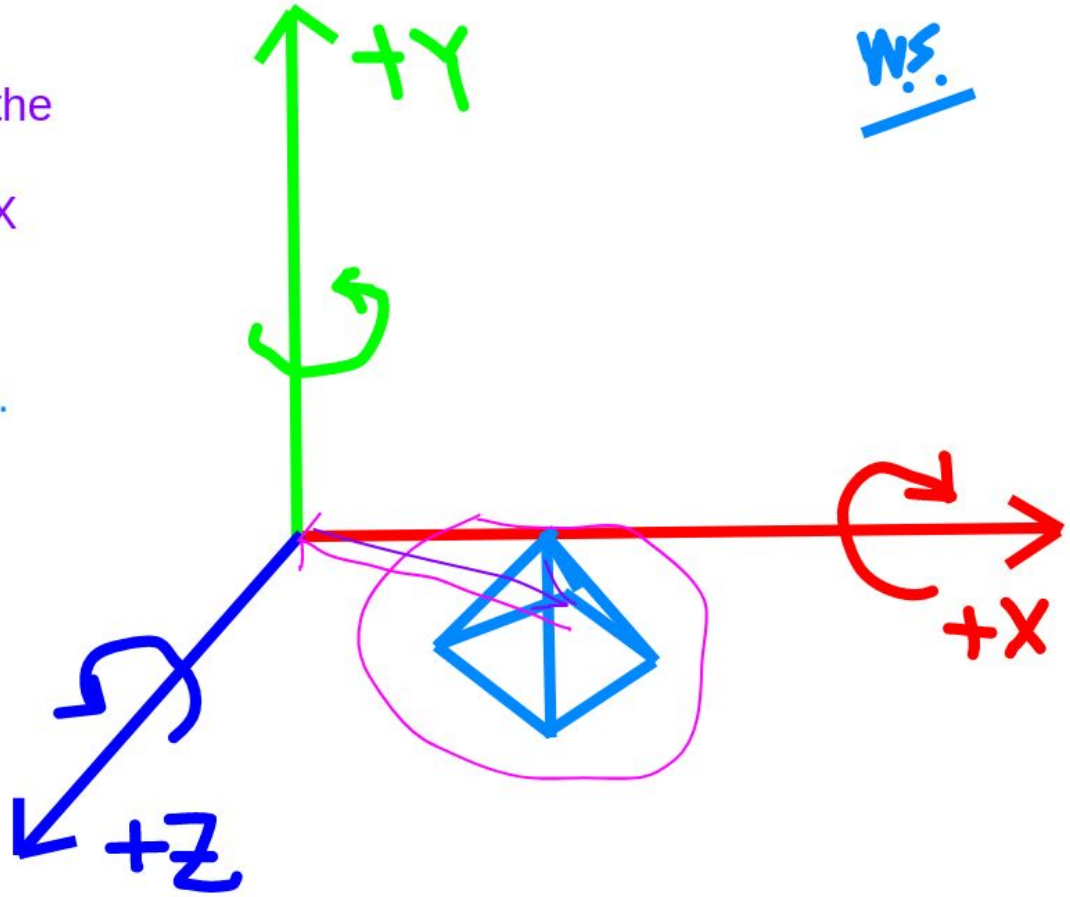


Euler rotation is done in WORLD SPACE: (1) translate the center of the object to the origin, (2) do the three rotations, about the Z, then Y, then X axes, (3) translate the center of the object back to its original position.

All of this is done in WORLD SPACE. These are general transforms that are done BEFORE we convert to SCREEN SPACE.

(Line clipping, on the other hand, is done in SCREEN SPACE.)

(We want to know that the lines we are drawing are going to fit in the space allocated for the screen.)



- find center — $\Theta(n)$
- compute r — $\Theta(1)$
- compute e — $\Theta(1)$
- transform each point — $\Theta(n)$

(n is # pts)

18·n ops

$\Theta(n)$

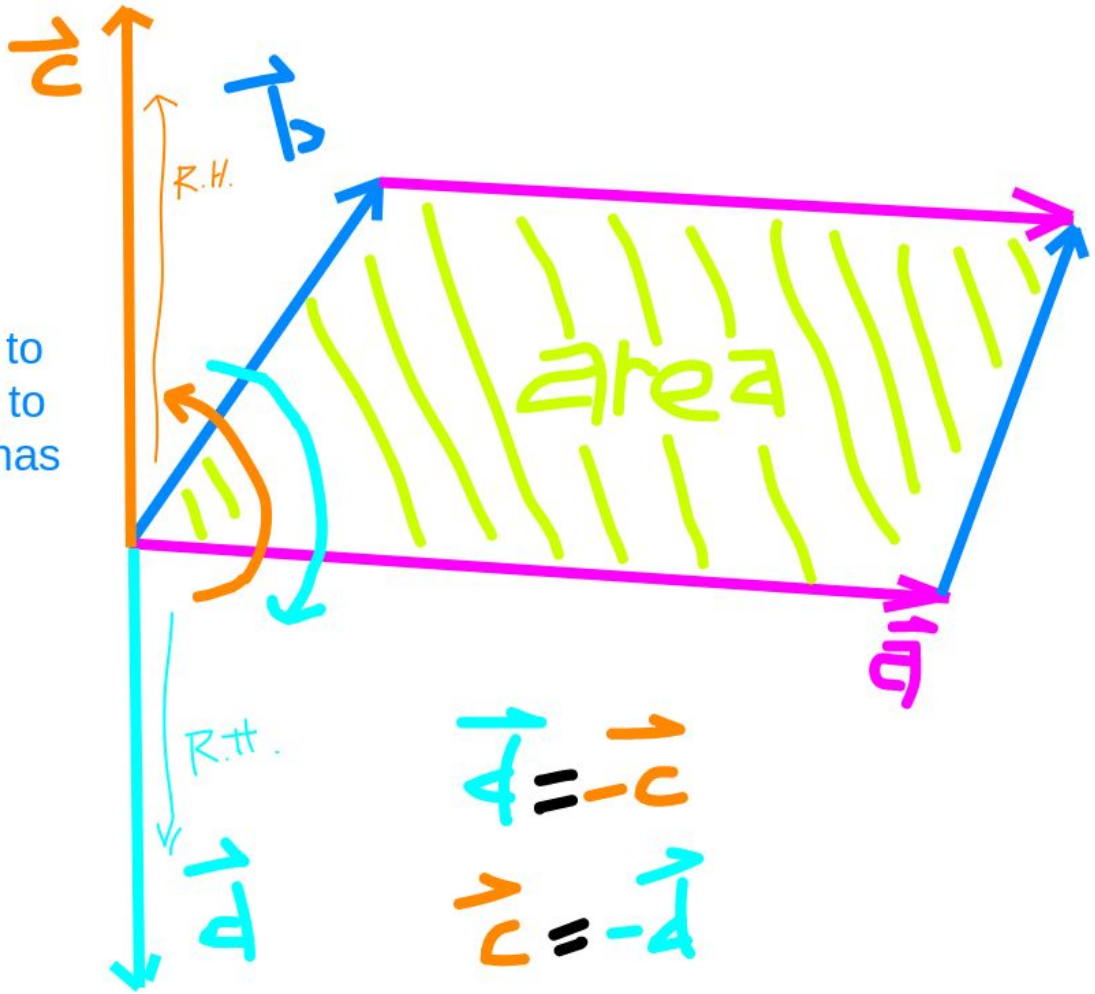
Euler rotation time complexity.

The cross product of two vectors is the Right-Hand perpendicular vector from the first to the second that has magnitude equal to the numeric value of the area bounded by the the first two vectors.

Reversing the order of the operands to the cross product causes the result to go in the opposite direction. It still has the same length, though.

$\vec{a} \times \vec{b} \rightarrow \vec{c}$

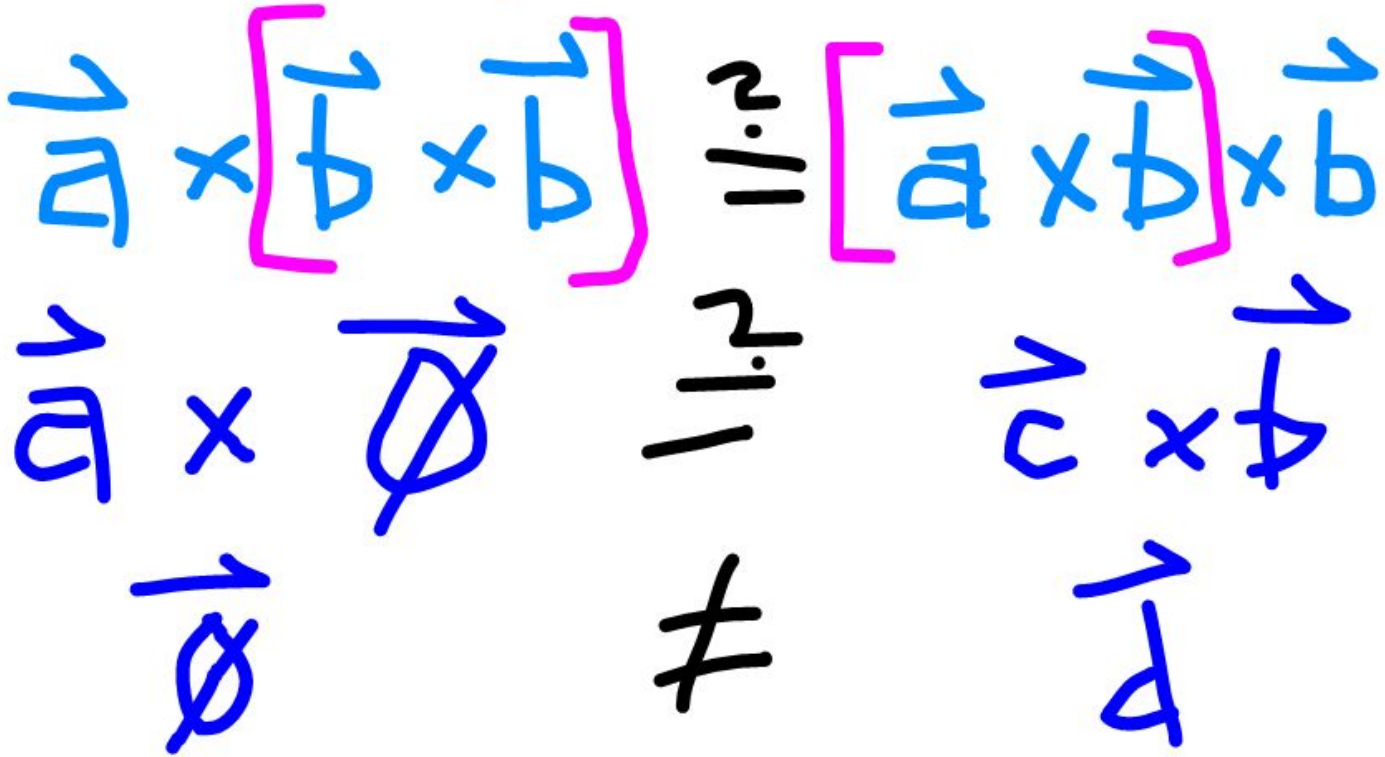
$\vec{b} \times \vec{a} \rightarrow \vec{d}$



$\vec{d} = -\vec{c}$

$\vec{c} = -\vec{d}$

Is the Cross Product operation Associative?



The Cross Product is NOT associative in general as this example shows.

The Cross Product of ANY vector with itself is always the Zero vector since there is no area between a vector and itself. The result therefore has zero magnitude.

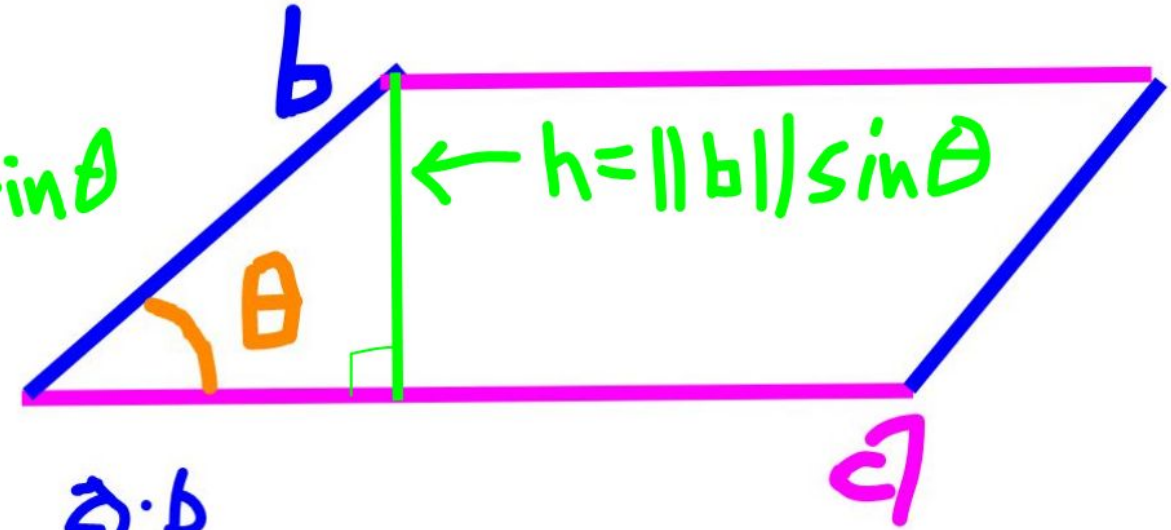
Area = ||a|| * ||b|| * sin(theta)

||aXb|| = ||a|| * ||b|| * sin(theta)

$$\sin\theta = \frac{\|a \times b\|}{\|a\| \cdot \|b\|}$$

$$A = w \cdot h$$

$$\|a\| \|b\| \sin\theta$$



$$\cos\theta = \frac{a \cdot b}{\|a\| \cdot \|b\|}$$

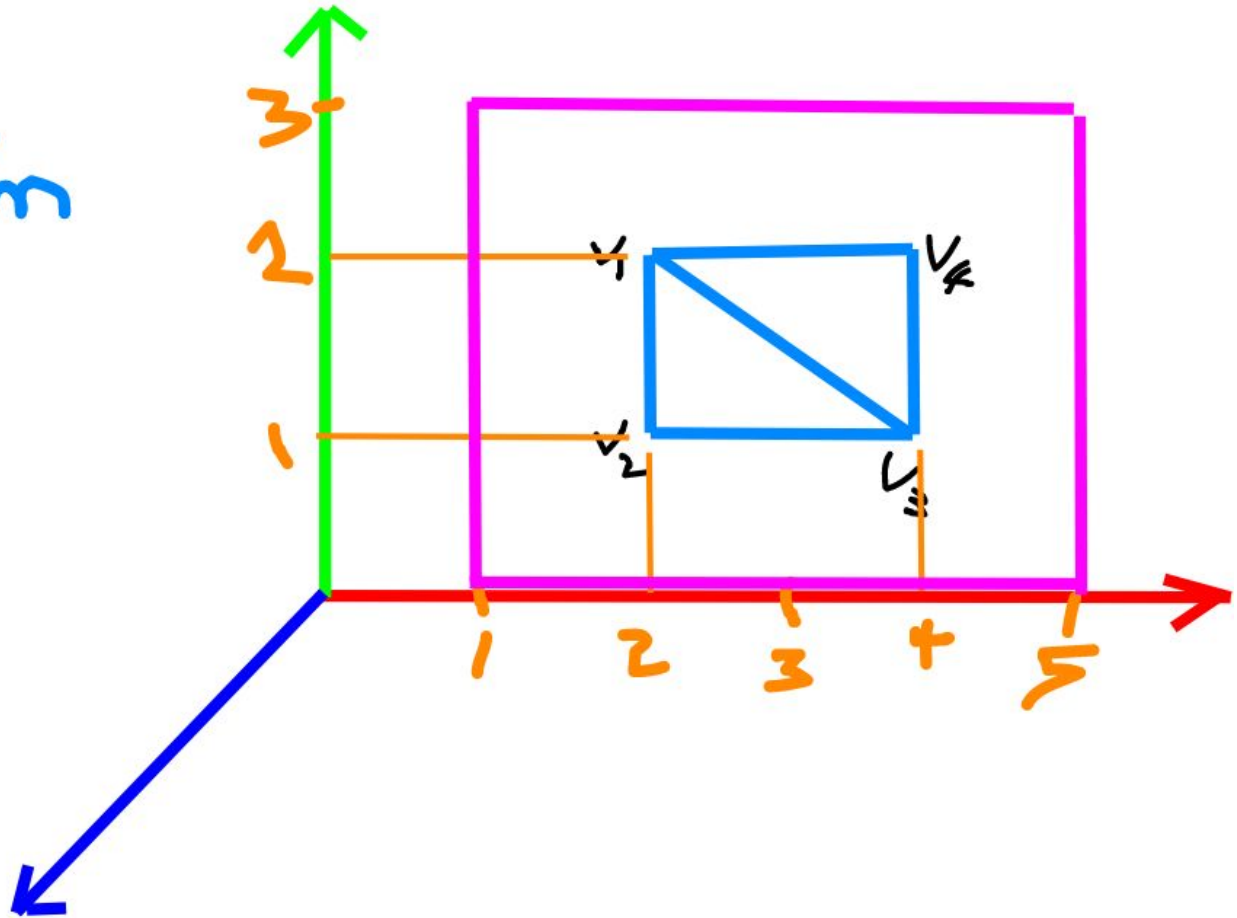
```
f  1  2  3
f  3  4  1

v  2.0  2.0  0.0
v  2.0  1.0  0.0
v  4.0  1.0  0.0
v  4.0  2.0  0.0
```

svfm

The world window range is

w 1.0 5.0 0.0 3.0

goes in param file

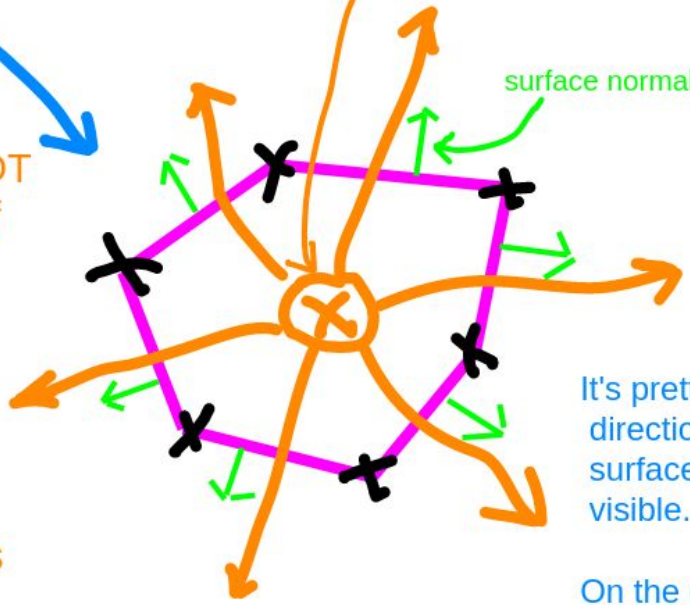


camera position and camera direction

(This camera is on the OUT-SIDE and therefore at the FRONT of the surfaces.)

A camera position that is on the INSIDE looking out (and therefore at the BACK of the surfaces).

surface normals



INSIDE looking out, the column is NOT visible; we're on the "wrong" side of it.

OUTSIDE looking towards it, the column IS visibile.

We detect this condition via the camera position and what's know as Surface Normals.

It's pretty clear that when the camera direction is "sort of OPPOSITE" the surface normal, the surface will be visible.

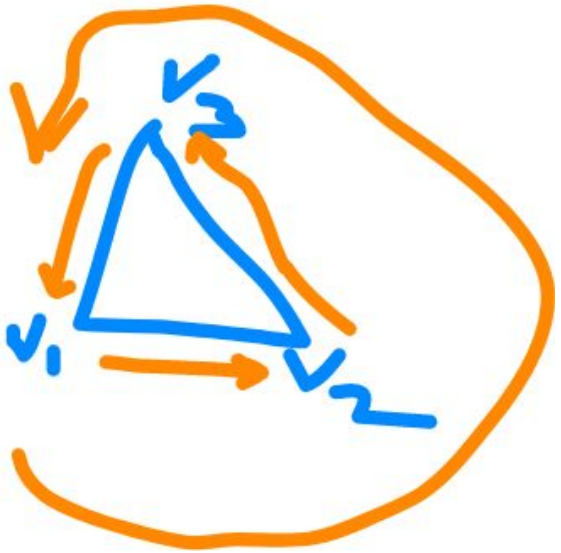
On the other hand, when the camera direction is "sort of ALIGNED" with the surface normal, the surface will be NOT visible.

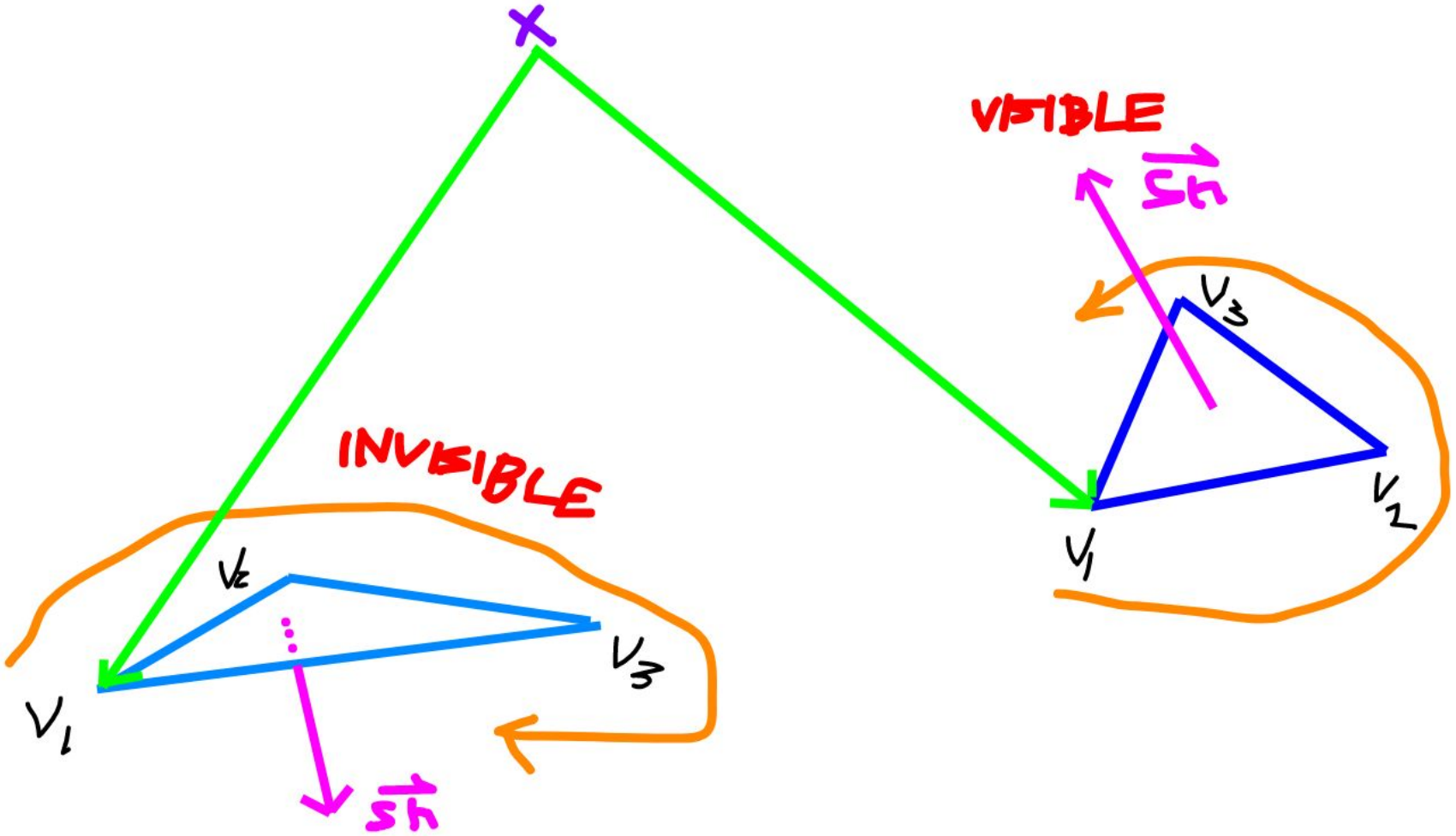
Backface Culling of Triangles

How do we know if a triangle should be drawn?

Triangles have two DIFFERENT sides, a front and a back.

We draw triangles ONLY if the FRONT of the triangle is facing the camera position. If the BACK of the triangle is facing the camera position, the triangle is INVISIBLE and should not be drawn.

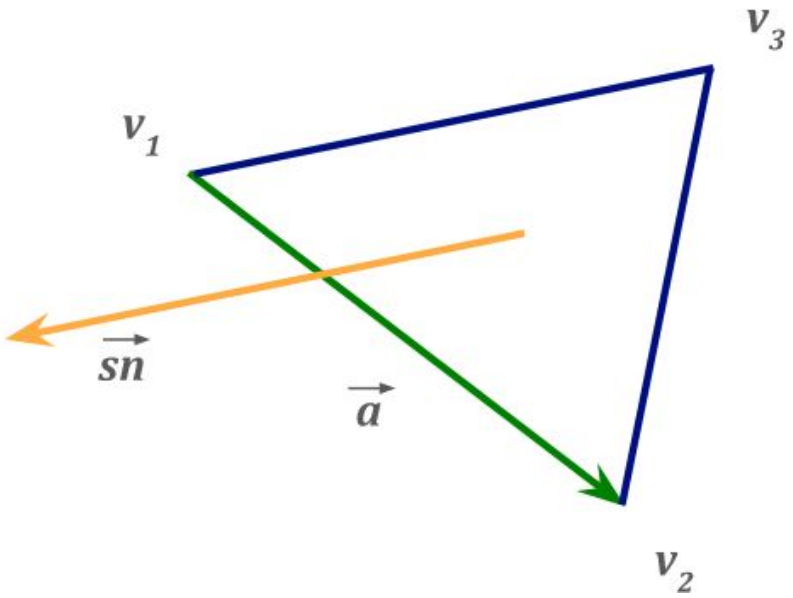



X
VISIBLE
$\vec{s_t}$
V_3
V_2
V_1
INVISIBLE
V_k
V_3
V_1
$\vec{s_h}$

Compute the Vector $\vec{a}$ from v_1 to v_2 …

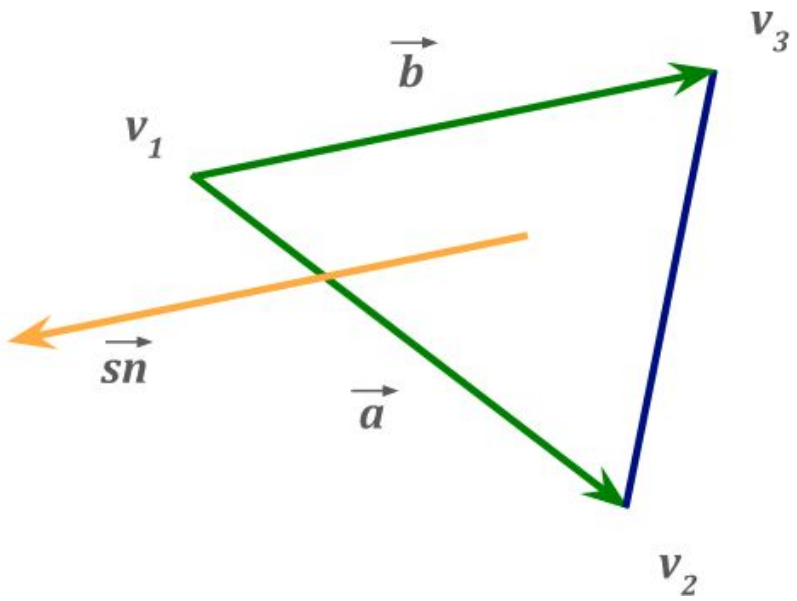
The vector a is the point v2 MINUS the point v1.

ax = v2x - v1x
ay = v2y - v1y
az = v2z - v1z
aw = 0 (which is v2w - v1w)



And the Vector $\vec{b}$ from v_1 to v_3 …

The vector b is v3 - v1 …



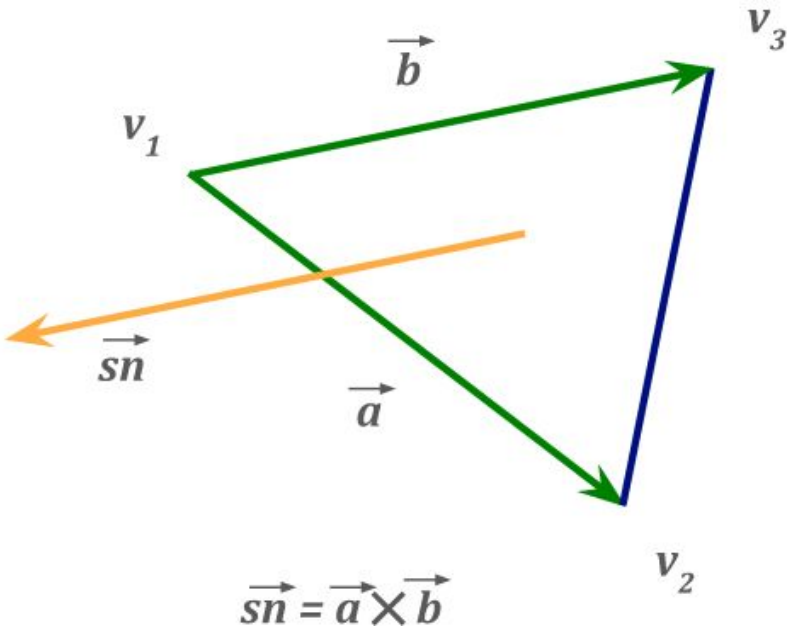
To get the Surface Normal.

Remember the definition of cross product:

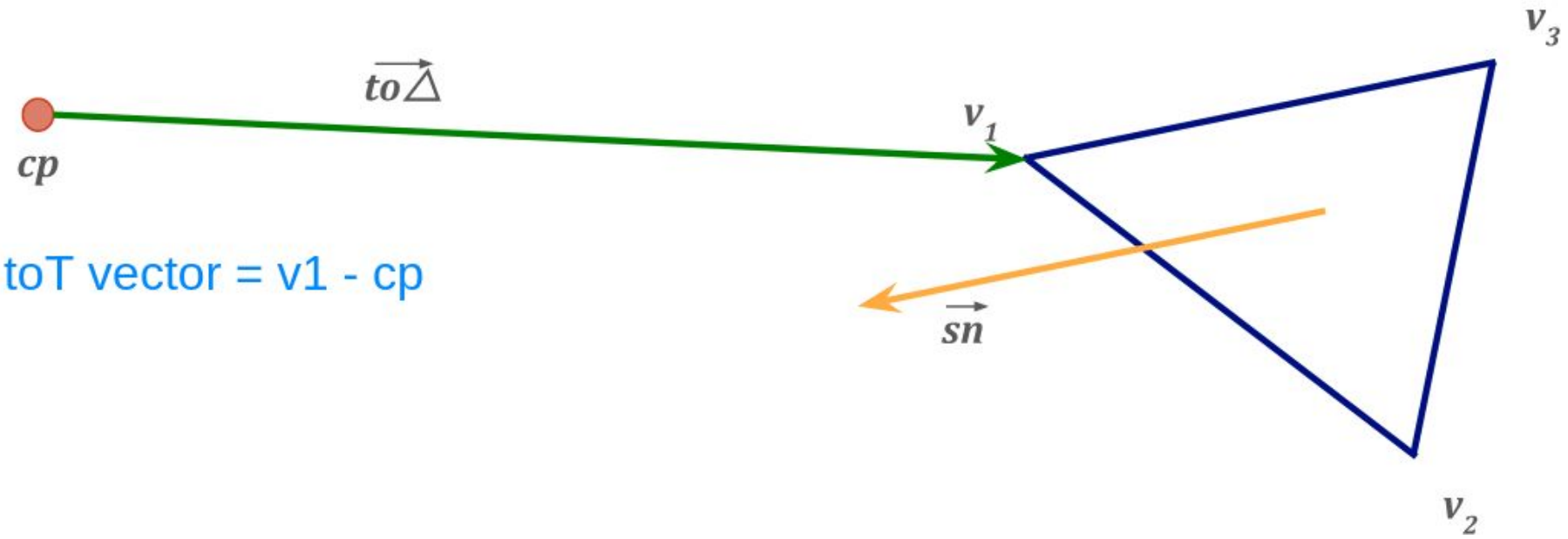
i	j	k
ax	ay	az
bx	by	bz

sn.x = ay*bz - az*by
sn.y = -(ax*bz - az*bx)
sn.z = ax*by - ay*bx

(The coefficients of i, j, and k, respectively, when the determinant of the matrix is calculated.)



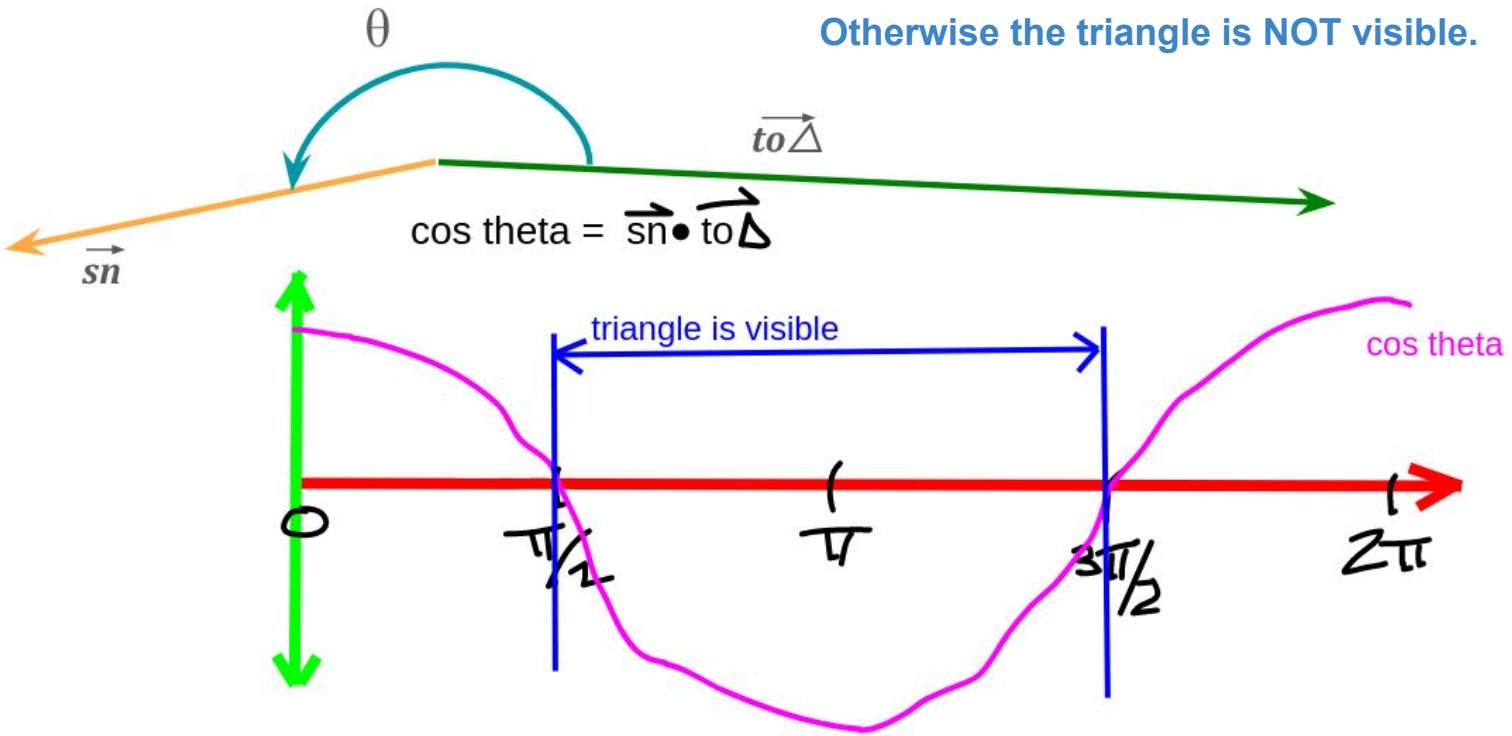
… Compute the Vector from the CP to v_1.



… To make the Angle θ more obvious.

If the dot product of the surface normal and the “to triangle” vector is STRICTLY less than zero, the triangle is VISIBLE.

Otherwise the triangle is NOT visible.



Backface Culling of a Triangle

1. Compute the vector a from v1 to v2.

2. Compute the vector b from v1 to v3.

3. Compute the surface normal sn = a X b.

4. Compute the toT vector from the camera position to v1.

5. Compute dp = toT dot product sn.

6. If dp < 0, the triangle is visible, otherwise the triangle is invisible.

This seems as if it will work OK as long as we have a camera position. (We need the camera position in step 4.)

What about when we have a PARALLEL projection and THERE IS NO CAMERA POSITION?

Well, PERSPECTIVE projection becomes PARALLEL projection as the camera position moves to +inf along the Z axis.

So, to compute toT vector when doing a PARALLEL projection, we ASSUME the camera position d = +inf (= DBL_MAX in C).

Finding min, max of a list of numbers?

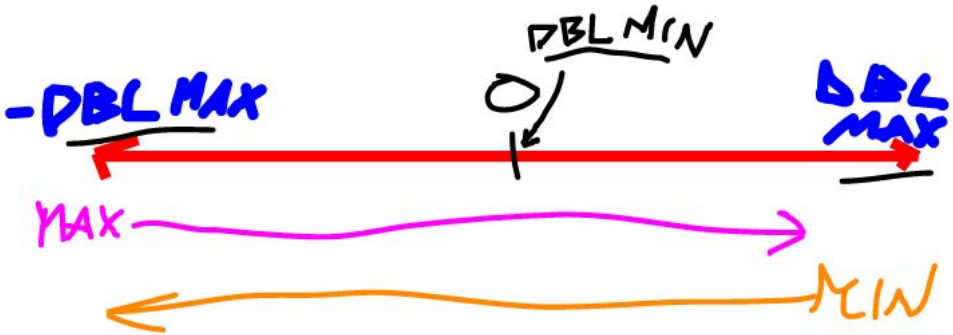
Well, if the list is unordered, we have to look at every item in the list, right? We compare each item to the currently known min and update min if necessary. We compare each item to the currently known max and update max if necessary.

What's the INITIAL value of min and max?

Well, what value is guaranteed to be >= EVERY POSSIBLE value?

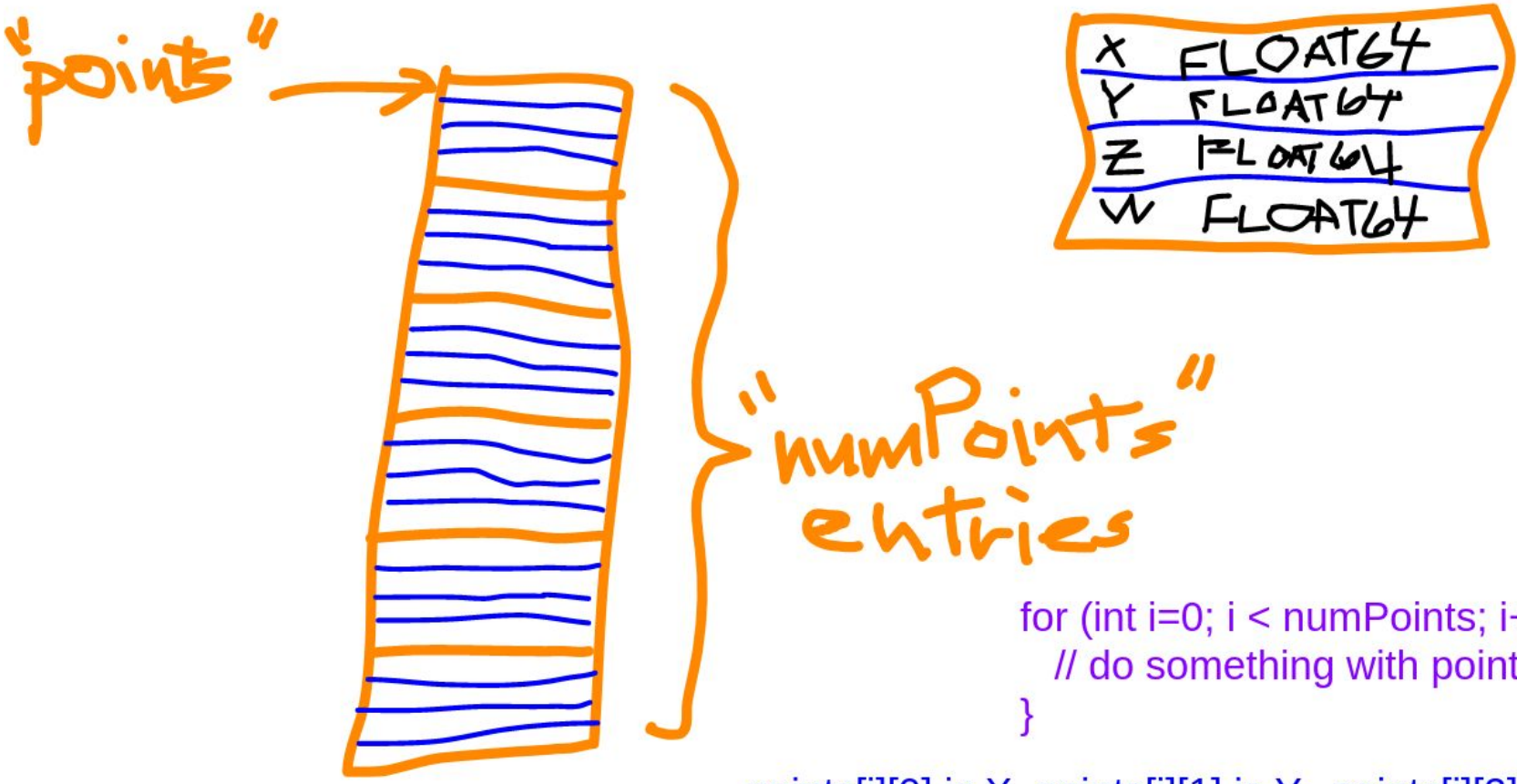
Therefore, min has to be initialized to DBL_MAX so it will be >= any possible user value.

Similarly, we initialize max to -DBL_MAX so it will be <= any possible user value.



The min and max are initialized to the extreme value in the OPPOSITE direction so they will be overridden by the user values.

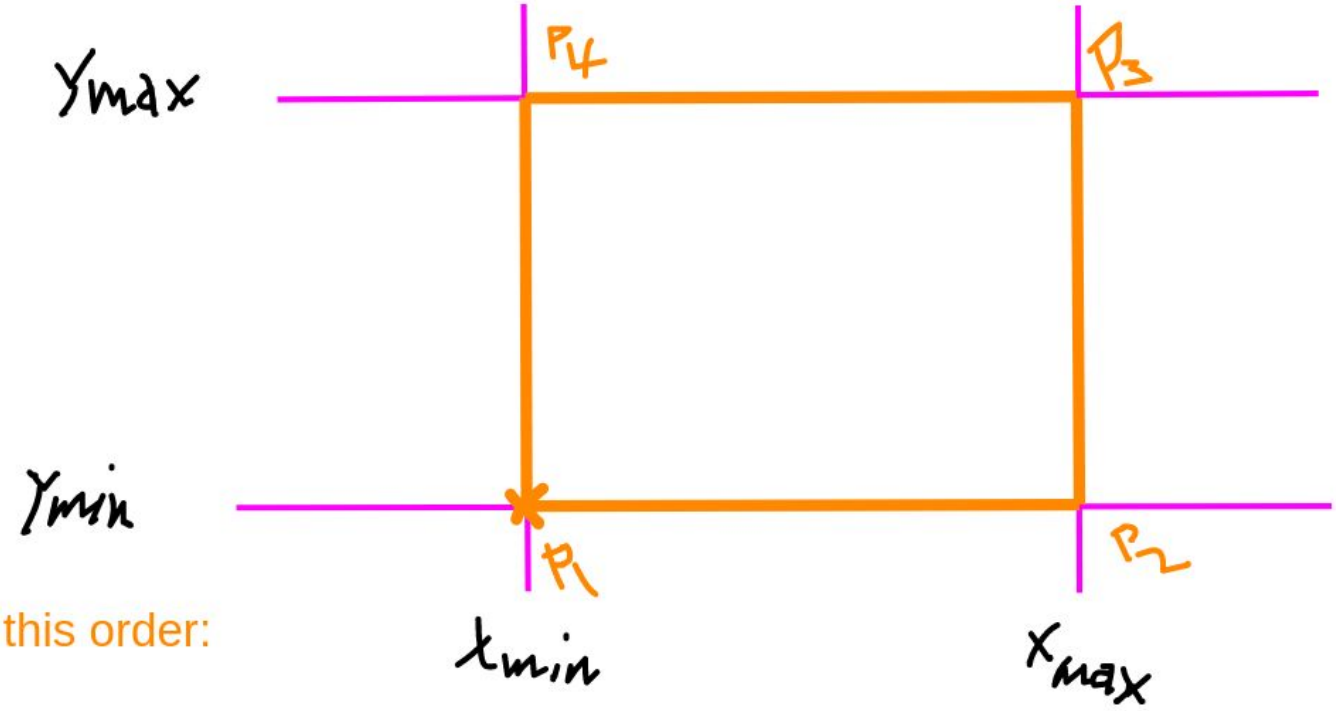
DON'T use DBL_MIN. DBL_MIN is the smallest POSITIVE fp number. It's the number that's as close to zero from the positive side as it's possible to get.




```
for (int i=0; i < numPoints; i++) {
   // do something with points[i]
}
```

points[i][0] is X, points[i][1] is Y, points[i][2] is Z, points[i][3] is W.

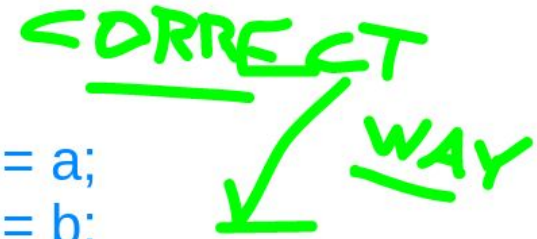
Drawing the Clip Region lines



Draw the lines in this order:

- p1 -> p2
- p2 -> p3
- p3 -> p4
- p4 -> p1

```
int a, b;

double aFP = a;
double bFP = b;

   ... some calculation using aFP
       and bFP ...
```



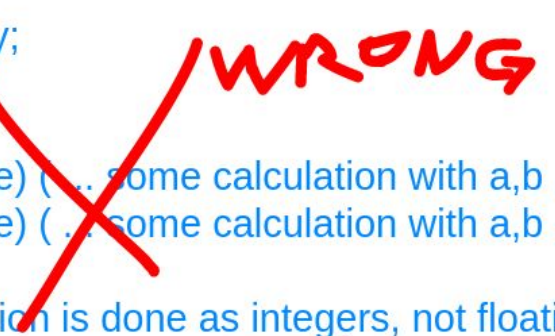
At end, convert back to int using the ROUND():

```
   a = ROUND(aFP);
   b = ROUND(bFP);
```

A common issue is to do the calculation as integers and do the conversion only at the end. This gets the wrong results usually.

For example,

```
double x, y;
int a, b;

x = (double) ( ... some calculation with a,b ... );
y = (double) ( ... some calculation with a,b ... );
```



The calculation is done as integers, not floating point.

The wrong result is therefore likely. Convert a,b to doubles FIRST and then do the calculation. Don't expect to get the correct answer if you do the calc as integer and then cast to floating point to the end.

"What Every Computer Scientist Should Know About Floating-Point Arithmetic", David Goldberg
ACM Computing Surveys, V23, N1, March 1991.

"Computational Methods" -- in general, this is a class about how to be accurate, precise, and repeatable in computation. (These are three separate criteria for calculations; you had better succeed at all three.)

We have IEEE-754 floating point, which defines the binary32 and binary64 representations.

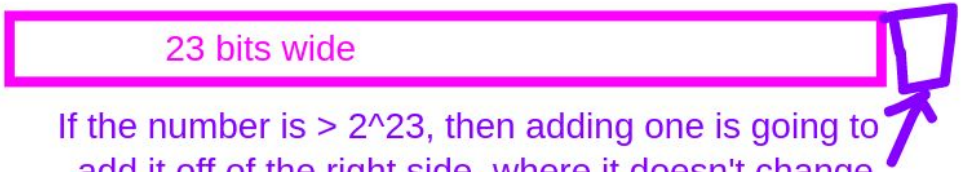
binary32 has one sign bit, 8 exponent bits, and 23 significand bits (and a hidden bit).

binary64 has one sign bit, 11 exponent bits and 52 significand bits (and a hidden bit).

These representations are only that precise, so FP arithmetic, depending on how one does it, can have different results, depending on the values.

One example is easy to see. Think about adding 1 to a binary32 FP number. If the number is > 2^23, adding 1 to it Does Not Change It.

Since the significand is only 23 bits wide, adding one to it is off the right side of the number and changes nothing.



If the number is > 2^23, then adding one is going to add it off of the right side, where it doesn't change the value of the binary32 number.

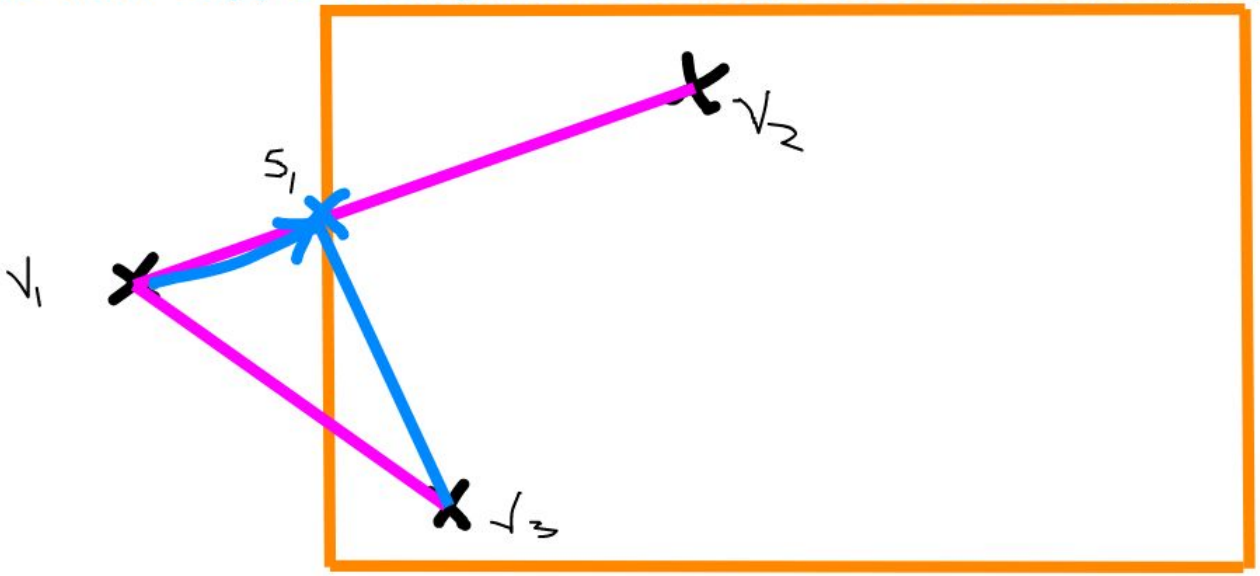
This is just an example of where FP arithmetic is not necessarily working the way one might think.

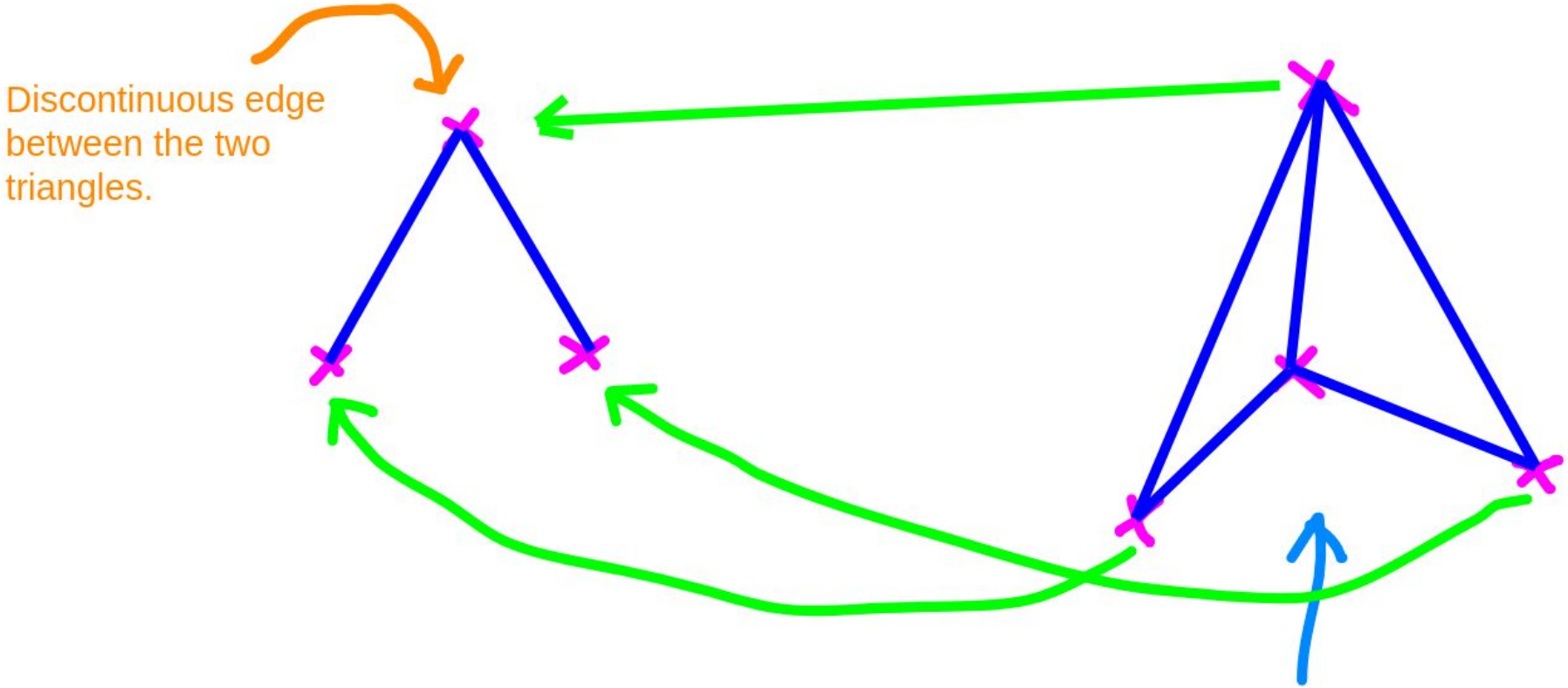
Another example is that (1.0/a)*b is not necessarily the same as b/a. It MIGHT be, but it's dependent on the exact values of a, b.

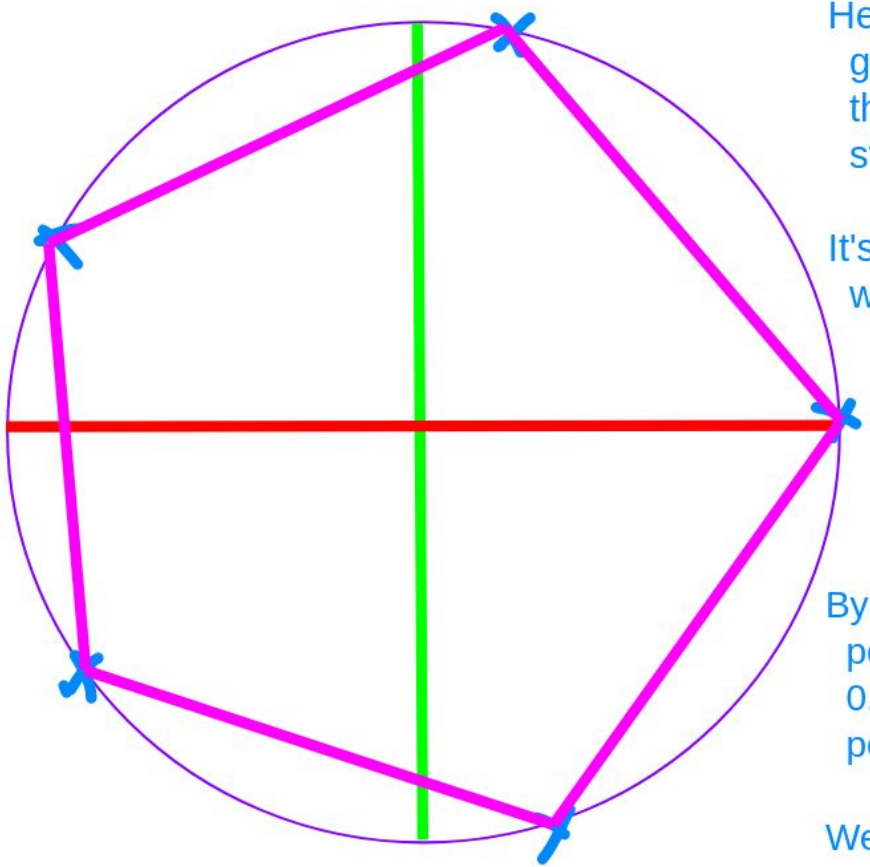
Think about 3.0/3.0 vs. (1.0/3.0)*3.0.

If v1 gets updated in the Point array when the line v1 to v2 is being drawn, an incorrect version of v1 (s1) gets used when trying to draw v1 to v3. The line s1 to v3 would get drawn instead.

Therefore, a "local" copy of each point needs to be used when calling clipLine().




Discontinuous edge between the two triangles.



Here the resolution is 5 -- we have generated 5 evenly spaced points on the IDEAL CIRCLE and then drawn straight lines between them.

It's the best approximation to a circle we can get with only 5 points.

By "evenly spaced" we mean those 5 points computed from t = 0.0, 0.2, 0.4, 0.6, 0.8 (the point 1.0 is the same as the point 0.0 since it's on a circle).

We compute x,y = C(t)
= cos 2*pi*t, sin 2*pi*t and then draw straight lines between adjacent points.

What's all this about NEGATIVE 0.0?

Negative Zero is still 0.0 but because of the way a calculation was done, the zero came from the negative side instead of the positive side.

Still, it's ZERO.

Why does this happen? Because the calculations were done in a different order or perhaps at a different precision.

What to do? Check your calculations to ensure that you're doing them correctly. It's possible to "get lucky" sometimes and some exercise cases might get the same answer even though the calcs are wrong.

That's why there are multiple exercises AND YOU SHOULD MAKE SOME EXERCISES OF YOUR OWN.

In general, ensure that you're doing math as double precision floating point (double or FLOAT64).

Ensure that you are not rounding in your culling calculations. (The only rounding spot is in the clip/display code, which you now have the reference solution for.)

Ensure that you are computing all of the vectors in the correct order. Remember, [redacted] subtraction is NOT commutative.

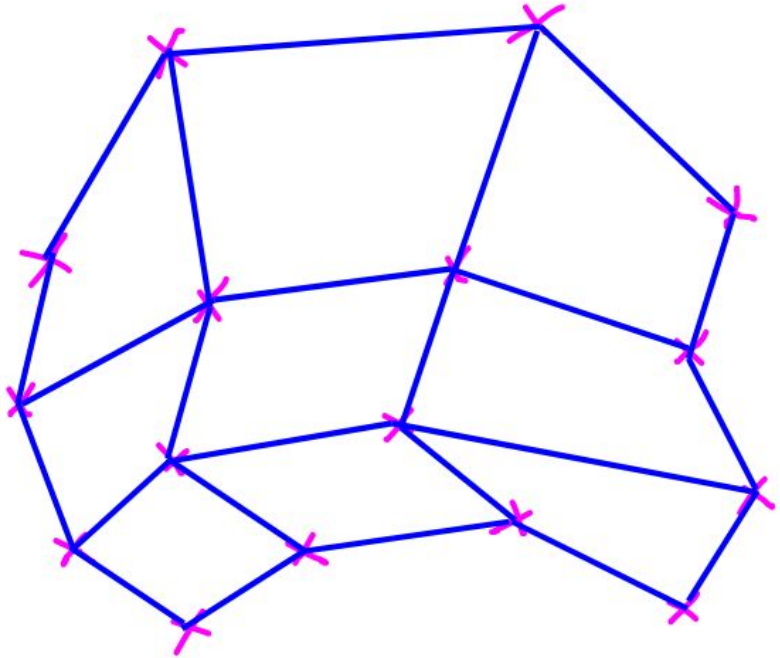
Ensure you're doing the cross product in the correct order. Remember, cross product is not commutative.

Bézier

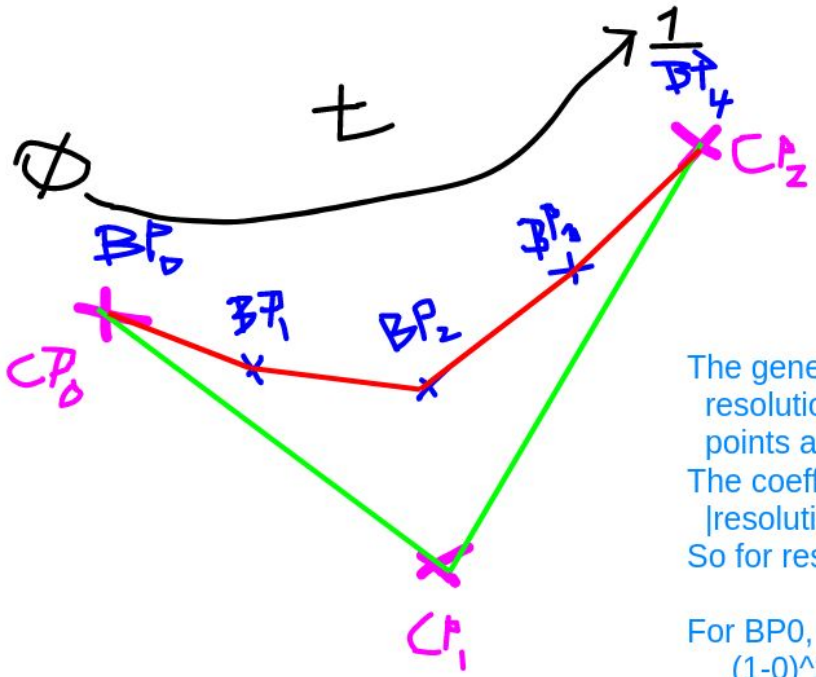
Bézier curves and surfaces are based on CONTROL POINTS. The higher the "order" of the curve / surface, the more control points it has.

For this class, all of our surfaces are what's known as "bicubic" -- meaning they are cubic in both dimensions, u and v.

(Remember, a cubic Bézier curve has four control points, so a bicubic Bézier surface has 16 control points, 4 x 4.)



Example of a set of 16 control points describing a bicubic Bézier surface.



$$B^2(t) = (1-t)^2 CP_0 + 2*(1-t)t\,CP_1 + t^2 CP_2$$

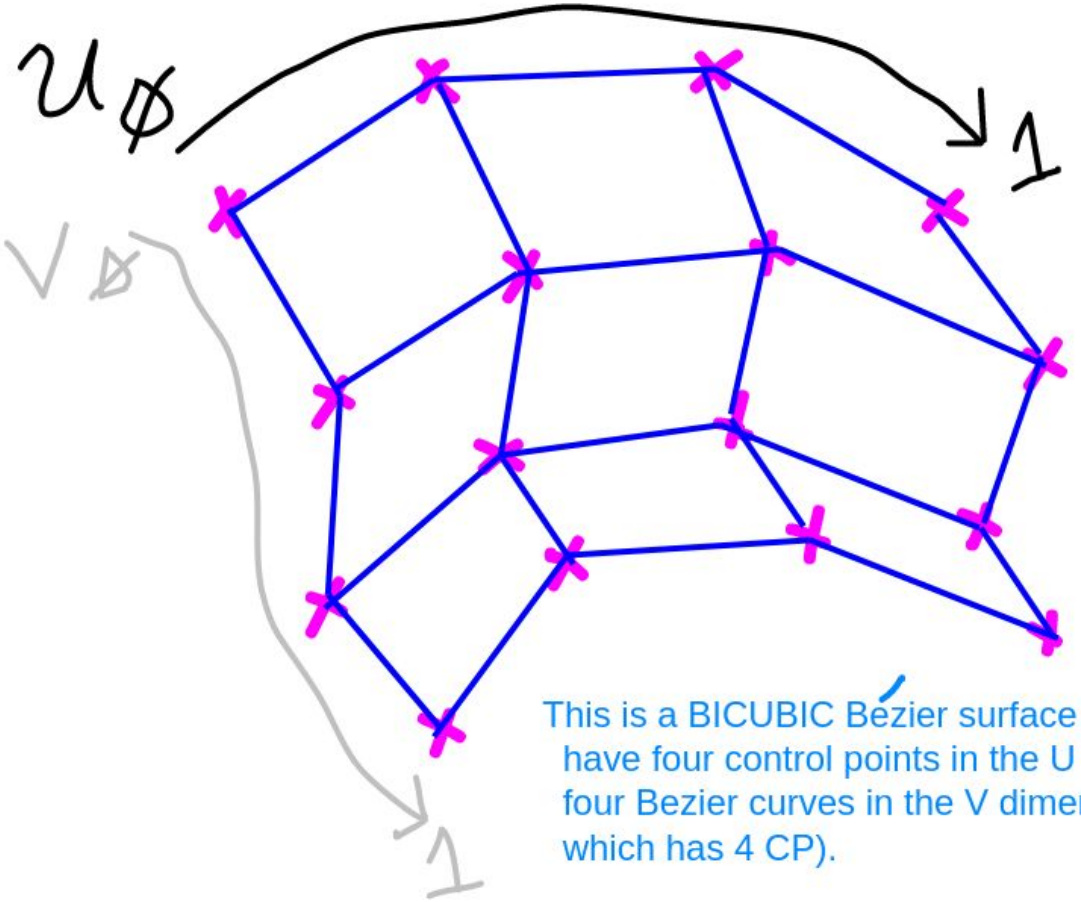
The generated points, BP0 .. BPn, where n is equal to the resolution-1, are each a linear combination of the control points and their coefficients.
The coefficients are computed using t, where t takes on |resolution| values ranging from 0 to 1, inclusive.
So for resolution 5, we have t = 0, 0.25, 0.50, 0.75, and 1.0.

For BP0, t=0, so BP0 is equal to :
(1-0)^2*CP0 + 2*(1-0)*0*CP1 + 0^2*CP2 = CP0.

For BP2, t=1/2, so BP2 is equal to :
(1-1/2)^2*CP0 + 2*(1-1/2)*1/2*CP1 + (1/2)^2*CP2 =
1/4*CP0 + 2*(1/2)*(1/2)*CP1 + 1/4*CP2 =
1/4*CP0 + 1/2*CP1 + 1/4*CP2

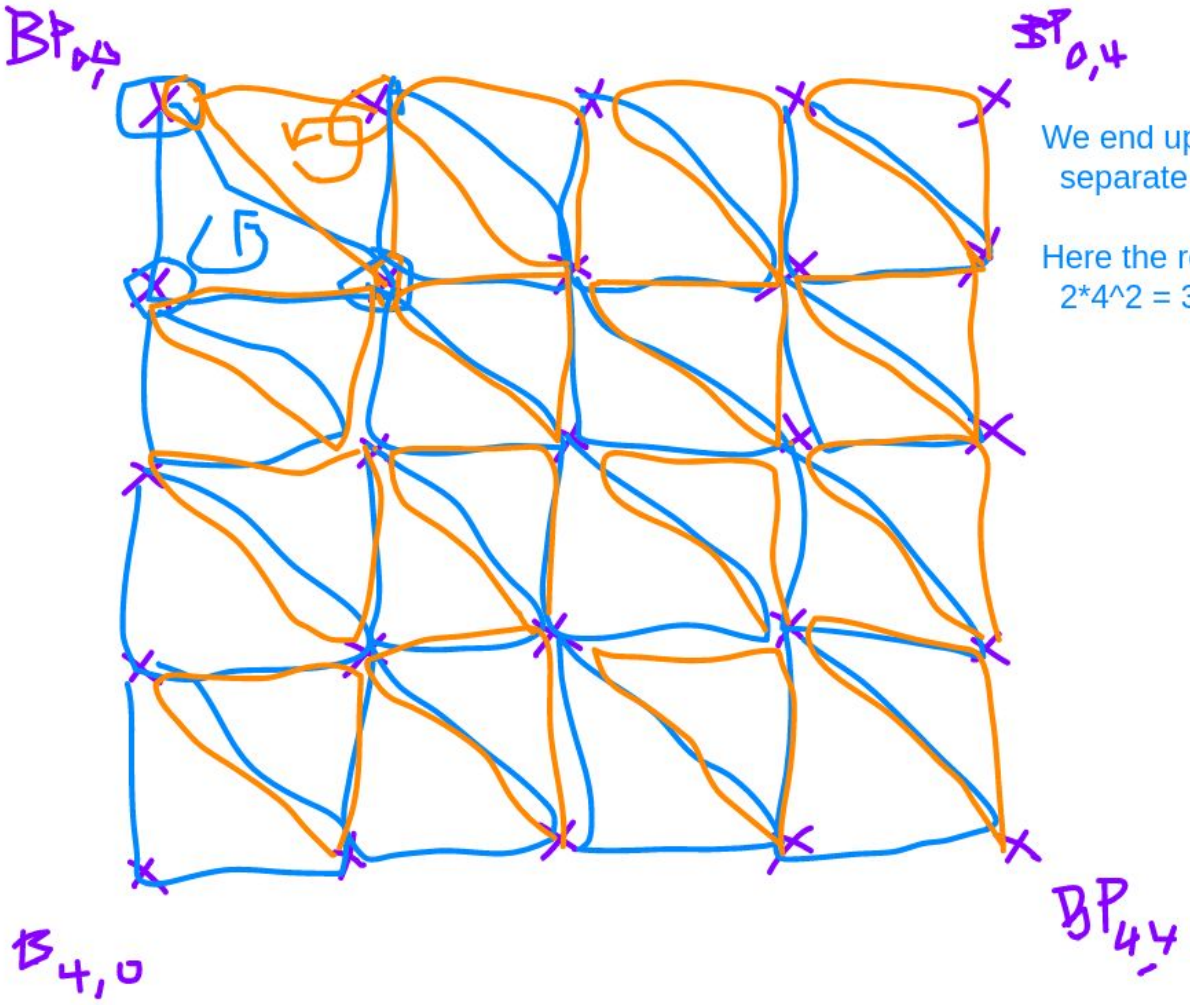
For BP4, t=1, so BP4 is equal to :
(1-1)^2*CP0 + 2*(1-1)*1*CP1 + 1^2*CP2 = CP2.

(You ought to be able to write what BP1 and BP3 are equal to using t = 0.25 and t = 0.75.)



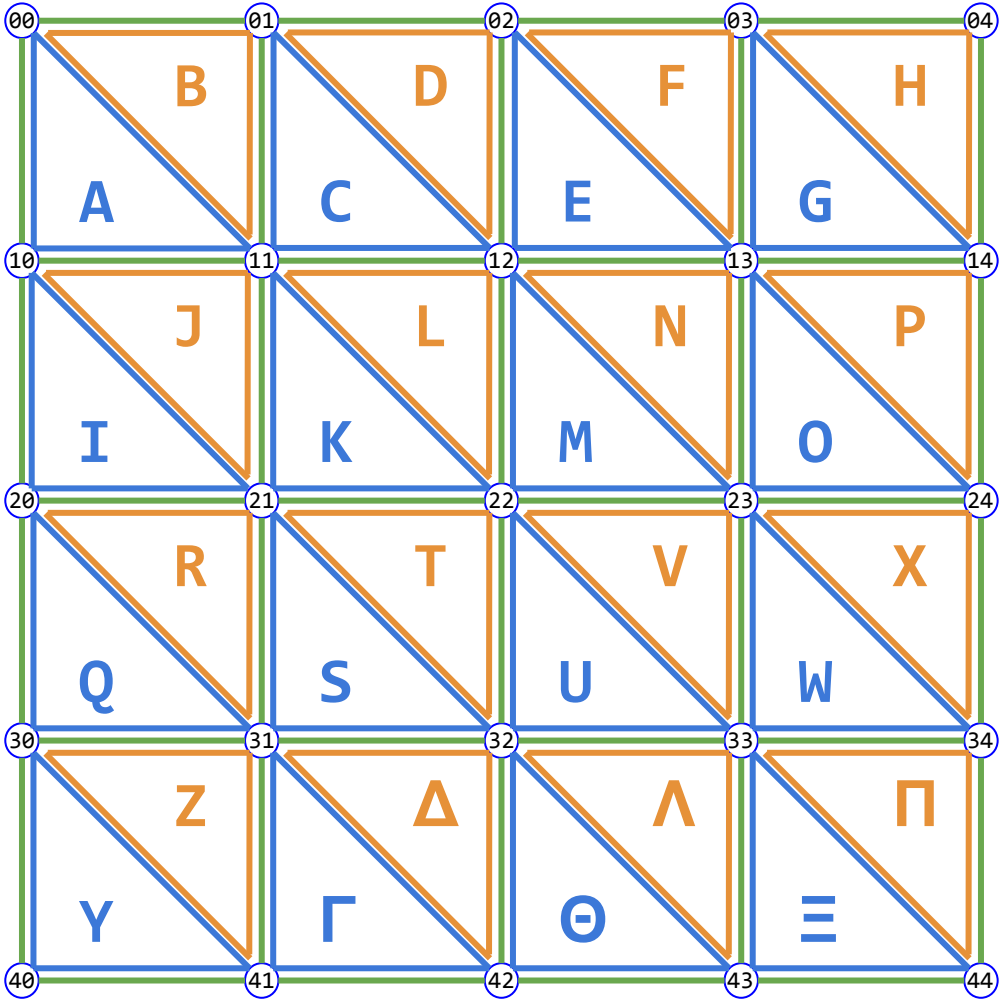
This is a BICUBIC Bezier surface because we have four control points in the U dimension and four Bezier curves in the V dimension (each of which has 4 CP).

Thus, we have 4 x 4 CP = 16 CP in total.



We end up drawing 2*(resolution-1)^2 separate triangles.

Here the resolution is 5, so 2*(5-1)^2 = 2*4^2 = 32 triangles are drawn.



The triangles are generated in pairs using the four vertices that surround each quad. Each triangle's vertices are listed in counter-clockwise order.

We count the rows from 0 to the resolution-2, here 5-2 = 3. We count the columns the same way, 0 .. 3 here.

The points are kept in a linear list, so they number from 0 to 24 in this example. The next vertex in the same column is *n* + resolution.

Each row starts at vertex rowStart = row*resolution. Each vertex in that row is here = rowStart + column. The vertex in the same column but one row down is there = here + resolution.

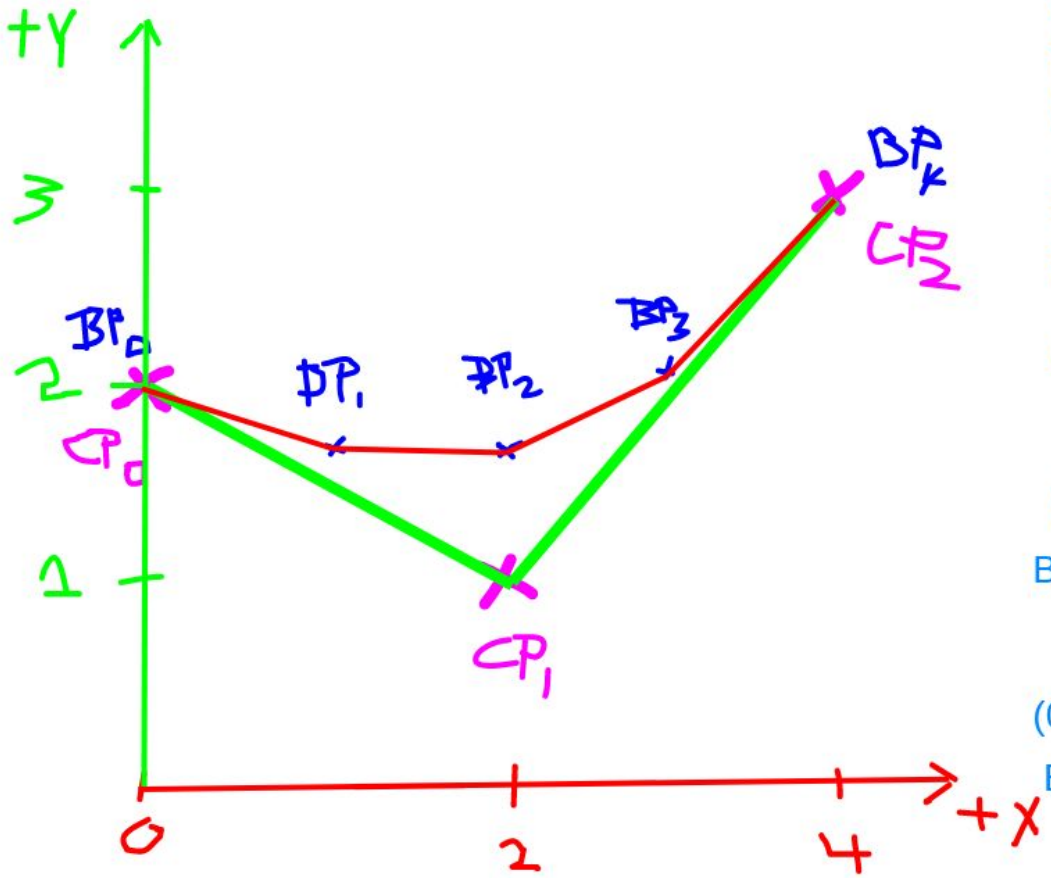
Let here = 0 (the first vertex in the first row). Then there = 5 (the first vertex in the second row).

△ A has vertices here (0), there (5), and there+1 (6), or 00, 10, 11. △ B has vertices there+1 (6), here+1 (1), and here (0), or 11, 01, 00.

Advance here to 1 (the second vertex in the first row). Then there = 6 (the second vertex in the second row).

△ C has vertices here (1), there (6), and there+1 (7), or 01, 11, 12. △ B has vertices there+1 (7), here+1 (2), and here (1), or 12, 02, 01.

And so forth …



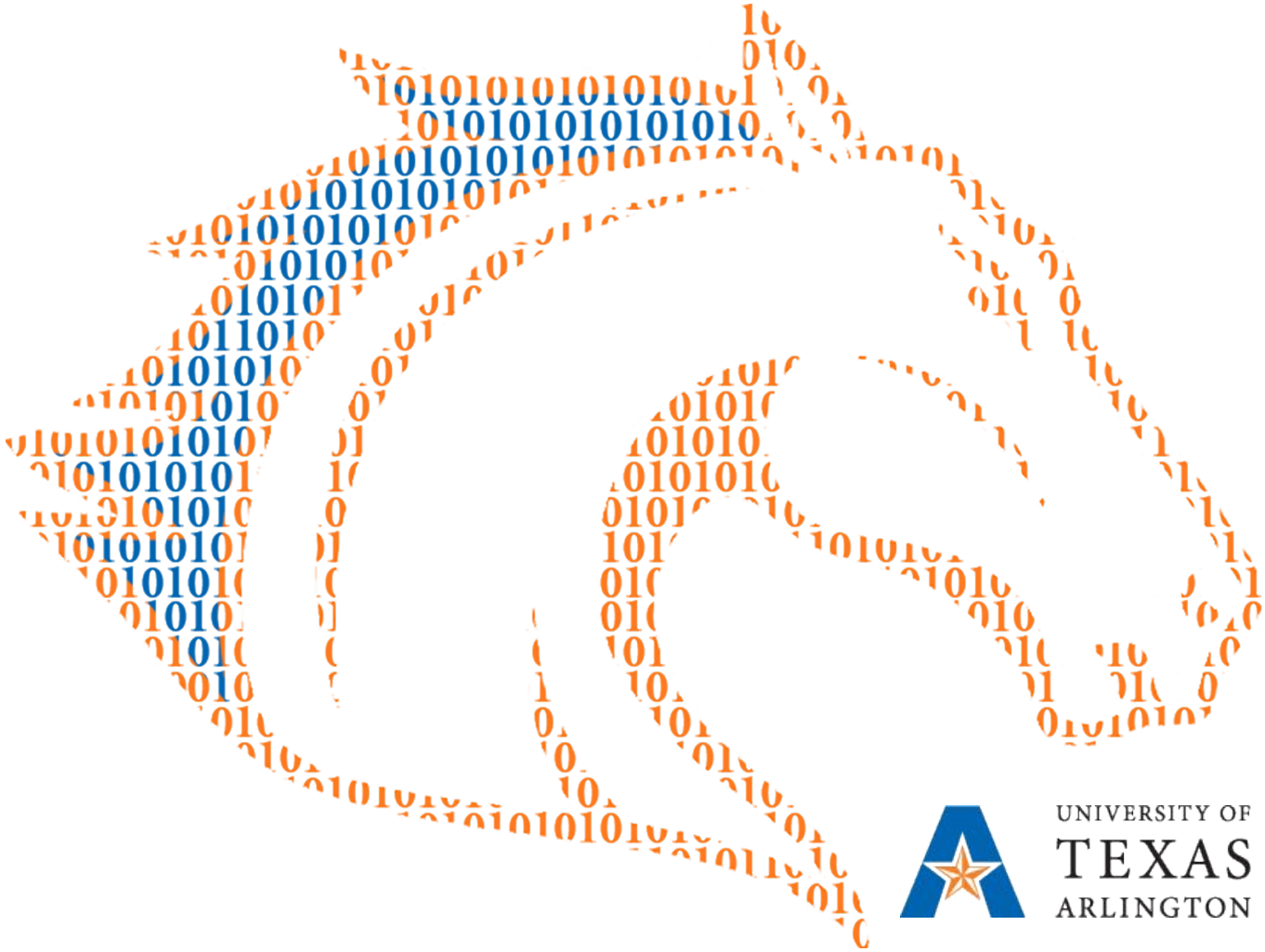
CP0 is (0,2), CP1 is (2,1), CP2 is (4,3).
Resolution is 5, so t is 0, 1/4, 1/2, 3/4, 1.
(1-t)^2*CP0 + 2*(1-t)*t*CP1 + t^2*CP2.

BP0 => t = 0 -> result is trivially CP0.
BP4 => t = 1 -> result is trivially CP2.

BP2 => t = 1/2, (1-1/2)^2 -> 1/4*CP0 +
2(1-1/2)1/2 -> 1/2*CP1 +
(1/2)^2 -> 1/4*CP2
(0,1/2) + (1,1/2) + (1,3/4) => (2,7/4)

BP1 => t = 1/4, (1-1/4)^2 -> 9/16*CP0 +
2(1-1/4)(1/4) -> 3/8*CP1 +
(1/4)^2 -> 1/16*CP2
(0,9/8) + (3/4,3/8) + (1/4,3/16) => (1,~1.7)

BP3 => t = 3/4, (1-3/4)^2 -> 1/16*CP0 +
2(1-3/4)(3/4) -> 3/8*CP1 +
(3/4)^2 -> 9/16*CP2
(0,1/8) + (3/4,3/8) + (9/4,27/16) => (3,~2.2)



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