Quantum Lattice Models and Anyonic Theories

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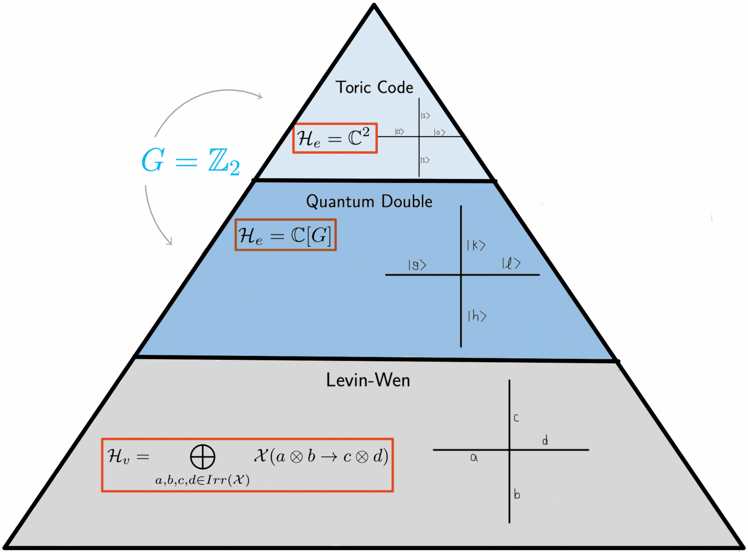
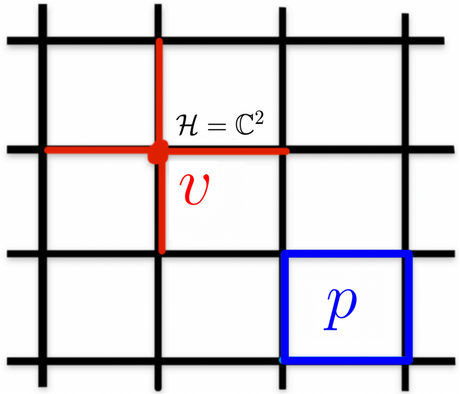
Quantum Lattice Models

What are Quantum Lattice Models?



Defining the Hilbert Space (Toric Code)



$$\mathcal{H}_{tot} = \bigotimes_{e \in \Gamma} \mathcal{H}_e$$

Defining the Operators

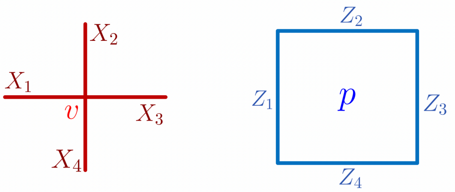
Pauli matrices:

$$\sigma^x = X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma^y = Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma^z = Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

We define the two operators, $\boxed{A_v = \prod_{e \in \Gamma} X_e}$ and $\boxed{B_p = \prod_{e \in \Gamma} Z_e}$.



Ground State and Excitations

Ground State

$$H = -\sum_{v\in\Gamma} A_v - \sum_{p\in\Gamma} B_p$$

- $H^\dagger = H$
- Lowest $H \to$ maximize A_v and $B_p \to$ eigenvalue $= 1$
  - Eigenvalues corresponds to energy levels

$$\boxed{A_v \left|\psi\right\rangle = B_p \left|\psi\right\rangle = \left|\psi\right\rangle}$$

Excited State

Excited state = Violations of H

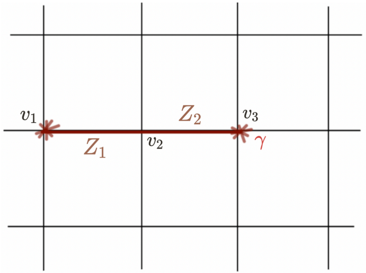
- **Violation** – Eigenvalue of operator is $\neq 1$
    - $A_v \left|\phi\right\rangle = -\left|\phi\right\rangle \implies \left|\phi\right\rangle$ is a violation of A_v
    - $A_v \left|\phi\right\rangle = \left|\phi\right\rangle \implies \left|\phi\right\rangle$ is **not** a violation of A_v (*satisfies* A_v)

- **Anyon** – Lowest energy excited state
- **Flux** – Violation of A_v operator
- **Charge** – Violation of B_p operator
- **Dyons** – Violation of both A_v and B_p operators

Interacting with Anyons

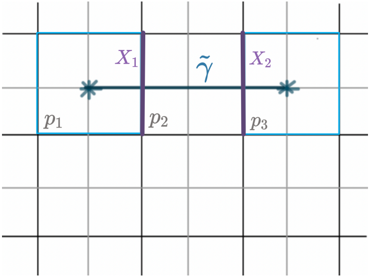
Creating Anyons

Fluxes



$$\boxed{S^Z_\gamma := \prod_{e\in\gamma} Z_e}$$

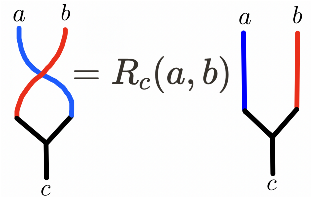
Charges



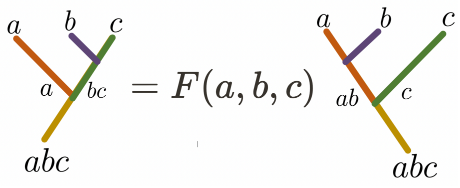
$$\boxed{S^X_{\tilde{\gamma}} := \prod_{e\in\tilde{\gamma}} X_e}$$

Braiding and Fusing Anyons

R-symbol



F-symbol



Accomplishments and Next Steps

What we did...

- For a given group, found a universal unitary movement operator for any arbitrary anyon
- Utilized that definition to calculate the categorical data for non-abelian anyons

...and what's to come

- Exploring interactions of anyons with our definition in various and more generalized models (Quantum Double, Levin-Wen)

References

1 David Green, Peter Huston, Kyle Kawagoe, David Penneys, Anup Poudel, Sean Sanford. (2024) Enriched string-net models and their excitations. arXiv:2305.14068

2 "Topological Order in Quantum Systems." MIT, n.d., https://xgwen.mit.edu/sites/default/files/documents/topo.pdf.

3 "Topological Order and Quantum Computation." Topological Condensed Matter Theory Group, n.d., https://topocondmat.org/w12_manybody/topoorder.html.

Thank you!

Special thanks to my collaborators and mentors,
the Ohio State University Department of Mathematics, and
the ROMUS program for this invaluable experience.