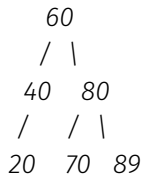


Question 1:

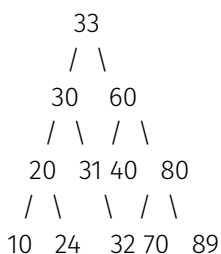
Each of the following key-value pairs in the list below represents a document (the integer) and a word in that document (the letter) from your P01 data set. Insert the key:value pairs sequentially from (20:O) to (32:E) into an initially empty AVL tree. Use the integer as the key to insert. *Note:* this will not produce an inverted index like you're creating for the practical. You only need to provide the balanced state of the tree after inserting (70:E) and (32:E).

[(20:O), (40:S), (60:T), (80:R), (89:N), (70:E), (30:T), (10:N), (33:A), (31:H), (24:R), (32:E)]

After inserting 70:E



After inserting 32:E



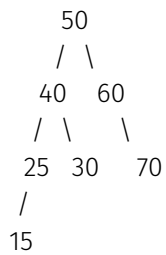
1. When inserting 60, the tree became imbalanced at 20 and was found to be a case RR.
2. When inserting 89, the tree became imbalanced at node 60 and is case RR.
3. When inserting 70, the tree became imbalanced at node 40 and is case RL.
4. When inserting 30, the tree became imbalanced at node 40 and is case LR.
5. When inserting 31, the tree became imbalanced at node 40 is case LL
6. When inserting 32, the tree became imbalanced at node 60 and is case LR.

Provide a list of all insertions that caused an imbalance, at what node the imbalance was found, and what imbalance case was found. You can use the following as a template:

When inserting _____, the tree became imbalanced at _____ and was found to be a case _____. (you can use the number 1, 2, 3, 4 or letter version LL, LR, RL, RR)

Question 2:

Draw an AVL Tree of height=4 (a tree with 4 levels) with the *minimum* number of nodes possible. You can draw circles to represent the nodes; they do not need to contain values.



Question 3:

Insert the tuples of the form (A:B) from question 1 into a hash table of size 10. The hash table should use separate chaining for collision resolution.

Version A: Use the hash function $h(A) = A \bmod 10$.

[(20:O), (40:S), (60:T), (80:R), (89:N), (70:E), (30:T), (10:N), (33:A), (31:H), (24:R), (32:E)]

Index | Keys

0		10 → 20 → 30 → 40 → 60 → 70 → 80
1		31
2		32
3		33
4		24
5		
6		
7		
8		
9		89

Version B: Use the $h(B) = \text{lookupASCII}(B) \bmod 10$. ASCII is a character encoding that maps a single character to an integer value. For example, the ASCII code of the letter M is 77. You can find an ASCII chart > [here](#) <.

[(20:O), (40:S), (60:T), (80:R), (89:N), (70:E), (30:T), (10:N), (33:A), (31:H), (24:R), (32:E)]

Index | Keys

0	
1	
2	(80:R), (31:H), (24:R)
3	(40:S)
4	(60:T), (30:T)
5	(33:A)
6	
7	
8	(89:N), (10:N)
9	(20:O), (70:E), (32:E)

Question 4:

Insert the following list of integers into a B+ Tree where $M = 3$. That is, each internal node contains max 3 keys and 4 children and each leaf node contains max three keys. When splitting a node due to overflow, leave 1 element in the left node and move 2 elements to the newly created right node.

[48, 65, 91, 90, 14, 13, 87, 74, 51, 92, 41, 70, 47, 64, 38, 29, 50, 21]

Inserting 48, 65, 91

[0048, 0065, 0091]

Node Split at 90, Height of Tree Increases to 1

```
      [0090]
     /      \
[0048, 0065] [0090, 0091]
-----
```

Inserting 14

```
          [0090]
         /  \
[0014, 0048, 0065] [0090, 0091]
```

Inserting 13, Node Split at 13, Height of Tree Stays the Same

```
      [0048, 0090]
     /      |      \
[0013, 0014] -> [0048, 0065] -> [0090, 0091]
```

Inserting 87

```
      [0048, 0090]
     /      |      \
[0013, 0014] -> [0048, 0065, 0087] -> [0090, 0091]
```

Inserting 74, Node Split at 74, Height of Tree Stays the Same

```
      [0048, 0074, 0090]
     /      |      |      \
[0013, 0014] [0048, 0065] [0074, 0087] [0090, 0091]
```

Inserting 51, 92, 41

```
          [0048, 0074, 0090]
         /      |      |      \
[0013, 0014, 0041] [0048, 0051, 0065] -> [0074, 0087] [0090, 0091, 0092]
```

Inserting 70, Node Splits at 70, Height of Tree Increases to 2

```
      [0074]
     /      \
```

```

      [0048,0065] [0090]
      /  |  \  /  \
[0013,0014,0041] [0048,0051] [0065,0070] [0074,0087] [0090,0091,0092]

```

Inserting 47, Node Splits at 47, Height of Tree Stays the Same

```

      [0074]
      /  \
    [0041,0048,0065] [0090]
    /  |  |  \  /  \
[0013,0014] [0041,0047] [0048,0051] [0065,0070] [0074,0087] [0090,0091,0092]

```

Inserting 64, 38

```

      [0074]
      /  \
    [0041,0048,0065] [0090]
    /  |  |  \  /  \
[0013,0014, 0038] [0041,0047] [0048,0051, 0061] [0065,0070] [0074,0087] [0090,0091,0092]

```

Inserting 29, Node Splits at 29, Height of Tree Stays the Same

```

      [0048,0074]
      /  |  \
    [0029,0041] [0065] [0090]
    /  |  \  /  \  /  \
[0013,0014] [0029,0038] [0041,0047] [0048,0051,0064] [0065,0070] [0074,0087] [0090,0091,0092]

```

Inserting 50, Node Splits at 50, Height Stays the same

```

      [0048,0074]
      /  |  \
    [0029,0041] [0051,0065] [0090]
    /  |  \  /  |  \  /  \
[0013,0014] [0029,0038] [0041,0047] [0048,0050] [0051,0064] [0065,0070] [0074,0087] [0090,0091,0092]

```

Insert 21

```

                        [0048,0074]
                        /  |  \
                    [0029,0041] [0051,0065] [0090]
                /  |  \  /  |  \  /  \
[0013,0014, 0021] [0029,0038] [0041,0047] [0048,0050] [0051,0064] [0065,0070] [0074,0087] [0090,0091,0092]

```

Which insertions triggered a node split? _____

1. 90, 13, 74, 70, 47, 29, 50

Which insertions increased the height of the tree by 1 level? _____

1. 90, 70